Saturation, coherence and exclusive final states

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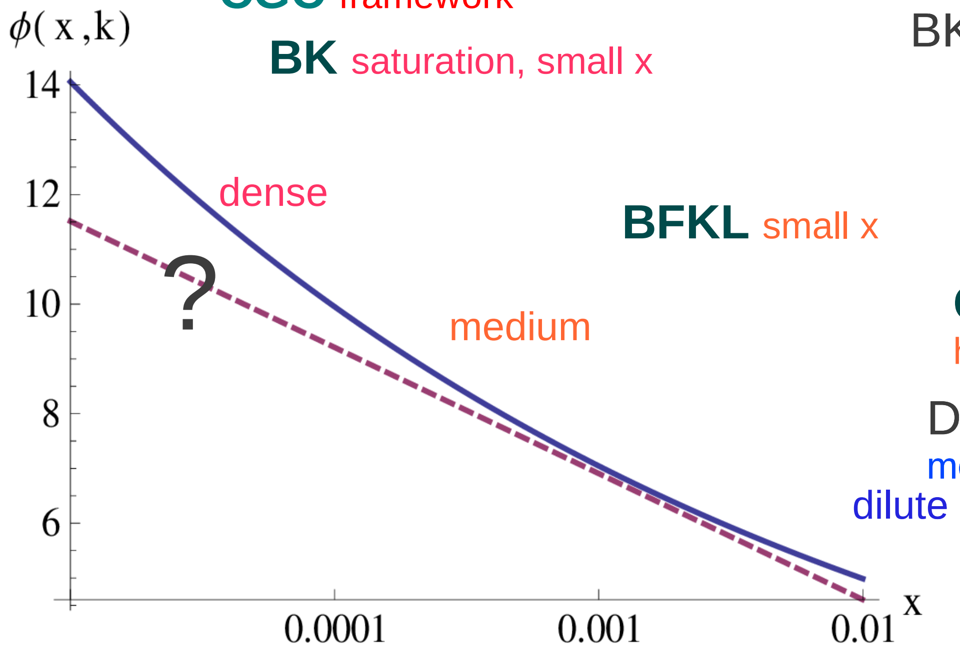
The motivation – to investigate gluon at low x

JIMWLK multi gluon, saturation, small x

CGC framework

BKP multi gluon, small x

BK saturation, small x

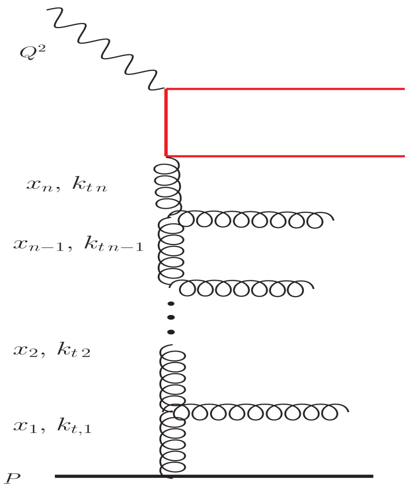


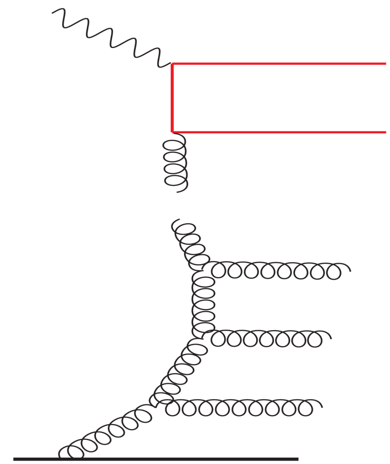
BFKL small x

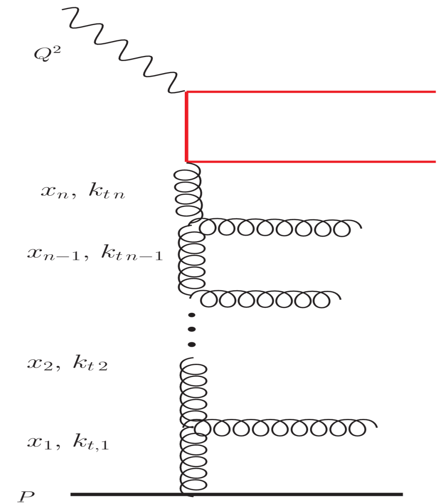
CCFM
hard processes, small x and large x

DGLAP hard processes, large x,
medium x

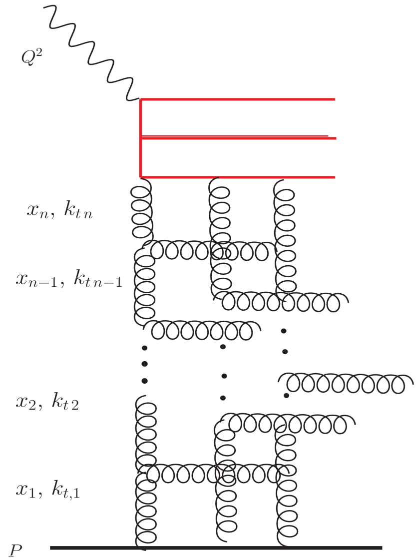
Schematic illustration of evolution scheems in pQCD

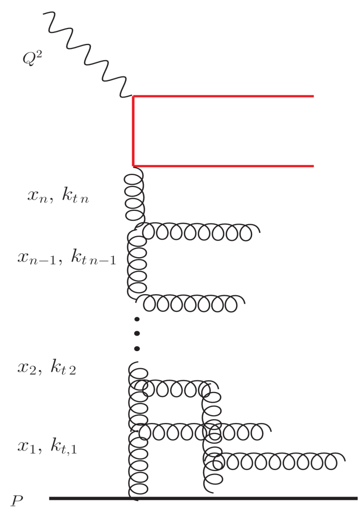


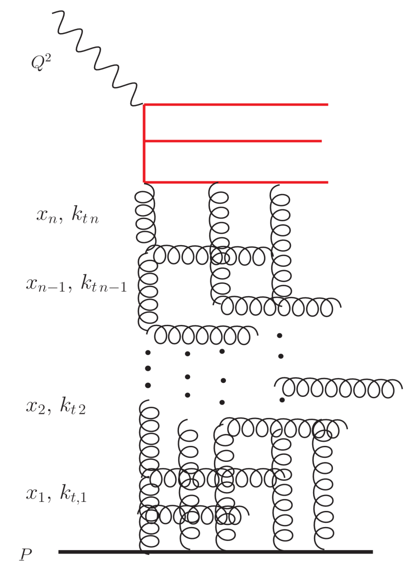




DGLAP (ordering in hard scale) CCFM (ordering in angle) BFKL (ordering in x)







BKP (multi gluons ord. In x)

BK (ordering in x, fusion) JIMWLK (multigluon,fusion)

The motivation – to understand gluon at low x with the help of exclusive processes

JIMWLK multi gluon, saturation, small x

CGC framework

BK saturation, small x

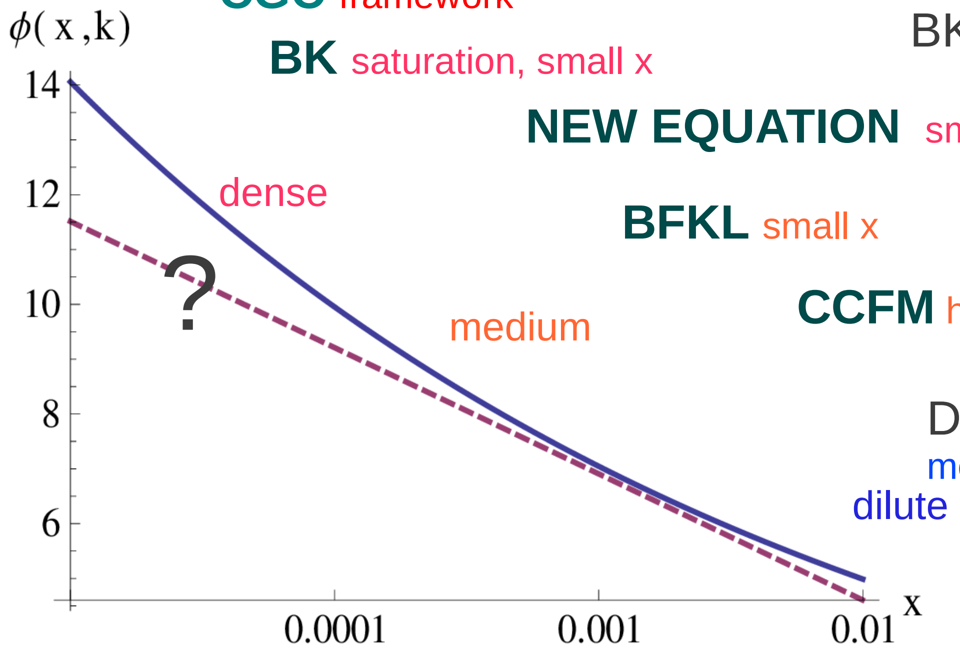
BKP multi gluon, small x

NEW EQUATION small x, large x, hard process

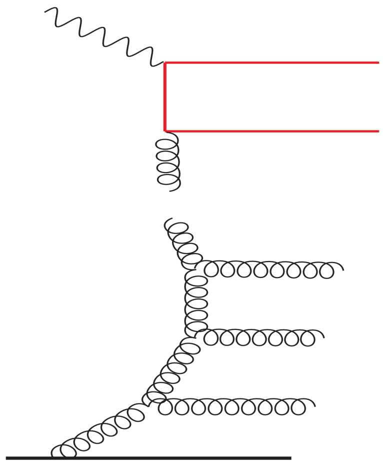
BFKL small x

CCFM hard processes, small and large x

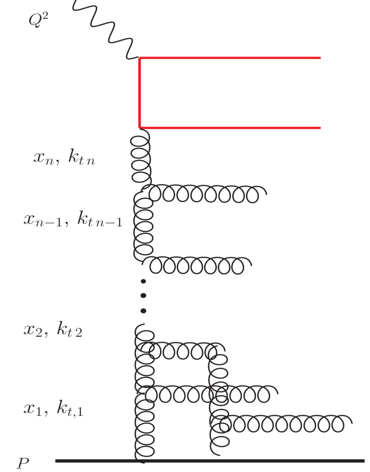
DGLAP hard processes, large x, medium x



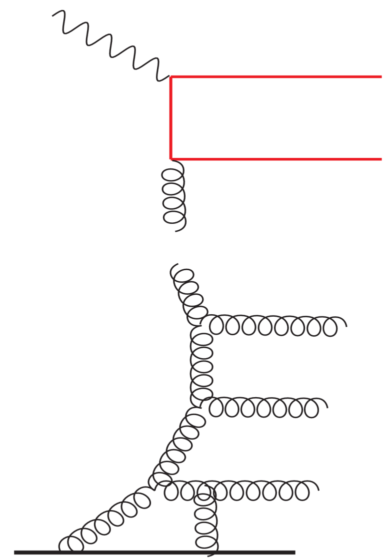
Schematic illustration of proposed recently new equation combining BK and CCFM



