Di-hadron azimuthal angular correlations in high energy collisions

Jamal Jalilian-Marian
Baruch College, New York NY

PA at the LHC, ECT*, Trento, May 6th – 10th, 2013

Di-hadron correlations

Rapidity correlations

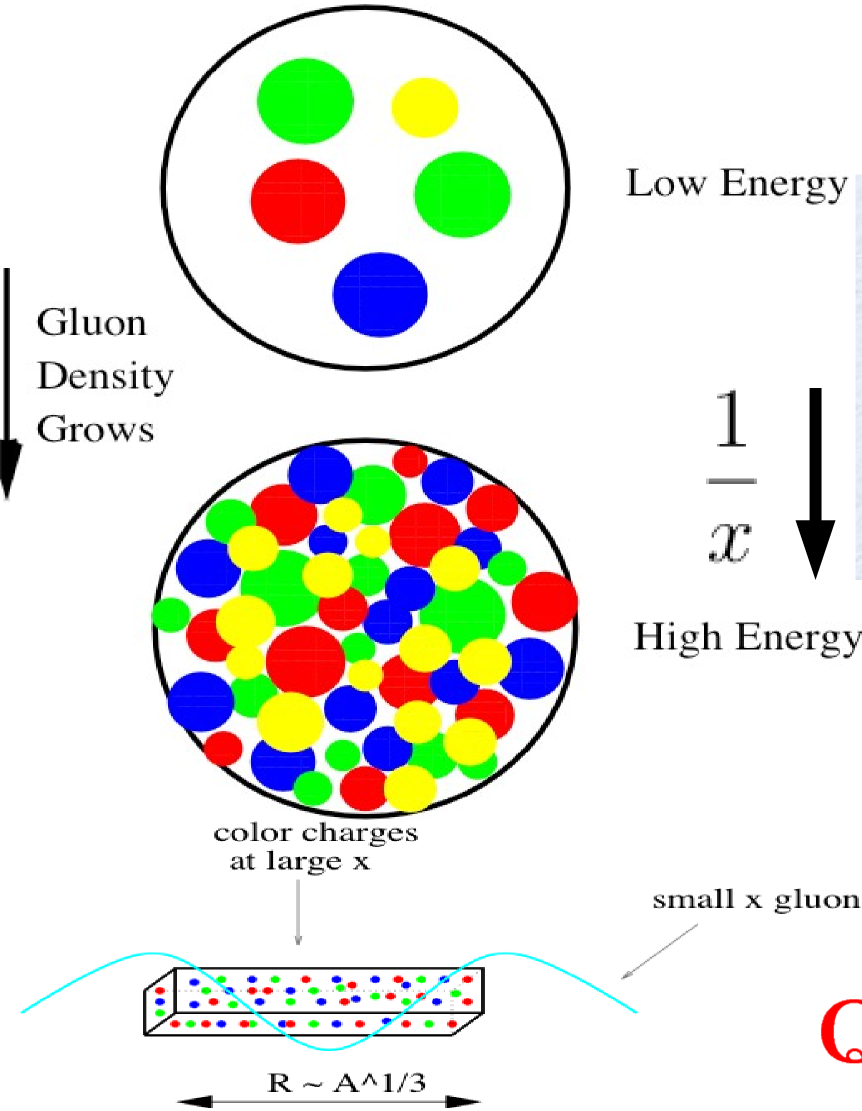
- ridge (near side)
  - nucleus-nucleus collisions
  - proton-proton collisions

Angular correlations (away side)

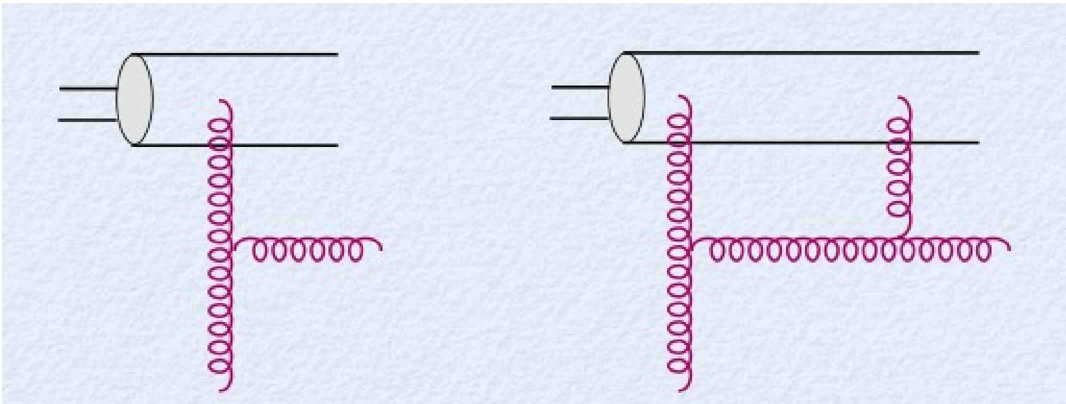
- large x (high p_t): pQCD
- small x: CGC
  - forward pA (dilute-dense) collisions

Gluon saturation

Gribov-Levin-Ryskin
Mueller-Qiu



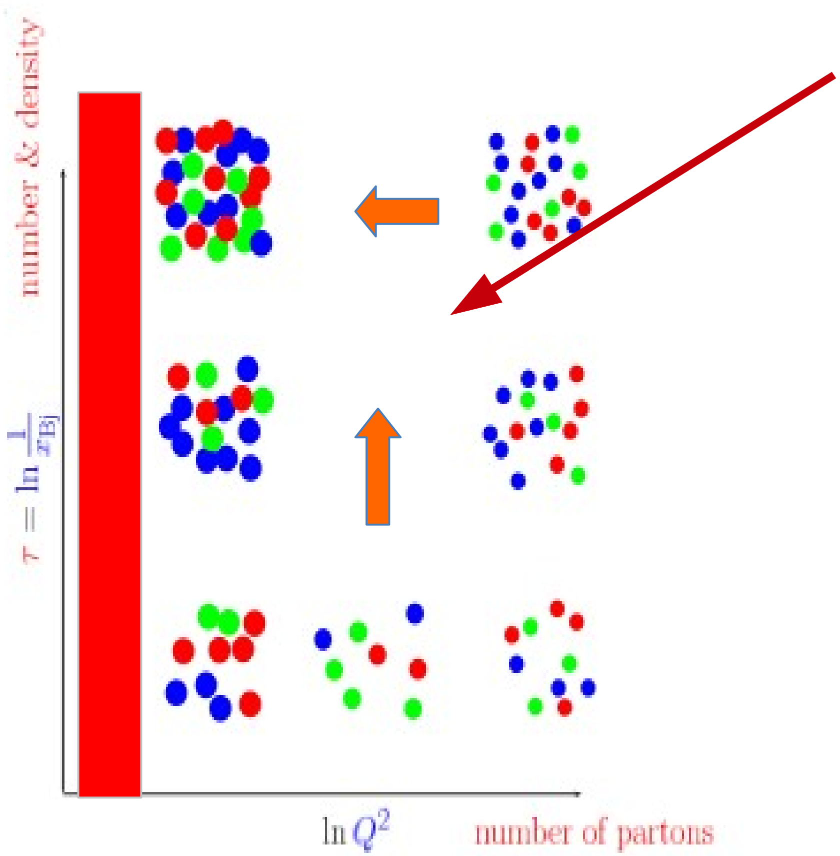
"attractive" bremsstrahlung
vs. "repulsive" recombination



$$\frac{\alpha_s}{Q^2}\,\frac{xG(x, b_t, Q^2)}{S_\perp} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3}\,\left(\frac{1}{x}\right)^{0.3}$$

CGC: universal gluonic matter



How does this happen ?

