

Parton saturation and particle production in pA collisions

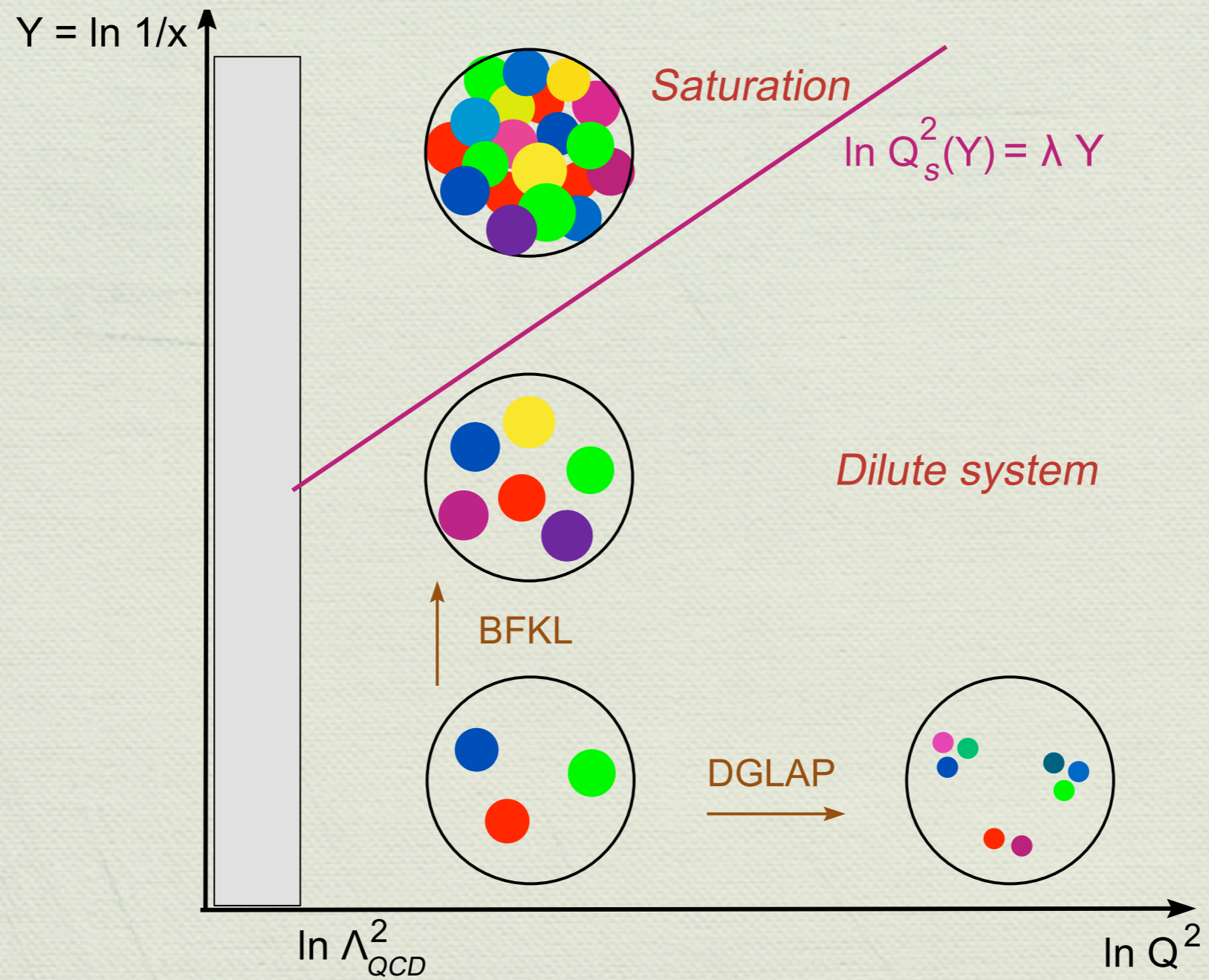
Dionysios Triantafyllopoulos
ECT* and FBK, Trento, Italy

pA collisions at the LHC, Trento, May 2013

Outline

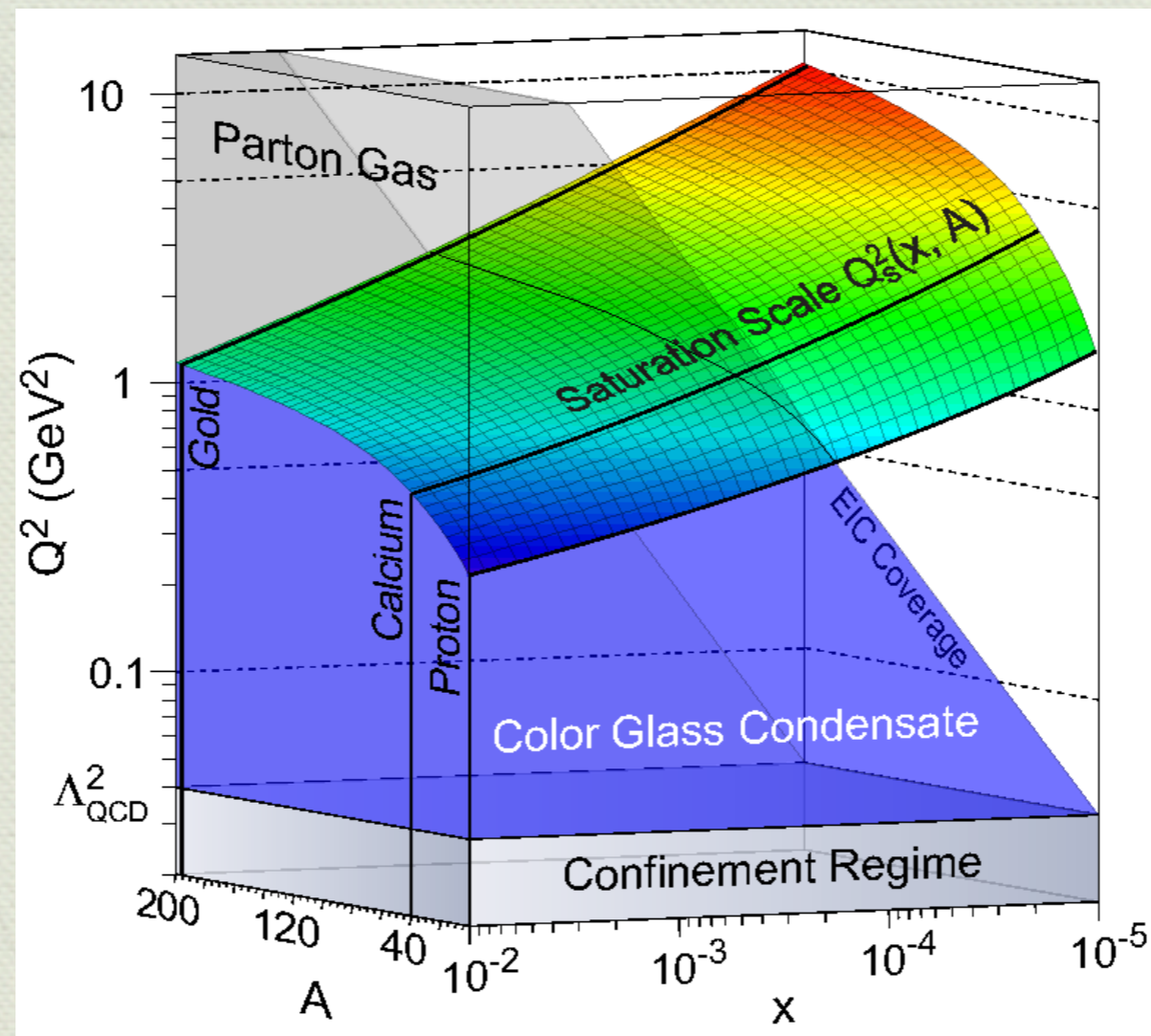
- The partonic “phase diagram” and saturation
- High energy scattering and Wilson lines
- Dihadron production in forward region in pA collisions
- JIMWLK evolution of dipoles, quadrupoles, ...
- The Gaussian approximation
- Production at different rapidities

Partonic "phase diagram"



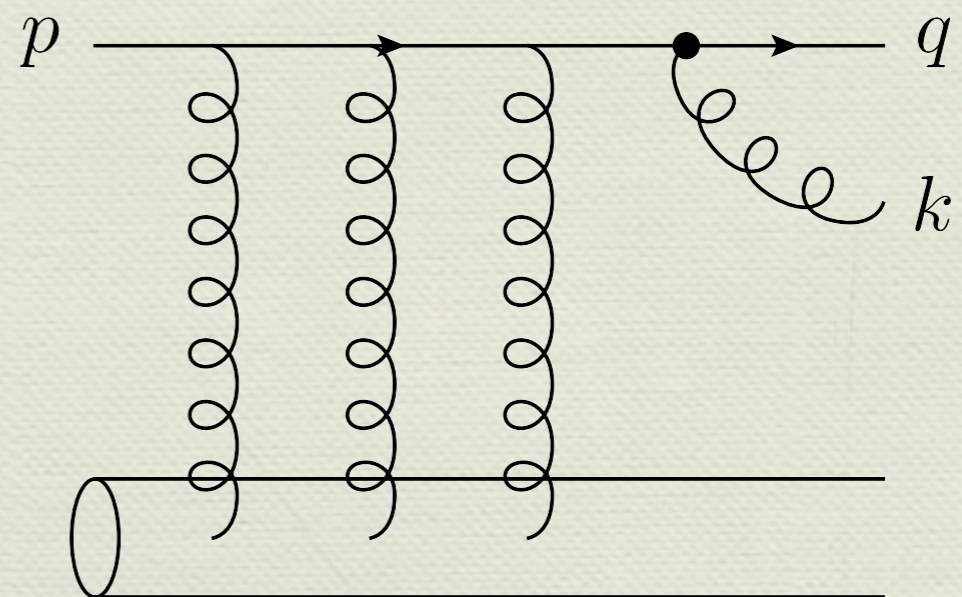
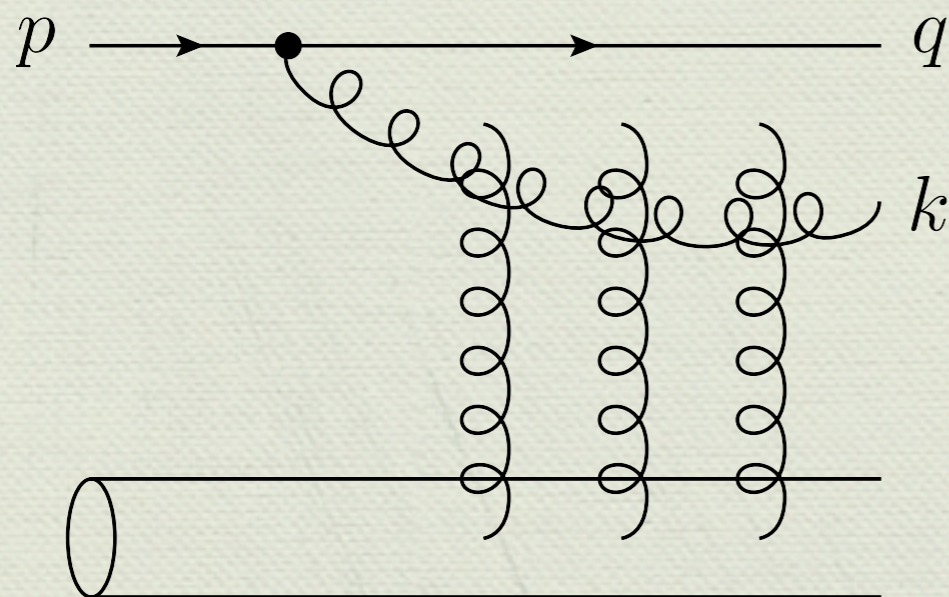
Saturation momentum

- Saturation when $\frac{xg(x, Q_s^2)}{Q_s^2 R^2} \sim \frac{1}{\alpha_s}$
- $Q_s^2(x, A) \sim Q_0^2 A^{1/3} \left(\frac{x_0}{x}\right)^\lambda$ with $\lambda = 0.2 \div 0.3$



The process

- Large- x quark from proton splits into quark-gluon pair
- Interacts with soft components of nucleus
- Quark-gluon pair “measured” in forward region



The outgoing state

- Mixed representation: transverse momenta \rightarrow coordinates
- Nucleus viewed as large classical color field
- Eikonal interaction \rightarrow Wilson lines: $V_{\mathbf{x}}^{\dagger} = P \exp \left[i g \int dx^{-} t^a \mathcal{A}_{\mathbf{x}}^{+a}(x^{-}) \right]$

$$\begin{aligned}
 |\Psi_{\text{out}}\rangle &= \int D\mathcal{A}^+ \Phi_Y[\mathcal{A}^+] \int_{\mathbf{x}, \mathbf{b}} dz p^- g e^{i\mathbf{p} \cdot \mathbf{b}} \sum_{j\beta c\lambda} \phi_{\alpha\beta}^{\lambda}(p, zp^-, \mathbf{x} - \mathbf{b}) \\
 &\quad [T^d V(\mathbf{b}) \tilde{V}^{dc}(\mathbf{x}) - V(\mathbf{b} + z(\mathbf{x} - \mathbf{b})) T^c]_{ij} \\
 &\quad |(1-z)p^-, \mathbf{b}, j, \beta; zp^-, \mathbf{x}, c, \lambda\rangle \otimes |\mathcal{A}^+\rangle
 \end{aligned}$$

The cross section

□ From $\langle \Psi_{\text{out}} | N_q(q) N_g(k) | \Psi_{\text{out}} \rangle$ calculate cross section

$$\frac{d\sigma^{qA \rightarrow qgX}}{d^3k d^3q} = \frac{\alpha_s N_c}{2} \int_{\mathbf{x} \mathbf{x}' \mathbf{b} \mathbf{b}'} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}') + i(\mathbf{q} - \mathbf{p}) \cdot (\mathbf{b}' - \mathbf{b})}$$