CCFM



BK



New equation

QCD at high energies – high energy factorization

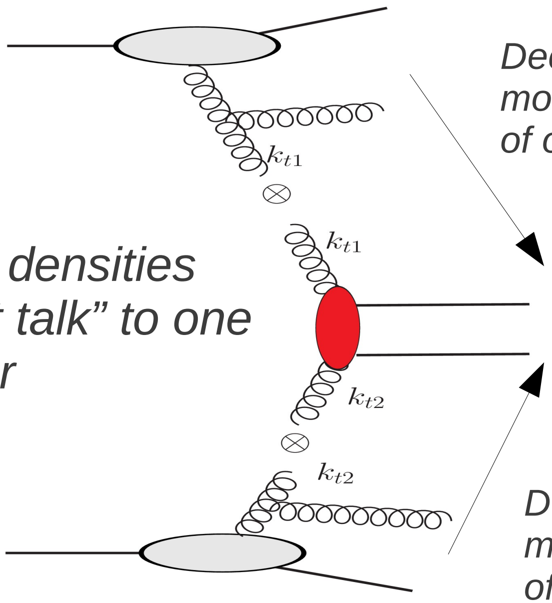
$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,b,c,d} \int \frac{d^2k_{1t}}{\pi}\frac{d^2k_{2t}}{\pi}\frac{1}{16\pi^2(x_1x_2S)^2}\overline{|\mathcal{M}_{ab\to cd}|}^2 \delta^2(\vec{k}_{1t}+\vec{k}_{2t}-\vec{p}_{1t}-\vec{p}_{2t})$$
$$\times\, \mathcal{F}_{a/A}(x_1,k_{1t}^2,\mu^2)\,\mathcal{F}_{b/B}(x_2,k_{2t}^2,\mu^2)\,\frac{1}{1+\delta_{cd}}$$

$$k_1^\mu = x_1 P_1^\mu + \bar{x}_1 P_2^\mu + k_{1t}^\mu \qquad k_2^\mu = x_2 P_2^\mu + \bar{x}_2 P_1^\mu + k_{2t}^\mu$$

$$\cancel{\bar{x}_1 = \frac{k_1^2+\mathbf{k}^2}{Sx_1}} \qquad \cancel{\bar{x}_2 = \frac{k_2^2+\mathbf{k}^2}{Sx_2}}$$

Parton densities “do not talk” to one another

$$|\mathcal{M}_{ab\to cd}|^2 = \frac{2x_1 k_1^{\mu_1} k_1^{\nu_1}}{k_1^2}\frac{2x_2 k_2^{\mu_2} k_2^{\nu_2}}{k_2^2}\mathcal{M}_{ab\to cd\,\mu_1\nu_1}\mathcal{M}^*_{ab\to cd\,\mu_2\nu_2}$$



Decreasing longitudianl momentum fractions of off-shell partons

Decreasing longitudianl momentum fractions of off-shell partons

Gribov, Levin, Ryskin '81
Ciafaloni, Catani, Hautman '93

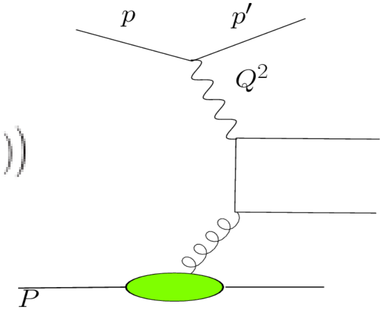
Originally derived for quarks in final state.
Lipatov provided general framework.

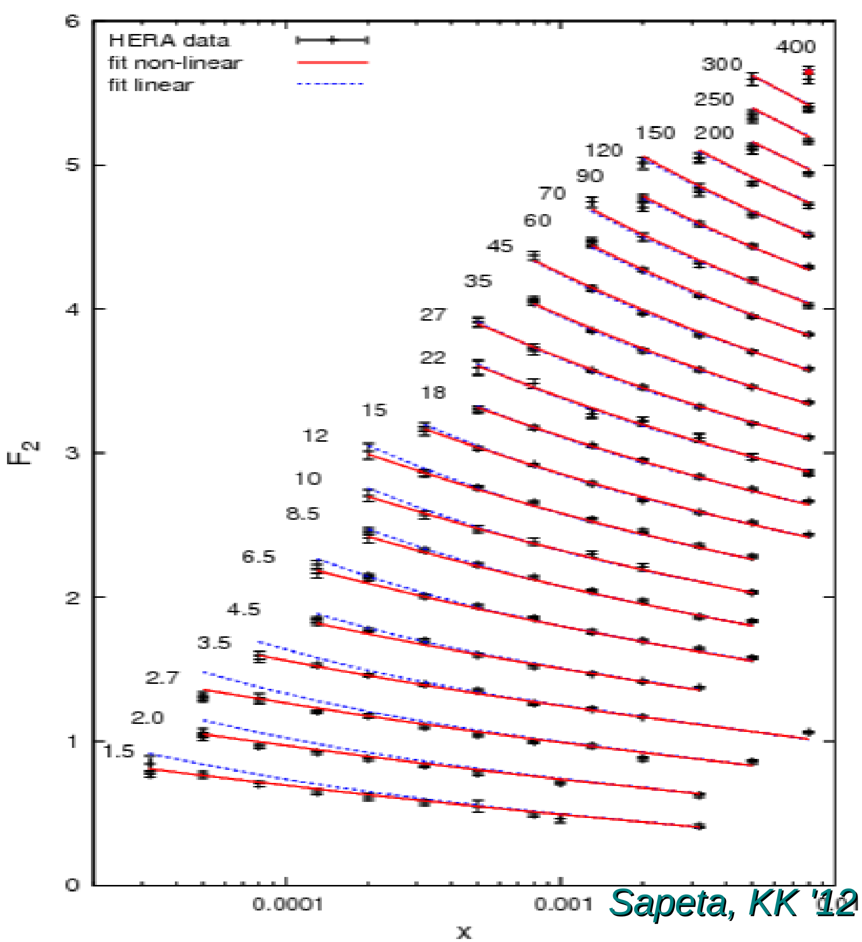
Recently new approach consistent with Lipatov's action allowed for formulation and numerical calculation of any tree level amplitude with off-shell gluons in initial state
Van Hameren, Kotko, KK '12

Attempts to generalize to p-A.
Dominguez, Huan, Marquet,Xiao '10

BFKL applied to DIS - some recent results

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2}\alpha_s \sum_q e_q^2 \int d^2k\, \mathcal{F}(x,k^2)\left(S_L(k^2,Q^2,m_q^2) + S_T(k^2,Q^2,m_q^2)\right)$$





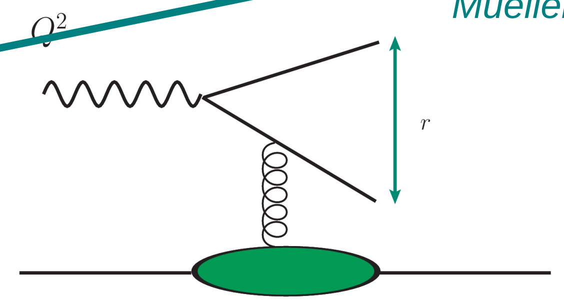
Sapeta, KK '12

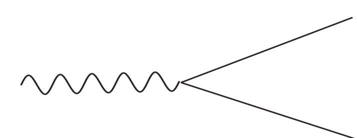
In the dipole formalism

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\int d^2b \int_0^1 dz \int d^2r \left(|\psi_L(z,r)|^2 + |\psi_T(z,r)|^2\right) N(x,r,b)$$

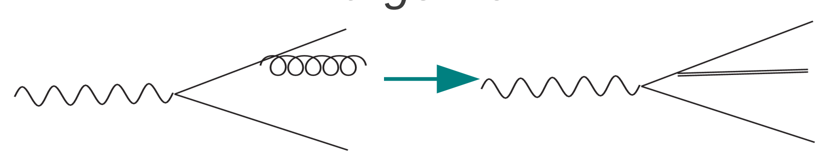
Mueller, Patel '95

Forward scattering amplitude





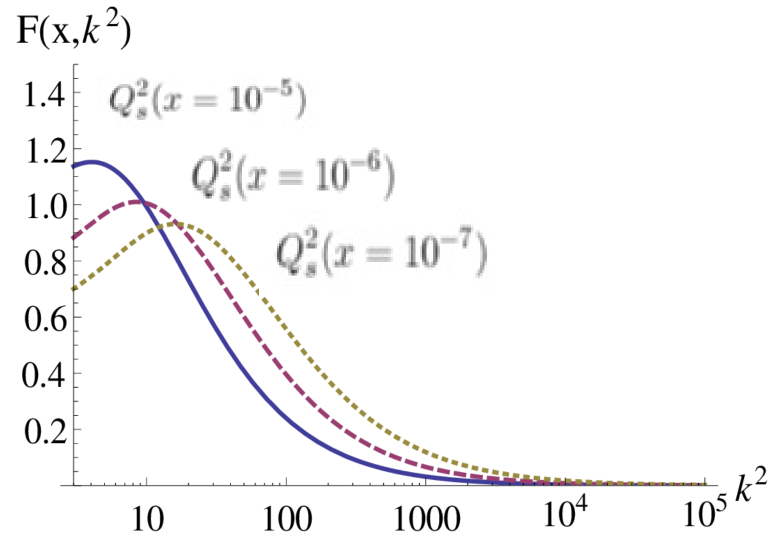
Large Nc



$$\mathcal{F}(x,k^2) = \frac{N_c}{\alpha_s(2\pi)^3}\int d^2b \int d^2r e^{ik\cdot r}\nabla_r^2 N(r,b,x)$$

