

Towards Exclusive HVM Production at NLO

$$\sigma(\gamma p \rightarrow p + J/\psi) \text{ \& \ } d\sigma/dy(pp \rightarrow p + J/\psi + p)$$

UNIVERSITY OF
LIVERPOOL

Stephen Jones

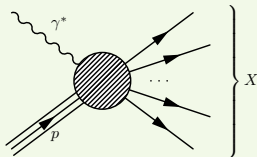
SJ, A. Martin, M. Ryskin, T. Teubner
JHEP **1311** (2013) 085, arXiv:1307.7099 [hep-ph]
JPG **41** (2014) 055009, arXiv:1312.6795 [hep-ph]

Workshop on Photon-induced
Collisions at the LHC

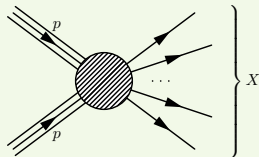
2nd June

Forward physics: Exclusive & Diffractive Processes

Inclusive Processes - Included in global analyses

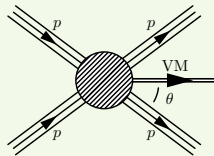
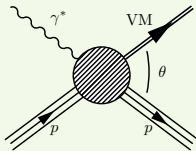


DIS



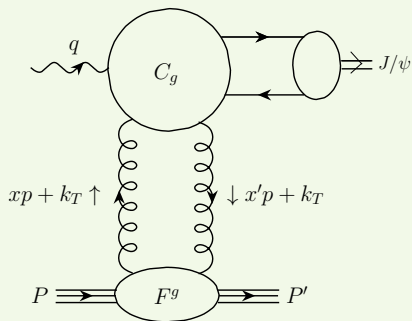
Inclusive pp Scattering

Exclusive & Diffractive Processes



Exclusive HVM Electroproduction Ultrapерipheral HVM production

J/ψ Photo(electro)-production



General Setup & Assumptions

- Factorises: $\phi_{c\bar{c}}^\gamma \otimes T_{c\bar{c}+p} \otimes \phi_{c\bar{c}}^{J/\psi}$
- Non-relativistic J/ψ wave function $\propto \Gamma_{ee}$
- 'Maximal skew': $x' = 0$, outgoing gluon carries only k_T^μ
- Compute Im, restore Re via dispersion relation
- $Q^2 = -q^2$ - photon virtuality
- W - CM energy of $\gamma^* p$
- $x = (Q^2 + M_{J/\psi}^2)/(W^2 + Q^2)$
- Scale: $\bar{Q}^2 = (Q^2 + M_{J/\psi}^2)/4$

Amplitude (k_T Factorisation)

$$A \propto \int \frac{dk_T^2}{2k_T^4} \left(\frac{1}{\bar{Q}^2} - \frac{1}{\bar{Q}^2 + k_T^2} \right) \alpha_s(k_T^2) f(x, k_T^2, \mu^2)$$

LO Result (LLA, Im Part)

- Leading log approximation (LLA): $k_T^2 \ll \bar{Q}^2$
- $\int dk_T^2/k_T^2 f(x, k_T^2, \mu^2) \rightarrow xg(x, \mu^2)$

$$\left. \frac{d\sigma}{dt}(\gamma^* p) \right|_{t=0} = \frac{\Gamma_{ee} M_{J/\psi}^3 \pi^3}{48\alpha} \frac{\alpha_s(\bar{Q}^2)^2}{\bar{Q}^8} [R_g xg(x, \bar{Q}^2)]^2 \left(1 + \frac{Q^2}{M_{J/\psi}^2} \right)$$

[Ryskin 1993]

- R_g - skewing factor [Shuvaev et. al 1999]

(Mandelstam) t dependence

- Assume form $\exp(-bt)$, fit slope parameter b from experiment
- $b(W) = (4.9) + 4(0.06) \ln(W/90 \text{ GeV}) \text{ GeV}^{-2}$