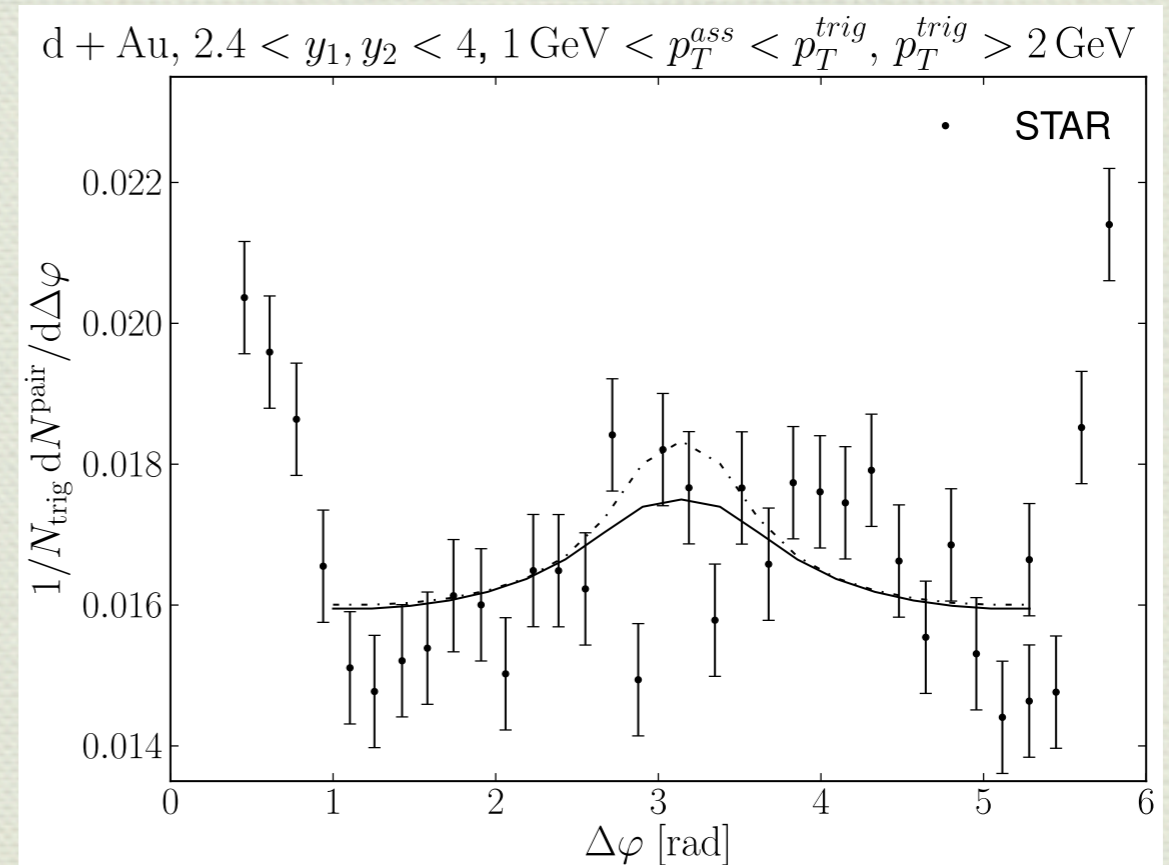
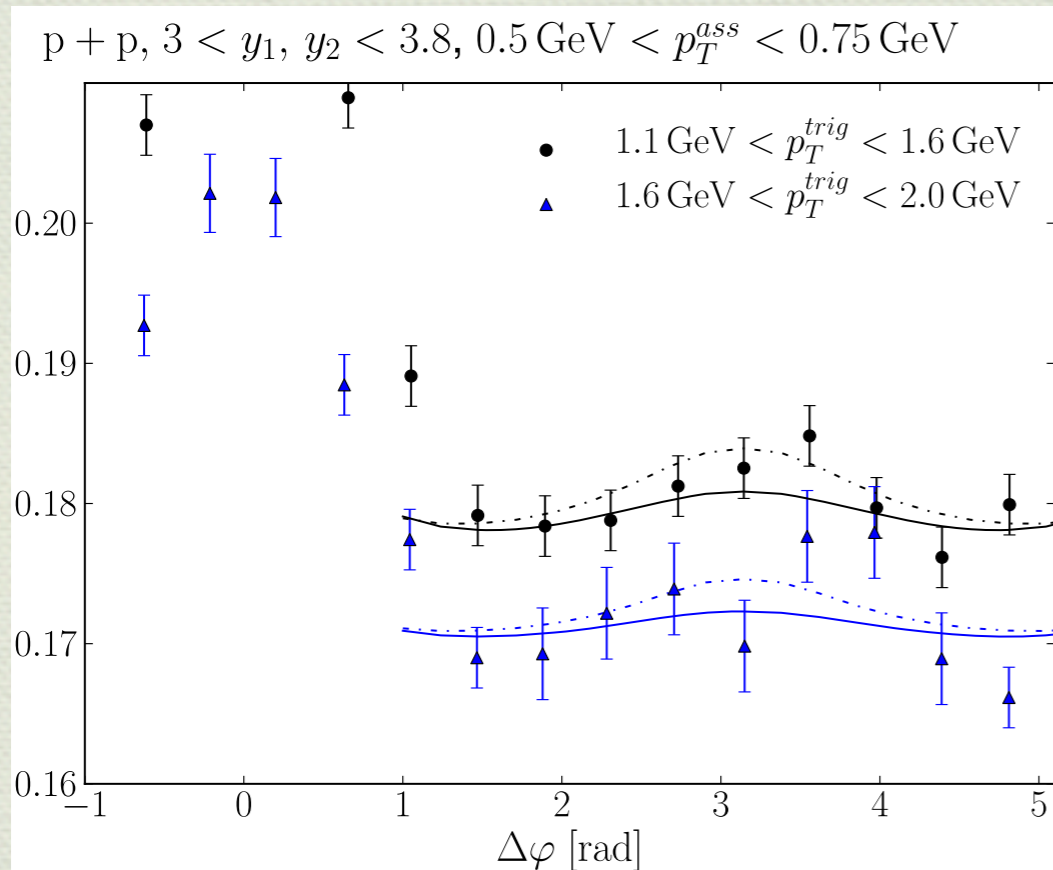
$$\sum_{\lambda \alpha \beta} \phi_{\alpha\beta}^{\lambda*}(p, zp^-, \mathbf{x}' - \mathbf{b}') \phi_{\alpha\beta}^{\lambda}(p, zp^-, \mathbf{x} - \mathbf{b})$$

$$\left\langle \frac{1}{N_c} \text{tr}[V^\dagger(\mathbf{x}) V(\mathbf{b}) V^\dagger(\mathbf{b}') V(\mathbf{x}')] \frac{1}{N_c} \text{tr}[V^\dagger(\mathbf{x}') V(\mathbf{x})] + \dots \right\rangle_Y$$

□ QCD dynamics in $\langle \dots \rangle_Y = \int D\mathcal{A}^+ W_Y[\mathcal{A}^+] \dots$

□ $e^{-Y} = x = \frac{|\mathbf{k}| e^{-y_k} + |\mathbf{q}| e^{-y_q}}{\sqrt{s}} \ll 1$ in forward region

Di-hadron azimuthal correlations



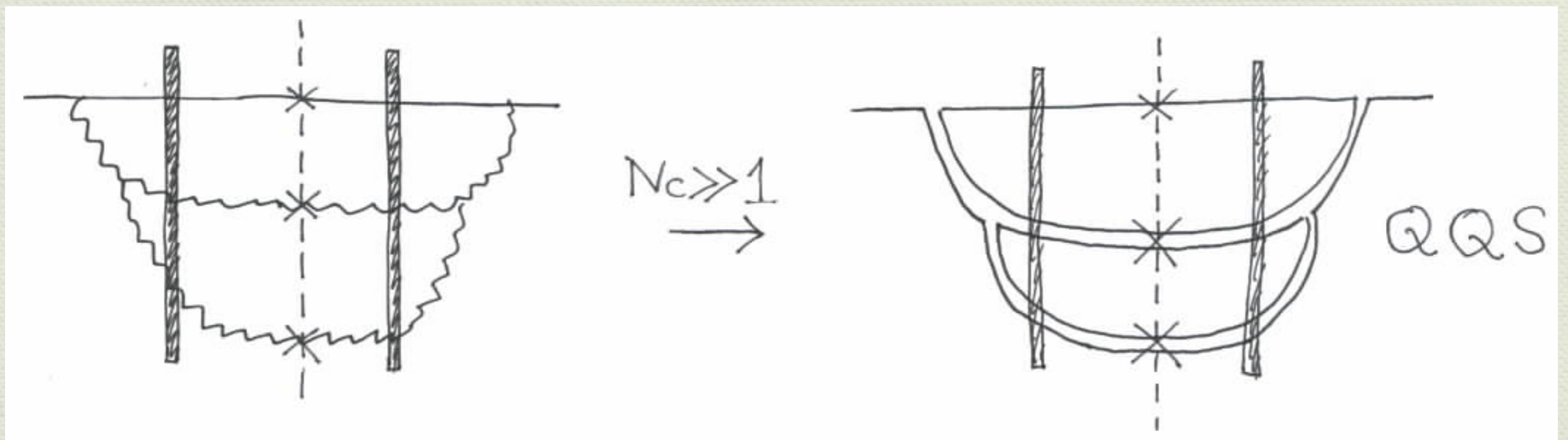
Lappi, Mantysaari '12

- For $p_{1T}, p_{2T} > Q_s$, width in $\Delta\phi \sim Q_s / p_{iT}$
- For $p_{1T}, p_{2T} \ll Q_s$, IR divergent:
subtracted and added as DPS in a collinear treatment

Wilson line correlators

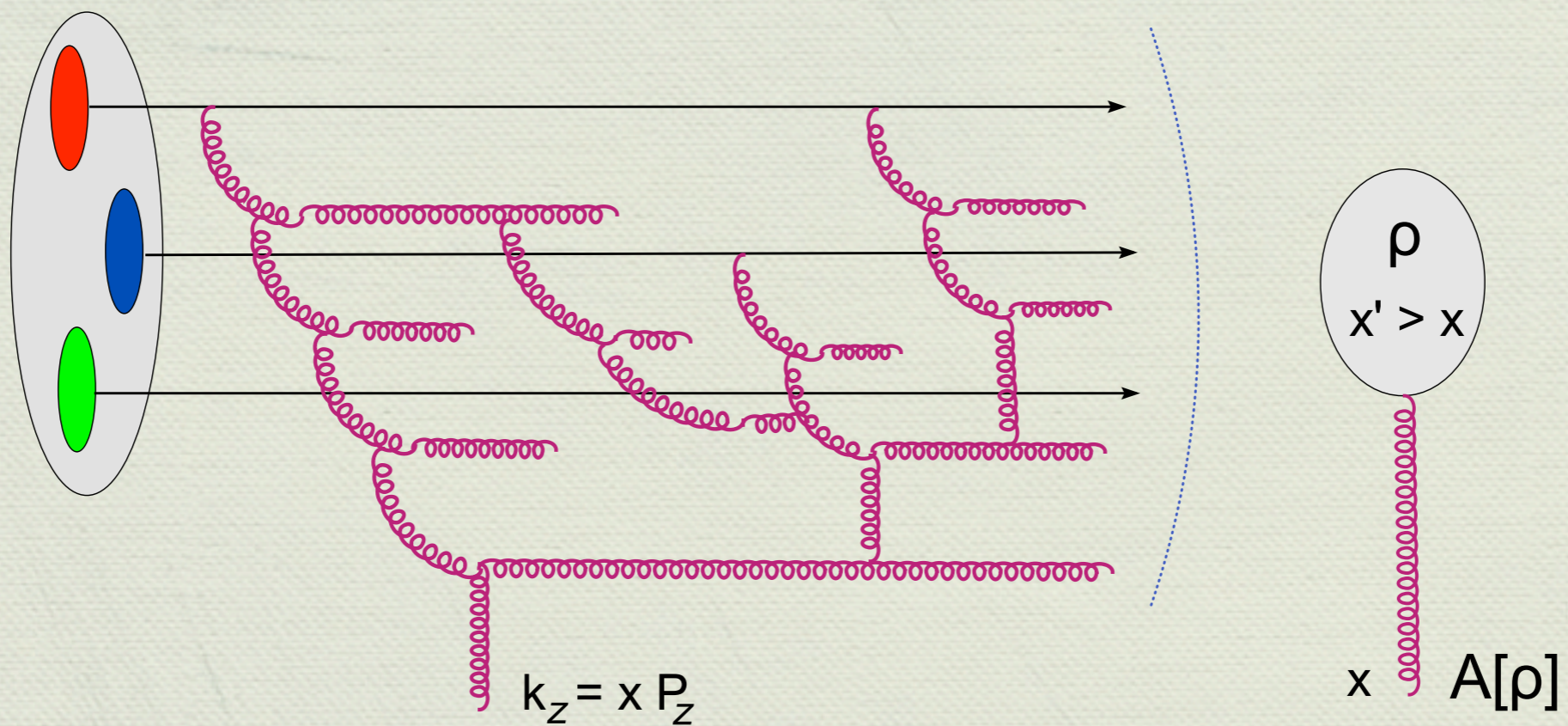
- Dipole $\hat{S}_{12} = \frac{1}{N_c} \text{tr}(V_1^\dagger V_2)$, quadrupole $\hat{Q}_{1234} = \frac{1}{N_c} \text{tr}(V_1^\dagger V_2 V_3^\dagger V_4)$
- At large- N_c can factorize $\langle \hat{Q} \hat{S} \rangle_Y = \langle \hat{Q} \rangle_Y \langle \hat{S} \rangle_Y$
- S not enough for at least two particles measured.
But S and Q is all we need for any number of particles.

Kovner, Lublinsky '06 / Dominguez, Marquet, Stasto, Xiao '12



Color Glass Condensate

- QCD, frozen sources, occupation numbers of order $1/\alpha_s$
- All orders in $\alpha_s \ln 1/x$ and classical field $A_a^\mu \sim \mathcal{O}(1/g)$



First idea : McLerran, Venugopalan '93

JIMWLK evolution of correlators

- QCD dynamics encoded in JIMWLK Hamiltonian

$$H = -\frac{1}{16\pi^3} \int_{uvz} \mathcal{M}_{uvz} [1 + \tilde{V}_u^\dagger \tilde{V}_v - \tilde{V}_u^\dagger \tilde{V}_z - \tilde{V}_z^\dagger \tilde{V}_v]^{ab} \frac{\delta}{\delta\alpha_u^a} \frac{\delta}{\delta\alpha_v^b}$$

- Evolution of expectation value of arbitrary correlator

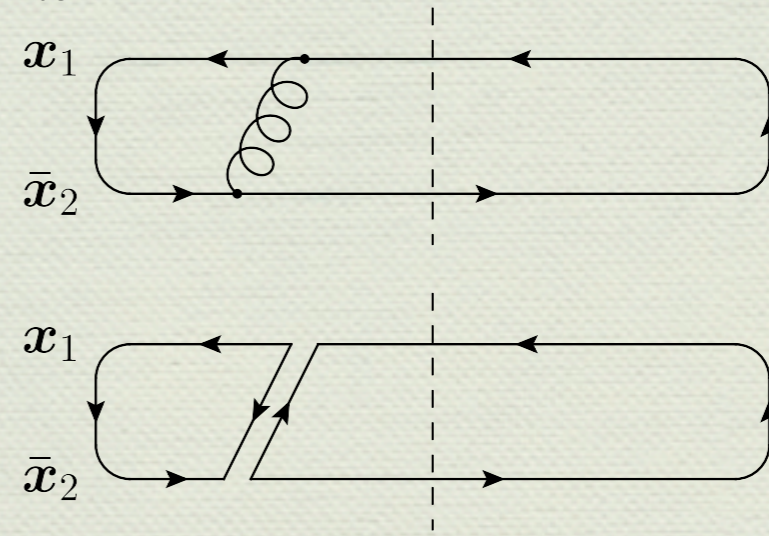
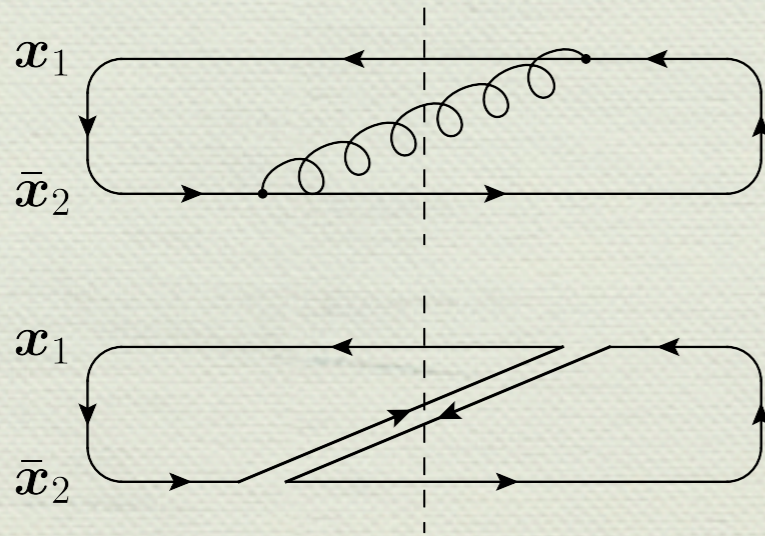
$$\frac{\partial W_Y[\alpha]}{\partial Y} = H W_Y[\alpha] \quad \Longrightarrow \quad \frac{\partial \langle \hat{\mathcal{O}} \rangle_Y}{\partial Y} = \langle H \hat{\mathcal{O}} \rangle_Y$$

- Easy to work out: act on end-point, use Fierz identities.