How do correlation functions of these evolve ?

Is there a universal fixed point for the RG evolution of d.o.f

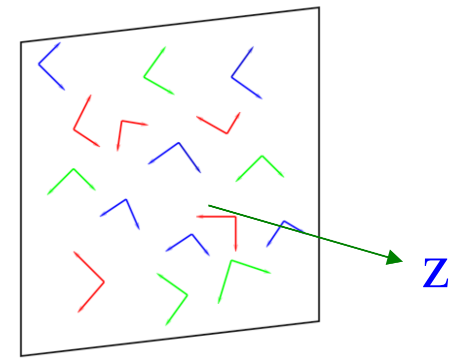
How does the coupling run ?

How does saturation transition to chiral symmetry breaking and confinement

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}}$$

QCD at low x: CGC

two main effects: *"multiple scatterings"*
evolution with ln (1/x)



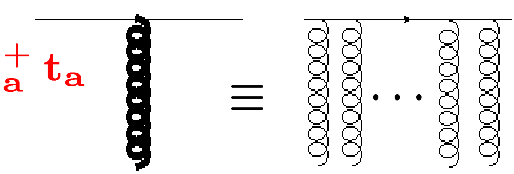
$$\mathbf{A^{\mu}_{a}(x_t, x^-) \sim \delta^{\mu +}\, \delta(x^-)\, \alpha_a(x_t)}$$

$$\mathbf{\alpha^a(k_t) = g\, \rho^a(k_t)/k_t^2}$$

CGC observables: $< \mathrm{Tr}\, V \cdots\cdots V^{\dagger} >$ with

propagation of quarks and gluons in the background of the classical field

$$\mathbf{V(x_t) = \hat{P} e^{ig \int dx^- A^+_a t_a}}$$



gluon distribution: $\sim \int^{\mathbf{Q^2}} \frac{\mathbf{d^2k_t}}{\mathbf{k_t^2}}\, \phi(\mathbf{x, k_t})$ with $\phi(\mathbf{k_t^2}) \sim < \rho^{\star}_{\mathbf{a}}(\mathbf{k_t})\, \rho_{\mathbf{a}}(\mathbf{k_t}) >$

<u>pQCD with collinear factorization:</u> *single scattering*
evolution with ln Q²

JIMWLK evolution equation

re-sum ln (1/x)

$$\frac{d}{d\,ln 1/x}\langle O\rangle = \frac{1}{2}\left\langle \int d^2x\, d^2y\, \frac{\delta}{\delta\alpha_x^b}\,\eta_{xy}^{bd}\,\frac{\delta}{\delta\alpha_y^d}\,O\right\rangle$$

$$\eta_{xy}^{bd} = \frac{1}{\pi}\int \frac{d^2z}{(2\pi)^2}\,\frac{(x-z)\cdot(y-z)}{(x-z)^2(y-z)^2}\left[\underbrace{1+U_x^\dagger U_y}_{\text{virtual}} - \underbrace{U_x^\dagger U_z - U_z^\dagger U_y}_{\text{real}}\right]^{bd}$$

U is a Wilson line in adjoint representation

Color Glass Condensate

Advantages:

A systematic, first-principle approach to high energy scattering in QCD

Controlled approximations

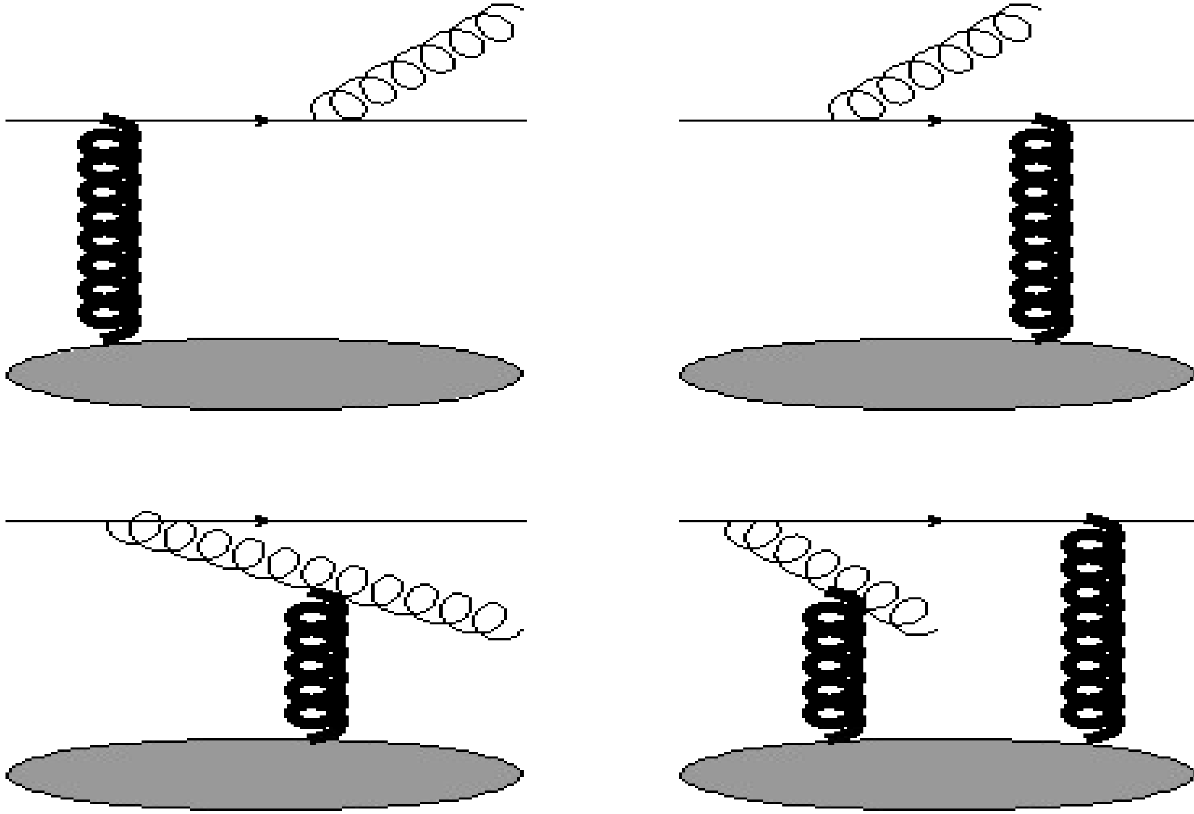
Same formalism can describe a wide range of phenomena

Disadvantages:

Applicable at low x (high x, Q2 missing)

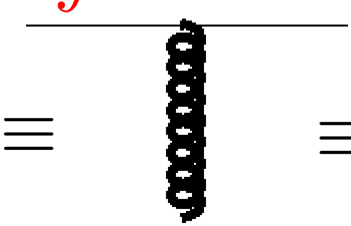
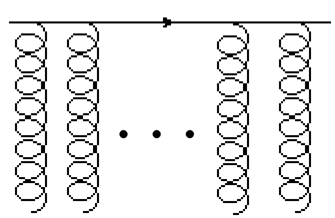
Di-jet production: pA