Beyond LO Result

- Above IR scale $Q_0^2 = 1 \text{ GeV}^2$ up to kinematic upper bound perform explicit k_T^2 integration in the last step of the evolution

$$\text{Im}A \sim \text{IR Part} + \int_{Q_0^2}^{(W^2 - M_{J/\psi}^2)/4} \frac{dk_T^2}{k_T^2} \frac{\alpha_s(\mu^2)}{\bar{Q}^2(\bar{Q}^2 + k_T^2)} f(x, k_T^2, \mu^2)$$

- Use 'Unintegrated' PDF

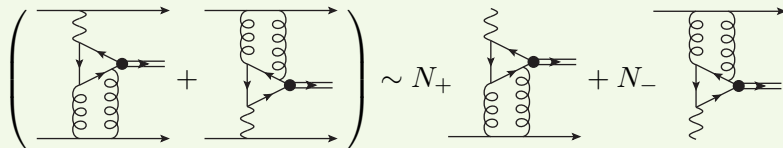
$$f(x, k_T^2, \mu^2) = \frac{\partial [R_g xg(x, k_T^2) \sqrt{T(k_T^2, \mu^2)}]}{\partial \ln k_T^2}$$

- T - Sudakov factor (hard gluon emits no additional partons)
- Disclaimer:** Not an NLO matrix element!

Ultraperipheral Production (pp)

- Ultraperipheral cross-section vs rapidity y receives contributions from two γp CM energies: $(W_{\pm})^2 = M_{J/\psi}^2 \sqrt{s} \exp(\pm|y|)$

$$\frac{d\sigma(pp)}{dy} = S_+^2 N_+ \sigma_+(\gamma p) + S_-^2 N_- \sigma_-(\gamma p)$$



- $N_{\pm} = k_{\pm} (dn/dk)_{\pm}$ - photon flux (EPA)
- S_{\pm}^2 - gap survival factors (KMR Model) [Khoze et al. 2002]

Extracting Small- x Gluon

- Try two different ansätze for small- x gluon

LO Approach

- Power law: $xg(x, \mu^2) = Nx^{-\lambda}$ with $\lambda = a + b \ln(\mu^2/0.45\text{GeV}^2)$

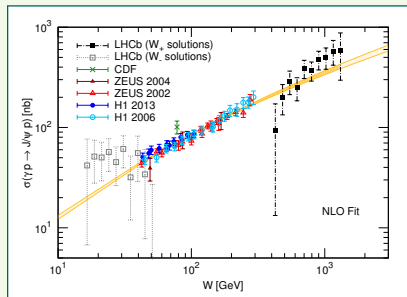
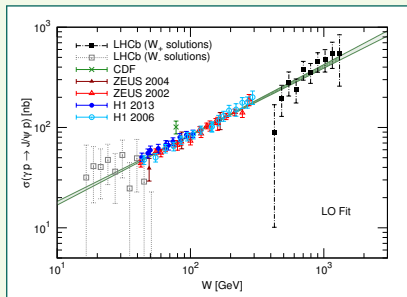
Beyond 'LO' Approach

- Resum leading $(\alpha_s \ln(1/x) \ln \mu^2)^n$ contributions
- $xg(x, \mu^2) = Nx^{-a}(\mu^2)^b \exp \left[\sqrt{16N_c/\beta_0 \ln(1/x) \ln(G)} \right]$
- $G = \ln(\mu^2/\Lambda_{\text{QCD}}^2) / \ln(Q_0^2/\Lambda_{\text{QCD}}^2)$ with $\Lambda_{\text{QCD}} = 200 \text{ MeV}$

Fitting Procedure

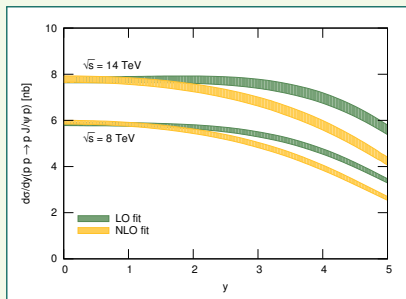
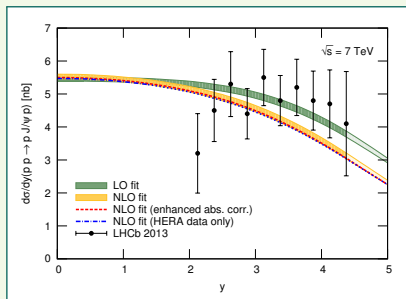
- Non-linear χ^2 fit to data
- Obtain best fit N, a, b and full covariance matrix for error estimate