The Dipole

Balitsky '96 / Kovchegov '99

□ Well-known eqn:
$$\frac{\partial \langle \hat{S}_{12} \rangle_Y}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int_z \mathcal{M}_{12z} \langle \hat{S}_{1z} \hat{S}_{z2} - \hat{S}_{12} \rangle_Y$$



- Weak scattering: linear in $T = 1 - S \ll 1$, BFKL, easy to solve
- Strong scattering, assume large N_c : linear in S

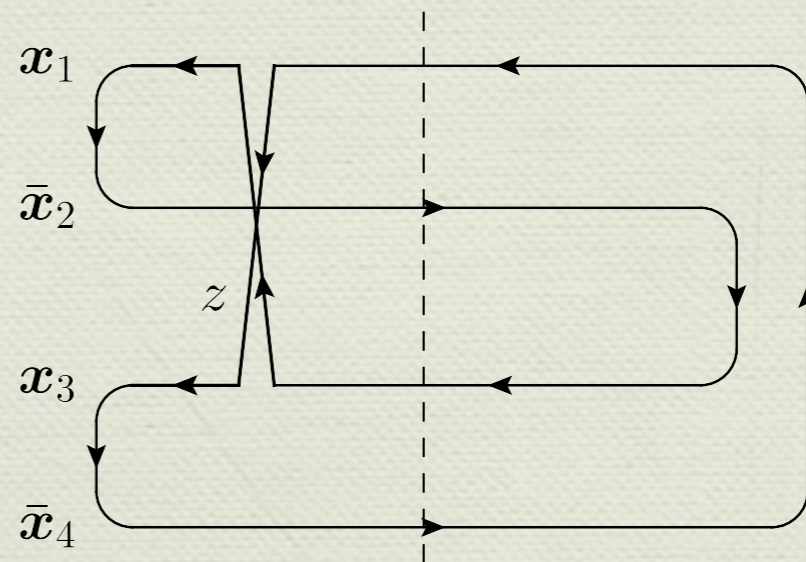
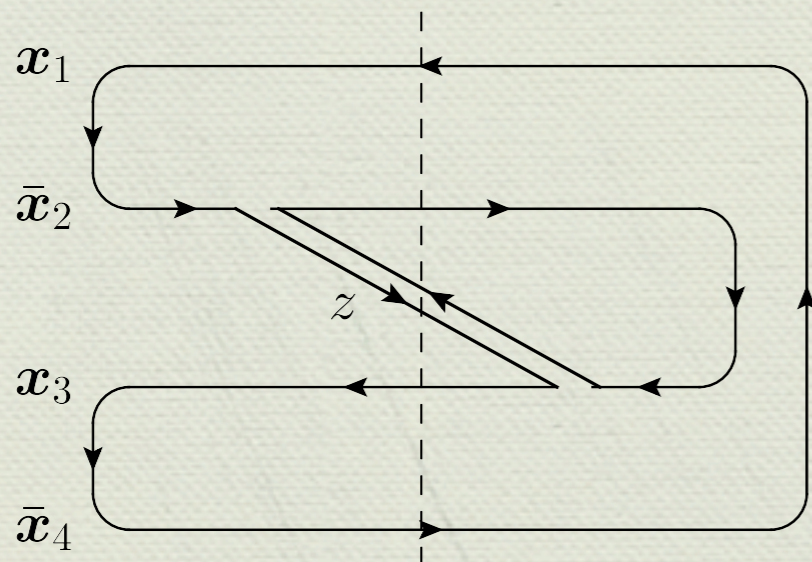
$$\frac{\partial \langle \hat{S}_{12} \rangle_Y}{\partial Y} = -\bar{\alpha}_s \int_{1/Q_s^2}^{r_{12}^2} \frac{dz^2}{z^2} \langle \hat{S}_{12} \rangle_Y = -\bar{\alpha}_s \ln(r_{12}^2 Q_s^2) \langle \hat{S}_{12} \rangle_Y$$

- Local in S , trivially solved if we know $Q_s(Y)$

The Quadrupole

$$\begin{aligned}
 \frac{\partial \langle \hat{Q}_{1234} \rangle_Y}{\partial Y} = & \frac{\bar{\alpha}_s}{4\pi} \int_z (\mathcal{M}_{12z} + \mathcal{M}_{14z} - \mathcal{M}_{24z}) \langle \hat{S}_{1z} \hat{Q}_{z234} \rangle_Y \\
 & + (\mathcal{M}_{12z} + \mathcal{M}_{23z} - \mathcal{M}_{13z}) \langle \hat{S}_{z2} \hat{Q}_{1z34} \rangle_Y \\
 & + (\mathcal{M}_{23z} + \mathcal{M}_{34z} - \mathcal{M}_{24z}) \langle \hat{S}_{3z} \hat{Q}_{12z4} \rangle_Y \\
 & + (\mathcal{M}_{14z} + \mathcal{M}_{34z} - \mathcal{M}_{13z}) \langle \hat{S}_{z4} \hat{Q}_{123z} \rangle_Y \\
 & - (\mathcal{M}_{12z} + \mathcal{M}_{14z} + \mathcal{M}_{23z} + \mathcal{M}_{14z}) \langle \hat{Q}_{1234} \rangle_Y \\
 & - (\mathcal{M}_{12z} + \mathcal{M}_{34z} - \mathcal{M}_{13z} - \mathcal{M}_{24z}) \langle \hat{S}_{12} \hat{S}_{34} \rangle_Y \\
 & - (\mathcal{M}_{14z} + \mathcal{M}_{23z} - \mathcal{M}_{13z} - \mathcal{M}_{24z}) \langle \hat{S}_{14} \hat{S}_{23} \rangle_Y
 \end{aligned}$$

Jalilian-Marian, Kovchegov '04 (in factorized form)



Quadrupole in limiting cases

- Weak scattering, expand Wilson lines, 2-gluon exchange

$$\hat{Q}_{1234} \simeq 1 - \hat{T}_{12} + \hat{T}_{13} - \hat{T}_{14} - \hat{T}_{23} + \hat{T}_{24} - \hat{T}_{34}$$

- Evolving like “six BFKL’s”

- Strong scattering, assume large N_c , keep quadratic terms, local

$$\begin{aligned} \frac{\partial \langle \hat{Q}_{1234} \rangle_Y}{\partial Y} \simeq & -\frac{\bar{\alpha}_s}{2} [\ln(r_{12}^2 Q_s^2) + \ln(r_{34}^2 Q_s^2) + \ln(r_{14}^2 Q_s^2) + \ln(r_{23}^2 Q_s^2)] \langle \hat{Q}_{1234} \rangle_Y \\ & -\frac{\bar{\alpha}_s}{2} [\ln(r_{12}^2 Q_s^2) + \ln(r_{34}^2 Q_s^2) - \ln(r_{13}^2 Q_s^2) - \ln(r_{24}^2 Q_s^2)] \langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{34} \rangle_Y \\ & -\frac{\bar{\alpha}_s}{2} [\ln(r_{14}^2 Q_s^2) + \ln(r_{23}^2 Q_s^2) - \ln(r_{13}^2 Q_s^2) - \ln(r_{24}^2 Q_s^2)] \langle \hat{S}_{32} \rangle_Y \langle \hat{S}_{14} \rangle_Y, \end{aligned}$$

- Given $Q_s(Y)$ and dipole, can solve for quadrupole, but better ...

Look for functional form

- Write logs in terms of log-derivative of dipole
- Leads to functional form: Quadrupole in terms of dipole
- Better than log-accuracy
- Extends to running coupling
- An, *a priori*, unexpected result
- Ordinary 1st order inhomogeneous differential equation

Solution to the quadrupole

$$\langle \hat{Q}_{1234} \rangle_Y = \sqrt{\langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{32} \rangle_Y \langle \hat{S}_{34} \rangle_Y \langle \hat{S}_{14} \rangle_Y} \left[\frac{\langle \hat{Q}_{1234} \rangle_{Y_0}}{\sqrt{\langle \hat{S}_{12} \rangle_{Y_0} \langle \hat{S}_{32} \rangle_{Y_0} \langle \hat{S}_{34} \rangle_{Y_0} \langle \hat{S}_{14} \rangle_{Y_0}}} + \frac{1}{2} \int_{Y_0}^Y dy \frac{\langle \hat{S}_{13} \rangle_y \langle \hat{S}_{24} \rangle_y}{\sqrt{\langle \hat{S}_{12} \rangle_y \langle \hat{S}_{32} \rangle_y \langle \hat{S}_{34} \rangle_y \langle \hat{S}_{14} \rangle_y}} \frac{\partial}{\partial y} \frac{\langle \hat{S}_{12} \rangle_y \langle \hat{S}_{34} \rangle_y + \langle \hat{S}_{14} \rangle_y \langle \hat{S}_{32} \rangle_y}{\langle \hat{S}_{13} \rangle_y \langle \hat{S}_{24} \rangle_y} \right]$$

- Expanding solution for small T : correct result.
Linear Hamiltonian, Q linear in T for small T
- Valid in two limits, not exact at transition but cannot be bad
- Can integrate over y for simple configurations
- Local in Y under reasonable assumptions

The Gaussian approximation

□ Same approximation at the level of the Hamiltonian:

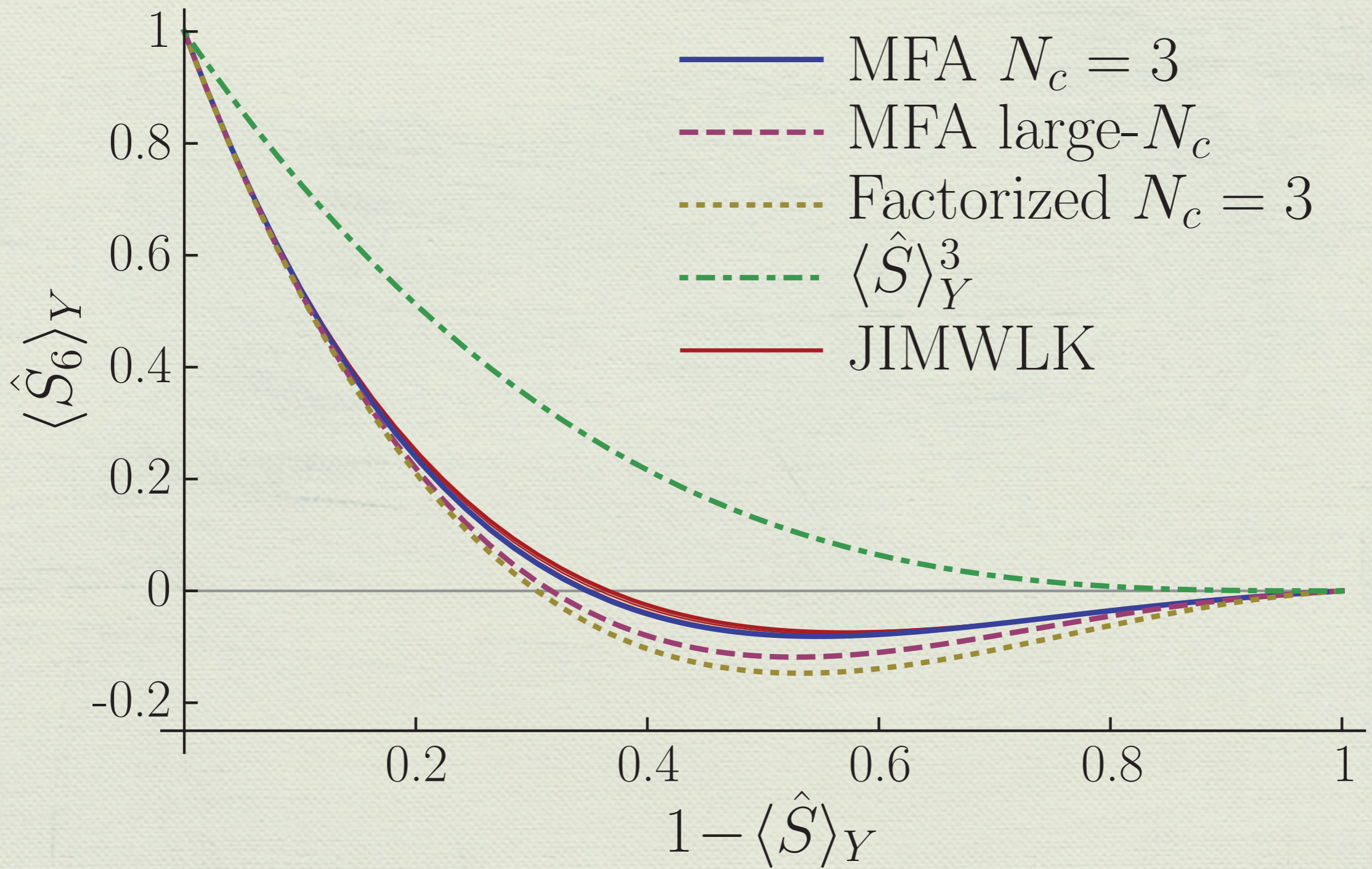
At saturation, dropping real terms, cutoff z -integration

$$H_{\text{sat}} \simeq -\frac{1}{8\pi^2} \int_{\mathbf{u}\mathbf{v}} \ln [(\mathbf{u} - \mathbf{v})^2 Q_s^2(Y)] \left(1 + \tilde{V}_{\mathbf{u}}^\dagger \tilde{V}_{\mathbf{v}}\right)^{ab} \frac{\delta}{\delta\alpha_{\mathbf{u}}^a} \frac{\delta}{\delta\alpha_{\mathbf{v}}^b}$$