$\mathbf{q(p)\,T \to q(q)\,g(k)}$



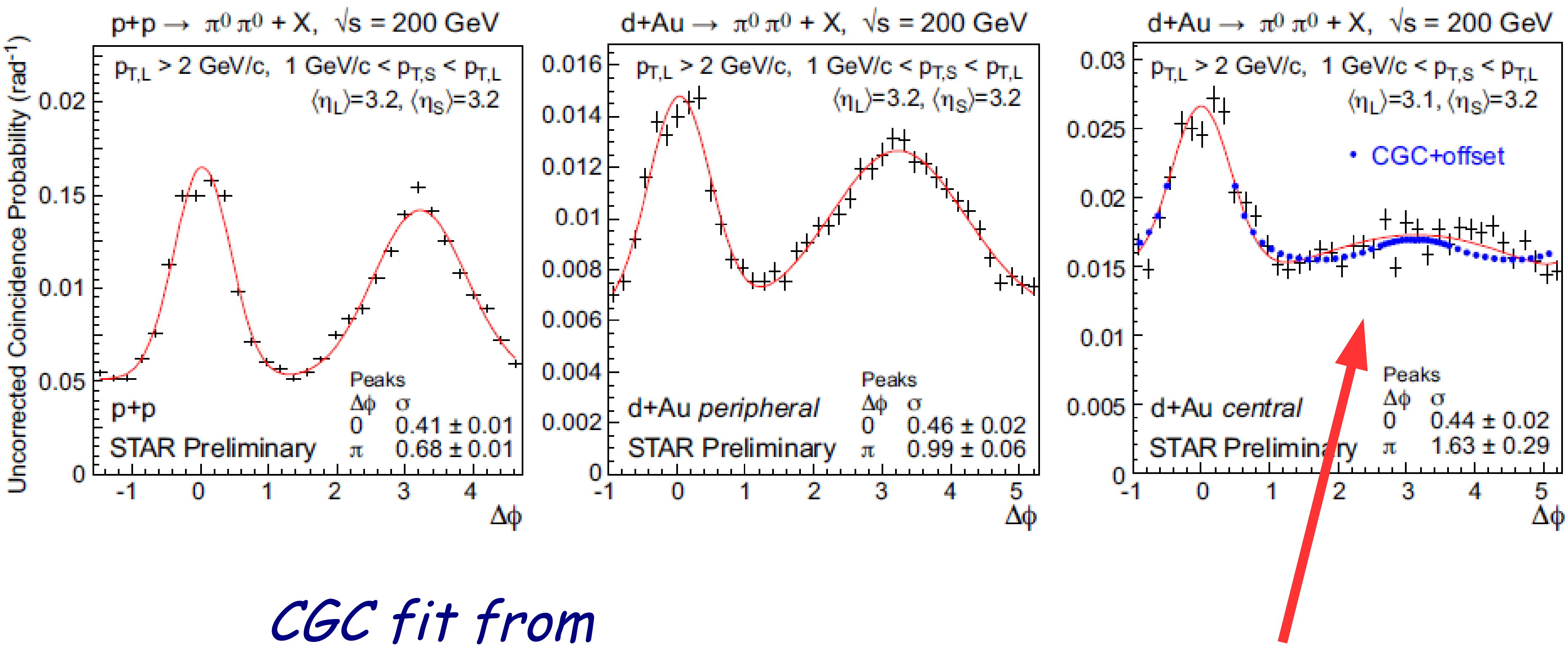
J. Jalilian-Marian, Y. Kovchegov
PRD70 (2004) 114017
C. Marquet, NPA796 (2007)
....

$$\mathbf{d}\sigma \sim \int \mathbf{K} \otimes \left[< \mathbf{Tr} \mathbf{V}\, \mathbf{V}^\dagger > + < \mathbf{Tr} \mathbf{V}\, \mathbf{V}^\dagger\, \mathbf{V}\, \mathbf{V}^\dagger > + \cdots \right]$$

$\mathbf{V} \equiv$  $\equiv$  $\sim \mathbf{1} + \mathbf{O}(\mathbf{g}\,\rho) + \mathbf{O}(\mathbf{g^2}\,\rho^2)$

disappearance of back to back jets

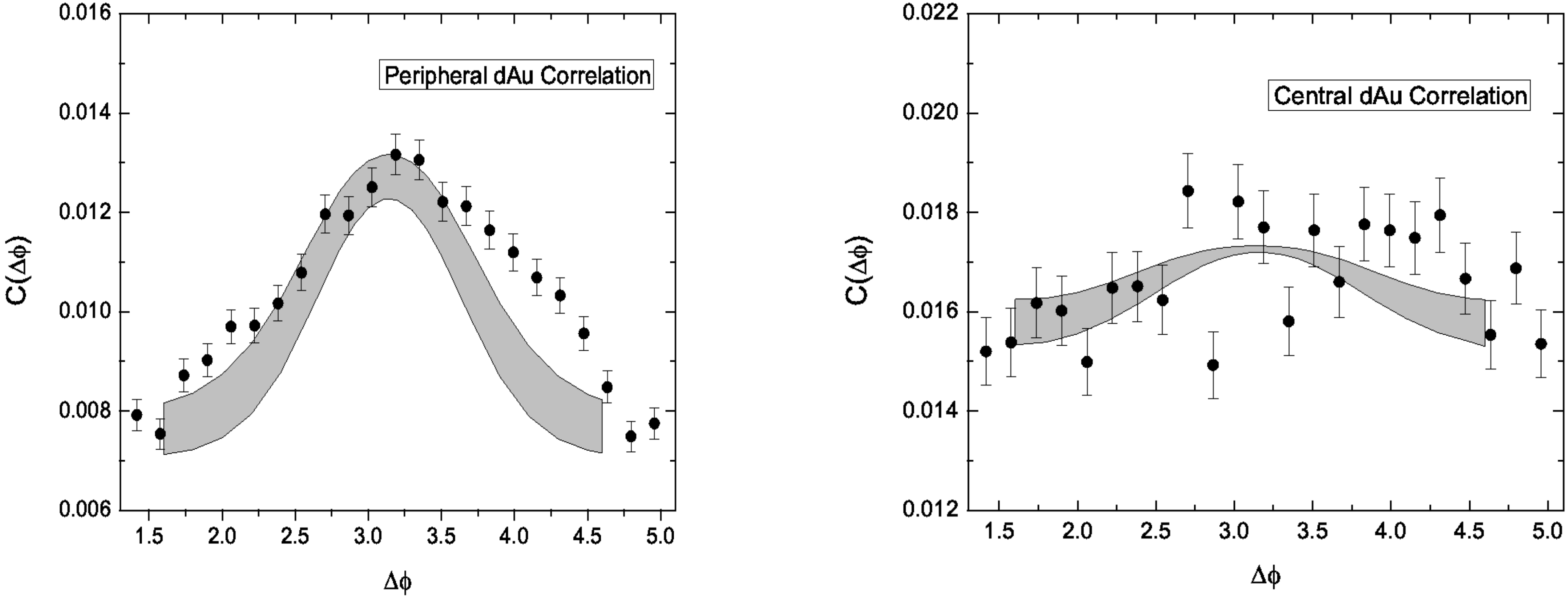
Recent STAR measurement (arXiv:1008.3989v1):



