Non-perturbative (Lattice) QCD in B Physics

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1. Introduction

Searching for New Physics via precise measurements/SM predictions of heavy flavor observables.

Constraining possible NP models.

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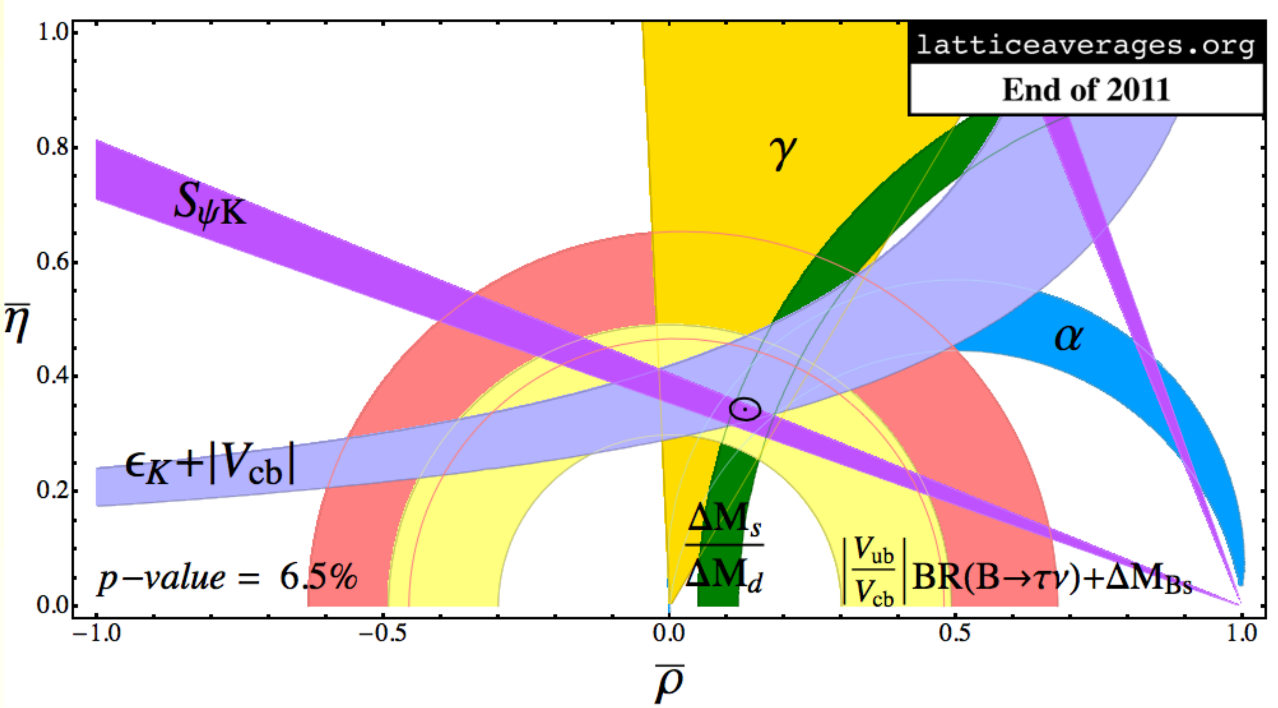
Laiho,Lunghi,Van de Water PRD81:034503 (2010)



Example: UT fits

Error bands are still dominated by theory errors, in particular due to hadronic matrix elements

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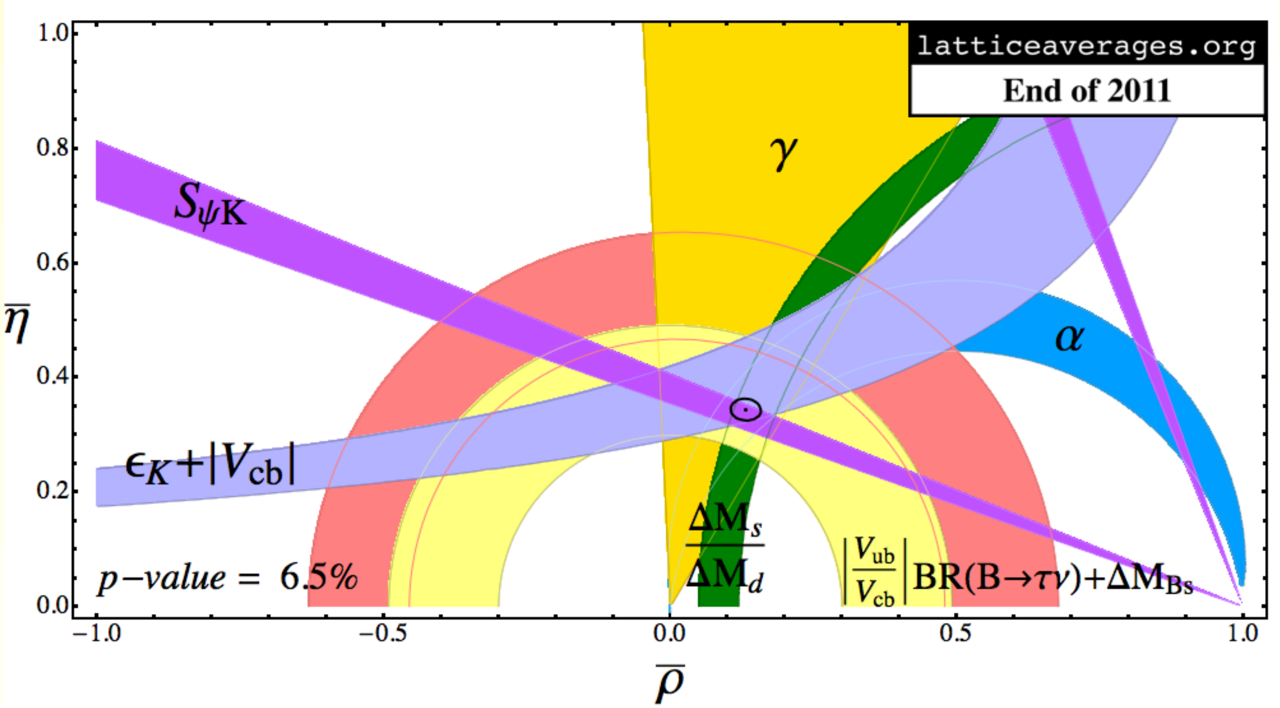

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Error bands are still dominated by theory errors, in particular due to hadronic matrix elements → use **lattice QCD**

1. Introduction: Lattice QCD

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Goal: Precise calculations ($\leq 5\%$ error)

* Control over systematic errors:

** Unquenched calculations: $N_f = 2+1$ or $N_f = 2+1+1$.

** Discretization: improved actions + simulations at several $a's \rightarrow$ continuum limit

** Chiral extrapolation: simulate at several m_π and extrapolate to $m_\pi^{\rm phys}$ using `ChPT`.

** Renormalization: non-perturbative, perturbative.

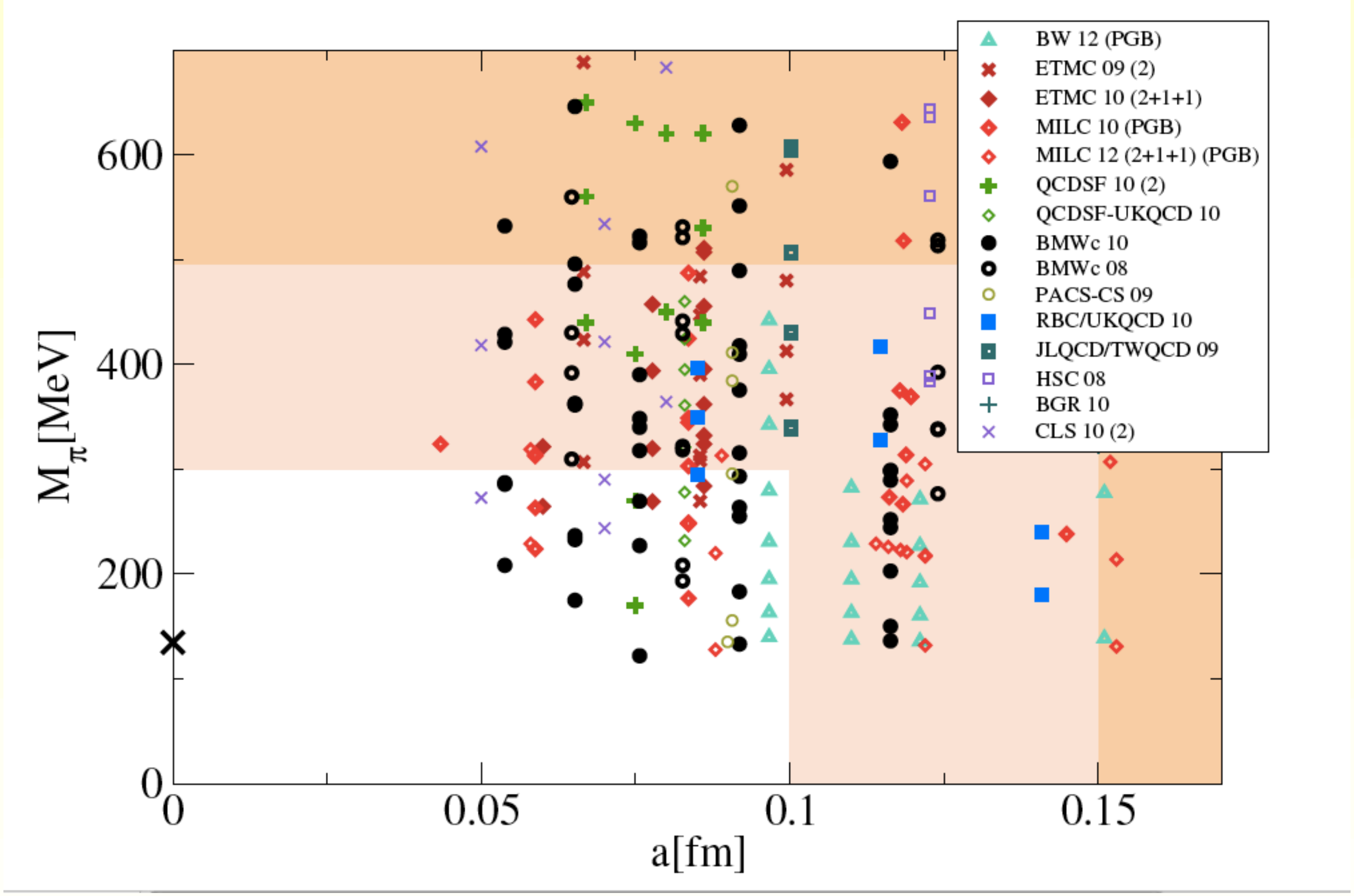
** Tuning lattice scale and masses

** Finite volume, isospin effects, electromagnetic effects, ...

Systematically improvable

1. Introduction: Overview of simulations parameters

Several $N_f = 2+1$ and even $N_f = 2+1+1$, and **physical quark masses**.