Modify the “Sudakov” kernel to extend at low density

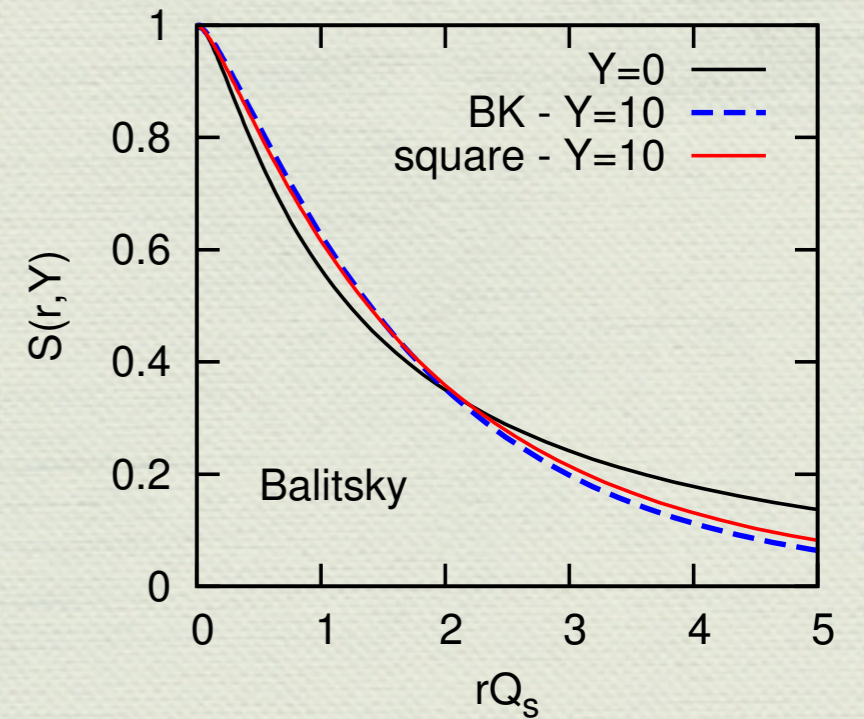
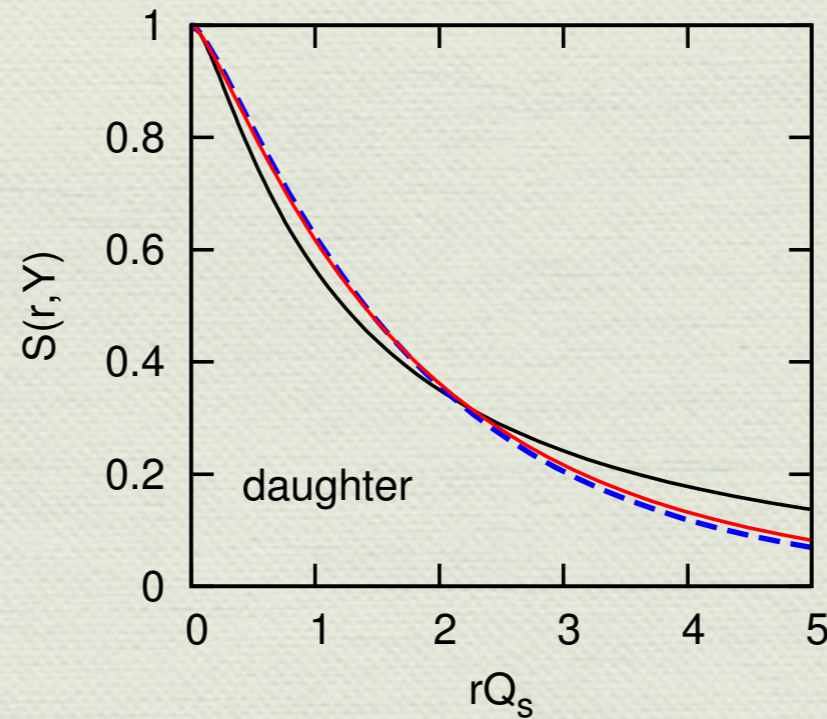
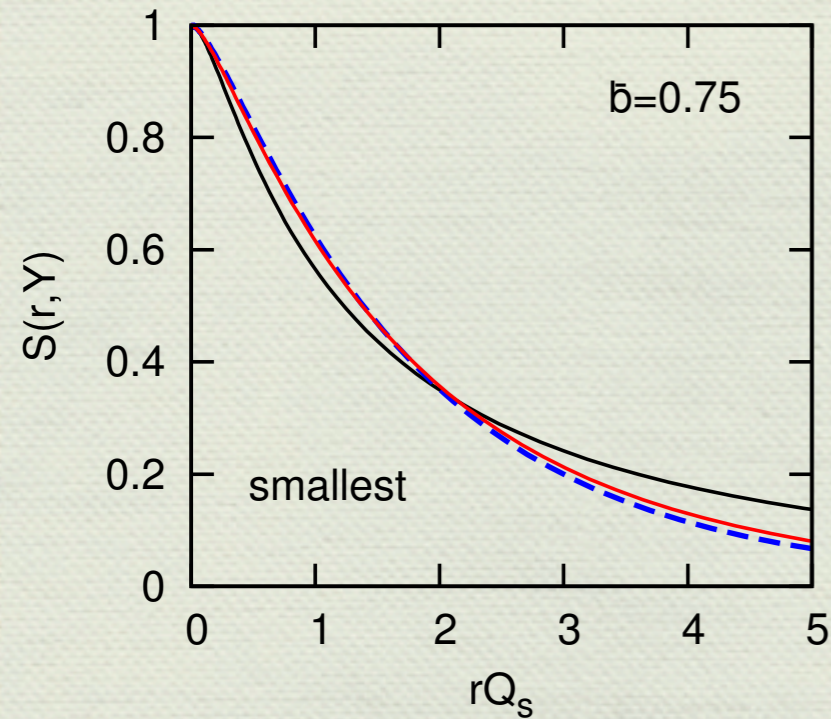
$$\frac{1}{4\pi^2} \ln [(\mathbf{u} - \mathbf{v})^2 Q_s^2(Y)] \rightarrow \gamma_Y(\mathbf{u}, \mathbf{v}) = -\frac{1}{2g^2 C_F} \frac{\partial \ln \langle \hat{S}_{\mathbf{u}\mathbf{v}} \rangle_Y}{\partial Y}$$

The solution is a Gaussian wavefunction



Iancu, DNT '11

with data from Dumitru, Jalilian-Marian, Lappi, Schenke, Venugopalan '11



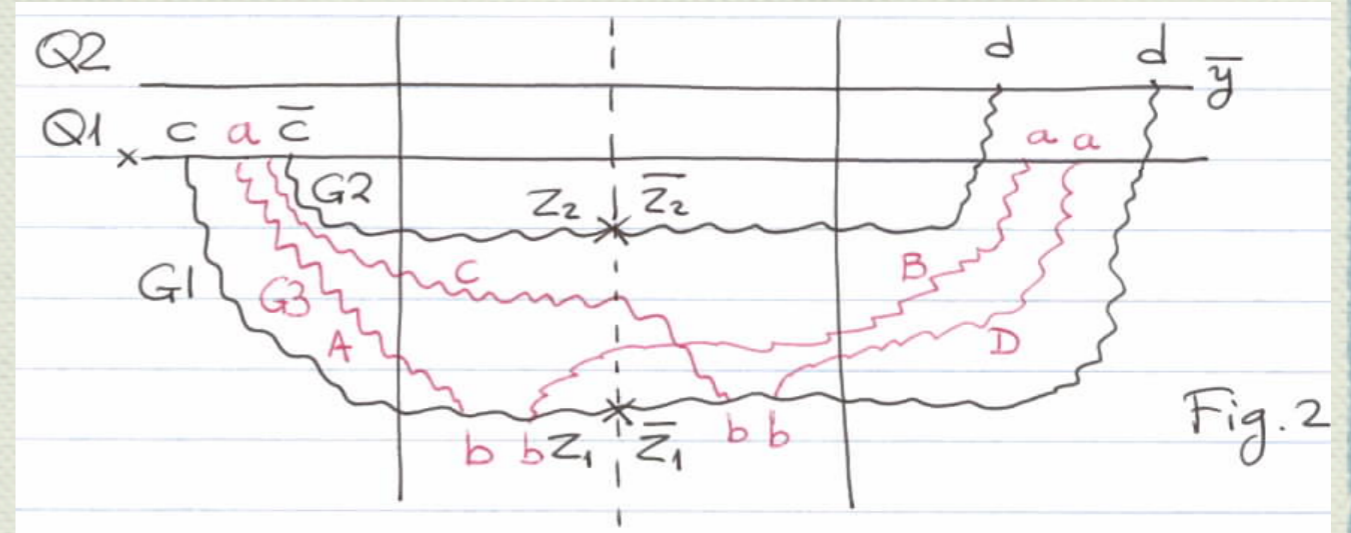
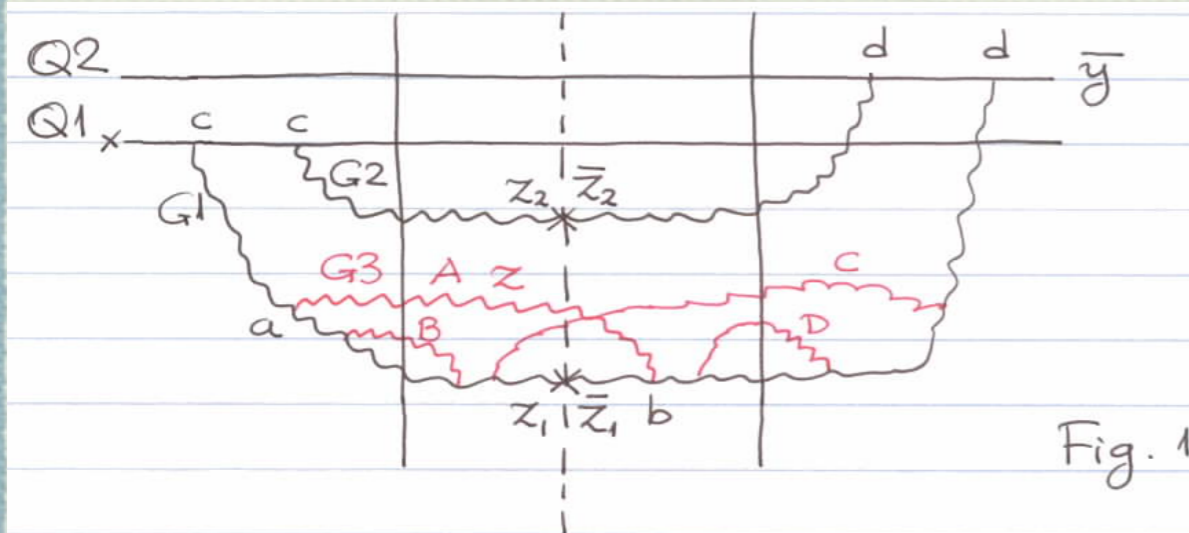
Gaussian average quadrupole equation for given configuration

Solve for S , compare to BK

Various running coupling scenarios

(Verified Levin-Tuchin law at saturation and extracted Q_s)

2g production with $Y_1 \gg Y_2$



Without extra gluon: Scattering of gluonic quadrupole

Fig1: Easy to systematize : “real” JIMWLK evolution of G1

Fig2: Complicated: “real” evolution of source of G1

Virtual terms: without source evolution, G1 evolution IR-divergent

Two particle production with $Y_1 \gg Y_2$

- Need to describe wavefunction squared of projectile:
two types of Wilson lines
- Physical quark is math. dipole: $S_{12}(\mathbf{x}\bar{\mathbf{x}}) = \frac{1}{N_c} \text{tr}(\bar{V}_{\bar{\mathbf{x}}} V_{\mathbf{x}}^\dagger)$
- Generating functional. Evolve, produce gluon, set $\bar{V} = V$

$$H_p = \frac{1}{4\pi^3} \int_{uv} \frac{\mathbf{y}-\mathbf{u}}{(\mathbf{y}-\mathbf{u})^2} \cdot \frac{\bar{\mathbf{y}}-\mathbf{v}}{(\bar{\mathbf{y}}-\mathbf{v})^2} \left[\bar{R}_v^b (\bar{W}_{\bar{\mathbf{y}}} W_{\mathbf{y}}^\dagger)^{ba} R_u^a + \bar{L}_v^a L_u^a - \bar{R}_v^b \bar{W}_{\bar{\mathbf{y}}}^{ba} L_u^a - \bar{L}_v^b W_{\mathbf{y}}^{\dagger ba} R_u^a \right]$$

- Solution?

Langevin form of JIMWLK

Blaizot, Iancu, Weigert '02

Nucleus R-mover, first approximation $\sim \delta(x^-)$

Wilson lines built stochastically adding layers in x^-

$$V_{\mathbf{x}}^\dagger(n\varepsilon + \varepsilon) = \exp(i\varepsilon\alpha_{L\mathbf{x}}^a t^a) V_{\mathbf{x}}^\dagger(n\varepsilon) \exp(-i\varepsilon\alpha_{R\mathbf{x}}^a t^a)$$

Iancu, DNT '11

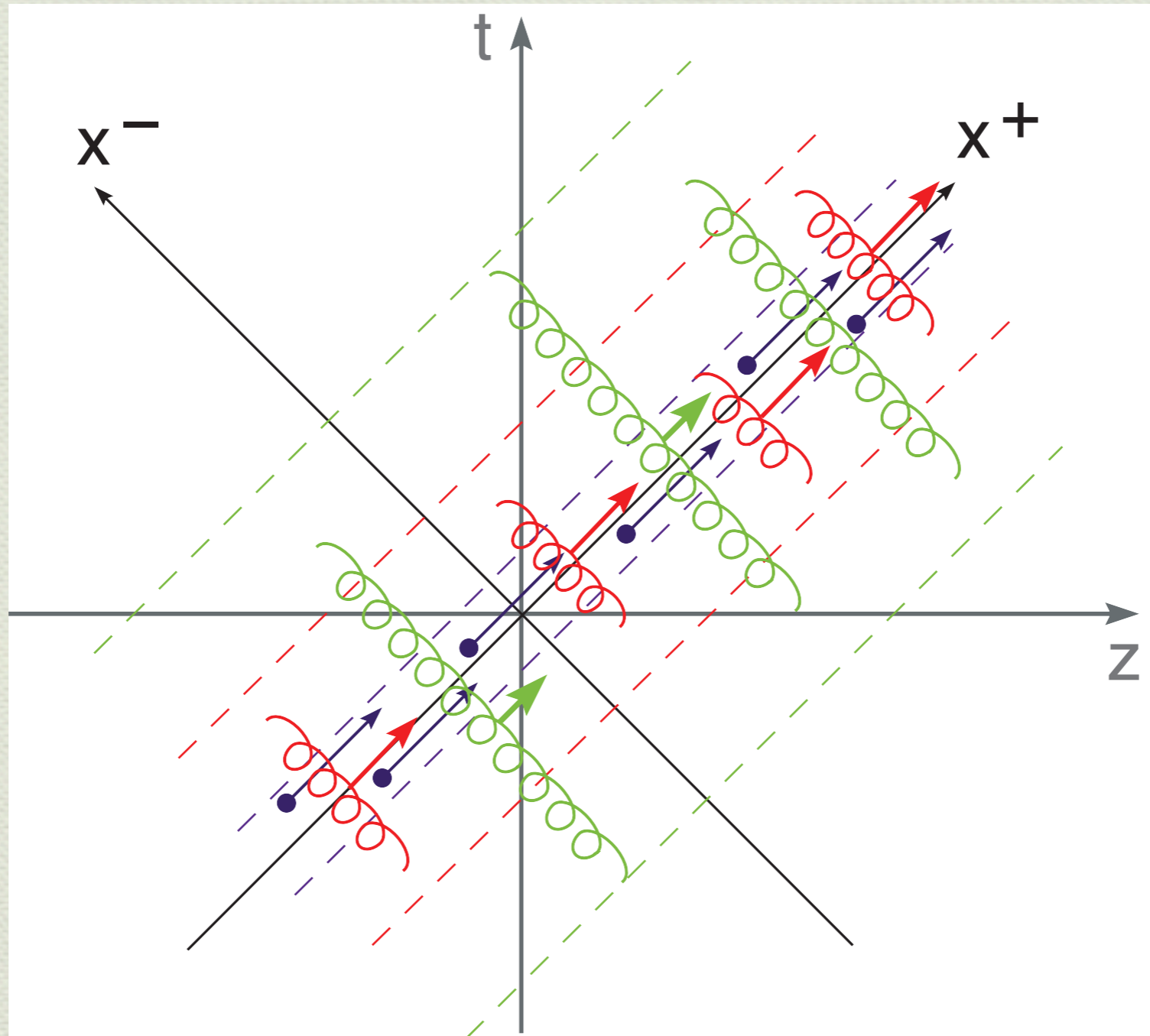
$$\alpha_{L\mathbf{x}}^a = \frac{\sqrt{\alpha_s}}{\pi} \int d\mathbf{z} \mathcal{K}_{\mathbf{xz}}^i \nu_z^{ia} \quad \alpha_{R\mathbf{x}}^a = \frac{\sqrt{\alpha_s}}{\pi} \int d\mathbf{z} \mathcal{K}_{\mathbf{xz}}^i \nu_z^{ib} U_z^{ba}$$

Lappi, Mantysaari '12

K is WW kernel, ν random color charge, U leads to cascade

Gaussian noise $\langle \nu_{\mathbf{x}}^{ia}(m\varepsilon) \nu_{\mathbf{y}}^{jb}(n\varepsilon) \rangle = \frac{1}{\varepsilon} \delta_{mn} \delta_{\mathbf{xy}} \delta^{ij} \delta^{ab}$

Symmetric longitudinal expansion



Langevin form of WFS

As before for V . In complex conjugate amplitude:

$$\bar{V}_{\mathbf{x}}^\dagger(n\varepsilon + \varepsilon) = \exp(i\varepsilon \bar{\alpha}_{L\mathbf{x}}^a t^a) \bar{V}_{\mathbf{x}}^\dagger(n\varepsilon) \exp(-i\varepsilon \bar{\alpha}_{R\mathbf{x}}^a t^a)$$

$$\bar{\alpha}_{L\mathbf{x}}^a = \frac{\sqrt{\alpha_s}}{\pi} \int d\mathbf{z} \mathcal{K}_{\mathbf{x}\mathbf{z}}^i \nu_z^{ia} \quad \bar{\alpha}_{R\mathbf{x}}^a = \frac{\sqrt{\alpha_s}}{\pi} \int d\mathbf{z} \mathcal{K}_{\mathbf{x}\mathbf{z}}^i \nu_z^{ib} \bar{U}_z^{ba}$$

Single noise, same as before

How to treat functional derivatives?