CGC fit from
Albacete + Marquet, PRL (2010)
using running coupling BK solution,
Also by Tuchin, NPA846 (2010)

multiple scatterings
de-correlate the hadrons

disappearance of back to back jets



CGC fit from
A. Stasto, B-W. Xiao, F. Yuan, arXiv:1109.1817
Also by T. Lappi et al. 2012

alternative idea: shadowing + energy loss (M. Strikman et al.)
Z. Kang, I. Vitev and H. Xing, PRD85 (2012) 054024

di-jet production in pA

$$O_2(r,\bar{r}) \equiv Tr V_r\, V^\dagger_{\bar{r}}$$ **dipole** → **F2 in DIS, single hadron in pA**

$$O_4(r,\bar{r}:s) \equiv Tr V^\dagger_r\, t^a\, V_{\bar{r}}\, t^b\, [U_s]^{ab} = \frac{1}{2}\Big[Tr V^\dagger_r\, V_s\;\; Tr V_{\bar{r}}\, V^\dagger_s - \frac{1}{N_c} Tr V^\dagger_r\, V_{\bar{r}}\Big]$$

$$O_6(r,\bar{r}:s,\bar{s}) \equiv Tr V_r\, V^\dagger_{\bar{r}}\, t^a\, t^b\, [U_s\, U^\dagger_{\bar{s}}]^{ba} = \frac{1}{2}\Big[Tr V_r\, V^\dagger_{\bar{r}}\, V_{\bar{s}}\, V^\dagger_s\, Tr V_s\, V^\dagger_{\bar{s}} - \frac{1}{N_c} Tr V_r\, V^\dagger_{\bar{r}}\Big]$$

quadrupole

calculations: classical

how about quantum corrections (energy dependence) ?

energy (rapidity) dependence from JIMWLK evolution of O's

evolution of a dipole is well known: BK eq.

how does a quadrupole evolve?

Mean field + large N_c :Balitsky-Kovchegov eq.

$$\frac{\mathbf{d}}{\mathbf{dy}}\mathbf{S}(\mathbf{r}-\bar{\mathbf{r}}) = \frac{\bar{\alpha}_{\mathbf{s}}}{\mathbf{2}\pi}\int \mathbf{d^2z}\,\frac{(\mathbf{r}-\bar{\mathbf{r}})^{\mathbf{2}}}{(\mathbf{r}-\mathbf{z})^{\mathbf{2}}(\bar{\mathbf{r}}-\mathbf{z})^{\mathbf{2}}}\,[\mathbf{S}(\mathbf{r}-\mathbf{z})\,\mathbf{S}(\bar{\mathbf{r}}-\mathbf{z})-\mathbf{S}(\mathbf{r}-\bar{\mathbf{r}})]$$

with $\mathbf{S}(\mathbf{r}-\bar{\mathbf{r}}) \equiv \frac{\mathbf{1}}{\mathbf{N_c}} < \mathbf{Tr}\,\mathbf{V}(\mathbf{r})\,\mathbf{V}^\dagger(\bar{\mathbf{r}}) >$ and

$$\mathbf{dP}_{\mathbf{d}\to\mathbf{d}\,\mathbf{d}} = \frac{\bar{\alpha}_{\mathbf{s}}}{\mathbf{2}\pi}\frac{(\mathbf{r}-\bar{\mathbf{r}})^{\mathbf{2}}}{(\mathbf{r}-\mathbf{z})^{\mathbf{2}}(\bar{\mathbf{r}}-\mathbf{z})^{\mathbf{2}}}\,\mathbf{d^2z}$$ dipole splitting probability

all n-point correlators are expressed in terms of the dipoles

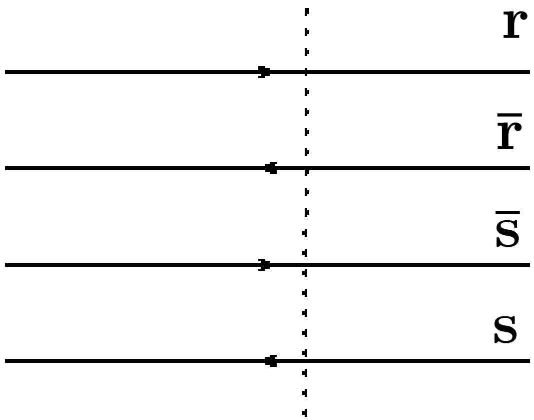
NLO: Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)

Evolution of quadrupole from JIMWLK

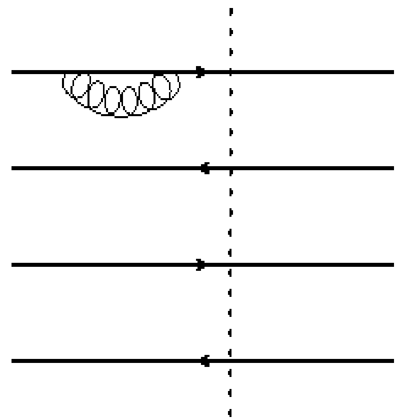
$$Q(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s}) \equiv \frac{1}{N_c} < \mathrm{Tr}\, V(\mathbf{r})\, V^\dagger(\bar{\mathbf{r}})\, V(\bar{\mathbf{s}})\, V^\dagger(\mathbf{s}) >$$

radiation kernels
as in dipole

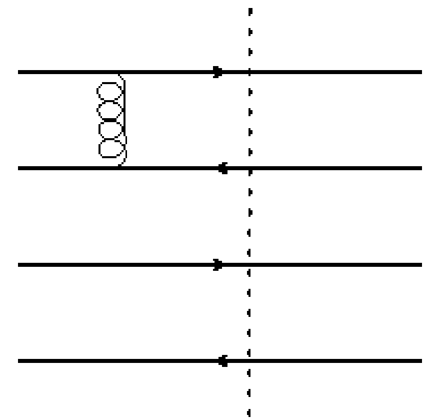
$$\frac{d}{dy}$$



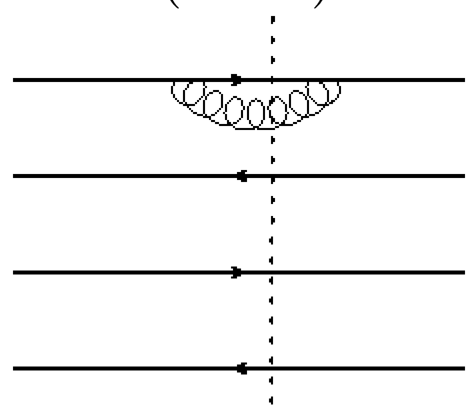
$$=$$

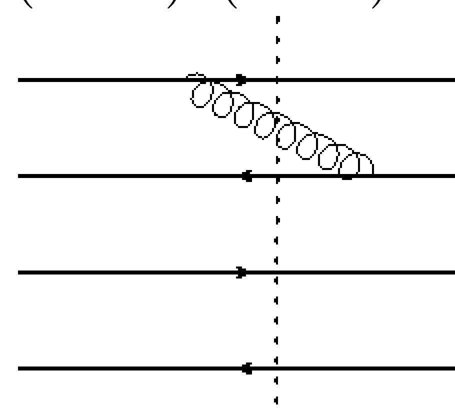


$$+$$



$$\frac{1}{(\mathbf{r}-\mathbf{z})^2} \qquad \frac{(\mathbf{r}-\mathbf{z})\cdot(\bar{\mathbf{r}}-\mathbf{z})}{(\mathbf{r}-\mathbf{z})^2(\bar{\mathbf{r}}-\mathbf{z})^2}$$