First results with simulations with physical light quark masses starting to appear.

plot by **C. Hoelbling**,

1. Introduction: Heavy quarks on the lattice

Problem is discretization errors ($\simeq m_Q a, (m_Q a)^2, \cdots$) if $m_Q a$ is large.

- **Effective theories:** Need to include multiple operators matched to full QCD `B-physics` $\surd$
  - HQET (static,...): sytematic expansion in $1/m_h$.
  - NRQCD: systematic (non-relativistic) expansion in (v_h/c).
  - Fermilab, RHQ, ...

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  - ** NRQCD: systematic (non-relativistic) expansion in (v_h/c).
  - ** Fermilab, RHQ, ...
- **Relativistic (improved) formulations:**
  - ** Allow accurate results for `charm` (especially twisted mass, HISQ (Highly improved staggered quarks)).
  - ** Advantages of having the same formulation for light and heavy: ratios light/heavy, PCAC for heavy-light, ... Also simpler tuning of masses.
  - ** Also for `bottom`: Results for $m_c \cdots \leq m_b$ and extrapolation to m_b (twisted mass, HISQ).

2. Tuning the parameters: m_b

HPQCD $N_f = 2+1$ NRQCD calculation (non-relativistic): 1302.3739

$$\bar{m}_b^{\overline{MS}}(\bar{m}_b) = 4.166(43)\ \mathrm{GeV}$$

- Direct determination from binding energy of both Υ and B_s mesons.
- Accurate through $\mathcal{O}(\alpha_s^2)$ (heavy quark energy shift at two-loops).

Still higher order perturbation theory dominates the error.

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HPQCD $N_f = 2+1$ HISQ calculation (relativistic): 1004.4285

$$\bar{m}_b^{\overline{MS}}(\bar{m}_b) = 4.164(23)\ \mathrm{GeV}$$

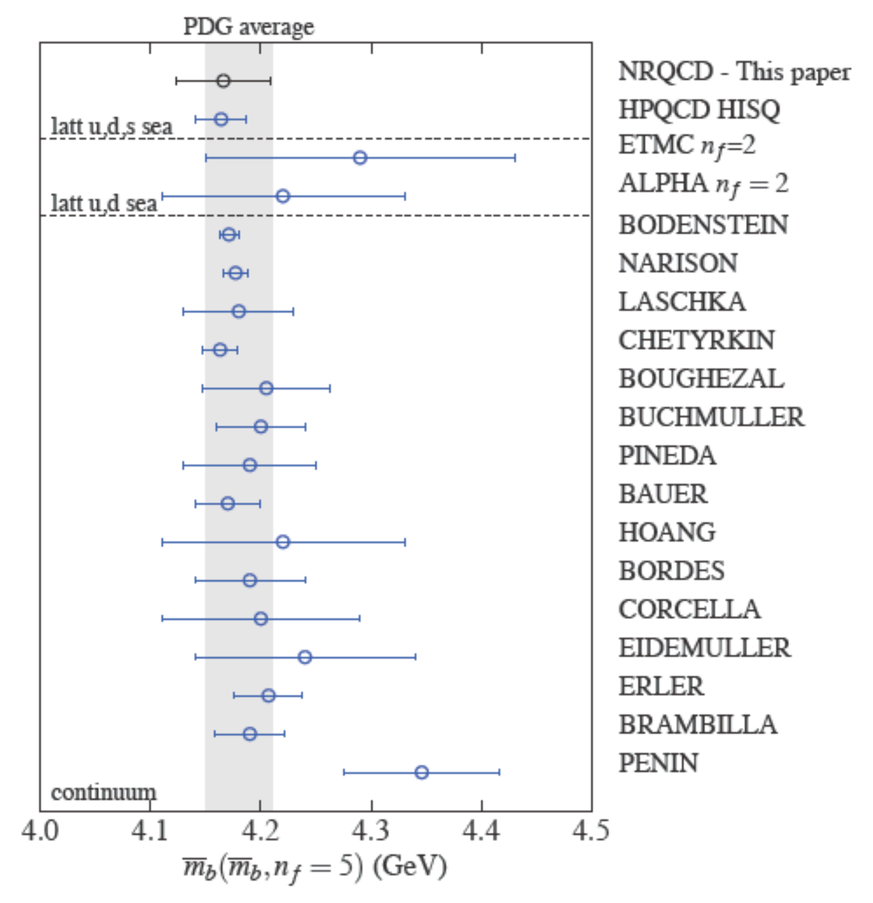
* η_b current-current correlators + high-order (α_s^3) continuum QCD perturbation theory:

* Simulate at $m_H < m_b$ and extrapolate to physical m_b

* Experimental input: M_{η_b} used to tune the lattice $b-quark$ mass.

* Eliminates renormalization and effective theory errors.

2. Tuning the parameters: m_b



plot from **A.J. Lee** *et al*, arXiv:1302.3739

3. B and B_s meson decay constants

Needed for processes potentially sensitive to NP: $B_{(s)} \to \mu^+\mu^-$.

Check agreement theory-experiment $Br(B^- \to \tau^- \bar{\nu}_\tau)$.

UT inputs.

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\# UT inputs.

Decay constants come from simple matrix elements

$\langle 0|A_0|B_q\rangle = M_{B_q} f_{B_q}$ → precise calculations on the lattice

(caveat: am_b discr. errors)

3. B and B_s meson decay constants

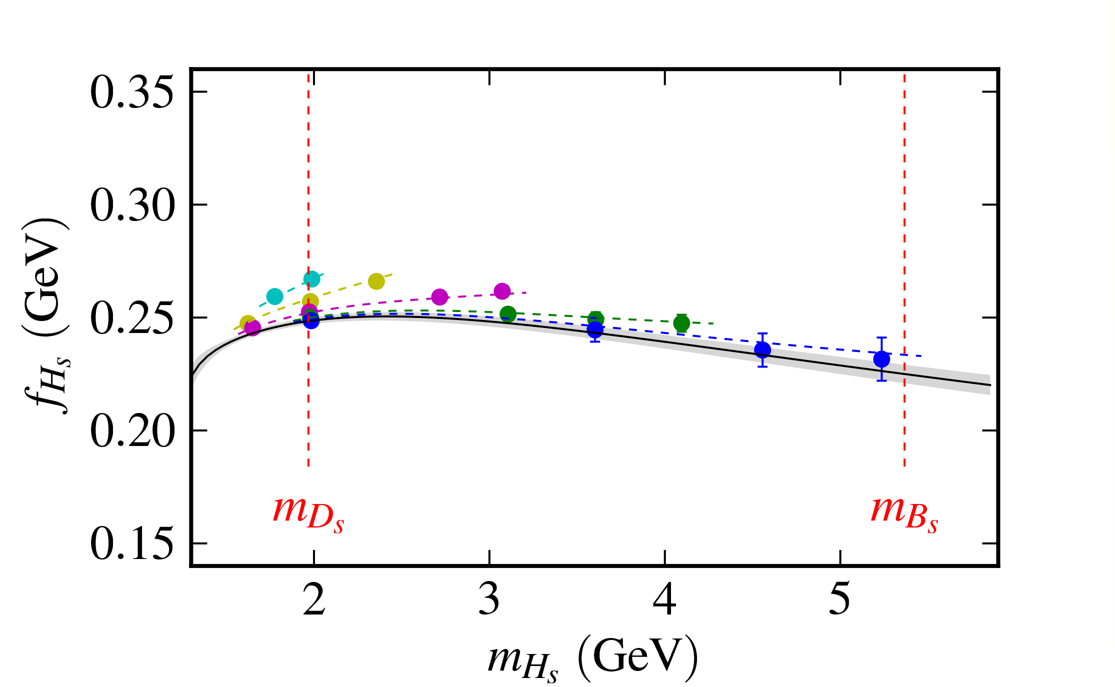
\# **HPQCD relativistic**, PRD 85 (2012) 031503: $N_f = 2+1$ with four a's.

* Using relativistic description (HISQ) for b reduce the error to 2%.

** No effective theory errors, no renormalization.

* Cross-checks: $m_b^{\overline{MS}}$, $m_{B_s} - m_{\eta_b}/2$, f_K, f_π.

* First empirical evidence for $1/\sqrt{m_{B_s}}$ depende predicted by HQET.