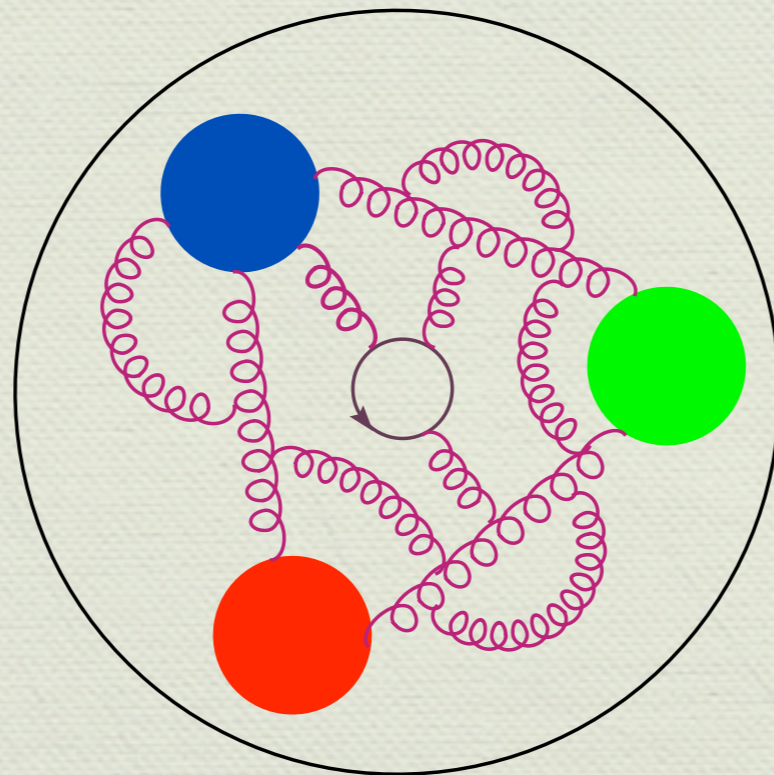
Conclusion

- Justify Gaussian approximation at finite N_c :
An analytical solution to JIMWLK (as function of S)
- Particle production at same rapidity needs only S and Q at large N_c (good enough)
- Particle production at different rapidities is more involved:
color charge sources of softest gluons also scatter
- Set-up a Langevin approach

BACKUP SLIDES

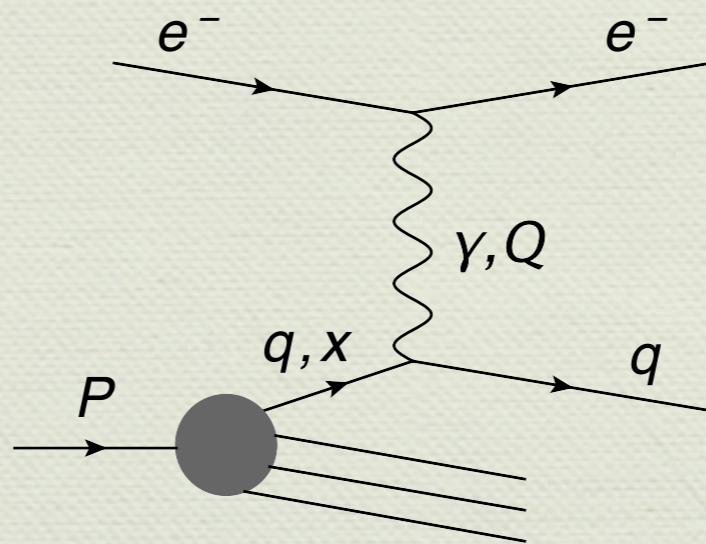
Constituents of a hadron

- Proton, or generic hadron, is complicated in rest frame
- Hadronic and vacuum fluctuations
- Non-perturbative with same lifetime $\Delta t_{\text{RF}} \sim 1/\Lambda_{\text{QCD}}$



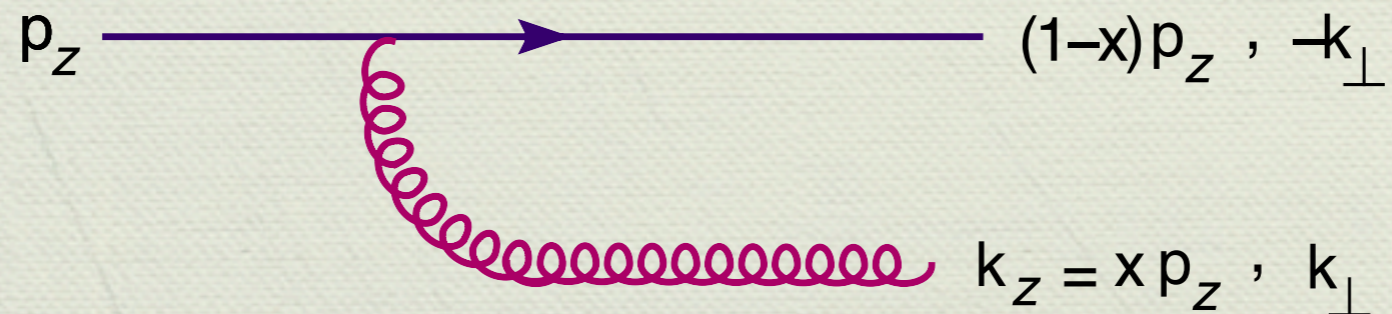
Infinite momentum frame and DIS

- IMF: Hadronic fluctuations live longer $\Delta t_{\text{IMF}} \sim \gamma/\Lambda_{\text{QCD}}$
- Longer than vacuum fluctuations
- Longer than collision time, e.g. in DIS $\Delta t_{\text{coll}} \sim 2xP/Q^2$
- Quark with $\Delta t_{\text{fluct}} \sim 2xP/k_{\perp}^2 \gtrsim \Delta t_{\text{coll}}$ seen by photon



Soft and collinear gluons

- $dP = C_R \frac{\alpha_s(k_\perp^2)}{\pi^2} \frac{d^2 k_\perp}{k_\perp^2} \frac{dx}{x}$
- Emission of soft and collinear gluons is favored
- Large logs can overcome smallness of coupling
- Source lives longer than emitted parton: frozen
- Gluons dominate at small-x



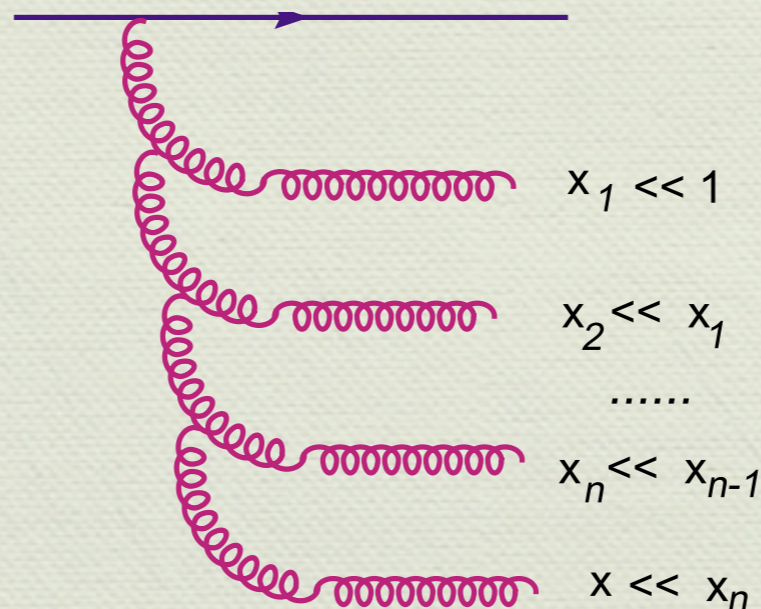
Cascades and evolution

- Successive emissions: DGLAP or BFKL cascade

$$\left(\frac{\alpha_s N_c}{\pi}\right)^n \int_x^1 \frac{dx_n}{x_n} \int_{x_n}^1 \frac{dx_{n-1}}{x_{n-1}} \dots \int_{x_2}^1 \frac{dx_1}{x_1} = \frac{1}{n!} \left(\bar{\alpha}_s \ln \frac{1}{x}\right)^n$$

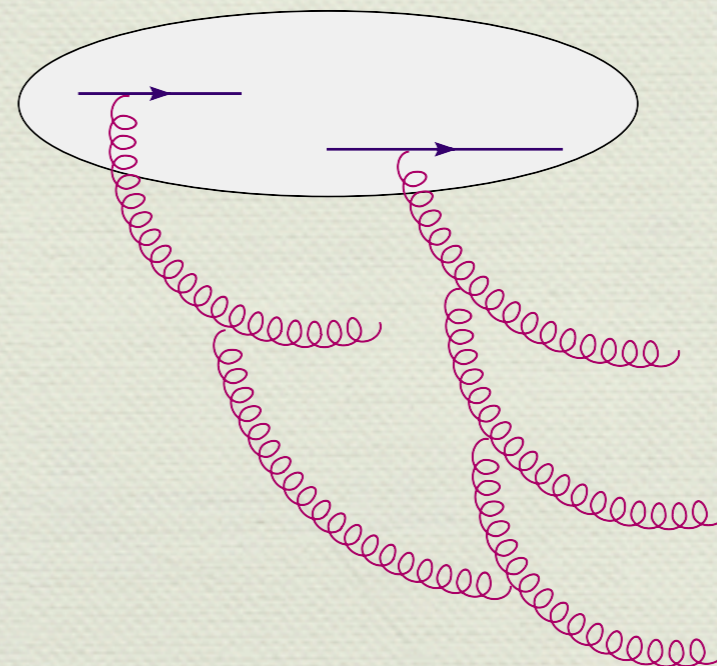
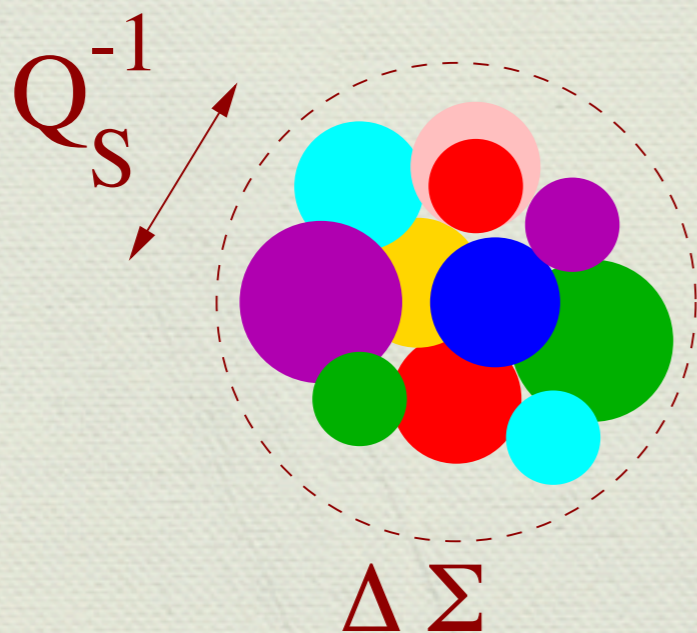
- Resum all diagrams $f_g = \frac{dN_g}{dY dk_{\perp}^2} \sim \frac{\alpha_s C_F}{\pi} \frac{1}{k_{\perp}^2} \exp(\omega \bar{\alpha}_s Y)$

- Evolution equation $\frac{d}{dY} f_g = \omega \bar{\alpha}_s f_g$



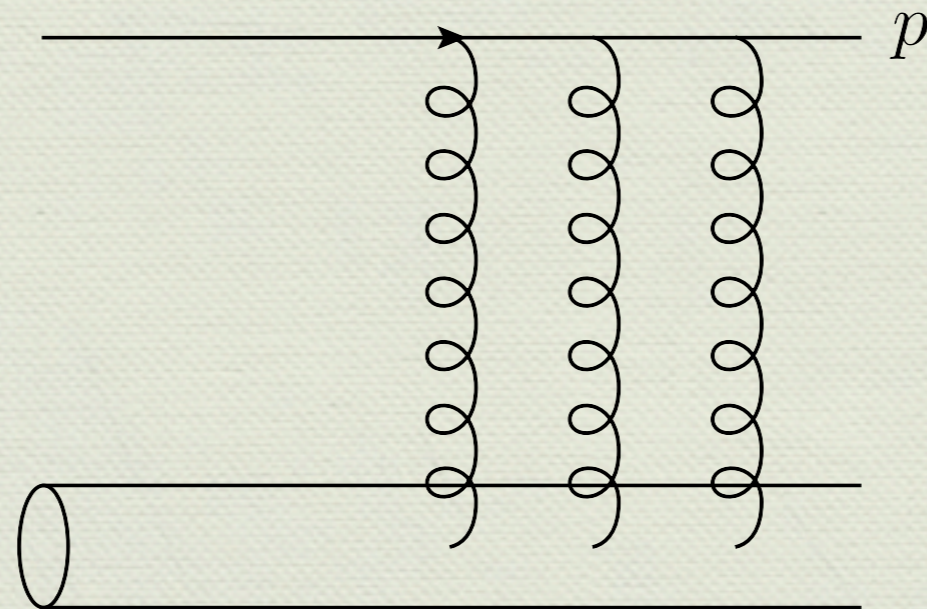
Kinematic regimes

- BFKL and DGLAP: linear, incoherent emissions
- DGLAP: smaller and smaller partons of size $1/Q^2$
- BFKL: typically same size partons
- Partons will “overlap”, coherent, non-linear evolution