J. Jalilian-Marian, Y. Kovchegov (2004)
Dominguez, Mueller, Munier, Xiao (2011)
J. Jalilian-Marian (2011)
D. Triantafyllopoulos (2011)

$$+ \cdots$$

Evolution of quadrupole from JIMWLK

$$
\begin{aligned}
&\frac{d}{dy}\langle Q(r,\bar r,\bar s,s)\rangle \\
=\;& \frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2z\Bigg\{\Bigg\langle\left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}\right]\,Q(z,\bar r,\bar s,s)\,S(r,z)\\
+\;&\left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\right]\,Q(r,z,\bar s,s)\,S(z,\bar r)\\
+\;&\left[\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(s-z)^2(\bar r-z)^2}\right]\,Q(r,\bar r,z,s)\,S(\bar s,z)\\
+\;&\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\right]\,Q(r,\bar r,\bar s,z)\,S(z,s)\\
-\;&\left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}\right]\,Q(r,\bar r,\bar s,s)\\
-\;&\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\right]\,S(r,s)\,S(\bar r,\bar s)\\
-\;&\left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}\right]\,S(r,\bar r)\,S(\bar s,s)\Bigg\rangle\Bigg\}
\end{aligned}
$$

$$\frac{d}{d\,y}Q=\int P_1\,[Q\,S]-P_2\,[Q]+P_3[S\,S]$$ with $$P_1-P_2+P_3=0$$

“approximate solution”: Iancu-Triantafyllopoulos, arXiv:1109.0302
“good news”: dipoles and quadrupoles only- Chirilli, Xiao, Yuan 2012

quadrupole evolution in the linear regime

define $\mathbf{T}(\mathbf{r}, \bar{\mathbf{r}}) \equiv \mathbf{1} - \mathbf{S}(\mathbf{r}, \bar{\mathbf{r}}) \qquad \mathbf{T_Q}(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s}) \equiv \mathbf{1} - \mathbf{Q}(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s})$

re-write the evolution eq. for T_Q rather than Q

expand in powers of gauge fields (or color charges)

ignore contribution of non-linear terms: T T and T_Q T

$\mathbf{O}(\alpha^{\mathbf{2}}) \qquad \mathbf{T_Q}(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s}) \to \mathbf{T}(\mathbf{r}, \bar{\mathbf{r}}) + \mathbf{T}(\mathbf{r}, \mathbf{s}) + \cdots$

with $\mathbf{T}(\mathbf{r}, \bar{\mathbf{r}}) \sim \alpha^{\mathbf{2}}(\mathbf{r}, \bar{\mathbf{r}})$

quadrupole evolution reduces to a sum of BFKL evolution eqs

Dominguez, Mueller, Munier, Xiao (2011)
J. Jalilian-Marian (2011)
D. Triantafyllopoulos (2011)

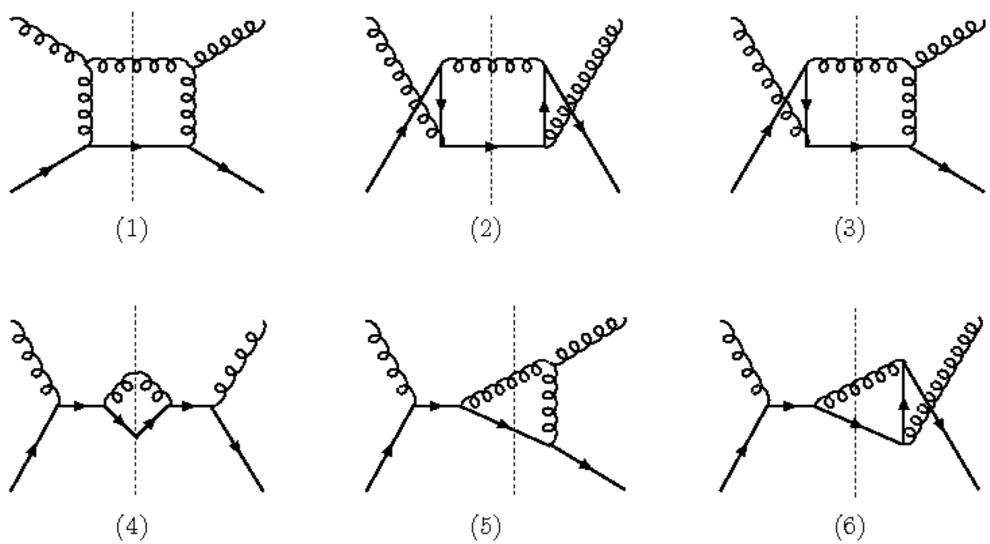
di-hadron correlations in the high p_t limit

$\mathbf{O}(\alpha^{\mathbf{2}})$

Dominguez, Marquet, Xiao, Yuan (2011)
Dominguez, Xiao, Yuan (2011)

factorization of target distribution functions and hard scattering matrix element

$$\mathbf{d}\sigma \sim \mathbf{\Phi} \otimes \frac{\mathbf{d}\sigma}{\mathbf{dt}}^{\mathbf{2\rightarrow 2}}$$



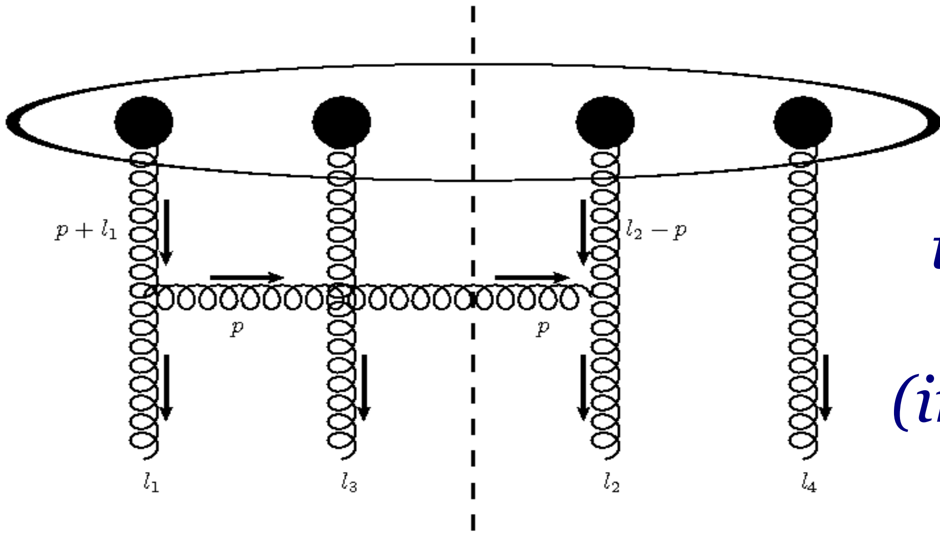
$$\frac{d\sigma}{dt}^{q\,g\rightarrow q\,g} \sim \frac{1}{s^2}\left[\frac{4}{9}\frac{s^2+u^2}{-s\,u} + \frac{s^2+u^2}{t^2}\right]$$