High energy factorizable gluon density with saturation

$$\mathcal{F}(x,k^2) = \frac{N_c}{\alpha_s(2\pi)^3}\int d^2b \int d^2r\, e^{ik\cdot r}\nabla_r^2 N(r,b,x)$$



$$\mathcal{F}(x,k^2) = \mathcal{F}_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z}\int_0^\infty \frac{dl^2}{l^2}\left[\frac{l^2\mathcal{F}(x/z,l^2) - k^2\mathcal{F}(x/z,k^2)}{|k^2-l^2|} + \frac{k^2\mathcal{F}(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right]$$
$$-\frac{2\alpha_s^2\pi}{N_c R^2}\int_{x/x_0}^1 \frac{dz}{z}\left\{\left[\int_{k^2}^\infty \frac{dl^2}{l^2}\mathcal{F}(x/z,l^2)\right]^2 + \mathcal{F}(x/z,k^2)\int_{k^2}^\infty \frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\mathcal{F}(x/z,l^2)\right\}$$

target's radius

Kwiecinski, KK '03
Nikolaev, Schaffer '04

$$\Phi_b(x,k^2) = \Phi_{0\,b}(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z}\int_0^\infty \frac{dl^2}{l^2}\left[\frac{l^2\Phi_b(x/z,l^2) - k^2\Phi_b(x/z,k^2)}{|k^2-l^2|} + \frac{k^2\Phi_b(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right] - \overline{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z}\Phi_b^2(x/z,k^2)$$

Balitsky 96, Kovchegov 99

$$\Phi_b(x,k^2) = \Phi(x,k^2)S(b) \qquad \int d^2\mathbf{b}\, S(b) = 1, \quad \int d^2b\, S^2(b) = \frac{1}{\pi R^2}$$

Interesting properties
travelling wave solution
Munier, Peschanski '03

$$\Phi(x,k^2) = \Phi_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z}\int_0^\infty \frac{dl^2}{l^2}\left[\frac{l^2\Phi(x/z,l^2) - k^2\Phi(x/z,k^2)}{|k^2-l^2|} + \frac{k^2\Phi(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right] - \frac{\overline{\alpha}_s}{\pi R^2}\int_{x/x_0}^1 \frac{dz}{z}\Phi^2(x/z,k^2)$$

$$\mathcal{F}(x,k^2) = \frac{N_c}{4\alpha_s\pi^2}k^2\nabla_k^2\Phi(x,k^2)$$

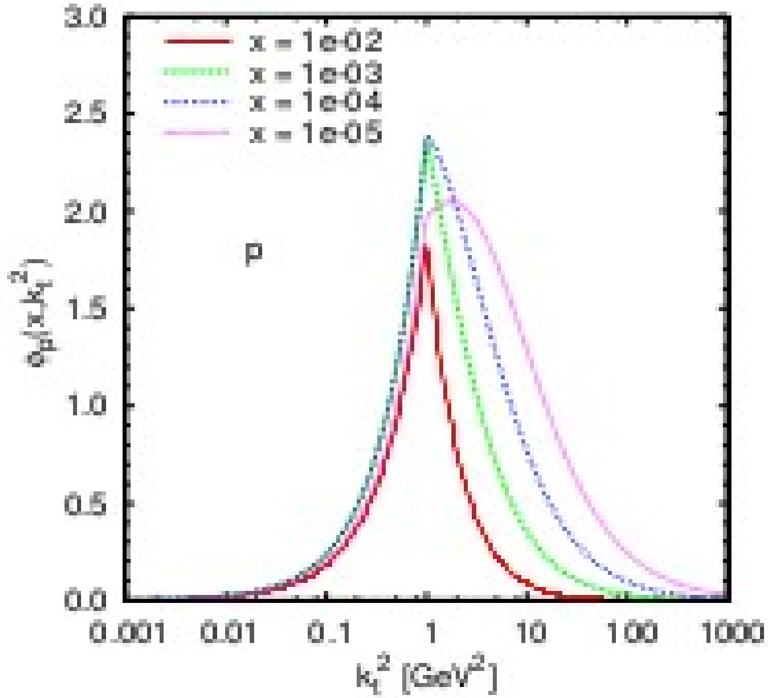
inversion

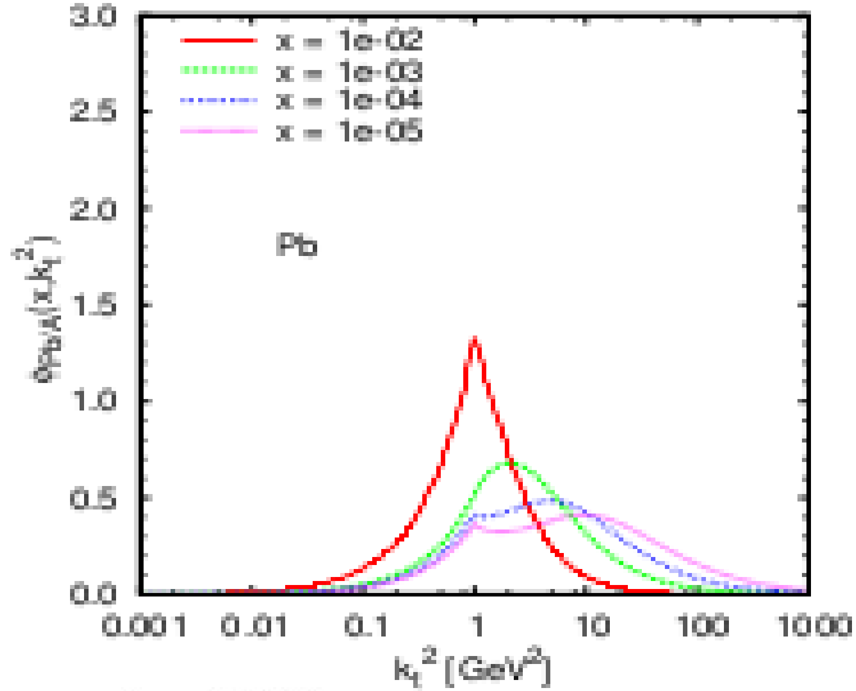
$$\Phi(x,k^2) = \frac{\alpha_s\pi^2}{N_c}\int_{k^2}^\infty \frac{dl^2}{l^2}\ln\frac{l^2}{k^2}\mathcal{F}(x,l^2)$$

density potential

Unique transformationpossible due to color transparency of the dipole amplitude

Glue in p vs. glue in Pb





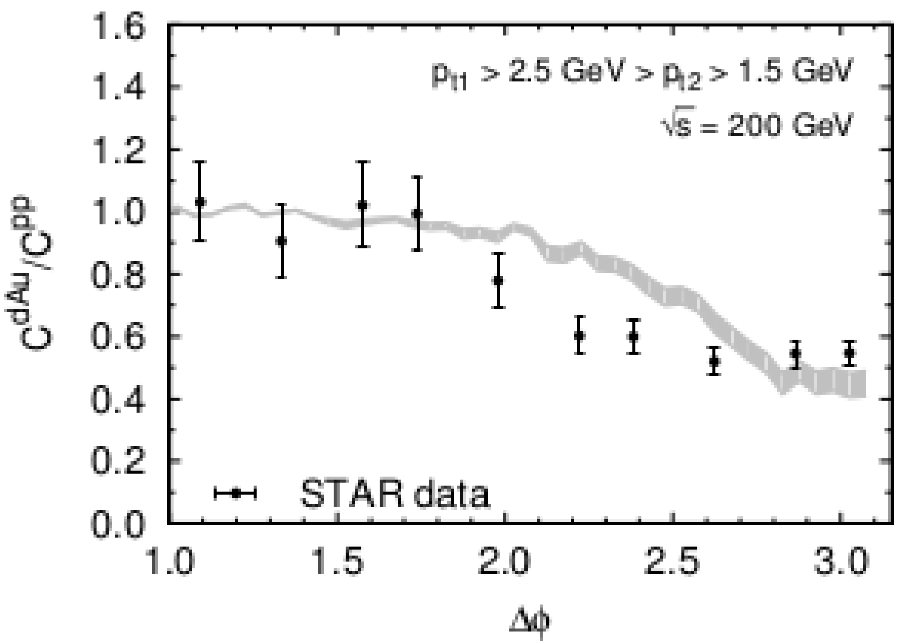
$R_{\rm A} = R\,A^{1/3}$

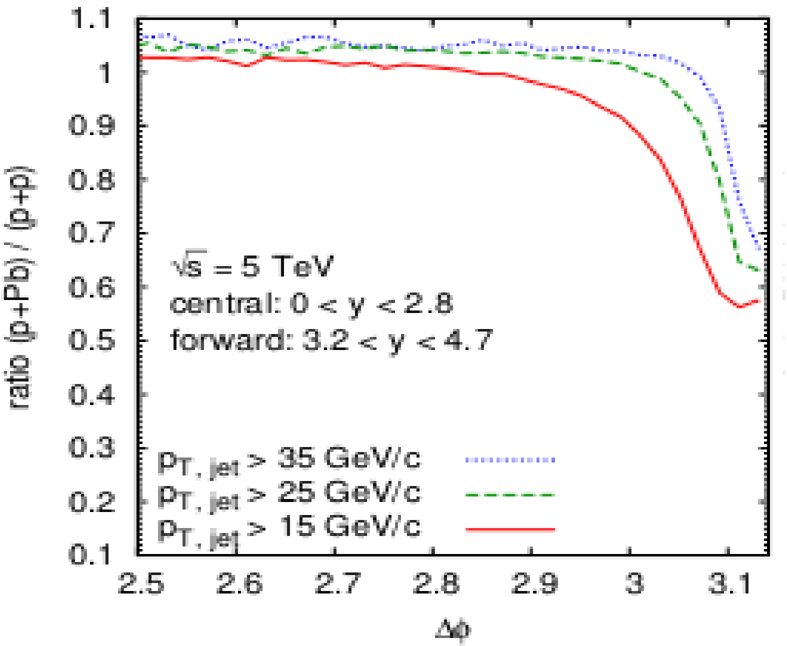
$$
\begin{aligned}
\phi_{\mathrm{HI}/A}(x,k^2) \;&=\; \phi^{(0)}_{\mathrm{HI}/A}(x,k^2) \\
+\; \frac{\alpha_s(k^2)N_c}{\pi}\int_x^1 \frac{dz}{z}\int_{k_0^2}^{\infty}\frac{dl^2}{l^2}&\left\{ \frac{l^2\phi_{\mathrm{HI}/A}(\frac{x}{z},l^2)\,\theta(\frac{k^2}{z}-l^2) \;-\; k^2\phi_{\mathrm{HI}/A}(\frac{x}{z},k^2)}{|l^2-k^2|} + \frac{k^2\phi_{\mathrm{HI}/A}(\frac{x}{z},k^2)}{|4l^4+k^4|^{\frac{1}{2}}}\right\} \\
+\; \frac{\alpha_s(k^2)}{2\pi k^2}\int_x^1 dz\,&\left[\left(P_{gg}(z)-\frac{2N_c}{z}\right)\int_{k_0^2}^{k^2} dl^2\,\phi_{\mathrm{HI}/A}\left(\frac{x}{z},l^2\right) + zP_{gq}(z)\Sigma_{HI/A}\left(\frac{x}{z},k^2\right)\right] \\
-\; \frac{2A^{1/3}\alpha_s^2(k^2)}{R^2}&\left[\left(\int_{k^2}^{\infty}\frac{dl^2}{l^2}\phi_{\mathrm{HI}/A}(x,l^2)\right)^2 + \phi_{\mathrm{HI}/A}(x,k^2)\int_{k^2}^{\infty}\frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\phi_{\mathrm{HI}/A}(x,l^2)\right],
\end{aligned}
$$