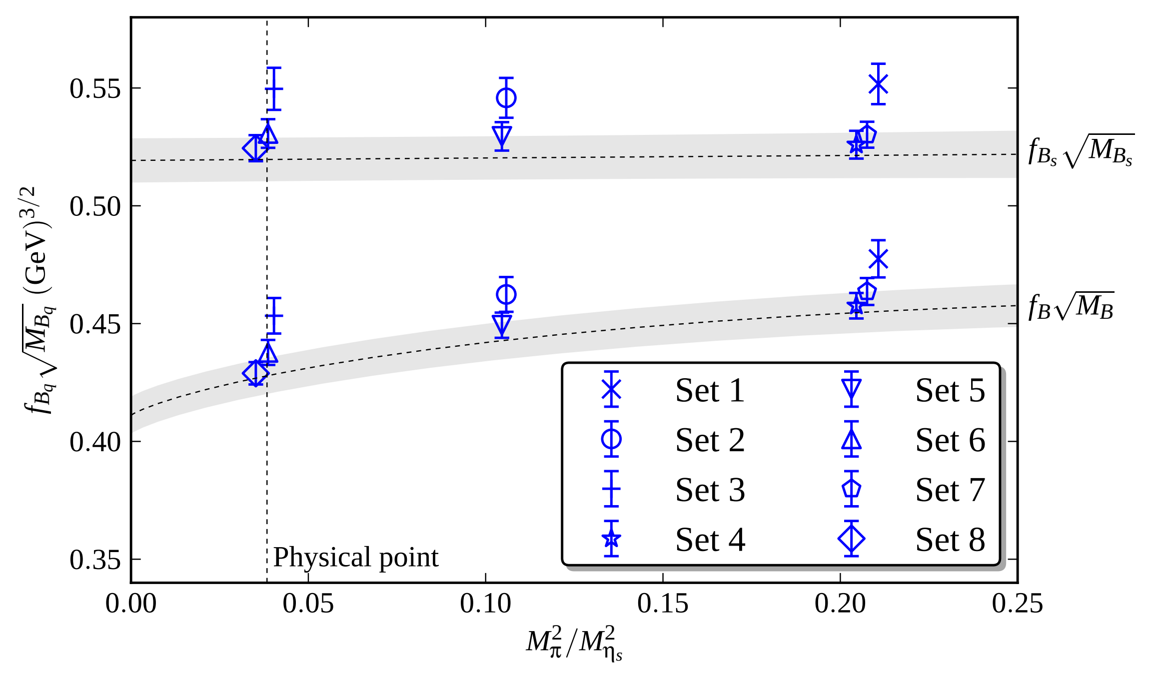


$$f_{B_s} = 224(4)\ \text{MeV}$$

3. B and B_s meson decay constants

\# First calculation with **physical light quark masses**: **HPQCD**, 1302.2644

- * $N_f = 2+1+1$ **MILC** configurations. Three a's.
- * NRQCD description of b quarks.
- * New estimate of matching errors: fit α_s^2 terms instead of power counting.

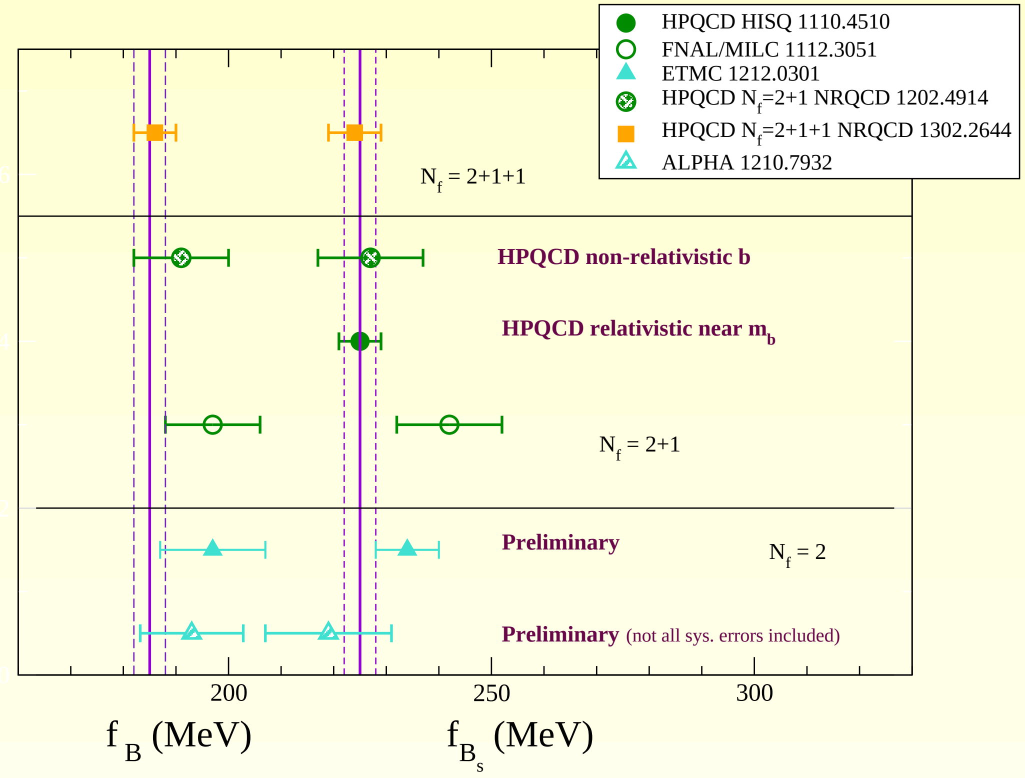


$$f_B = 186(4)\ \mathrm{GeV}$$

$$f_{B_s} = 224(5)\ \mathrm{GeV}$$

$$f_{B_s}/f_B = 1.205(7)$$

3. B and B_s meson decay constants



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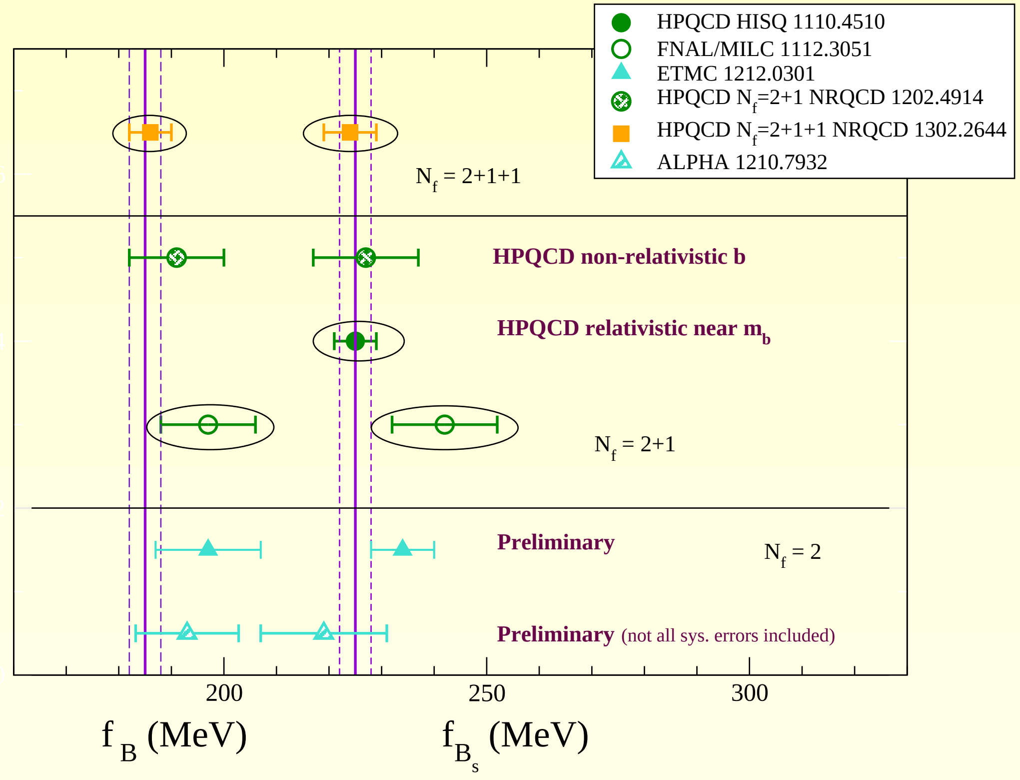

(for exp. see **Horii's talk**)

Averages from 1302.2644

$$f_B = (185 \pm 3)\ MeV$$

$$f_{B_s} = (225 \pm 3)\ MeV$$

$$f_{B_s}/f_B = 1.218(8)$$

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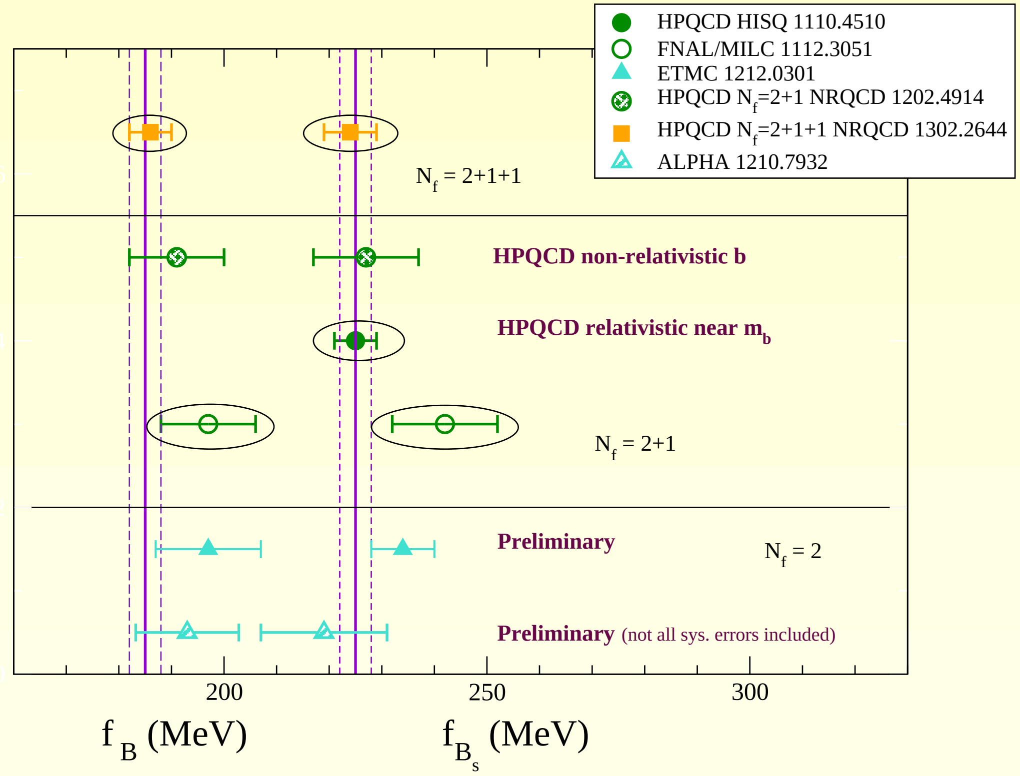