partons are back to back

quadrupole evolution in the linear regime

BJKP equation $\mathbf{O}(\alpha^4)$: 4-gluon exchange

J. Jalilian-Marian, PRD85 (2012) 014037



the color structure is identical on both sides of this eq. (independent of color averaging)

$$\frac{d}{dy}\hat{T}_4(l_1, l_2, l_3, l_4) = \frac{N_c\,\alpha_s}{\pi^2}\int d^2p_t\left[\frac{p^i}{p_t^2} - \frac{(p^i - l_1^i)}{(p_t + l_1)^2}\right]\cdot\left[\frac{p^i}{p_t^2} - \frac{(p^i - l_2^i)}{(p_t + l_2)^2}\right]$$
$$\hat{T}_4(p_t + l_1, l_2 - p_t, l_3, l_4) + \cdots$$
$$- \frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2p_t\left[\frac{l_1^2}{p_t^2(l_1 - p_t)^2} + \{l_1 \to l_2, l_3, l_4\}\right]\hat{T}_4(l_1, l_2, l_3, l_4)$$

this will <u>de-correlate</u> the produced partons at high $p_t > Q_s$

color structure

$$\mathbf{\hat{T}_4(l_1, l_2, l_3, l_4) \equiv \frac{1}{N_c} Tr\, \rho(l_1)\, \rho(l_2)\, \rho(l_3)\, \rho(l_4) = Tr\,(t^a\, t^b\, t^c\, t^d)\, \rho^a(l_1)\, \rho^b(l_2)\, \rho^c(l_3)\, \rho^d(l_4)}$$

$$\begin{aligned} Tr\,(t^a\, t^b\, t^c\, t^d) &= \frac{1}{4N_c}\left[\delta^{ab}\delta^{cd} - \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right] \\ &+ \frac{1}{8}\left[d^{abr}d^{cdr} - d^{acr}d^{bdr} + d^{adr}d^{bcr}\right] \\ &+ \frac{i}{8}\left[d^{abr}f^{cdr} - d^{acr}f^{bdr} + d^{adr}f^{bcr}\right] \end{aligned}$$

overall state is a singlet, how about pairwise?

for $N_c = 3$

$$\left[\delta^{\mathbf{ab}}\delta^{\mathbf{cd}} + \delta^{\mathbf{ac}}\delta^{\mathbf{bd}} + \delta^{\mathbf{ad}}\delta^{\mathbf{bc}}\right] = \mathbf{3}\left[\mathbf{d^{abr}d^{cdr} + d^{acr}d^{bdr} + d^{adr}d^{bcr}}\right]$$

can the exchanged pairs be in a bound state?

J. Bartels: YES!

the linear regime

$\mathbf{O}(\alpha^{\mathbf{3}})$: 3-gluon (odderon) exchange

$V\,V^\dagger\,V$ *Hatta, Iancu, Itakura, McLerran* **BJKP equation**

Kovchegov et al.

BJKP equation describes evolution of n-Reggeized gluons in a singlet state

JIMWLK (linear) and BJKP eqs. agree for n=2,3,4

non-linear interactions:

1) MV action (SJ+RV) with JIMWLK evolution

2) Triple (and more) pomeron vertices

Chirilli, Szymanowski, Wallon (2010)

QCD at high energy

Two distinct approaches:

1) CGC

McLerran-Venugopalan effective action
JIMWLK evolution

2) Reggeized-gluon exchange

BJKP equation
triple pomeron vertex

Conjecture: CGC contains BJKP + multi-pomeron vertices

quadrupole evolution: limits

$$< Q(r,\bar{r},\bar{s},s) >\equiv \frac{1}{N_c} < Tr\, V(r)\, V^{\dagger}(\bar{r})\, V(\bar{s})\, V^{\dagger}(s) >$$

calculated in a Gaussian model: JM-K, DMXY, IT

line config.: $r=\bar{s},\ \bar{r}=s,\ z\equiv r-\bar{r}$

square config.: $r-\bar{s}=\bar{r}-s=r-\bar{r}=\cdots\equiv z$

“naive” Gaussian: $Q=S^2$

Gaussian $$Q_{|}(z)\approx\frac{N_c+1}{2}[S(z)]^{2\frac{N_c+2}{N_c+1}}-\frac{N_c-1}{2}[S(z)]^{2\frac{N_c-2}{N_c-1}}$$

Gaussian + large Nc $$Q_{|}(z)\rightarrow S^2(z)[1+2\,log[S(z)]]$$

quadrupole evolution: limits

$$< Q(r, \bar{r}, \bar{s}, s) > \equiv \frac{1}{N_c} < Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s) >$$

Gaussian

$$Q_{sq}(z) = [S(z)]^2 \left[\frac{N_c+1}{2} \left(\frac{S(z)}{S(\sqrt{2}z)} \right)^{\frac{2}{N_c+1}} - \frac{N_c-1}{2} \left(\frac{S(\sqrt{2}z)}{S(z)} \right)^{\frac{2}{N_c-1}} \right]$$

Gaussian + large N_c

$$Q_{sq}(z) = \left[1 + 2 \ln \left(\frac{S(z)}{S(\sqrt{2}z)} \right) \right]$$

quadrupole evolution on lattice

a "random" (Gaussian) distribution of color charges (at initial rapidity y_o)

construct the Wilson line

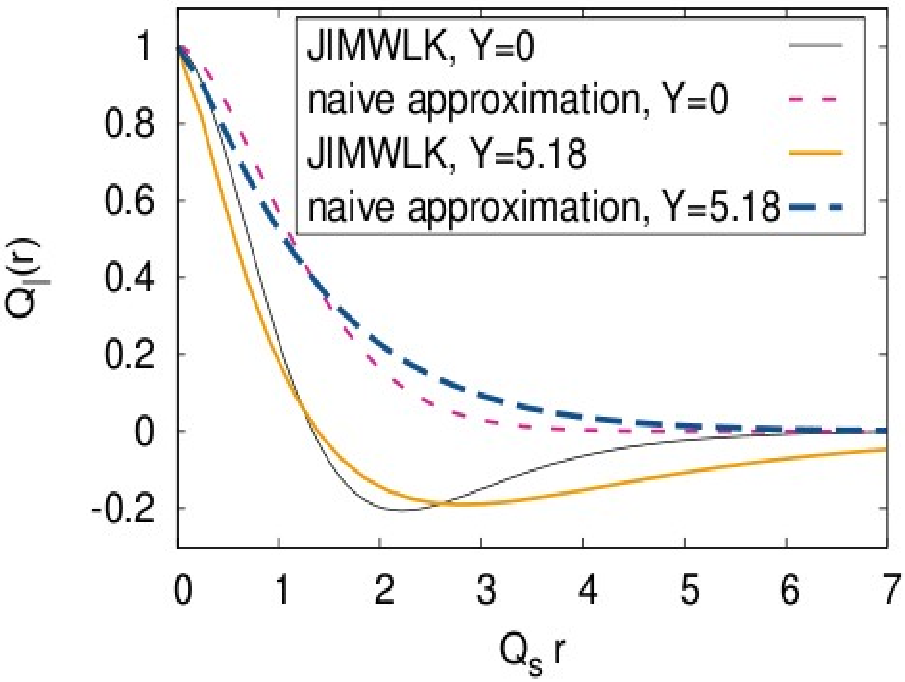
evolve the Wilson line to a higher rapidity y