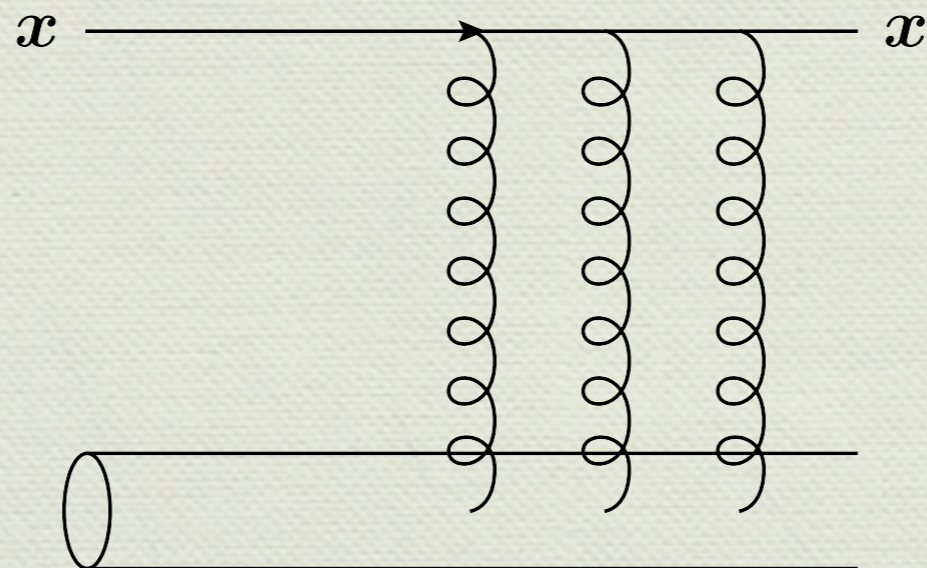
Single particle production

- Large- x quark from proton: eikonal trajectory
- Interacts with soft components of nucleus
- Quark “measured” in forward region



Wilson lines

- Mixed representation: transverse momenta \rightarrow coordinates
- Nucleus viewed as large classical color field
- Eikonal interaction \rightarrow Wilson lines: $V_x^\dagger = P \exp \left[i g \int dx^- t^a \mathcal{A}_x^{+a}(x^-) \right]$



The cross section

- Multiply by c.c. and F.T. to calculate cross section

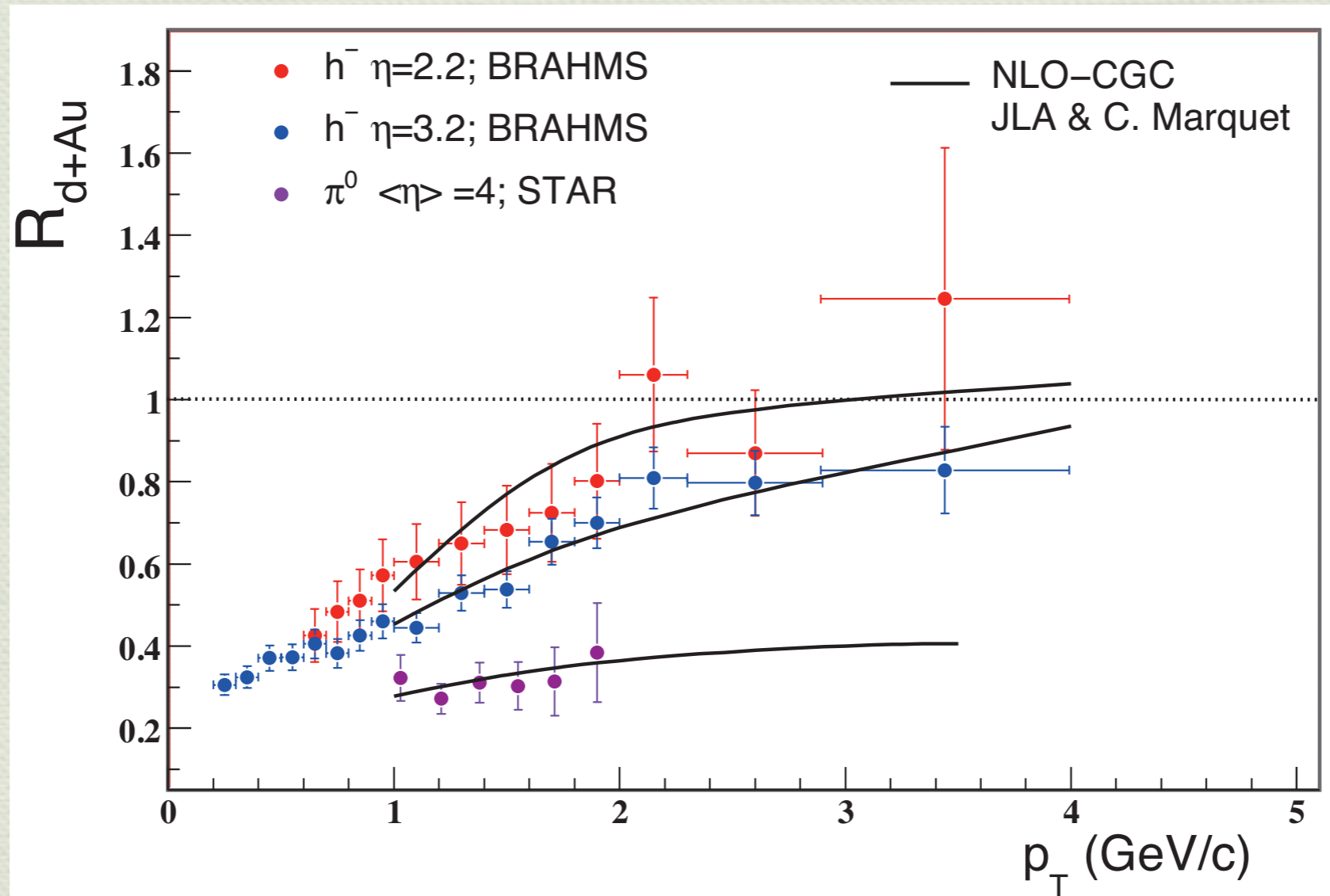
$$\frac{d\sigma^{qA \rightarrow qX}}{dyd^2\mathbf{p}} = x_1 f_q(x_1, \mathbf{p}^2) \int_{\mathbf{r}} e^{-i\mathbf{r} \cdot \mathbf{p}} \frac{1}{N_c} \left\langle \text{tr}[V(\mathbf{x})V^\dagger(\mathbf{y})] + \dots \right\rangle_Y$$

- Summing over final color, average initial: $1/N_c \text{tr} \dots$

- QCD dynamics in $\langle \dots \rangle_Y = \int D\mathcal{A}^+ W_Y[\mathcal{A}^+] \dots$

- $e^{-Y} = x_2 = \frac{|\mathbf{p}|e^{-y}}{\sqrt{s}} \ll 1$ in forward region, $x_1 = \frac{|\mathbf{p}|e^y}{\sqrt{s}}$

Comparing with data



$$R_{pA}(\eta, \mathbf{p}) \equiv \frac{1}{A^{1/3}} \frac{dN_h/d\eta d^2\mathbf{p}|_{pA}}{dN_h/d\eta d^2\mathbf{p}|_{pp}}$$

JIMWLK evolution

- Evolution for functional of color sources

$$\frac{\partial W_Y[\rho]}{\partial Y} = H_{\text{JIMWLK}} W_Y[\rho]$$

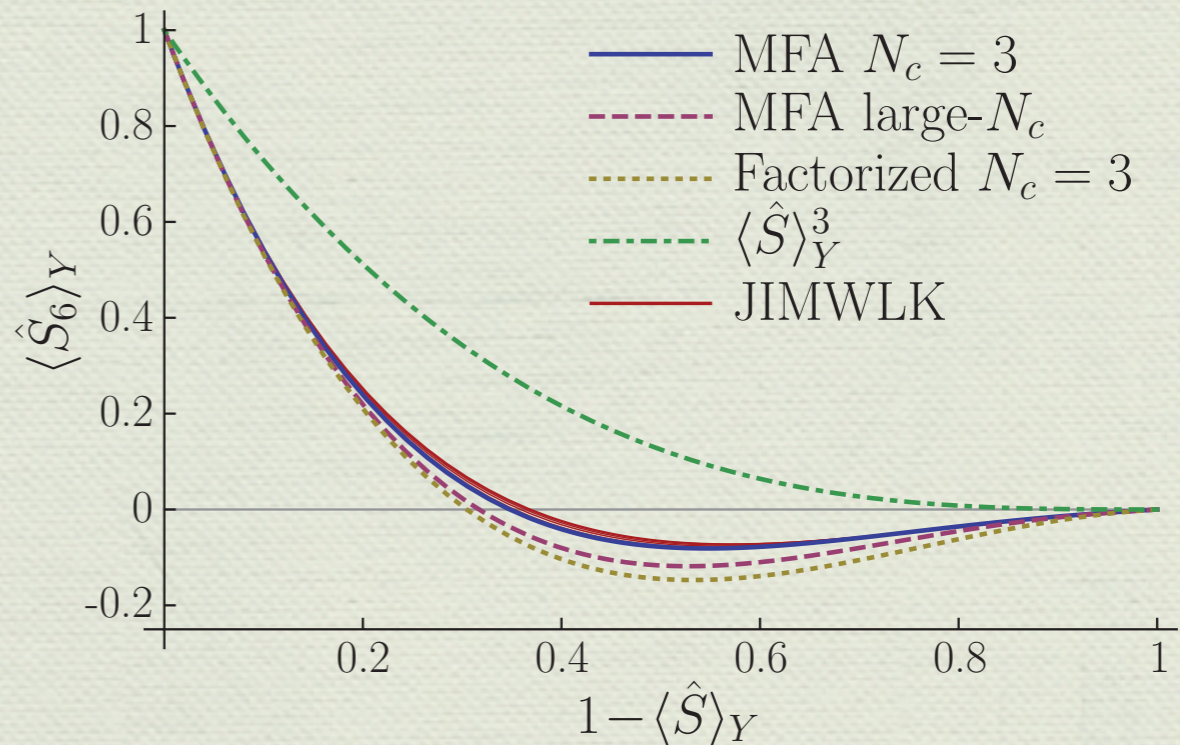
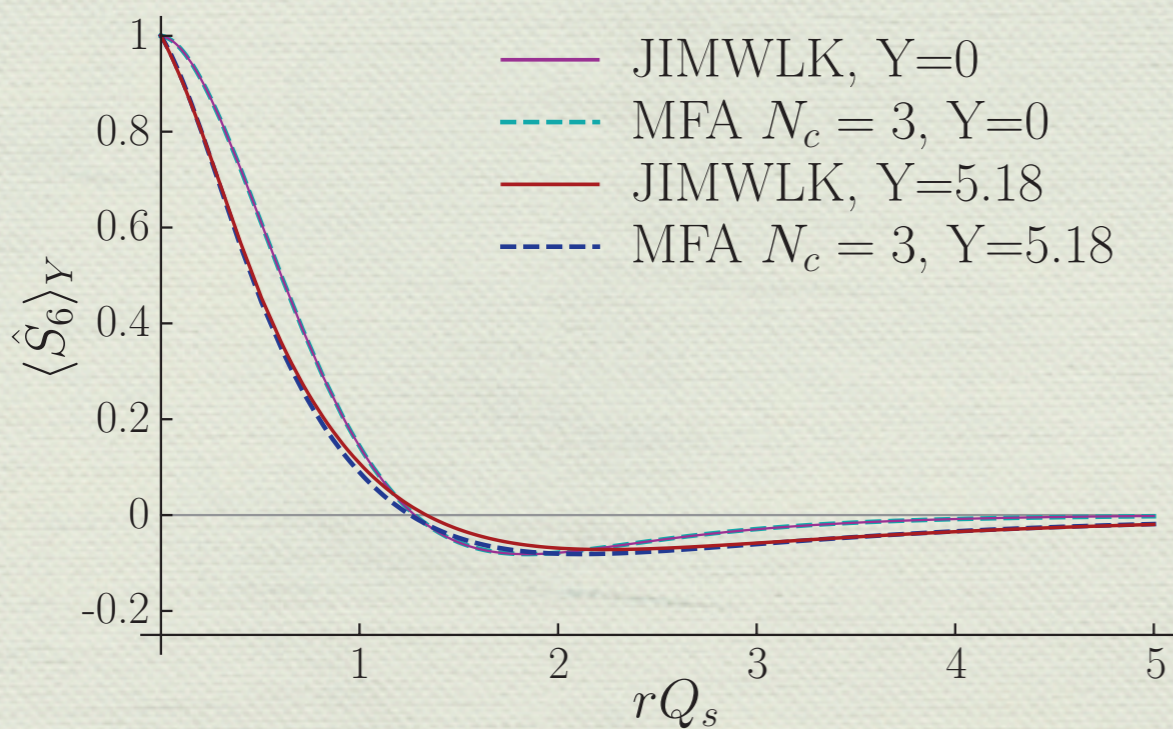
- Average over color sources

$$\langle \mathcal{O}[A] \rangle_Y = \int \mathcal{D}\rho W_Y[\rho] \mathcal{O}[A(\rho)]$$

- Solve classical Yang-Mills equations to get the field

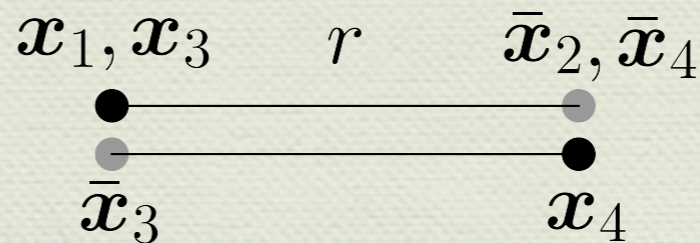
$$D_\nu^{ab} F_b^{\nu\mu} = \delta^{\mu+} \rho^a(x^-, \mathbf{x}_\perp)$$

vs numerical solution



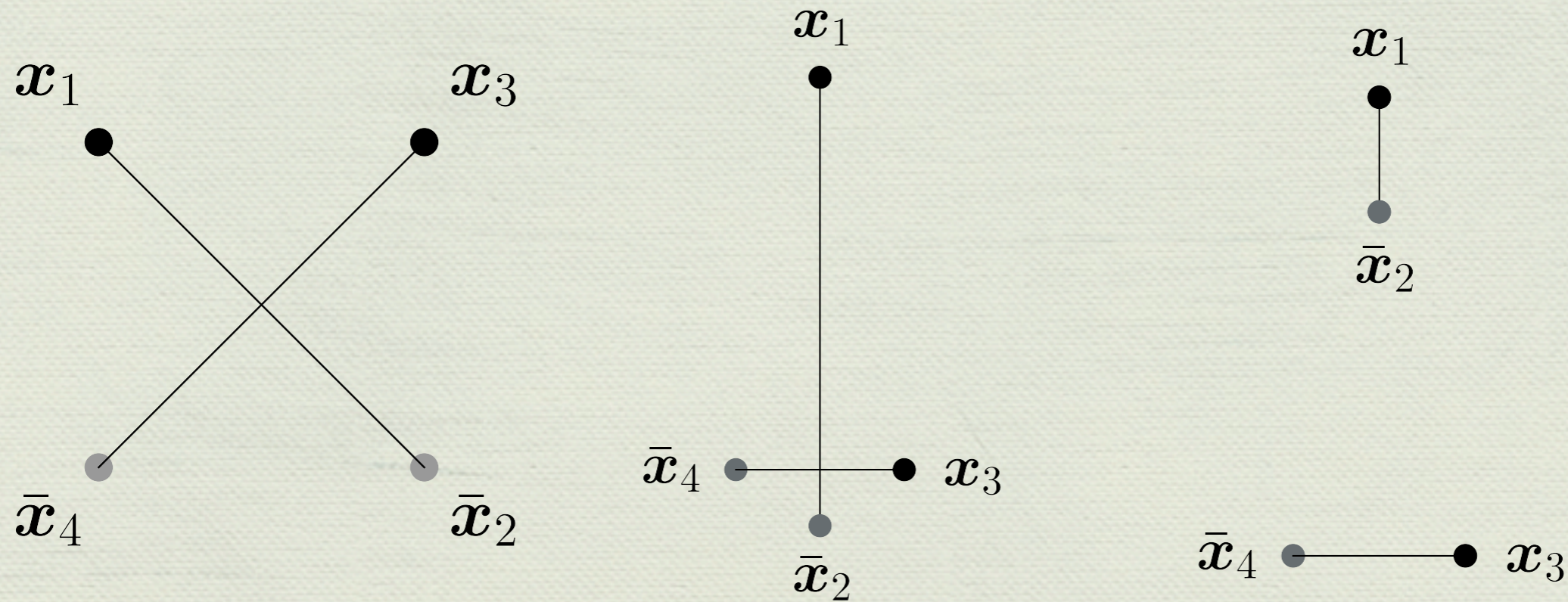
□ 6-point function $\hat{S}_6 \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_3 \mathbf{x}_4 = \frac{N_c^2}{N_c^2 - 1} \hat{Q}_{1234} \hat{S}_{43} - \frac{1}{N_c^2 - 1} \hat{S}_{12}$