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Belle, 1208.4678: $Br(B^+ \to \tau\nu)/|V_{ub}^{exc.}|^2 = 6.9 \pm 3.1$

$Br(B^+ \to \tau\nu)/|V_{ub}^{inc.}|^2 = 3.9 \pm 1.7$

Averages in, 1201.2401: $Br(B^+ \to \tau\nu)/|V_{ub}^{exc.}|^2 = 16.1 \pm 4.2$

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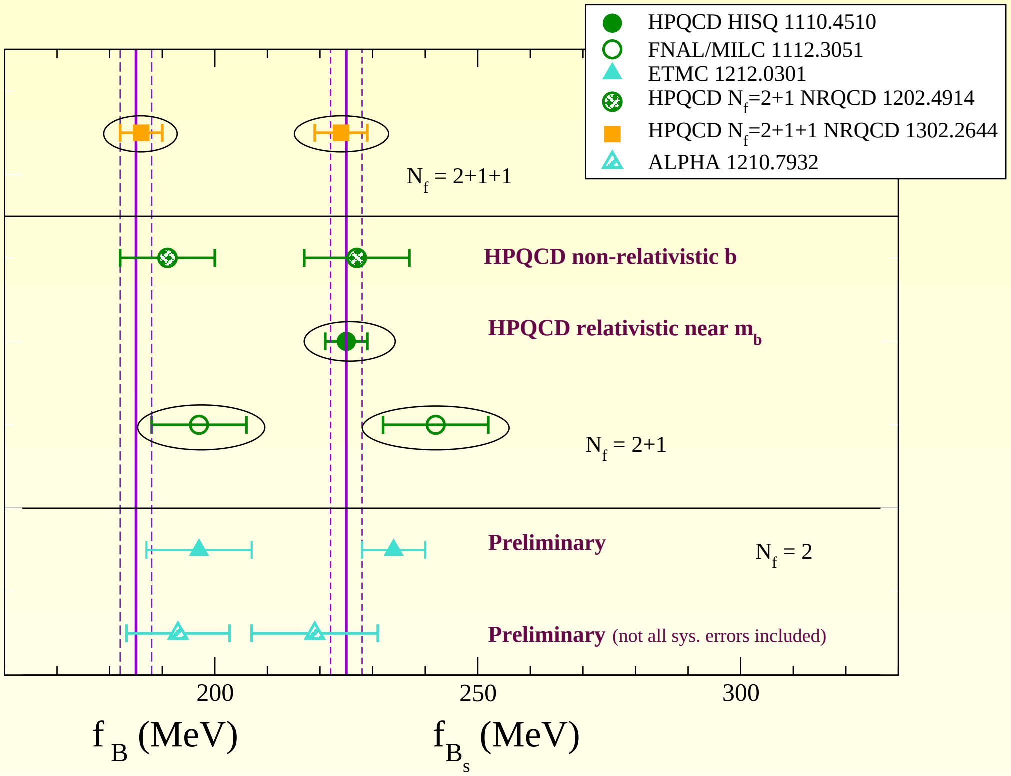

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\# In progress: **FNAL/MILC, ALPHA, ETMC, RBC/UKQCD**

4. Semileptonic decays

4.1. $B \to \pi l \nu$: Exclusive determination of $|V_{ub}|$

See Giulia Ricciardi's Talk

* No new calculations since 2009.

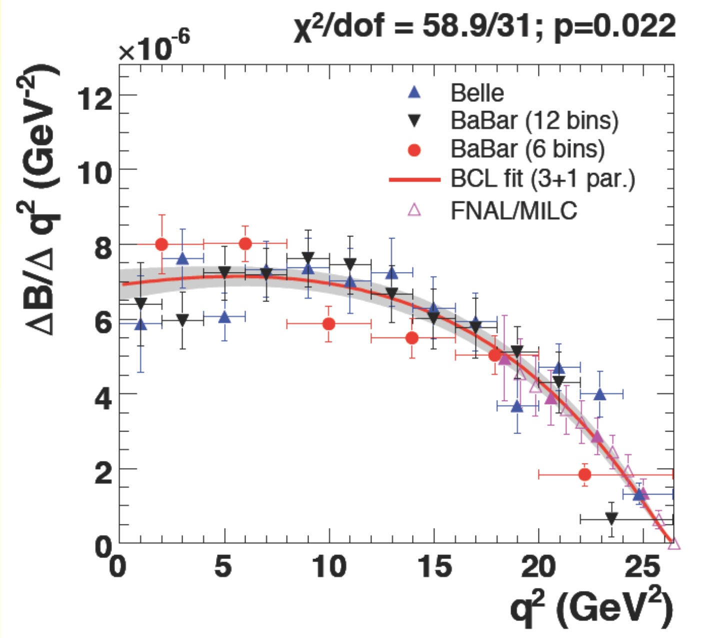
Combined fit of lattice data

FNAL/MILC, 0811.3604

and experimental data

HFAG 2012, from BaBar and Belle data

from different q^2 regions using z-expansion.



$$|V_{ub}^{exc.}| = (3.23 \pm 0.30) \times 10^{-3}$$

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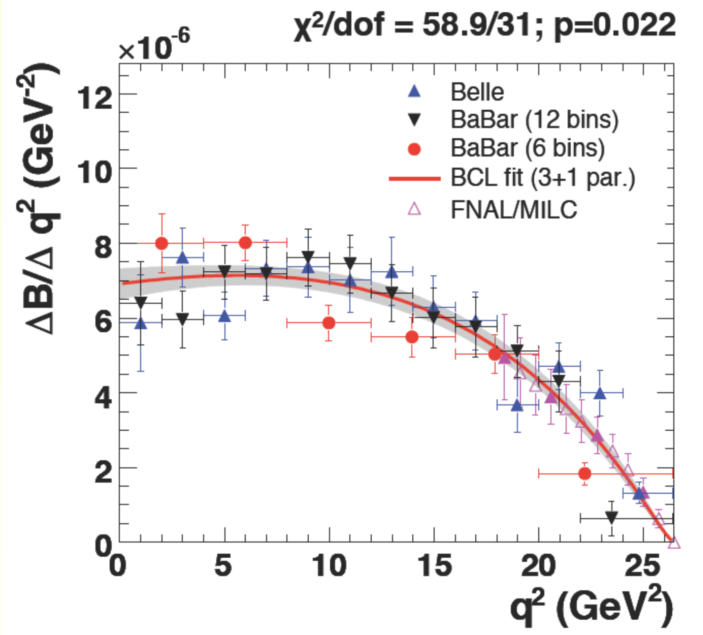

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\# **In progress:** Important improv. on existing $N_f = 2+1$ calculations

- * **FNAL/MILC** Many more data, smaller lattice spacings, improvements on parametrization of shape ...
- * **HPQCD**: several improvements in statistical errors (more data, smearing, random wall sources), better scale determination, `z-expansion . . .`

\# **In progress:** **RBC/UKQCD** ($N_f = 2+1$), **ALPHA** ($N_f = 2$)

4.2. $B_s \to K l \nu$: Exclusive determination of $|V_{ub}|$

Alternative to $B \to \pi l \nu$ to extract $|V_{ub}|$

* Experiment: Expect to be measured by **LHCb** and **Belle II**

* On the lattice: Corresponding form factors can be calculated with smaller errors (spectator quark is heavier (`strange`)

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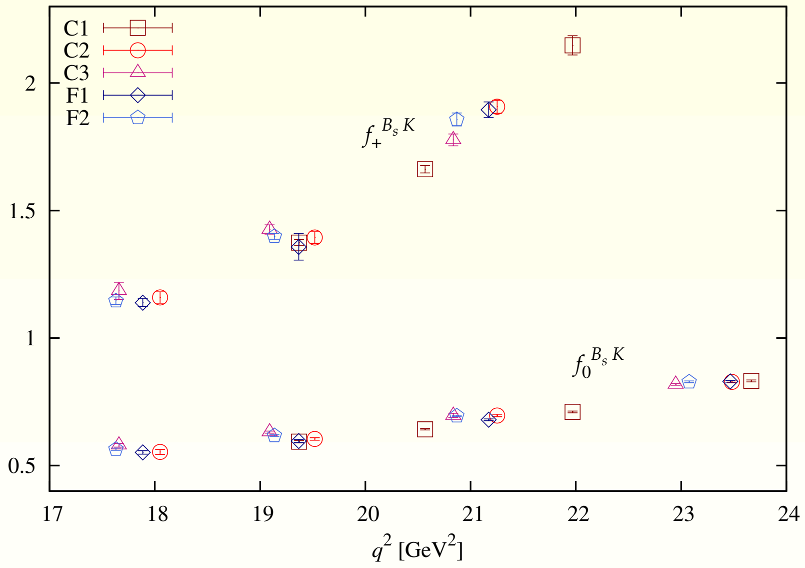
\# **HPQCD** ($N_f = 2 + 1$ calculation with NRQCD `b-quarks`)

Preliminary



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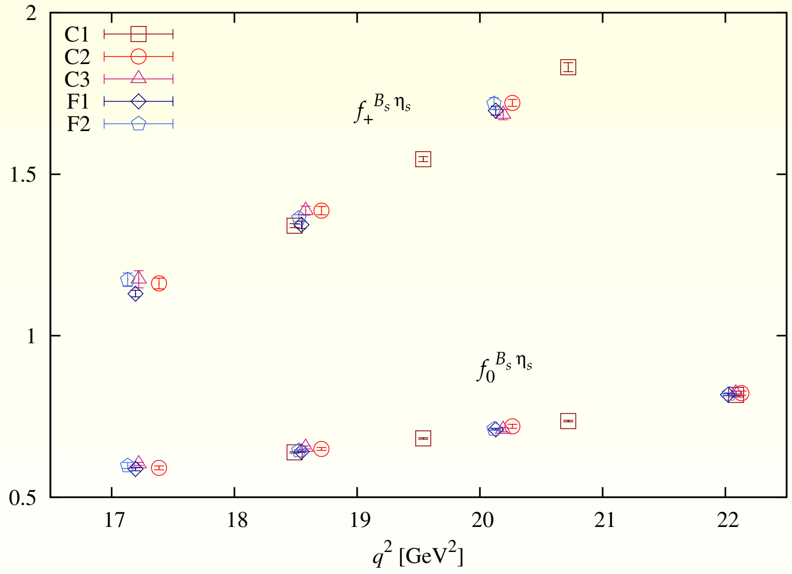