Application to some hadronic observables

Sapeta, KK '12

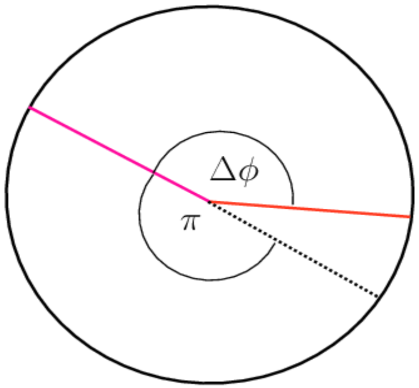
Sapeta, KK '13





Framework works well for di-hadron production at RHIC

also *Albacete, Marquet '10, Juan,Stasto,Xiao '11 ,...*



High Energy Factorization - matrix elements

- *General theory given by Lipatov effective action (Lipatov' 95; Antonov, Cherednikov, Kuraev,,Lipatov '05).*
- *So far there is no numerical tool which generates matrix elements directly from the effective action.*

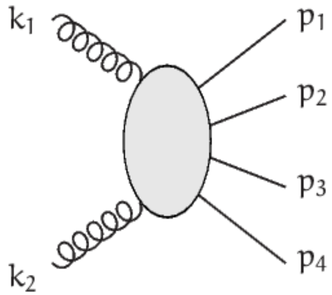
- *For collinear factorization there are: HELAC, Amagic++,AlpGen, MadGraph,...*

- *New framework which is equivalent to Lipatov effective action and which makes use of existing tools for evaluation of matrix elements*

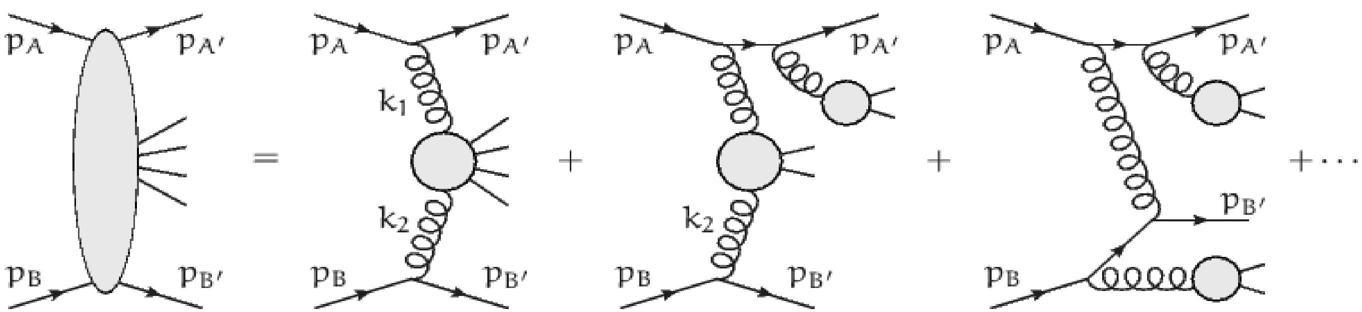
Van Hameren, Kotko, KK JHEP 1301 (2013) 078, Van Hameren, Kotko, KK JHEP 1212 (2012) 029 .

Kinematics of High Energy Factorization

Van Hameren, Kotko, KK JHEP 1301 (2013) 078



$$k_1 + k_2 = p_1 + p_2 + p_3 + p_4$$
$$k_1 = x_1 P_A + k_{\perp 1} \qquad k_2 = x_2 P_B + k_{\perp 2}$$
$$P_A \cdot k_{\perp 1} = P_A \cdot k_{\perp 2} = P_B \cdot k_{\perp 1} = P_B \cdot k_{\perp 2} = 0$$
$$P_A^2 = P_B^2 = 0$$
$$k_1^2 = k_{\perp 1}^2 \qquad k_2^2 = k_{\perp 2}^2$$



$$\ell_1 = (E, 0, 0, E) \qquad \ell_2 = (E, 0, 0, -E)$$
$$p_A - p_{A'} = k_1 = x_1 \ell_1 + k_{1\perp} + y_2 \ell_2 \qquad p_B - p_{B'} = k_2 = x_2 \ell_2 + k_{2\perp} + y_1 \ell_1$$

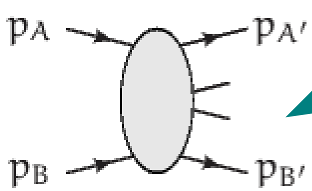
Needed to keep quarks on shell. Usually neglected

Towards automation of High Energy Factorization

Van Hameren, Kotko, KK
JHEP 1301 (2013) 078

Let us consider $\mathcal{A}(g^*g^* \to X)$

Must be gauge invariant



$$p_A = (\Lambda + x_1)\ell_1 + \kappa_{13}\ell_3$$
$$p_{A'} = \Lambda\ell_1 - \kappa_{14}\ell_4$$
$$p_A - p_{A'} = x_1\ell_1 + k_{1\perp}$$

$$p_B = (\Lambda + x_2)\ell_2 + \kappa_{24}\ell_4$$
$$p_{B'} = \Lambda\ell_2 - \kappa_{23}\ell_3$$
$$p_B - p_{B'} = x_2\ell_2 + k_{2\perp}$$

Introduce complex p_A, p_B, $p_{A'}$ $p_{B'}$

$$\ell_3^\mu = \tfrac{1}{2}\langle \ell_2 - |\gamma^\mu|\ell_1 - \rangle \qquad \ell_4^\mu = \tfrac{1}{2}\langle \ell_1 - |\gamma^\mu|\ell_2 - \rangle$$

$$p_A^\mu = (\Lambda + x_1)\ell_1^\mu - \frac{\ell_4 \cdot k_{1\perp}}{\ell_1 \cdot \ell_2}\ell_3^\mu \qquad p_{A'}^\mu = \Lambda\ell_1^\mu + \frac{\ell_3 \cdot k_{1\perp}}{\ell_1 \cdot \ell_2}\ell_4^\mu$$

$$p_B^\mu = (\Lambda + x_2)\ell_2^\mu - \frac{\ell_3 \cdot k_{2\perp}}{\ell_1 \cdot \ell_2}\ell_4^\mu \qquad p_{B'}^\mu = \Lambda\ell_2^\mu + \frac{\ell_4 \cdot k_{2\perp}}{\ell_1 \cdot \ell_2}\ell_3^\mu$$

$$p_A^\mu - p_{A'}^\mu = x_1\ell_1^\mu + k_{1\perp}^\mu \qquad p_B^\mu - p_{B'}^\mu = x_2\ell_2^\mu + k_{2\perp}^\mu$$

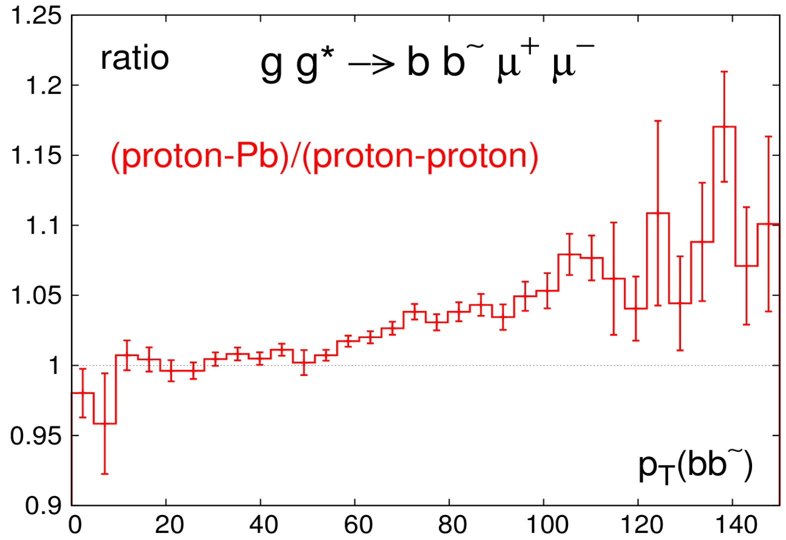
$$p_A^2 = p_{A'}^2 = p_B^2 = p_{B'}^2 = 0$$

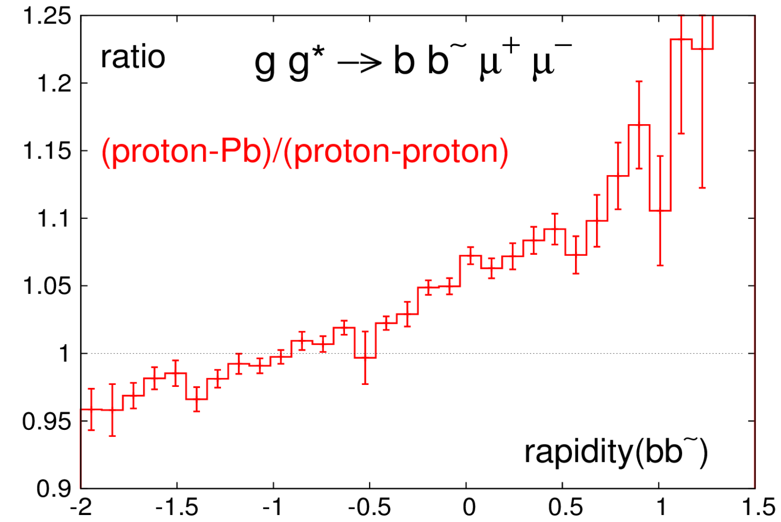
For a given process amplitude is evaluated numerically using helicity method.
In our approach the helicity method is extended for off shell initial state partons.