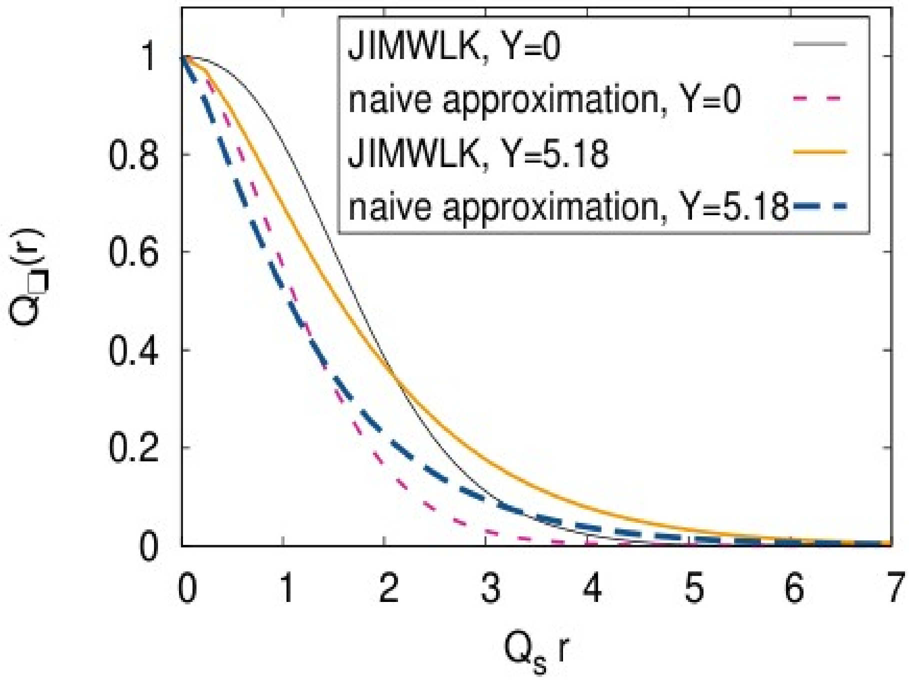
compute ensemble average of any number of Wilson lines at y