$B_s \to \eta_s l \nu$

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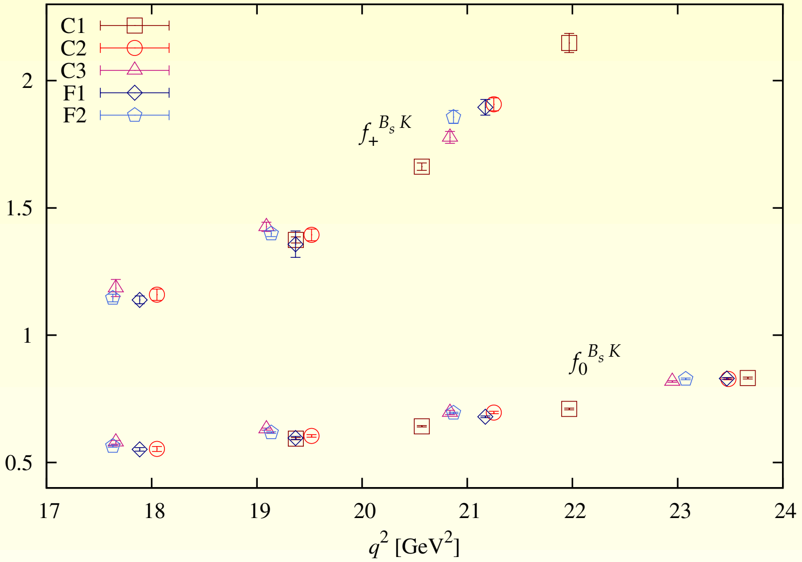

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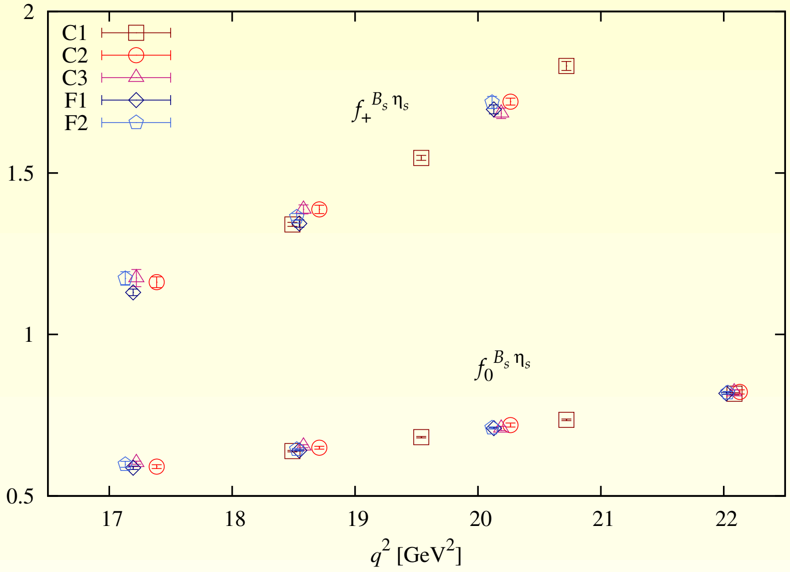

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* Also build ratio of form factors for $B_s \to Kl\nu$ and $B_s \to \eta_s l\nu$ (many systematic cancel partially or totally).

+ relativistic (HISQ) calculation of $B_s \to \eta_s l\nu$
→ precise determination of $B_s \to Kl\nu$ form factors.

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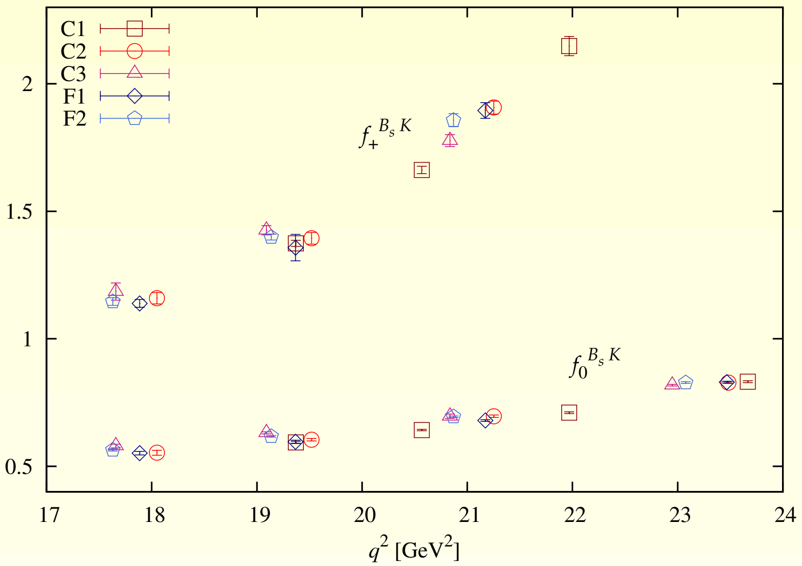

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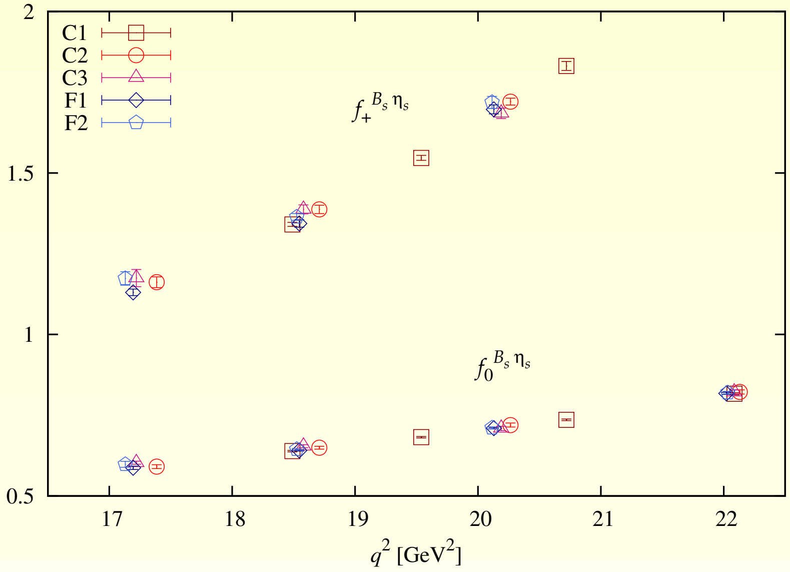

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\# FNAL/MILC also work in progress to get $B_s \to Kl\nu$ form factors.

4.3. B rare decays: $B \to K l^+ l^-$

\# **Preliminary** results from **Cambridge group**, 1101.2726, **FNAL/MILC**, 1211.1390 and **HPQCD**, Bouchard Lattice12

- * All calculations on **MILC** $N_f = 2 + 1$ configurations. Different light quark actions (Asqtad, HISQ) and heavy quark actions (Fermilab, NRQCD).
- * Need three form factors (vector, scalar, tensor).

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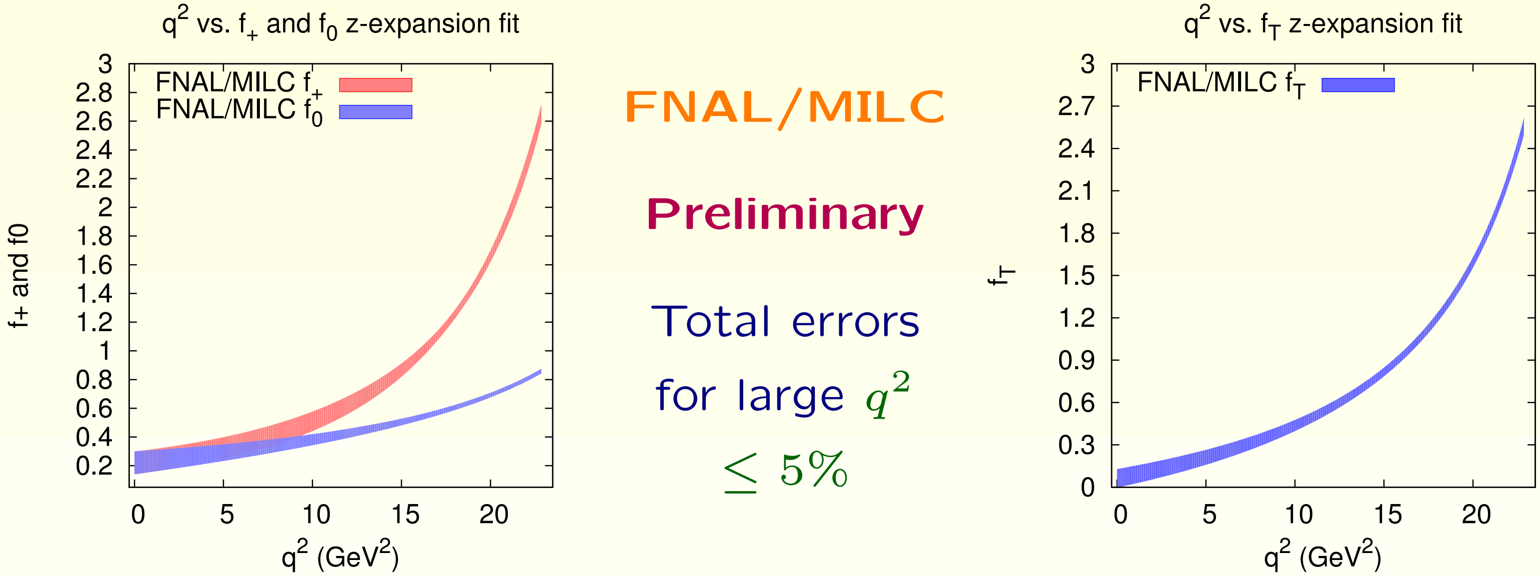