In agreement with effective action approach

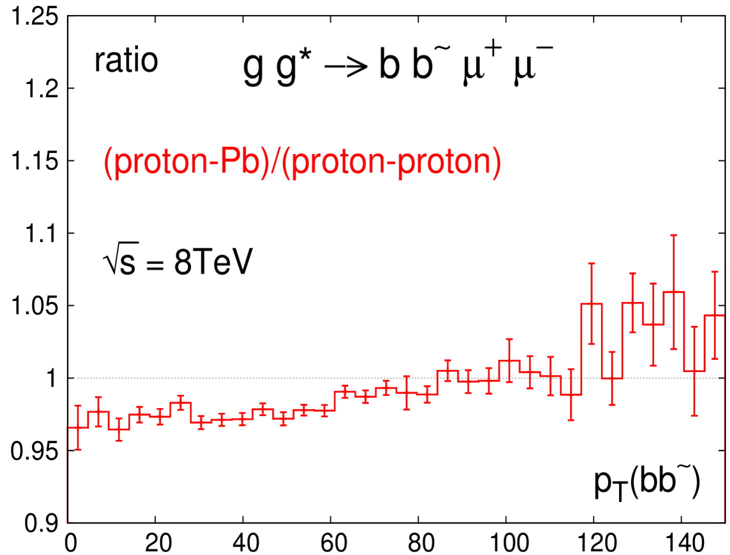
Z production in p-p vs. p-Pb

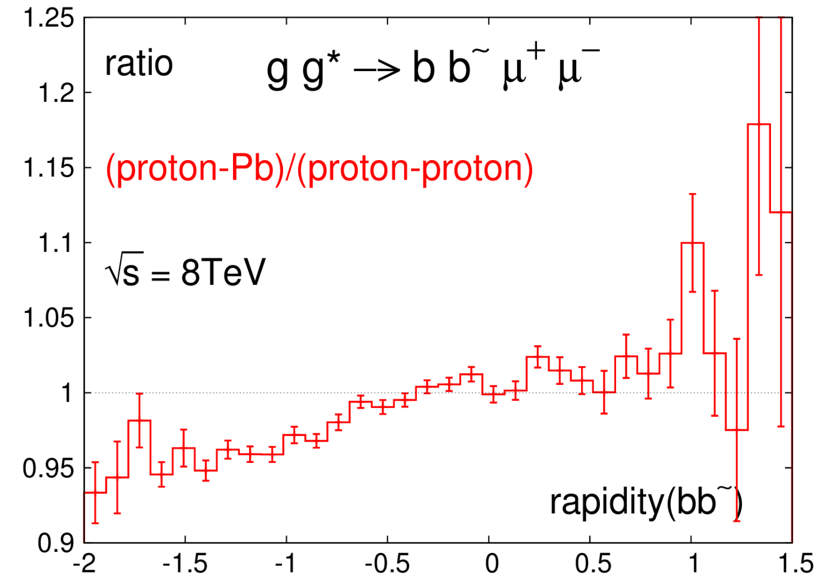
5 TeV





8 TeV





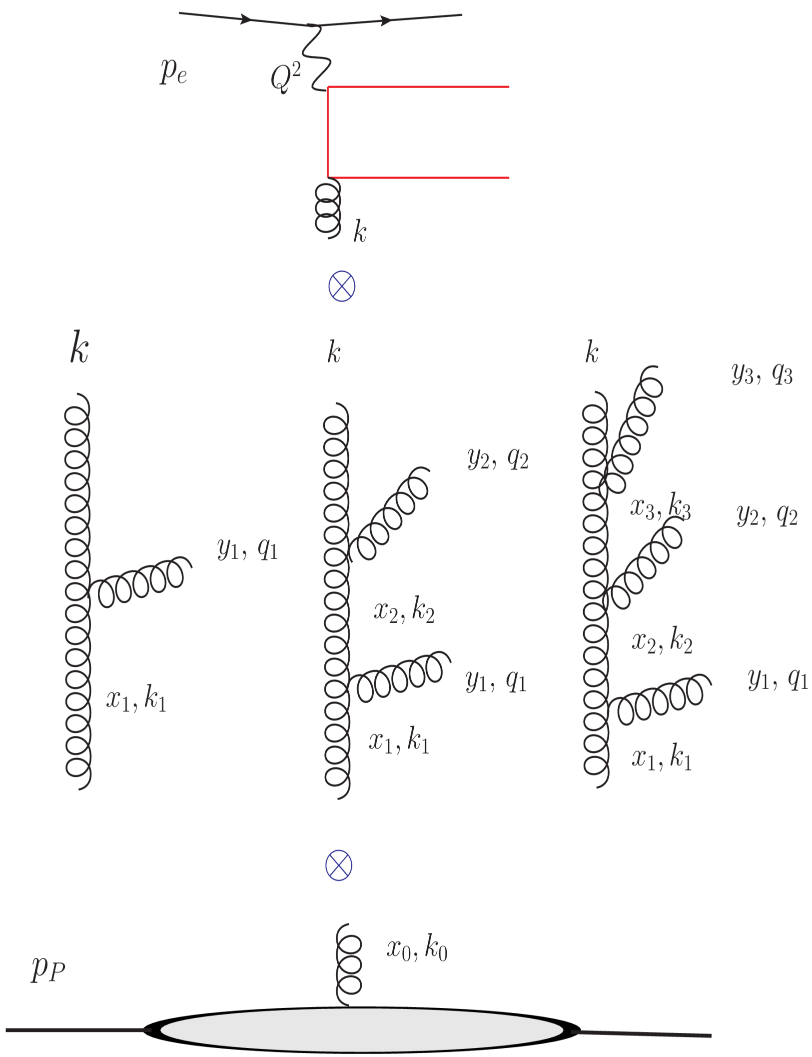
p_t of b barb > 20GeV, y of b, barb < 2.5
Pt of lepton >20GeV, y of lepton < 2.1
Et > 20 GeV

Work in progress with Andreas van Hameren and Piotr Kotko

CCFM evolution equation - evolution with observer

Catani,Ciafaloni,Fiorani Marchesin '88

Recent review: Avsar, Iancu '09



In x → 1region where emitted gluons are soft the dominant contribution to the amplitude comes from the angular ordered region.

$$\bar{\xi} > \xi_i > \xi_{i-1} > \ldots > \xi_1 > \xi_0$$

The same structure for x → 0 although the softest emitted gluons are harder than internal.

$$q_i = \alpha_i\, p_P + \beta_i\, p_e + q_{t\,i} \qquad s = (p_P + p_e)^2$$

$$\eta_i = \frac{1}{2}\ln(\xi_i) \equiv \frac{1}{2}\ln\left(\frac{\beta_i}{\alpha_i}\right) = \ln\left(\frac{|\mathbf{q}_i|}{\sqrt{s}\,\alpha_i}\right) \qquad \tan\frac{\theta_i}{2} = \frac{|\mathbf{q}_i|}{\sqrt{s}\,\alpha_i}$$

$$\bar{\xi} = p^2/(x^2 s)$$

$$z_i = x_i/x_{i-1}$$

$$dP_i^{\theta} = \frac{\alpha_S}{2\pi} dz_i \frac{d^2 q_i}{q_i^2} P_{gg}(z_i)\theta(q_i - z_{i-1}q_{i-1})(1 - z_i)$$

Implemented in CASCADE Monte Carlo H. Jung 02
New program CohRad developed by
M. Slawinska, S. Jadach, K.K

CCFM evolution equation - evolution with observer

$$\mathcal{A}(x,k^2,p)=\mathcal{A}(x,k^2,p)+\bar{\alpha}_s\int\frac{d^2\bar{\mathbf{q}}}{\pi\bar{q}^2}\int_x^{1-Q_0/|\bar{q}|}dz\,\theta(p-z\bar{q})P_{gg}(z,k^2,\bar{q})\mathcal{A}(x/z,k',\bar{q})$$

In DIS $p^2=\dfrac{Q^2}{z(1-z)}$

$\bar{q}=q/(1-z)$

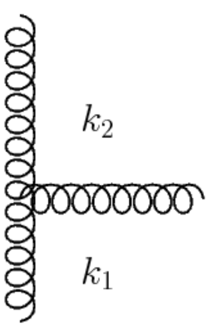
$$P_{gg}(z,k^2,p)=\frac{\alpha_S}{2\pi}2C_A\,\Delta_S(zq,p)\left(\frac{\Delta_{NS}(z,q,k^2)}{z}+\frac{1}{1-z}\right),$$

non-eikonal emission

eikonal emission

regulates 1/(1-z)

regulates 1/z



~ 1/z+1/(1-z)

$$\Delta_{ns}(z_i,q_i,k_i)=\exp\left(-\int_{z_i}^{1}dz'\frac{\bar{\alpha}_s}{z'}\int\frac{dq'^2}{q'^2}\theta(k_i-q')\theta(q'-z'q_i)\right)$$