□ Line configuration:



$$\langle \hat{Q} \hat{S} \rangle_Y = \frac{(N_c + 2)(N_c - 1)}{2N_c} \langle \hat{S} \rangle_Y^{\frac{3N_c - 1}{N_c - 1}} - \frac{(N_c + 1)(N_c - 2)}{2N_c} \langle \hat{S} \rangle_Y^{\frac{3N_c + 1}{N_c + 1}}$$

Other special configurations



$$\langle \hat{Q}_{1234} \rangle_Y = \langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{34} \rangle_Y$$

□ Factorization violations at finite N_c (at saturation)

$$\left\langle \hat{S}_{13} \hat{S}_{32} - \frac{1}{N_c^2} \hat{S}_{12} \right\rangle_Y = \frac{N_c^2 - 1}{N_c^2} \left[\frac{\langle \hat{S}_{13} \rangle_Y \langle \hat{S}_{32} \rangle_Y}{\langle \hat{S}_{12} \rangle_Y} \right]^{\frac{1}{(N_c^2 - 1)}} \langle \hat{S}_{13} \rangle_Y \langle \hat{S}_{32} \rangle_Y$$

The right-derivatives

- Functional derivatives act on upper endpoint: L-derivative

$$\frac{\delta}{\delta\alpha_u^a} V_x^\dagger = ig\delta_{xu} t^a V_x^\dagger$$

- Adjoint Wilson lines transform to R-derivatives: lower endpoint

$$(\tilde{V}^\dagger)^{ac} \frac{\delta}{\delta\alpha_u^a} V_x^\dagger = ig\delta_{xu} V_x^\dagger t^c$$

- Wilson lines expand symmetrically in longitudinal direction

$$V_{n+1}^\dagger(\mathbf{x}) = \exp[ig\epsilon\alpha_{n+1}(\mathbf{x})] V_n^\dagger(\mathbf{x}) \exp[ig\epsilon\alpha_{-(n+1)}(\mathbf{x})]$$

An even simpler expression

- Deep at saturation solve for dipole (fixed coupling)

$$\langle \hat{S}_{ij} \rangle_Y \simeq \exp \left[-\frac{1}{2\omega} \ln^2(r_{ij}^2 Q_s^2) \right]$$

- Possible to integrate over y

$$\begin{aligned} \langle \hat{Q}_{1234} \rangle_Y &= \frac{\ln[\langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{34} \rangle_Y / \langle \hat{S}_{13} \rangle_Y \langle \hat{S}_{24} \rangle_Y]}{\ln[\langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{34} \rangle_Y / \langle \hat{S}_{14} \rangle_Y \langle \hat{S}_{23} \rangle_Y]} \langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{34} \rangle_Y \\ &+ \frac{\ln[\langle \hat{S}_{14} \rangle_Y \langle \hat{S}_{23} \rangle_Y / \langle \hat{S}_{13} \rangle_Y \langle \hat{S}_{24} \rangle_Y]}{\ln[\langle \hat{S}_{14} \rangle_Y \langle \hat{S}_{23} \rangle_Y / \langle \hat{S}_{12} \rangle_Y \langle \hat{S}_{34} \rangle_Y]} \langle \hat{S}_{14} \rangle_Y \langle \hat{S}_{23} \rangle_Y \end{aligned}$$

Jalilian-Marian, Kovchegov '04

Dominguez, Marquet, Xiao, Yuan '11

- Still correct for small T , symmetric under exchange of 2 and 4

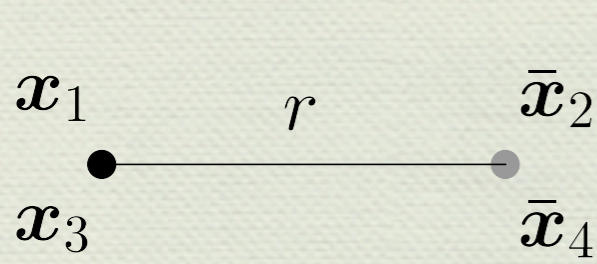
The Gaussian approximation

- Expression for Q first derived in MV model = special Gaussian

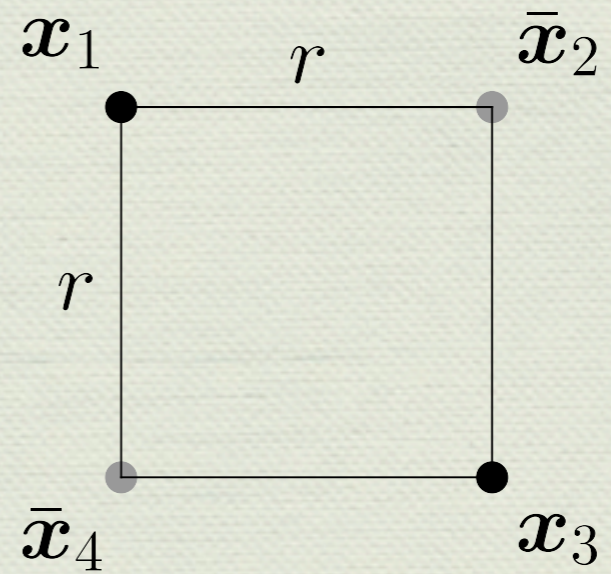
$$\langle \rho_a(x_1^-, \mathbf{x}_1) \rho_b(x_2^-, \mathbf{x}_2) \rangle = \delta_{ab} \Theta(x_0^- - |x_1^-|) \delta(x_1^- - x_2^-) \delta_{\mathbf{x}_1 \mathbf{x}_2} \lambda(x_1^-, \mathbf{x}_1)$$

- Virtual terms arise from Gaussian part of H
- All correlators in terms of 2-point function \rightarrow Gaussian kernel
- Valid at finite N_c
- At saturation drop last two terms of gluon emission
- At weak scattering only 2-point function and products

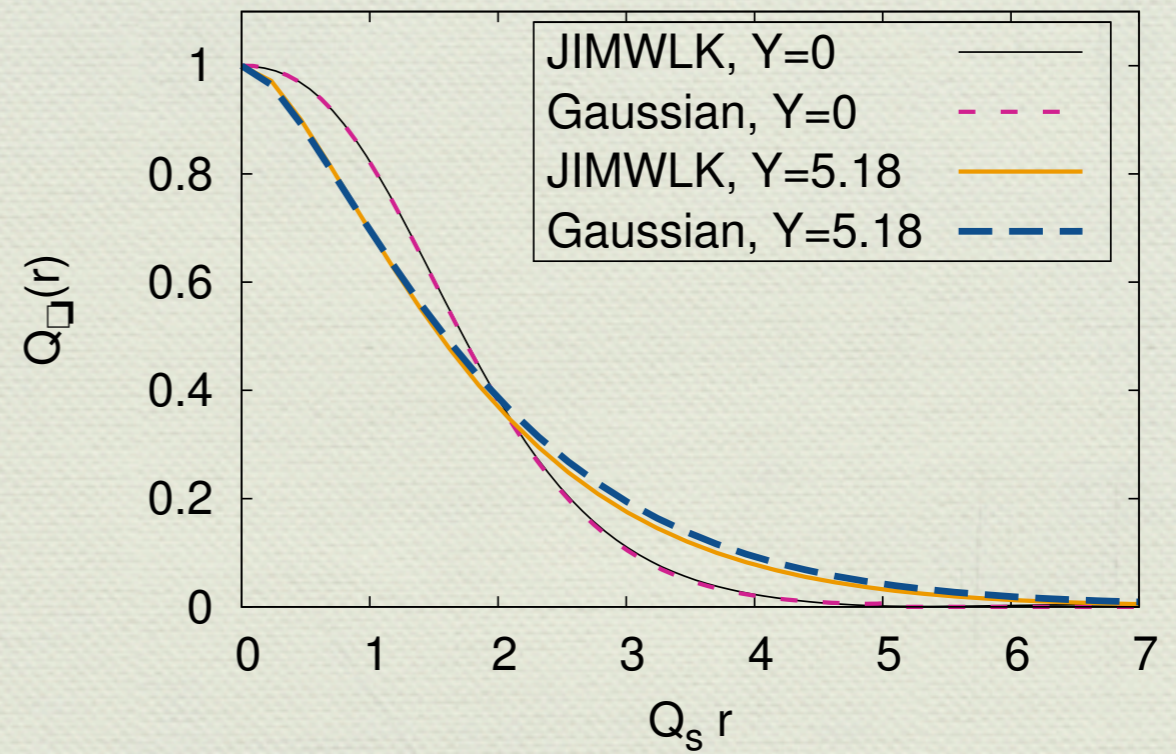
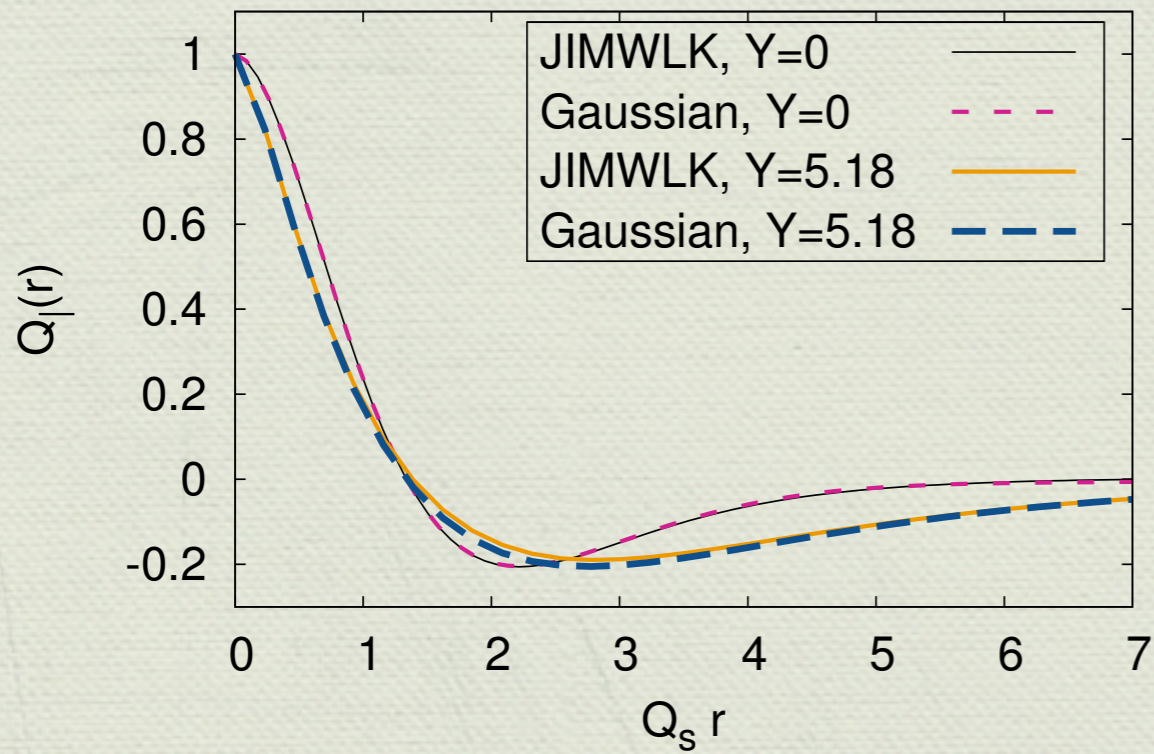
vs numerical solution



(a)