FNAL/MILC

Preliminary

Total errors

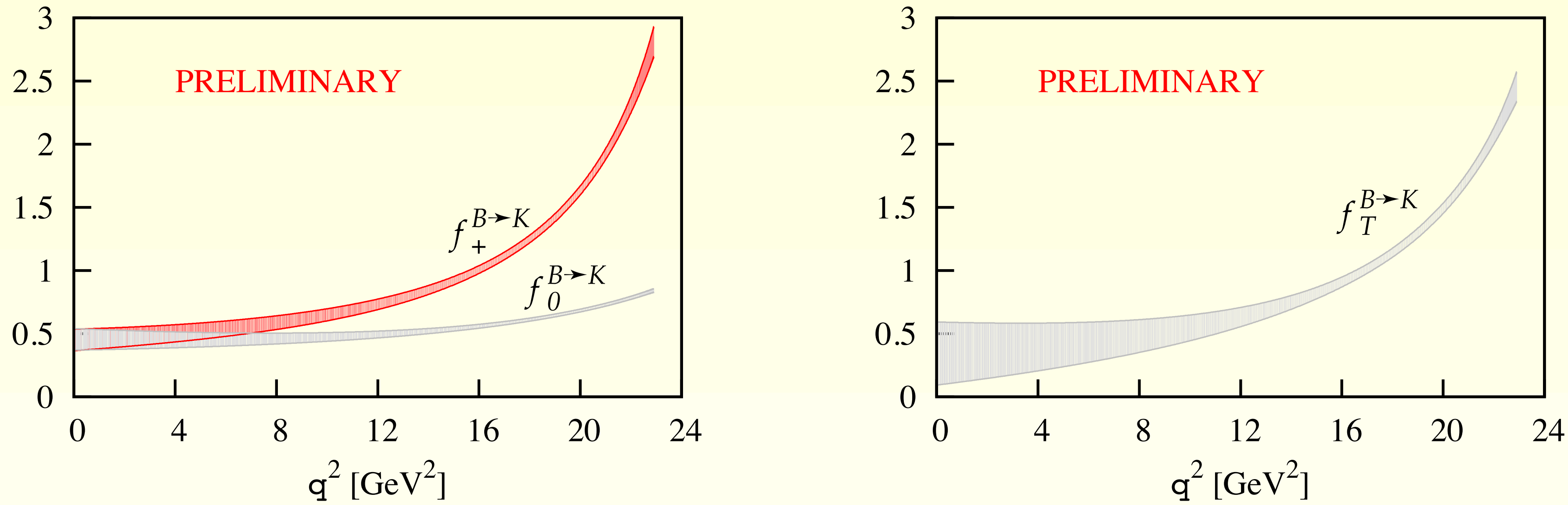
for large q^2

$\leq 5\%$

- **FNAL/MILC** shape from `z-expansion` (data at $q^2 \geq 15\ \mathrm{GeV}^2$) and systematic errors included. Four lattice spacings.

4.3. B rare decays: $B \to K l^+ l^-$

HPQCD shape from `z-expansion` (data at $q^2 \geq 17\ \mathrm{GeV}^2$) and systematic errors included. Two lattice spacings, NRQCD.



* All systematic errors included except for matching (dominant one)

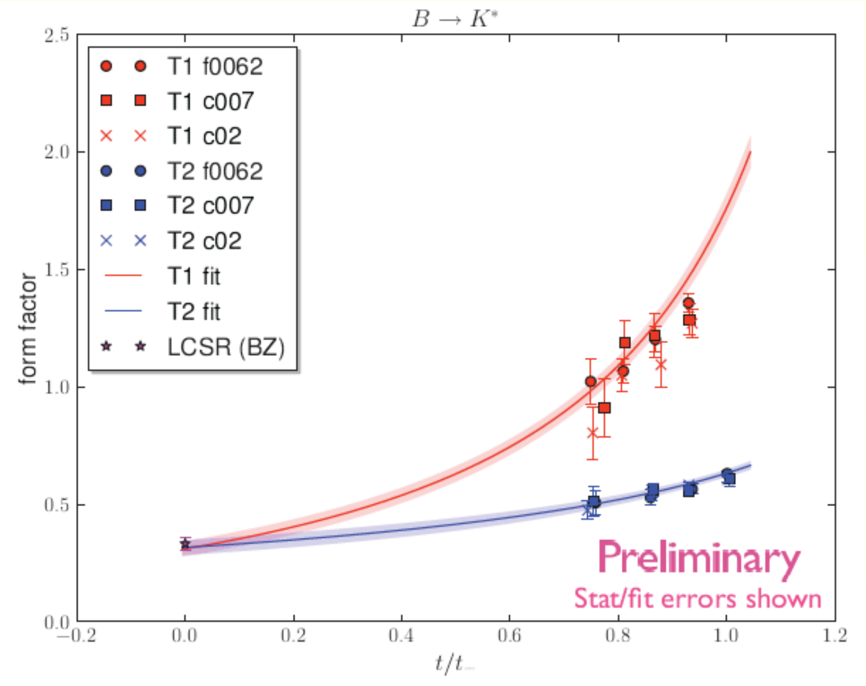
Experimental results so far consistent with SM predictions.

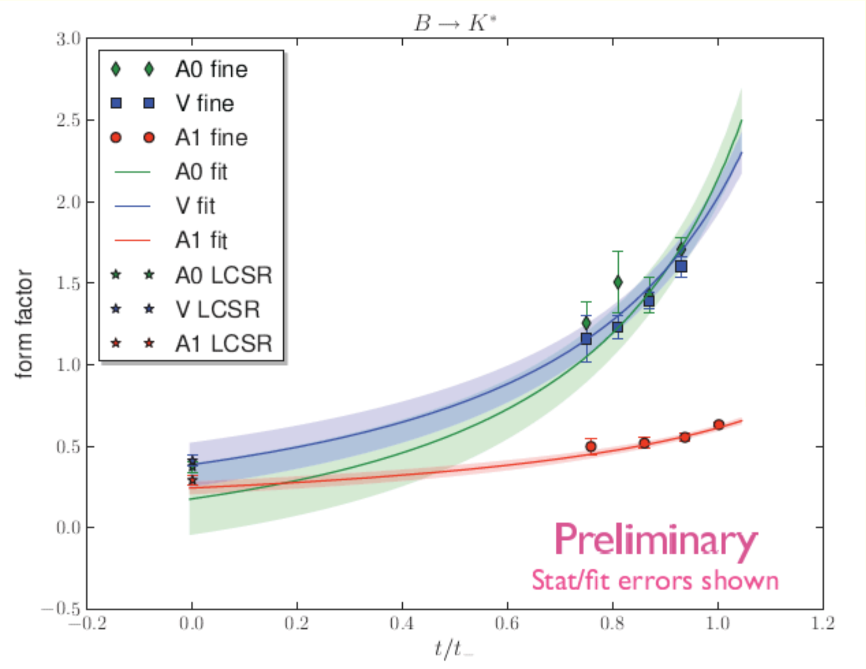
4.4. B **rare decays:** $B \to K^* l^+ l^-$

\# Coming soon: Update from **Cambridge group** for $B \to K(K^*) l\nu$.

- * Need seven form factors.
- * Simulate at q^2 away from charmonium resonances.

Preliminary: $B \to K^* ll$ form factors vs q^2/q^2_{max} **M. Wingate** Lattice2012





4.5. Exclusive determination of $|V_{cb}|$

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Some **preliminary** results:

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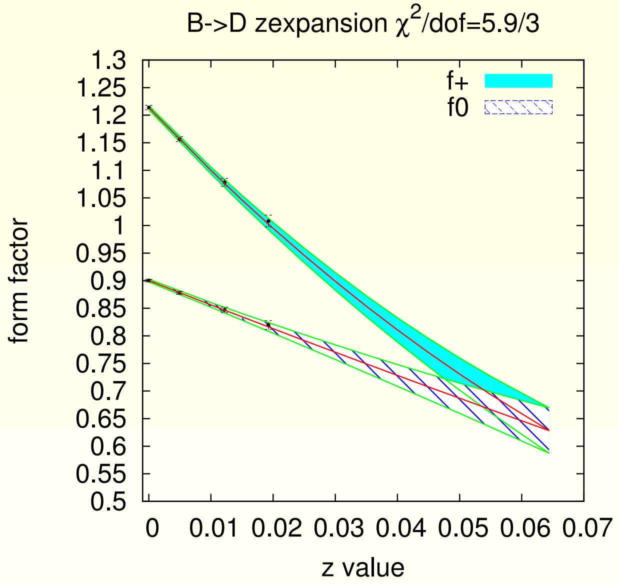
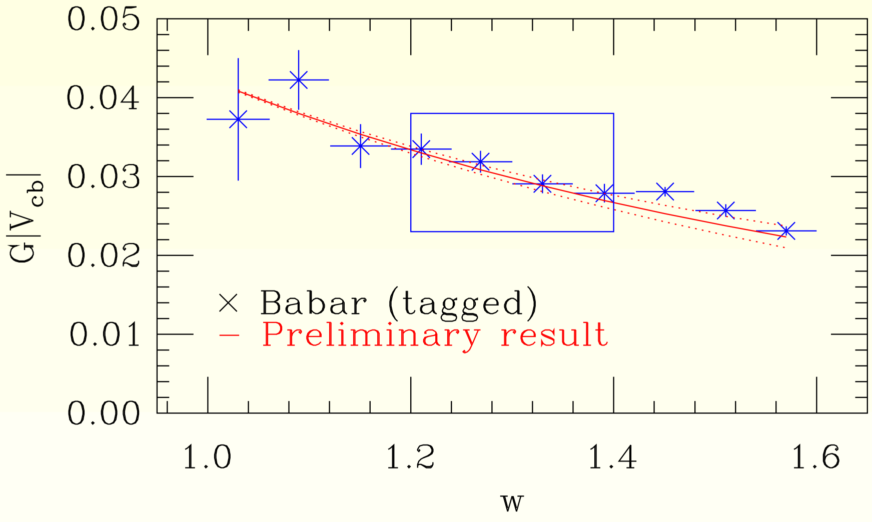
Some **preliminary** results:

Extraction from $B \to D l\nu$: need non-zero recoil to match $B \to D^* l\nu$ precision.