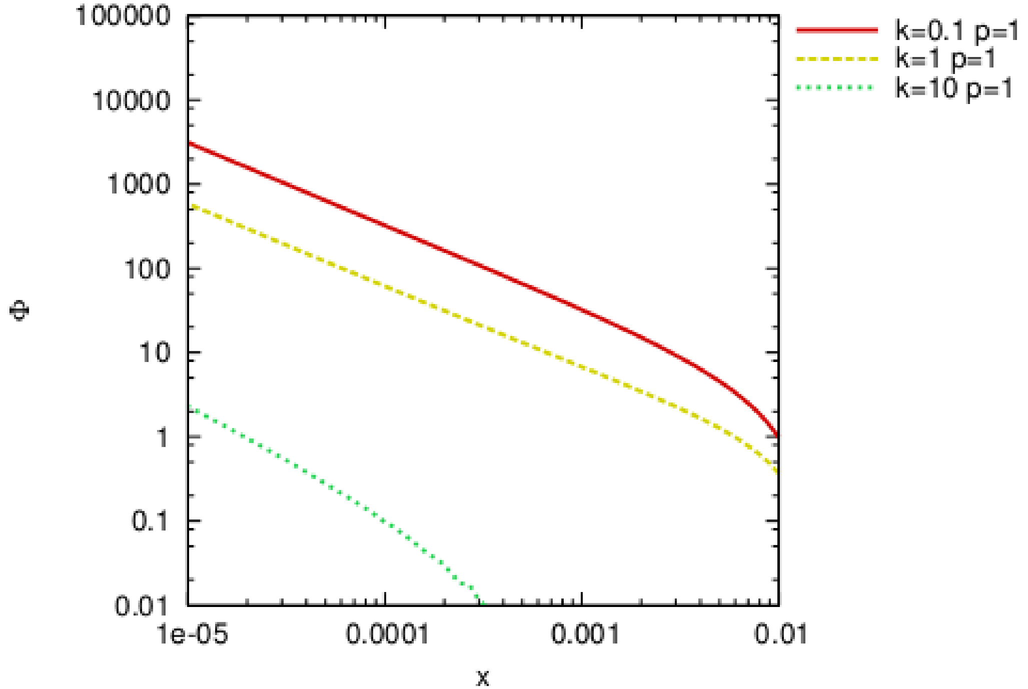
No emission of gluons with $x'=z'x_{i-1}$

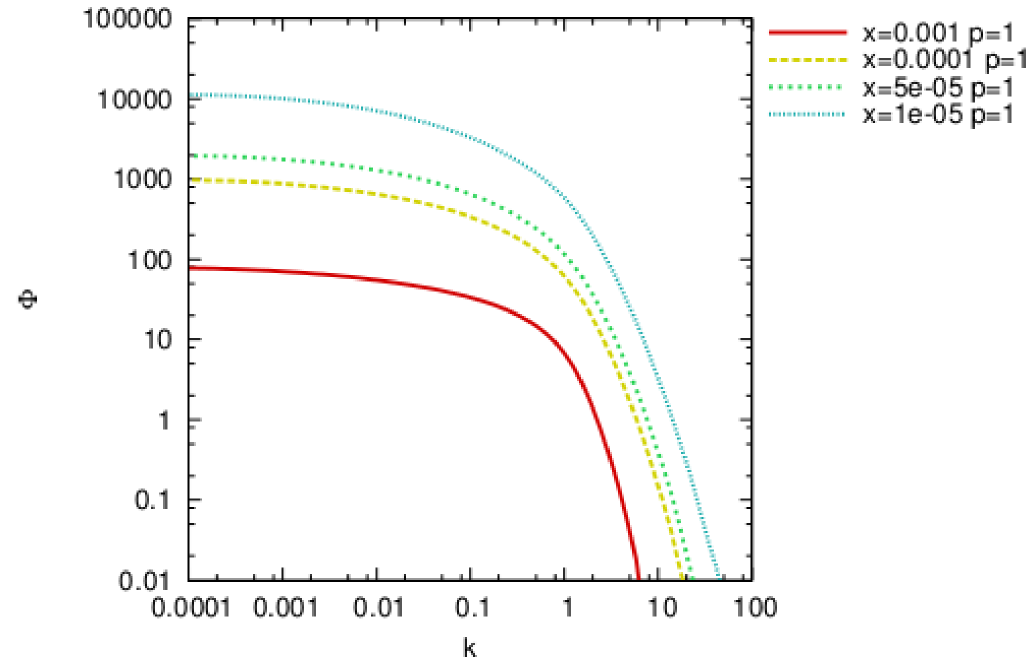
in region $x_i<x'<x_{i-1}$

and with momentum q' smaller than k_i

and with angle $\theta'>\theta_i$

New solution of the CCFM equation

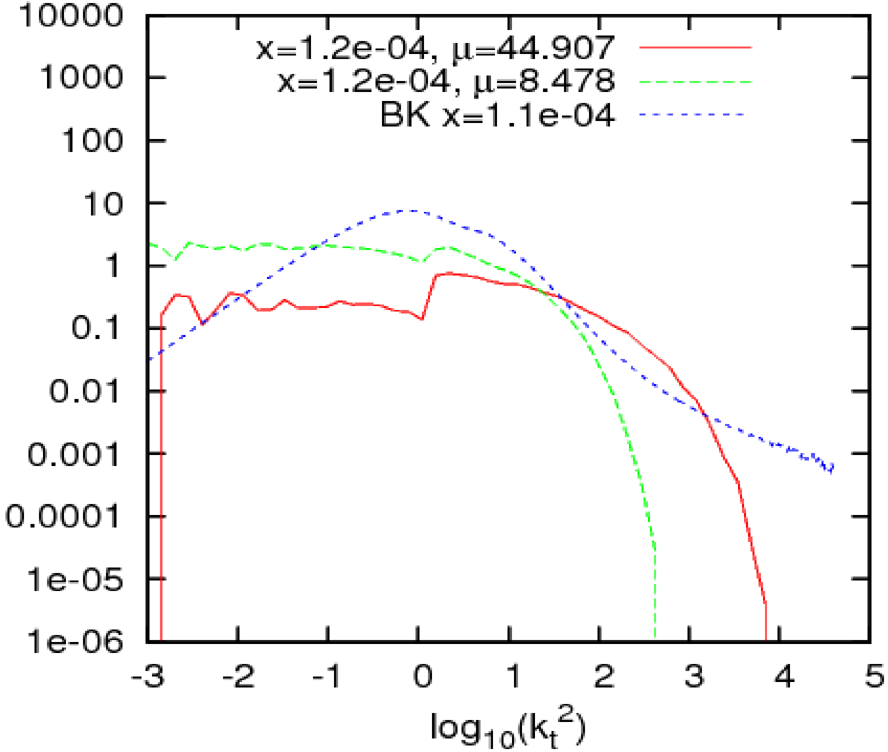


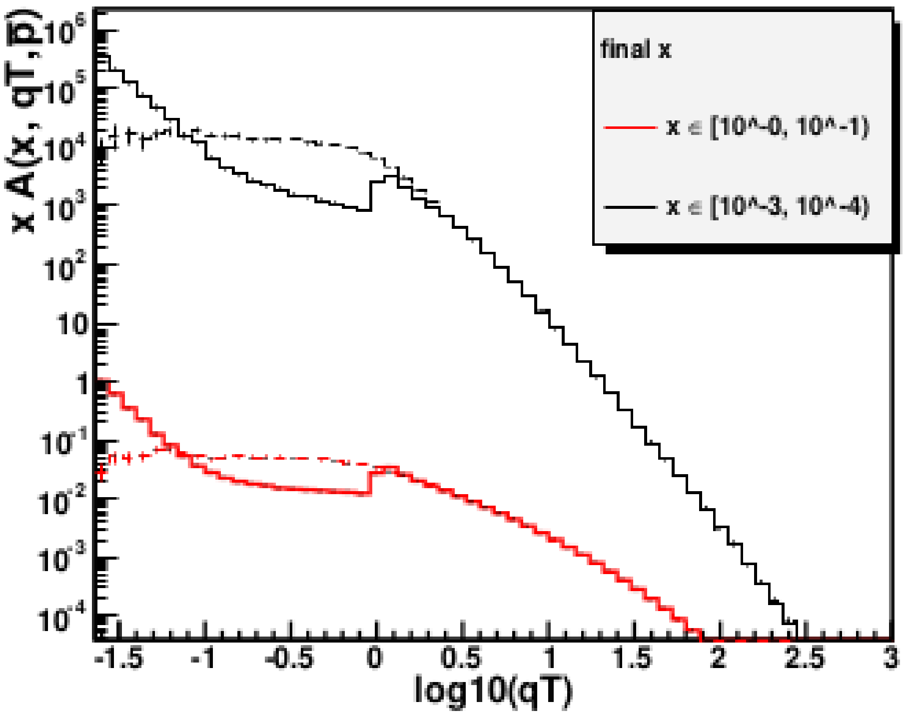


Calculated by postdoc
Dawid Toton

To simplify numerics only non-Sudakov and 1/z pole.

Forward physics as the way to constrain gluon both at large and small pt





Slawinska, Jadach, KK,Phys. Lett. B

- *Too flat behavior of at large k_t in BK*
- *No universality of CCFM distribution at small k_t*
- *Lack of saturation in CCFM at at small k_t*

Needed framework which unifies both correct behaviors

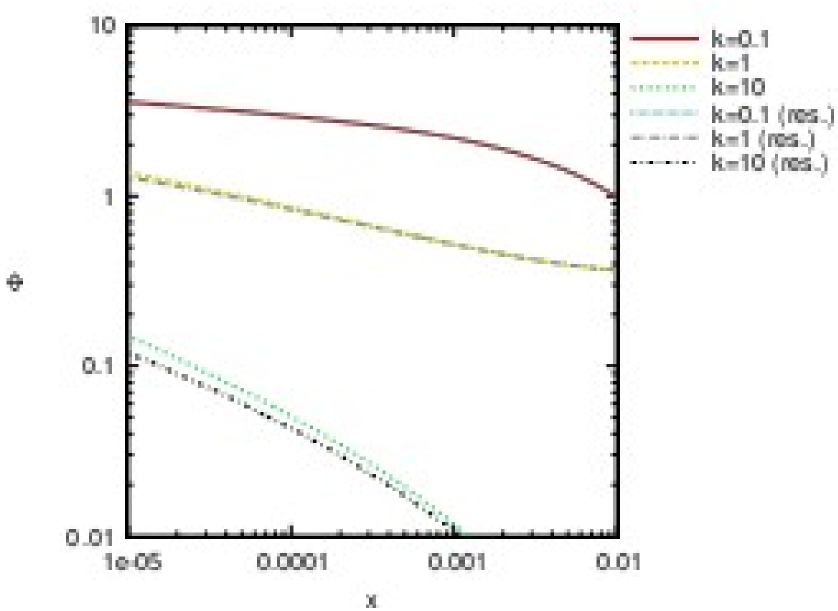
BK equation in the resummed exclusive form