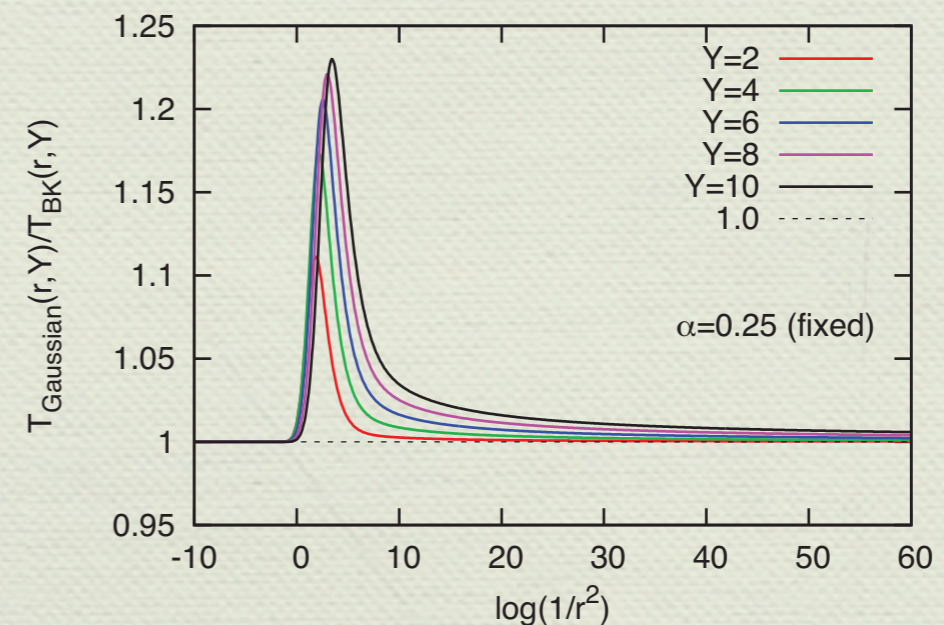
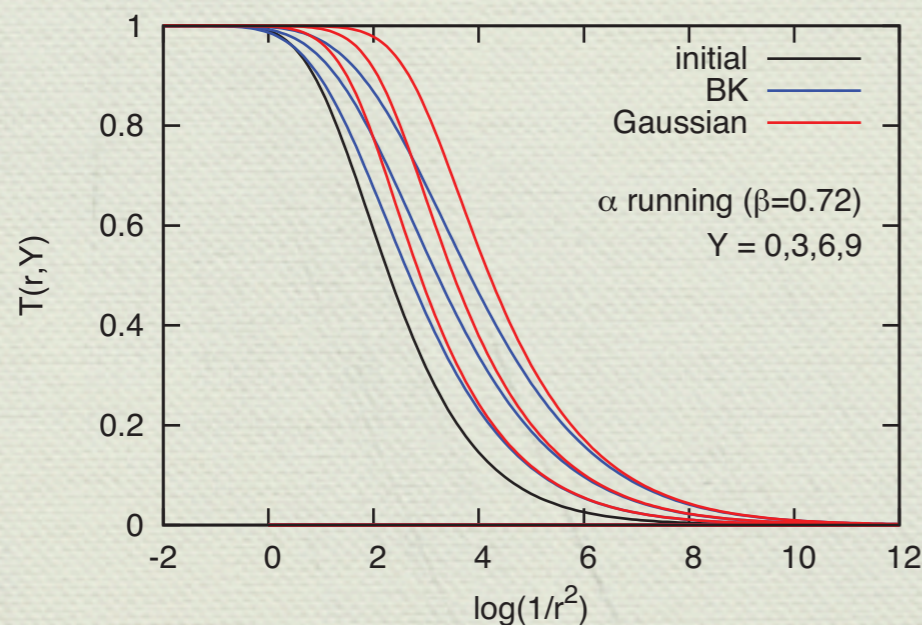
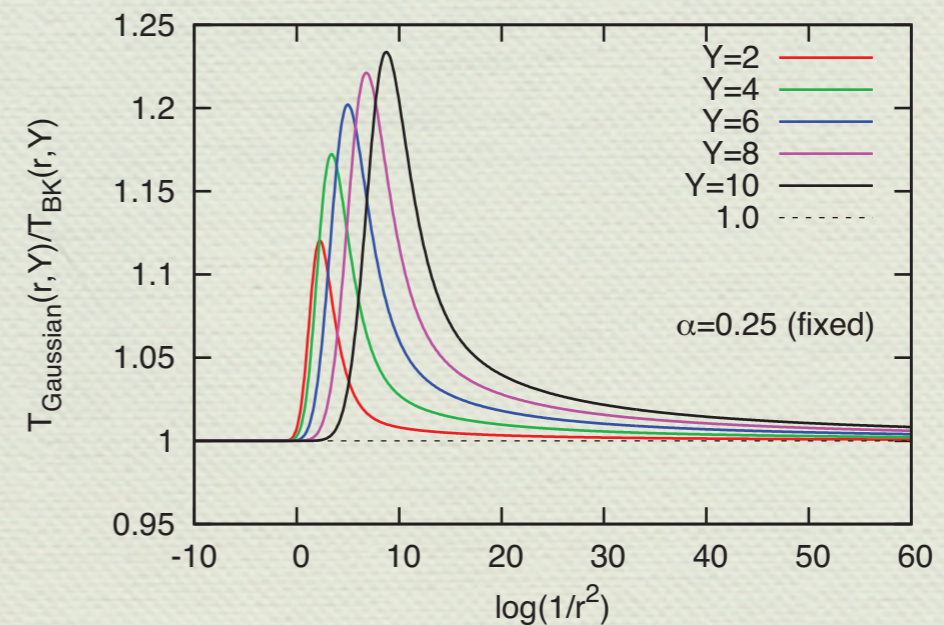
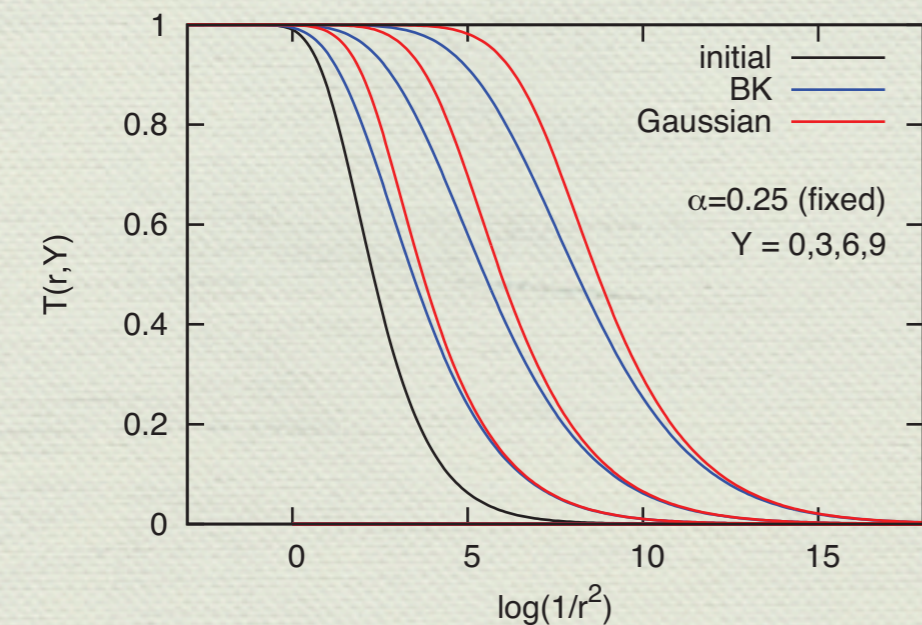
(b)



“Internal check”

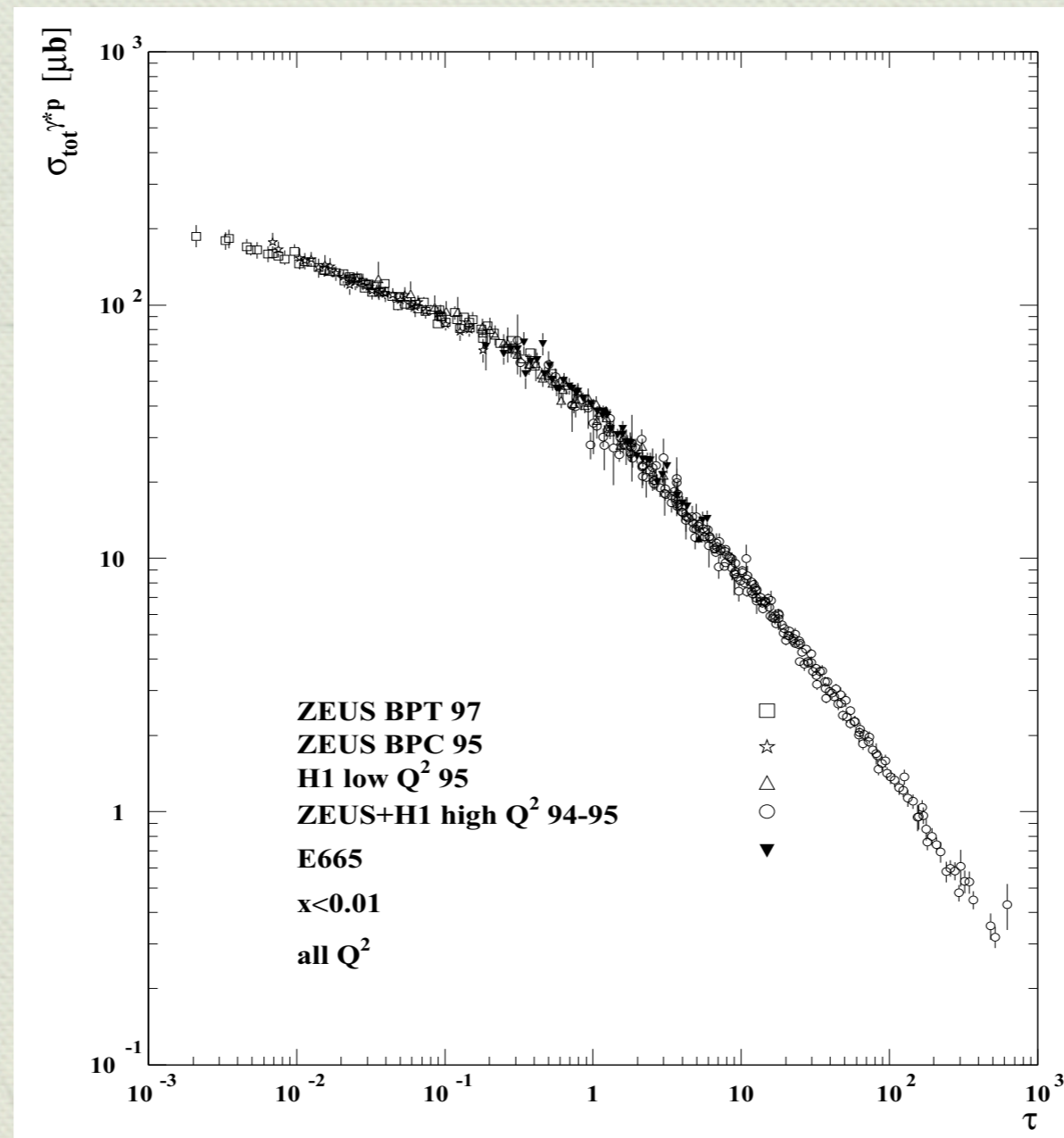
□ At large N_c for example:

Solve BK and MFA from averaging JIMWLK kernel



Scaling

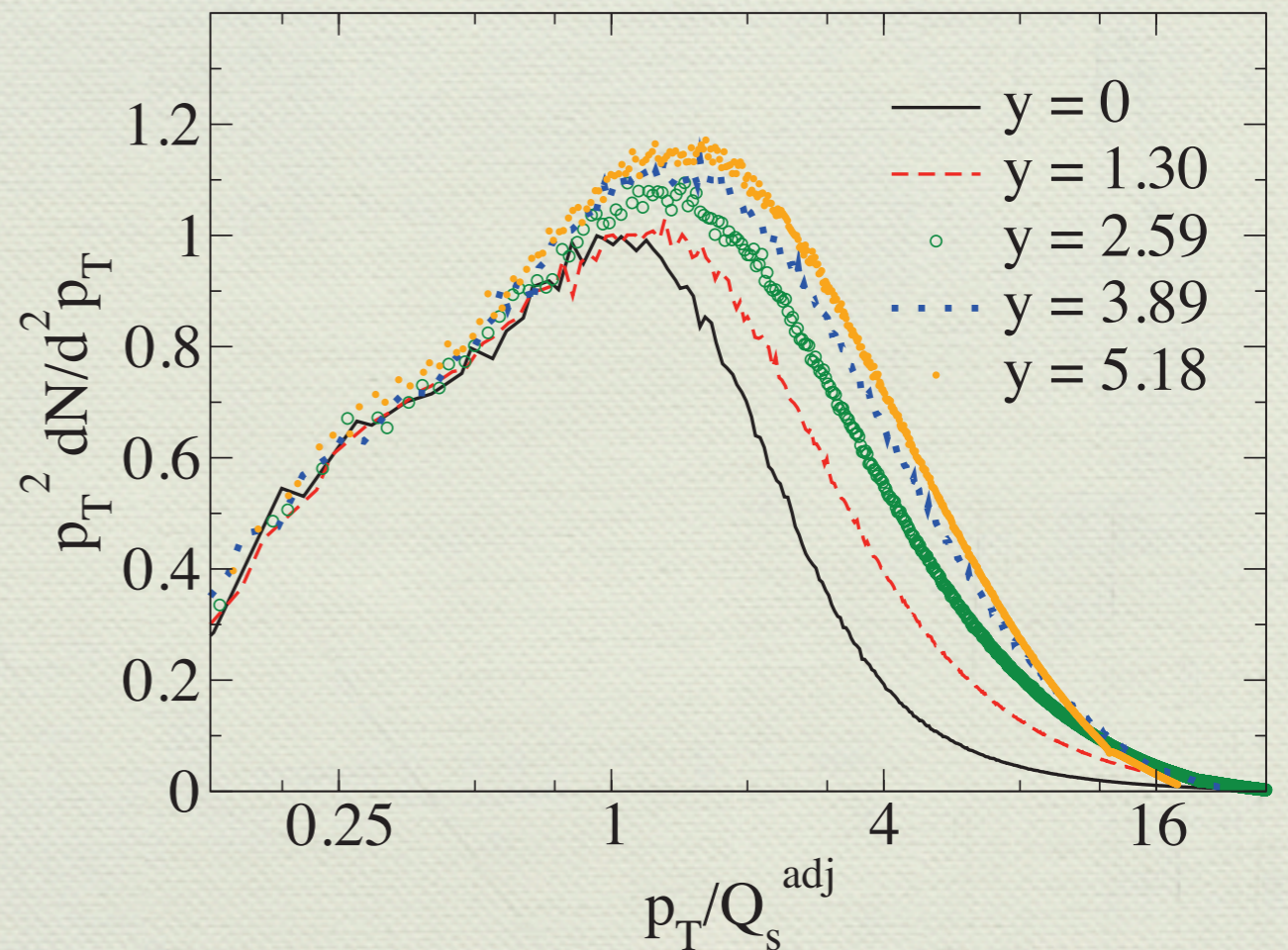
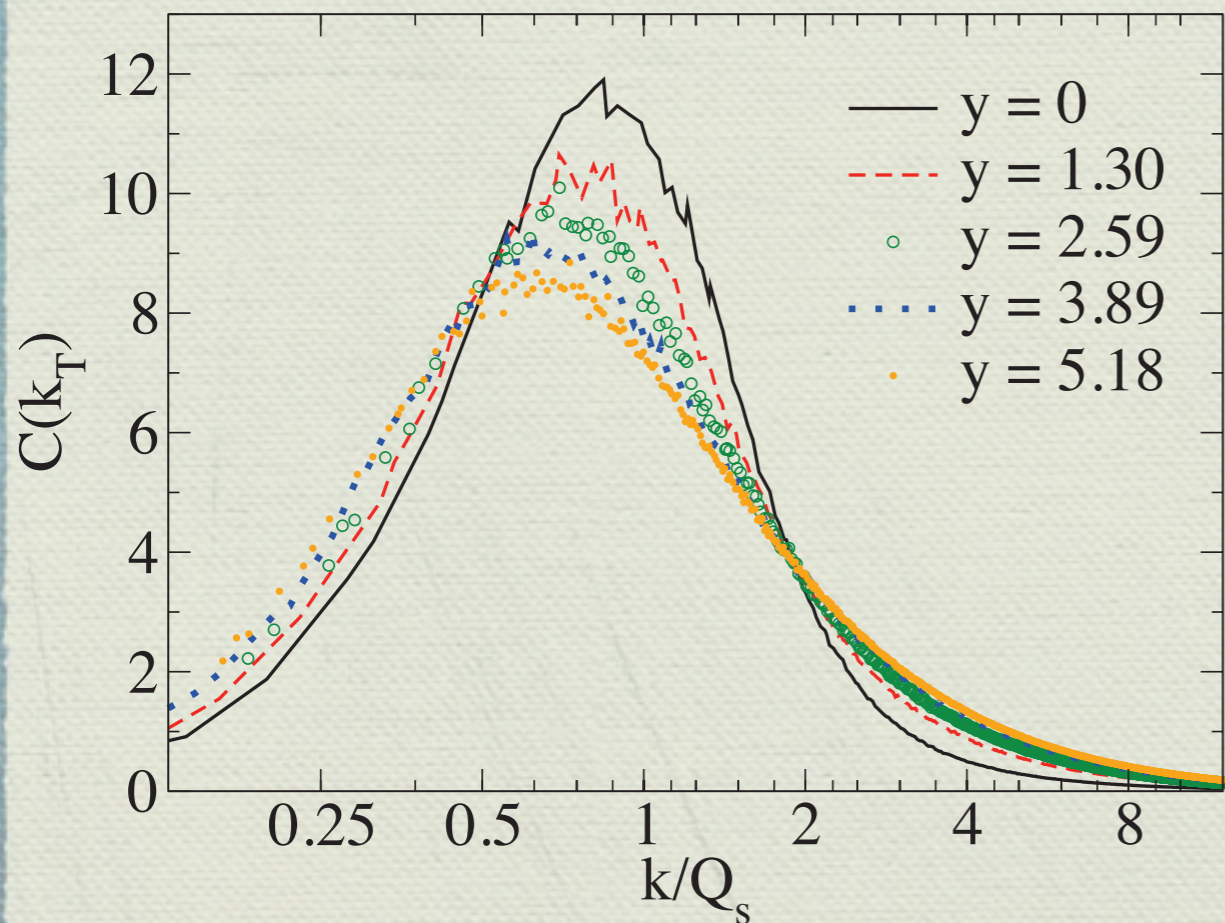
- $Q_s(Y)$ sets the scale: dependence only on $rQ_s(Y)$ around Q_s



Gluon distribution and gluon production in AA

□ Distribution
$$C(Y, \mathbf{p}) = \mathbf{p}^2 \int_r e^{-i\mathbf{r}\cdot\mathbf{p}} \frac{1}{N_c} \left\langle \text{tr}[V(\mathbf{x})V^\dagger(\mathbf{y})] + \dots \right\rangle_Y$$

□ Production: collide two “color glasses”



Multiplicities in AA

- Multiplicities should not be affected by final state