Preliminary FNAL/MILC $N_f = 2+1$, 1211.2247 four lattice spacings



Form factors parametrized by the `z-expansion`



(with $\omega = v_B \cdot v_D$)

Boxed region: smallest combined errors.

→ will allow complementary extraction of $|V_{cb}|$.

4.6. $B \to D\tau\nu$ and NP hints?

\# **BaBar** recently measured the ratio of branching fractions

$R(D) = \frac{\mathcal{B}r(B\to D\tau\nu)}{\mathcal{B}r(B\to Dl\nu)} = 0.440(72), \quad R(D^*) = 0.332 \pm 0.030$ PRL109 (2012)101802

Using form factors in **Kamenik,Mescia**, 0802.3790 (quenched lattice)

→ $(3.4)\sigma$ exclusion of SM PRL109 (2012)101802

(2σ exclusion with only $R(D)$)

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$N_f = 2+1$ form factor calculation by **FNAL/MILC**, PRL109 (2012)071802

$$\boxed{R(D) = 0.316(12)(7)} \to 1.7\sigma \text{ from experiment}$$

Becirevic, Kosnik, Tayduganov, 1206.4977: $R(D) = 0.31(2)$

* In progress: Analysis in the complete $N_f = 2+1$ **FNAL/MILC** data set
→ important reduction of errors in $R(D)$

* Another target: unquenched lattice calculation of $R(D^*)$

5. Neutral B-`meson` mixing

\# Hints of NP in neutral B-`meson` mixing: UTfit **1010.5089**, CKMfitter **1203.0238**, like-sign dimuon charge asymmetry **1106.6308** + UT tensions . . .

Not confirmed by recent analyses (B_s) **Lenz et al**, 1203.0238,
talk by Silvestrini

or by recent LHCb measurements. See talk by Vesterinen

Still room for important effects in B mixing. **Lenz et al**, 1203.0238

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Effective Hamiltonian describing neutral $B-$meson mixing.

$$\mathcal{H}_{eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{Q}_i \qquad \text{with}$$

$$Q_1^q = (\bar{b}^\alpha \, \gamma_\mu L \, q^\alpha) \left(\bar{b}^\beta \, \gamma^\mu L \, q^\beta\right) \qquad \textbf{SM}$$

$$Q_2^q = (\bar{b}^\alpha \, L \, q^\alpha) \left(\bar{b}^\beta \, L \, q^\beta\right) \qquad Q_3^q = \left(\bar{b}^\alpha \, L \, q^\beta\right) \left(\bar{b}^\beta \, L \, q^\alpha\right)$$

$$Q_4^q = (\bar{b}^\alpha \, L \, q^\alpha) \left(\bar{b}^\beta \, R \, q^\beta\right) \qquad Q_5^q = \left(\bar{b}^\alpha \, L \, q^\beta\right) \left(\bar{b}^\beta \, R \, q^\alpha\right)$$

$\tilde{Q}_{1,2,3} = Q_{1,2,3}$ with the replacement $L(R) \to R(L)$

5.1 Neutral B-meson mixing: SM

In the Standard Model the mass differences $\Delta M_{s(d)}$ depend on a single matrix element.

$$\Delta M_q|_{SM} = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tq}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_q}^2 \hat{B}_{B_q}$$

** Non-perturbative input $\frac{8}{3} f_{B_q}^2 B_{B_q}(\mu) M_{B_q}^2 = \langle \bar{B}_q^0 | O_1^q | B_q^0 \rangle (\mu)$

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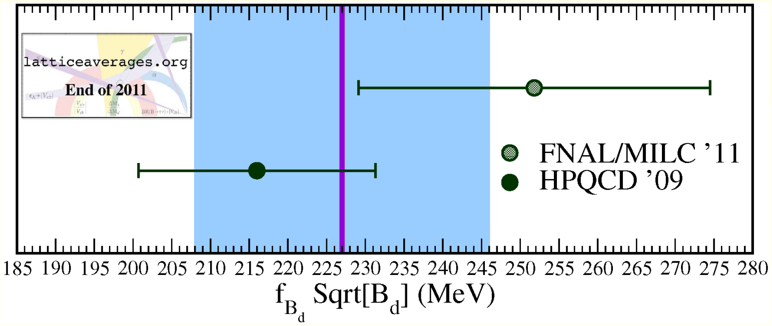

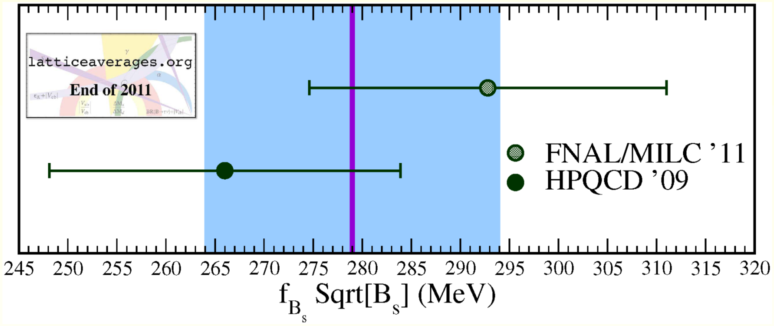

$$f_{B_s}\sqrt{\hat{B}_{B_s}}^{\,\mathbf{LLV}} = 279(15)\mathrm{MeV} \qquad f_{B_d}\sqrt{\hat{B}_{B_d}}^{\,\mathbf{LLV}} = 227(19)\mathrm{MeV}$$

5.1 Neutral B-meson mixing: SM

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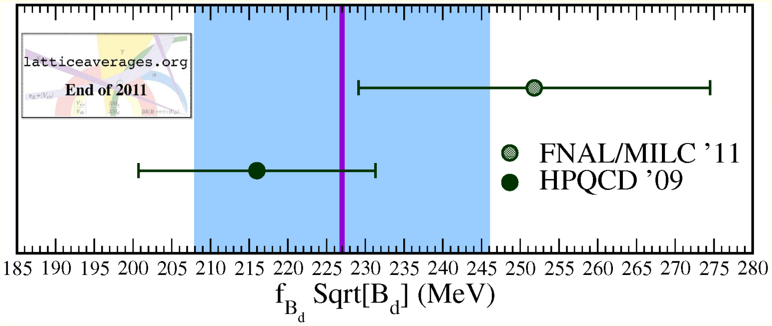

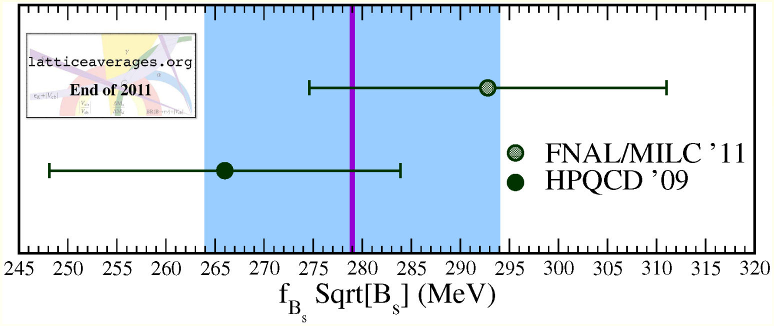

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$\Delta M_s^{SM} = (19.6 \pm 2.1) ps^{-1}$ **Lenz,Nierste** + above average

$\Delta M_s^{SM} = (16.9 \pm 1.2) ps^{-1}$ **Lenz,Nierste** + aver. f_{B_s} + B_{B_s} next slide

$\Delta M_s^{exp} = (17.768 \pm 0.023 \pm 0.006) ps^{-1}$ **LHCb** Moriond 2013 **preliminary**

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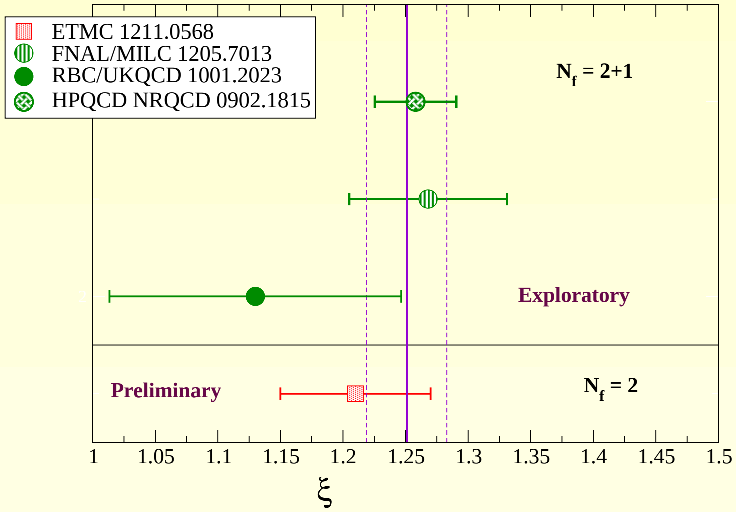

Results for $\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} \propto \sqrt{\frac{\Delta M_{B_s}}{\Delta M_{B_d}}}$

$$\boxed{\xi^{\mathbf{N_f=2+1}} = 1.251 \pm 0.032}$$

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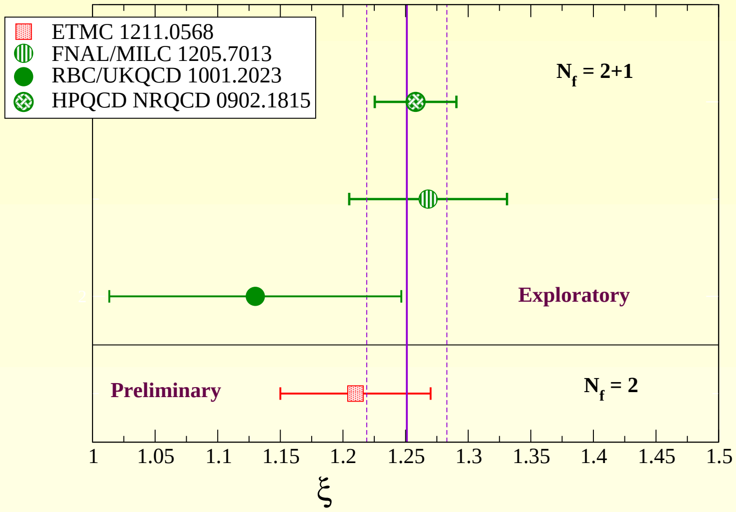