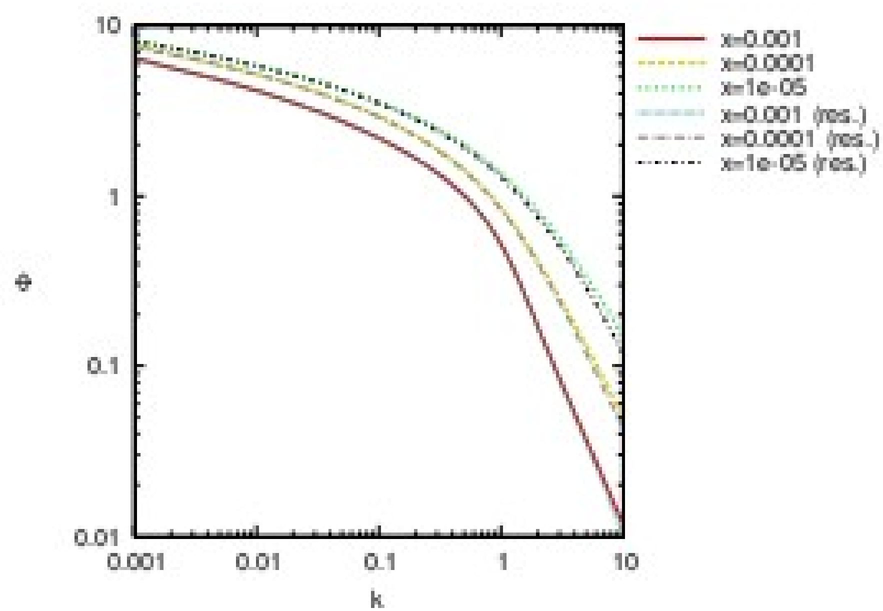
$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2)$$
$$+\,\overline{\alpha}_s \int_x^1 d\,z \int \frac{d^2\mathbf{q}}{\pi q^2}\,\theta(q^2-\mu^2)\,\frac{\Delta_R(z,k,\mu)}{z}\left[\Phi(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2) - q^2\delta(q^2-k^2)\,\Phi^2(\frac{x}{z},q^2)\right]$$

$$\Delta_R(z,k,\mu) \equiv \exp\left(-\overline{\alpha}_s \ln\frac{1}{z}\ln\frac{k^2}{\mu^2}\right)$$

- *The same resumed piece for linear and nonlinear*
- *Initial distribution also gets multiplied by the Regge form factor*
- *New scale introduced to equation. One has to check dependence of the solution on it*
- *Suggestive form to promote the CCFM equation to nonlinear equation which is more suitable for description of final states*

BK before and after resummation





K.K, W. Placzek, D. Toton, Arxiv 1303.0431

Agreement within a 1%

Extension of CCFM to non linear equation for gluon number density

K.K. '12
K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek '12

Unifies saturation and exclusiveness
Structure motivated by the form of resummed BK and CCFM

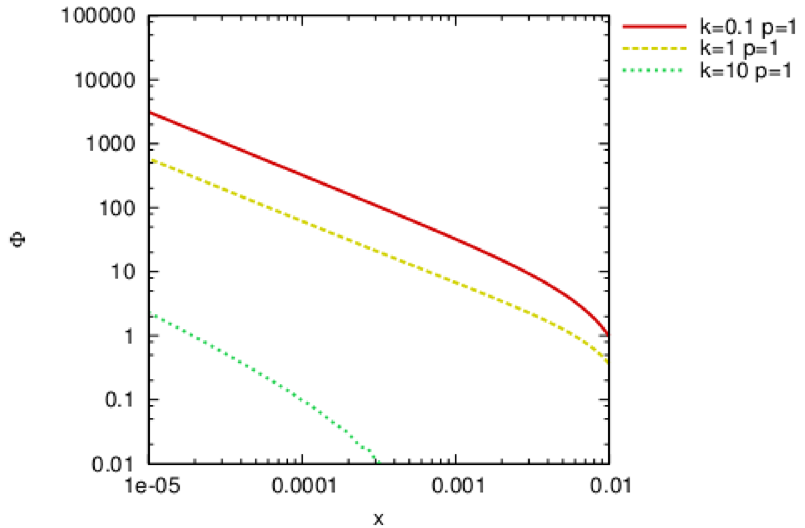
$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 d\,z \int \frac{d^2\mathbf{q}}{\pi q^2}\,\theta(q^2-\mu^2)\frac{\Delta_R(z,k,\mu)}{z}\left[\Phi(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2) - q^2\delta(q^2-k^2)\,\Phi^2(\frac{x}{z},q^2)\right]$$

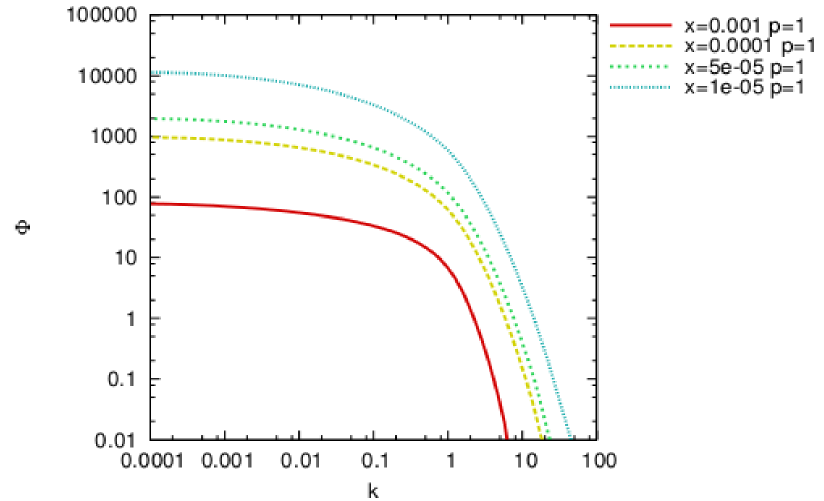
$$\mathcal{E}(x,k^2,p) = \mathcal{E}_0(x,k^2,p) + \int \frac{d^2\bar{\mathbf{q}}}{\pi \bar{q}^2}\int_x^{1-Q_0/|\bar{\mathbf{q}}|} dz\,\theta(p-z\bar{q})\Delta_s(p,z\bar{q})\left(\frac{\Delta_{ns}(z,k,\bar{q})}{z}+\frac{1}{1-z}\right)\left[\mathcal{E}\left(\frac{x}{z},k'^2,\bar{q}\right) - \bar{q}^2\delta(\bar{q}^2-k^2/(1-z)^2)\,\mathcal{E}^2(\frac{x}{z},\bar{q}^2,\bar{q})\right]$$

Solution of the KGBJS equation

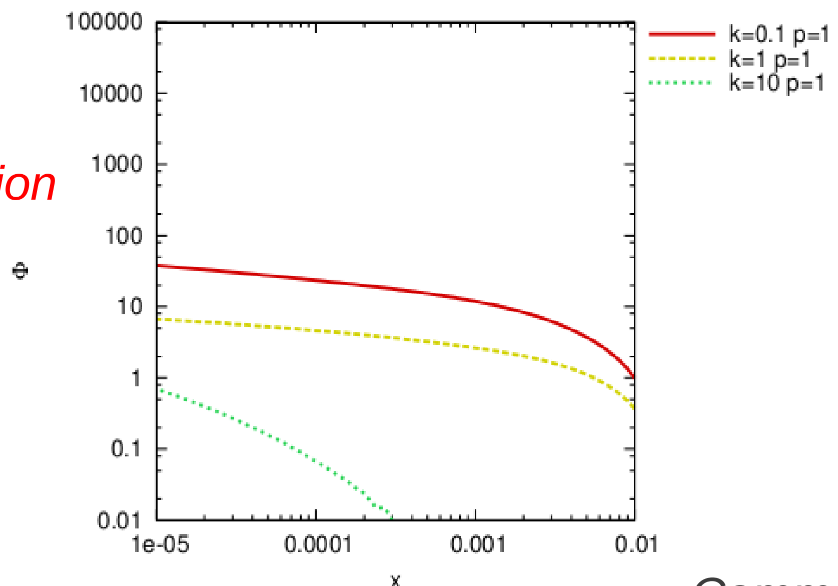
To be released soon

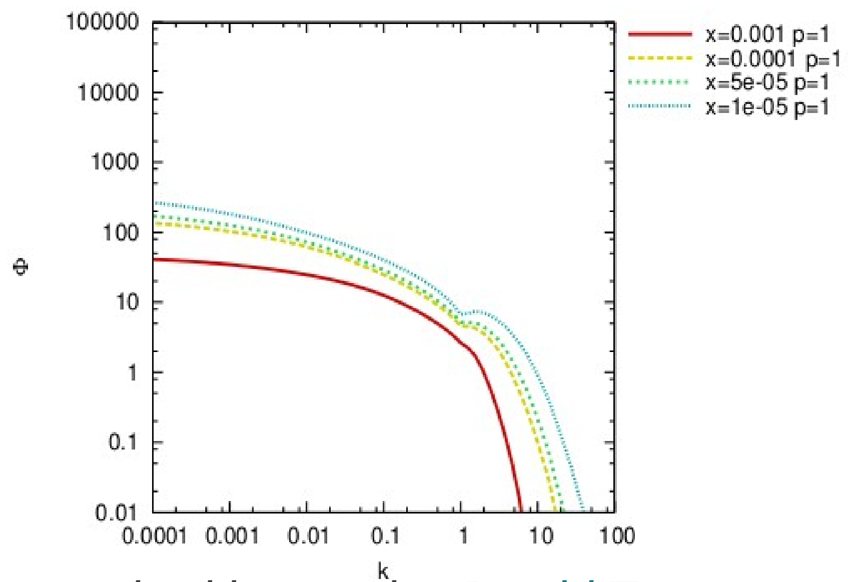
CCFM





New equation



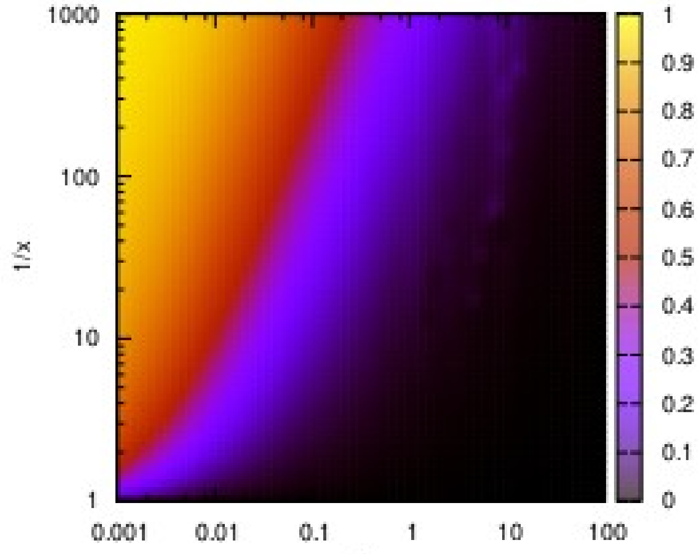


To simplify numerics only non-Sudakov and 1/z pole.