Dumitru-Jalilian-Marian-Lappi-Schenke-Venugopalan:PLB706 (2011) 219

Quadrupole evolution

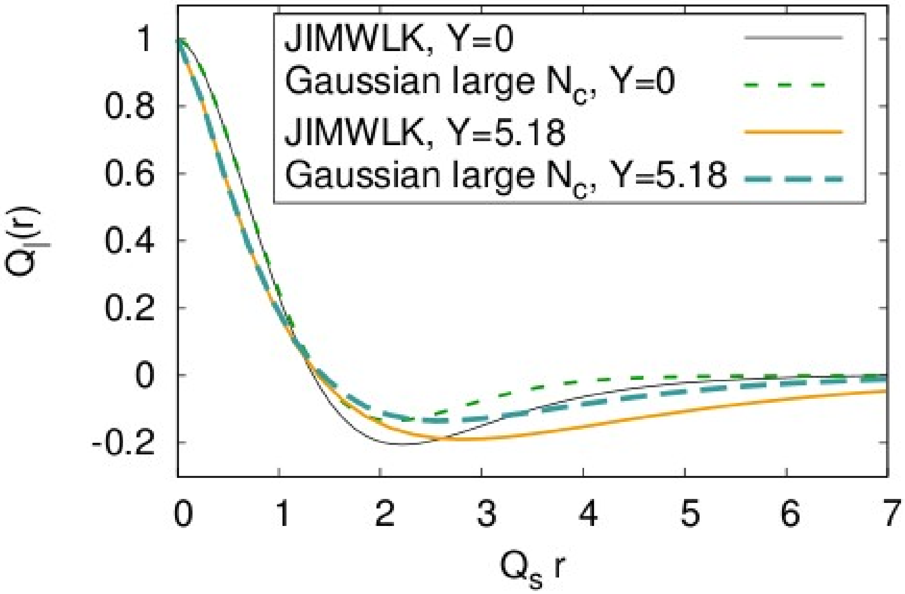
comparing with “naive” Gaussian

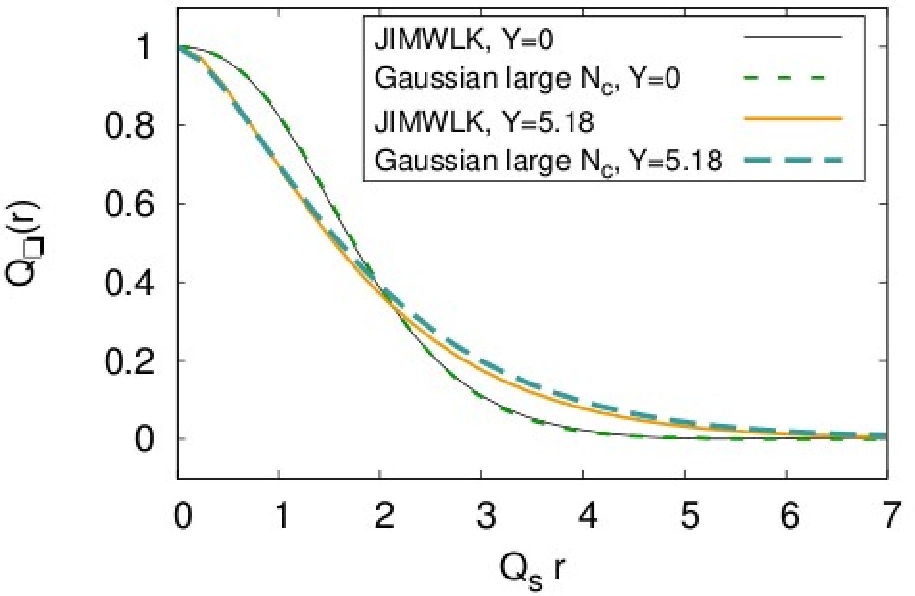




Quadrupole evolution

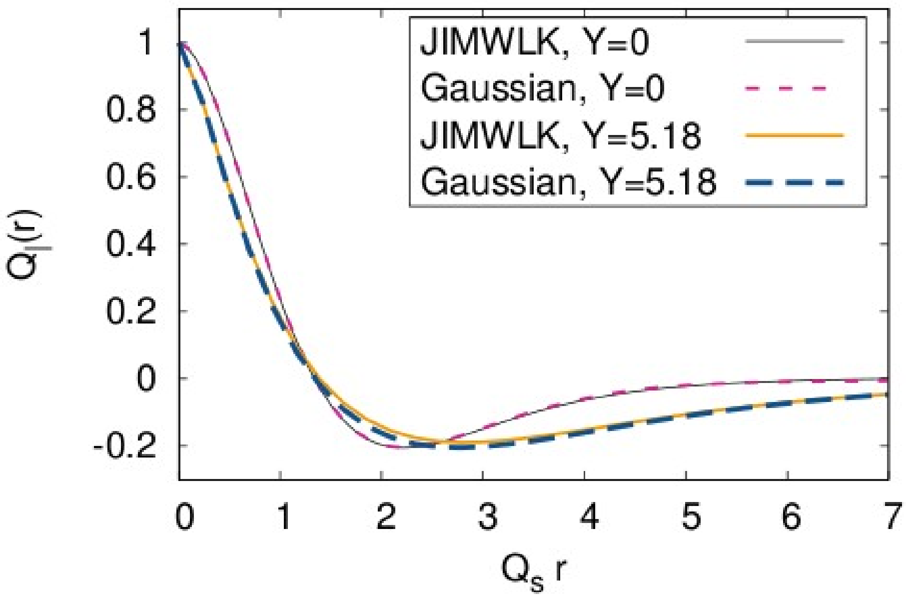
comparing with Gaussian + large N_c

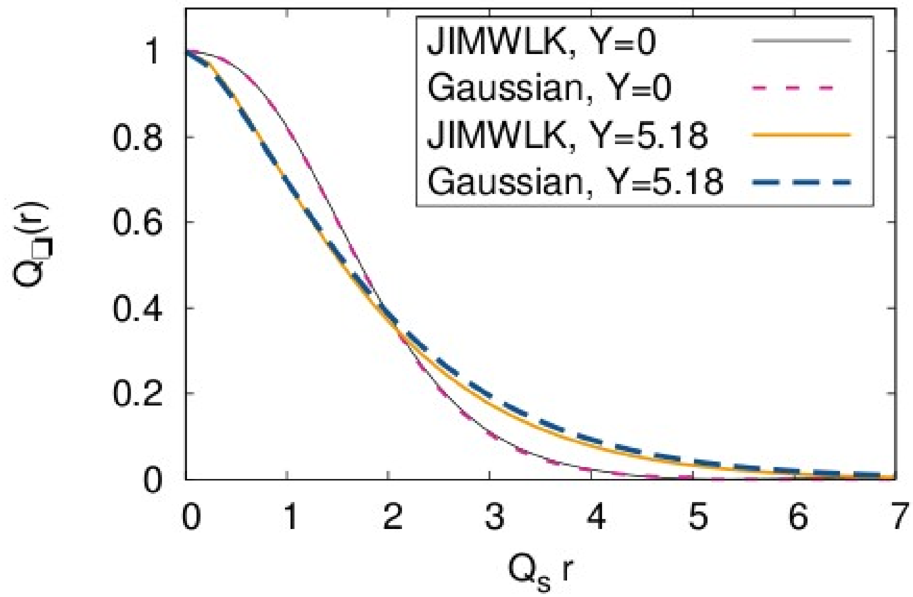




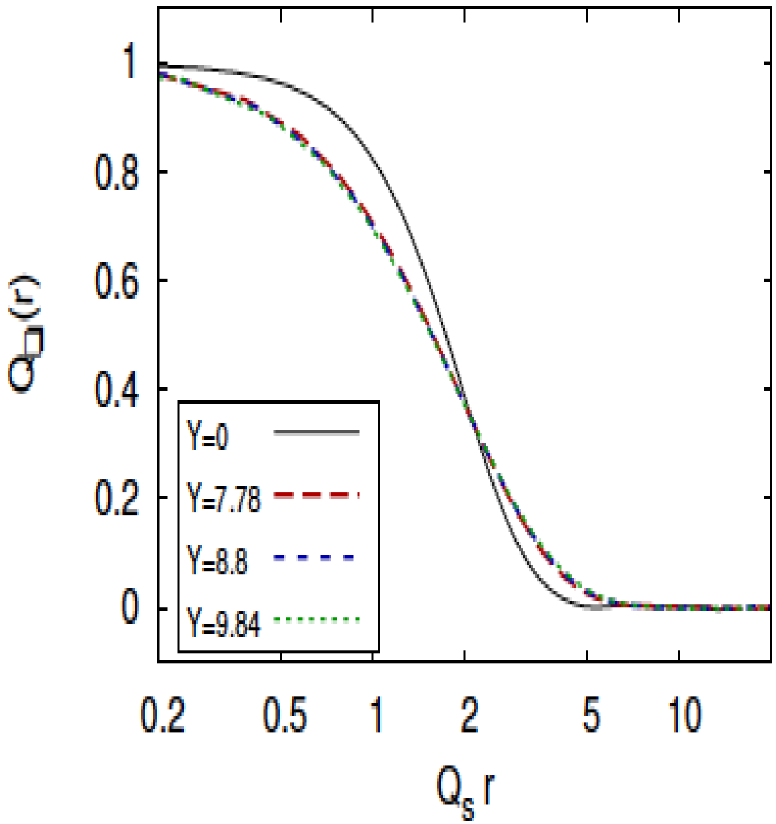
Quadrupole evolution

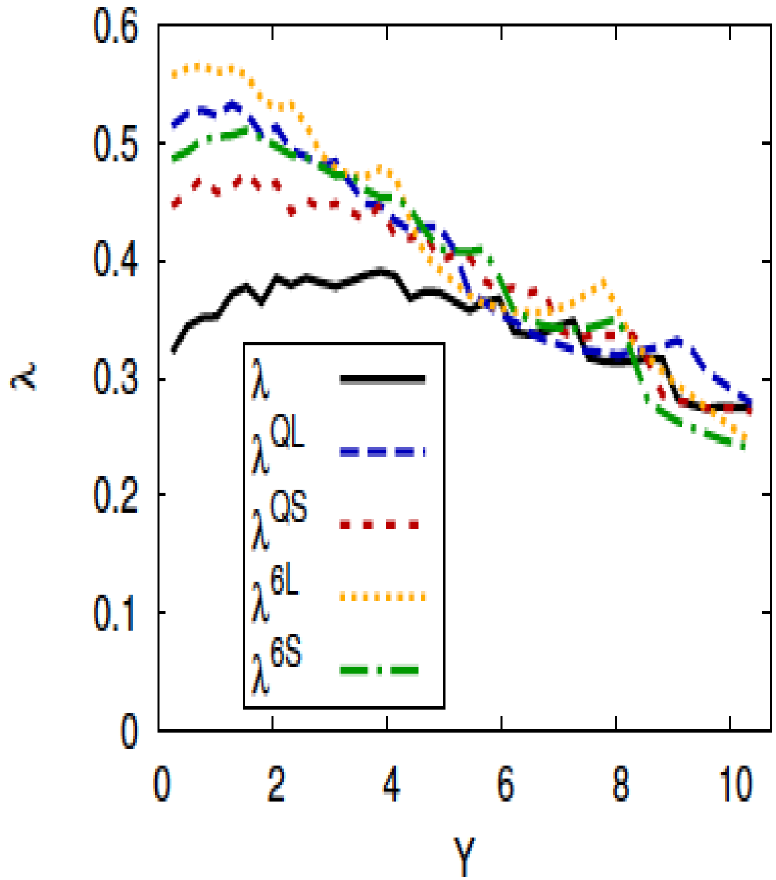
comparing with Gaussian





Quadrupole evolution





Geometric scaling also present in quadrupoles

Growth of the saturation scale

Two-hadron angular correlations

A unique window to dynamics of high energy QCD

We have just started to scratch the surface: there is much more to be understood