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Bag parameters

* **HPQCD**, 0902.1815: $\boxed{B_{B_s}^{\overline{MS}}(m_b) = 0.86(4)}$ $\boxed{B_{B_d}^{\overline{MS}}(m_b) = 0.82(7)}$

$$B_{B_s}/B_{B_d} = 1.05(7)$$

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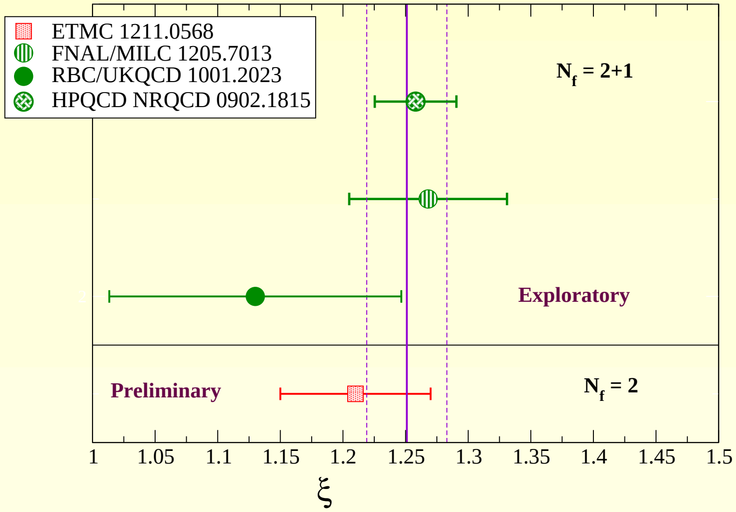

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* Bag parameters: **ETMC**, Lattice 2012 $N_f = 2$ **Preliminary**

$$B_{B_s}^{\overline{MS}}(m_b) = 0.90(5) \qquad B_{B_d}^{\overline{MS}}(m_b) = 0.87(5) \qquad \frac{B_{B_s}}{B_{B_d}} = 1.03(2)$$

In progress: **ETMC**, $N_f = 2+1+1$ calculation

5.2 Neutral B-meson mixing: BSM

\# SM predictions + BSM contributions = experiment

→ constraints on BSM building **Dobrescu and Krnjaic**, 1104.2893; **Altmannshofer and Carena**, 1110.0843; **Buras and Girrbach**, 1201.1302 ...

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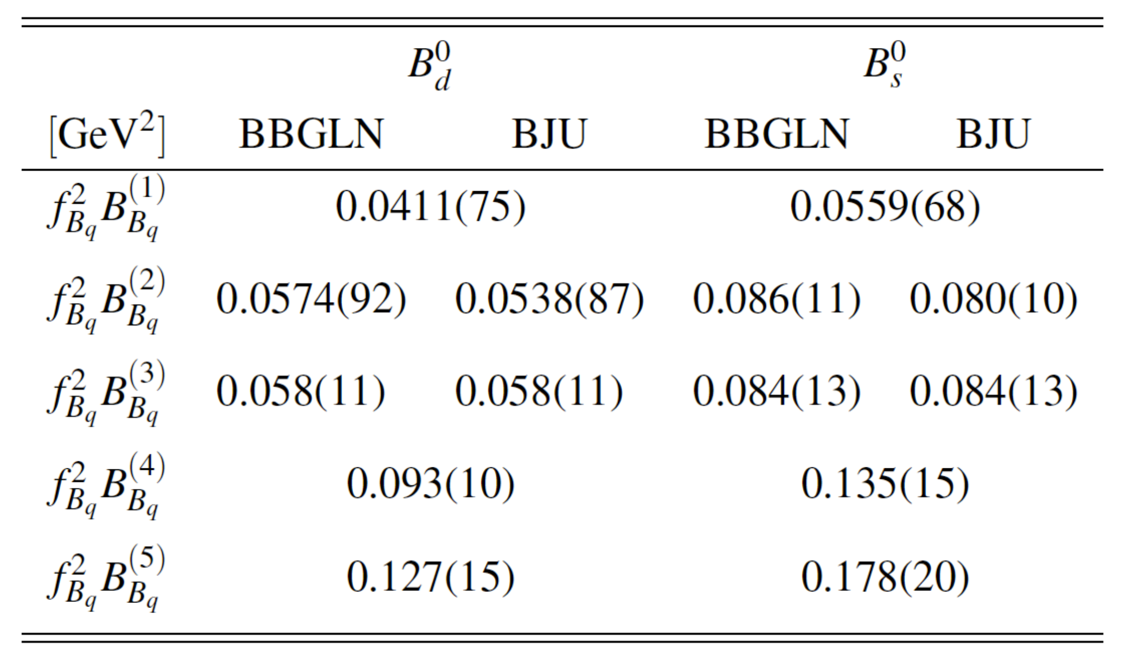

	B^0_d		B^0_s	
[GeV^2]	BBGLN	BJU	BBGLN	BJU
$f^2_{B_q} B^{(1)}_{B_q}$	0.0411(75)		0.0559(68)	
$f^2_{B_q} B^{(2)}_{B_q}$	0.0574(92)	0.0538(87)	0.086(11)	0.080(10)
$f^2_{B_q} B^{(3)}_{B_q}$	0.058(11)	0.058(11)	0.084(13)	0.084(13)
$f^2_{B_q} B^{(4)}_{B_q}$	0.093(10)		0.135(15)	
$f^2_{B_q} B^{(5)}_{B_q}$	0.127(15)		0.178(20)	

Preliminary results from **FNAL/MILC**, 1112.5642 $N_f = 2+1$

* $\langle Q_1 \rangle, \langle Q_3 \rangle$ will also allow new prediction for $\Delta\Gamma_s$.

$\Delta\Gamma_s^{SM} = (0.075 \pm 0.020) ps^{-1}$ **Nierste**, CKM2012 using **preliminary** results above

$\Delta\Gamma_s^{exp} = (0.106 \pm 0.011 \pm 0.007) ps^{-1}$ **LHCb**, Moriond 2013

6. Rare decays $\mathcal{B}r(B_{s(d)} \to \mu^+\mu^-)$

See talks by Altmannshofer, Girrbach, and Knegjens

\# Bag parameters describing B−`meson` mixing in the SM can be used for theoretical prediction of $\mathcal{B}r(B \to \mu^+\mu^-)$ **Buras**, hep-ph/0303060

$$\frac{\mathcal{B}r(B_q \to \mu^+\mu^-)}{\Delta M_q} = \tau(B_q)\, 6\pi \frac{\eta_Y}{\eta_B} \left(\frac{\alpha}{4\pi M_W sin^2\theta_W}\right)^2 m_\mu^2 \frac{Y^2(x_t)}{S(x_t)} \frac{1}{\hat{B}_q}$$

* Need to include the effects of a non-vanishing $\Delta\Gamma_s$ to compare with experiment **K. de Bruyn et al.**, 1204.1737

$$\mathcal{B}r(B_q \to \mu^+\mu^-)_{SM} \to \mathcal{B}r(B_q \to \mu^+\mu^-)_{y_s} \equiv \mathcal{B}r(B_q \to \mu^+\mu^-)_{SM} \times \frac{1}{1-y_s}$$

with $y_s \equiv \Delta\Gamma_s/(2\Gamma_s)$.

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* Using $\hat{B}_{B_s} = 1.33(6)$, $\hat{B}_{B_d} = 1.26(11)$ **HPQCD**, 0902.1815, $y_s = 0.087 \pm 0.014$ **LHCb**,1212.4140

$$\mathcal{B}r(B_s \to \mu^+\mu^-)_{y_s} = (3.71 \pm 0.17) \times 10^{-9}$$ **Buras et al.** 1303.3820

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Error dominated by uncertainty in the bag parameter **Buras et al.** 1303.3820

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\# Improved $f_{B_{s,d}}^{lattice}$ makes direct theoretical calculation competitive **Buras and Girrbach**,1204.5064

* Using the lattice averages giving in 1302.2644: $f_B = (185 \pm 3)\ MeV$ and $f_{B_s} = (225 \pm 3)\ MeV$.

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Dominant errors: $|V_{tb}^* V_{ts}|$ 4%, f_{B_s} 2.7%

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\# Most stringent experimental bounds **LHCb** Moriond 2013:

$$\mathcal{B}r(B_s \to \mu^+\mu^-) = \left(3.2^{+1.4+0.5}_{-1.2-0.3}\right) \times 10^{-9}$$

$\mathcal{B}r(B_d \to \mu^+\mu^-) < 9.4 \times 10^{-10}$ at 95% CL

7. Conclusions and outlook

- # Smallest errors achieved on the lattice for many quantities: decay constants, $|V_{ub,cb}|$ from exclusive decays, neutral meson mixing parameters $\ldots$

- # Progress using relativistic description of `b-quarks`

  → important reduction of error.

- # First results with all quark masses physical and $N_f = 2+1+1$.

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Expected in the next year:

* New calculations of $f_{B_{d,s}}$: **FNAL/MILC**, **ETMC** $(N_f = 2)$.

* First unquenched calculations of complete set of bag parameters describing B mixing in the SM and beyond

** Expect important reduction of errors in SM bag parameters

* Update on exclusive $|V_{ub}|$ determination from $B \to \pi l \nu$ (several groups) and exclusive $|V_{cb}|$ determination from $B \to D(D^*) l \nu$

* Exclusive $|V_{ub}|$ determination from $B_s \to K l \nu$ (several groups)

* First unquenched calculation of $B \to K l^+ l^-$ (several groups)