Common work with postdoc Dawid Toton.
*Solution also obtained by M. **Deak '12***

Saturation scale in KGBJS

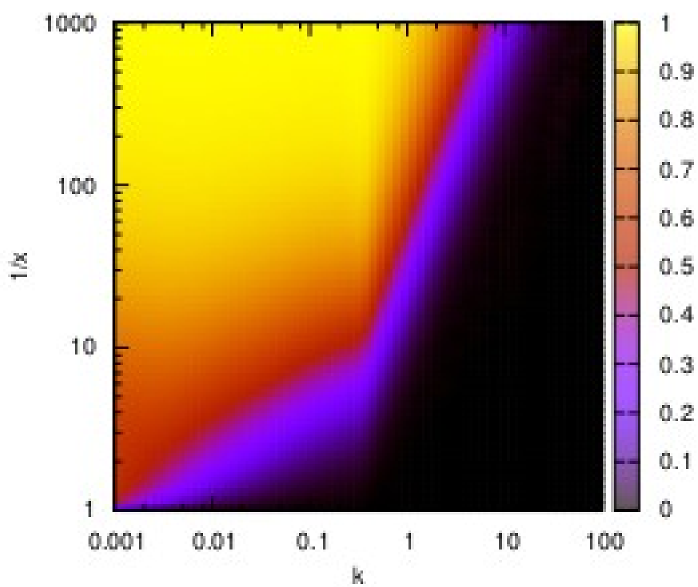
BK

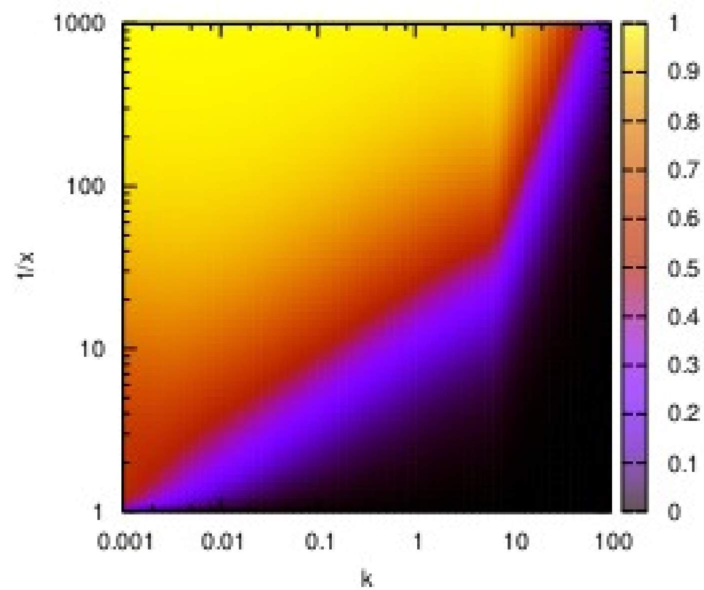


Relative differences between BFKL and BK

Relative differences between CCFM and KGBJS

KGBJS





p=1

p=10

Extension of CCFM to nonlinear equation for gluon momentum density

The same procedure of resummation can be applied to the high energy factorizable gluon density. The stucture of nonlinearity does not introduce complications:

KK JHEP 12

$$\mathcal{F}(x,k^2) = \tilde{\mathcal{F}}_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^{1} \frac{dz}{z} \Delta_R(z,k,\mu) \left\{ \int \frac{d^2\mathbf{q}}{\pi q^2} \theta(q^2-\mu^2) \mathcal{F}\left(\frac{x}{z}, |\mathbf{k}+\mathbf{q}|^2\right) - \frac{\pi\alpha_s^2}{4N_c R^2} k^2 \nabla_k^2 \left[\int_{k^2}^{\infty} \frac{dl^2}{l^2} \ln\frac{l^2}{k^2} \mathcal{F}(x,l^2) \right]^2 \right\}$$

And resummed TPV

$$\mathcal{V}_{\text{resummed}} = \frac{\pi\alpha_s^2}{4\,N_c\,R^2} k^2 \Delta_R(z,k,\mu) \nabla_k^2 \ln\frac{l_1^2}{k^2} \theta(l_1^2-k^2) \ln\frac{l_2^2}{k^2} \theta(l_2^2-k^2)$$

Include coherence

$$\mathcal{F}(x,k^2,p) = \tilde{\mathcal{F}}_0(x,k^2,p) + \overline{\alpha}_s \int \frac{d^2\mathbf{q}}{\pi q^2} \int_{x/x_0}^{1} \frac{dz}{z} \theta(p-q\,z) \Delta_{ns}(z,k,q) \left\{ \mathcal{F}\left(\frac{x}{z}, |\mathbf{k}+\mathbf{q}|^2, q\right) - \frac{\pi\alpha_s^2}{4N_c R^2} q^2 \delta(q^2-k^2) \nabla_q^2 \left[\int_{q^2}^{\infty} \frac{dl^2}{l^2} \ln\frac{l^2}{k^2} \mathcal{F}(x/z,l^2,l) \right]^2 \right\}$$

Can be further extended to include Sudakov and 1/(1-z) terms

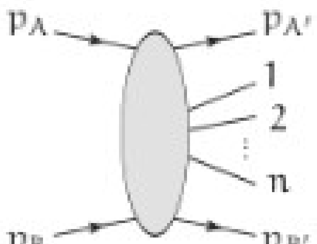
Conclusions and outlook

- *LHC gives oportunity to test parton densities both when the parton density is probed at low x and at low, medium and large kt at some external scale.*
- *The new representation of the BK equation has been found*
- *Well motivated anzatz for the equations which incorporate both saturation effects and coherence. Solutions are still to be understood*
  - *Efficient framework for evaluation of Matrix Elements with high energy projectors*
- *Prospects for direct Monte Carlo simulation of physics of saturation*

Prescription to get amplitude with off-shell gluons

1. Consider the process $q_A\, q_B \to q_A\, q_B\, X$, where q_A, q_B are distinguishable massless quarks not occurring in X, and with momentum flow as if the momenta p_A, p_B of the initial-state quarks and $p_{A'}, p_{B'}$ of the final-state quarks are given by

$$p_A^\mu = k_1^\mu \quad , \quad p_B^\mu = k_2^\mu \quad , \quad p_{A'}^\mu = p_{B'}^\mu = 0$$



2. Associate the number 1 instead of spinors with the end points of the A-quark line, interpret every vertex on the A-quark line as $g_s T^a_{ij}\, \ell_1^\mu$ instead of $-ig_s T^a_{ij}\, \gamma^\mu$, interpret every propagator on the A-quark line as $\delta_{ij}/\ell_1 \cdot p$ instead of $i\delta_{ij}/\not{p}$.

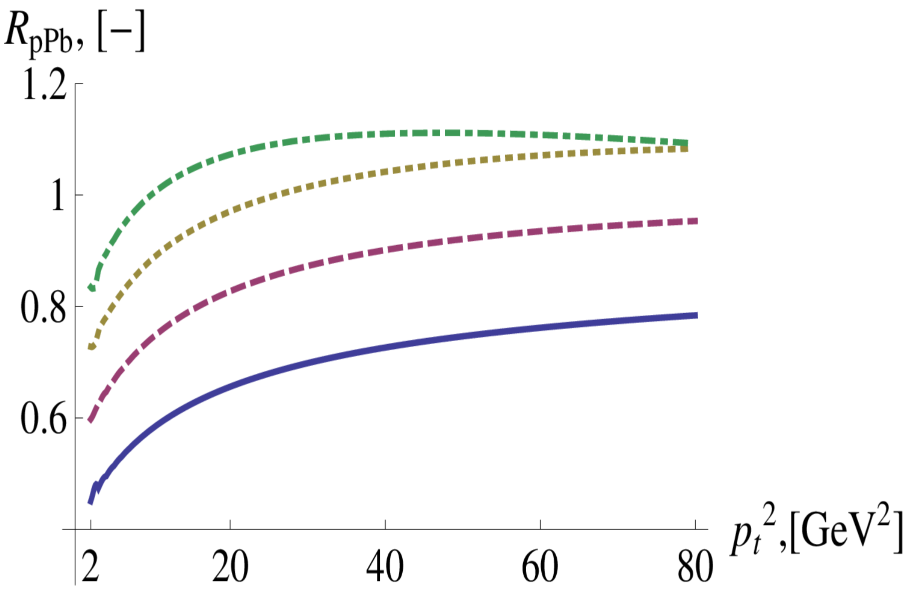
3. Associate the number 1 instead of spinors with the end points of the B-quark line, interpret every vertex on the B-quark line as $g_s T^a_{ij}\, \ell_2^\mu$ instead of $-ig_s T^a_{ij}\, \gamma^\mu$, interpret every propagator on the B-quark line as $\delta_{ij}/\ell_2 \cdot p$ instead of $i\delta_{ij}/\not{p}$.

4. Multiply the amplitude with $F = \dfrac{i\, x_1 \sqrt{-2k_{1\perp}^2}}{g_s} \times \dfrac{i\, x_2 \sqrt{-2k_{2\perp}^2}}{g_s}$.

5. For the rest normal Feynman rules apply.

In agreement with Lipatov's effective action.

Inclusive gluon production



Done by master student
Malgorzata Pikies

Inclusive gluon production at LHC
Consistent with known results: Albacete, Marquet; Kovchegov,Jalilian-Marian, Tuchin,Rezaian,..