J/ψ Fit HERA & LHCb 2013



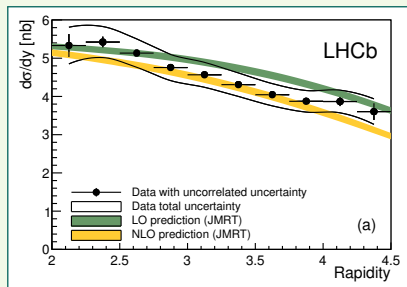
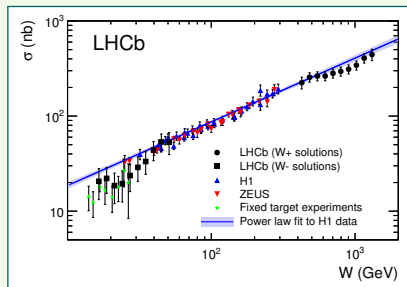
- Extends to $x \sim 10^{-6}$
- LHCb W_{\pm} points calculated with our S^2 , dn/dk , $\sigma(\gamma p)$
- **Recall:** LO and 'NLO' parameters have different meaning

	LO	'NLO'
N	1.20	0.29
a	0.05	-0.10
b	0.08	-0.20
$\chi_{d.o.f}^2$	0.5	0.6



- Can use fitted gluon for $d\sigma(pp)/dy$ prediction (stable at 14 TeV)
- W_- component accounts for $\mathcal{O}(30 - 40\%)$ of $d\sigma(pp)/dy$
- HERA fit is consistent with LHCb 2013 data (LHCb not constraining)

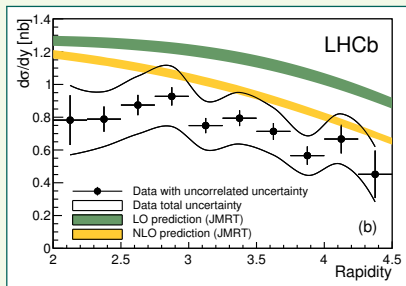
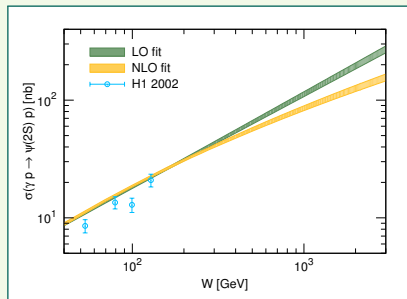
J/ψ Prediction vs LHCb 2014



- 2014 data ($\sqrt{s} = 7$ TeV) \sim halves error on integrated σ [LHCb 2014]
- Still consistent with HERA power law
- Good agreement with predictions without refitting (not quite distinguishing)

$\Psi(2S)$ Prediction vs LHCb 2014

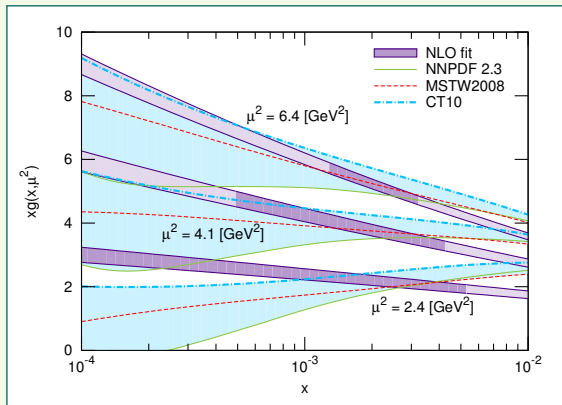
- Use gluon fit extracted from J/ψ data
- Γ_{ee} , M , S^2 for $\psi(2S)$
- $\sigma(\gamma p)$ related to $d\sigma(pp)/dy$ as for J/ψ

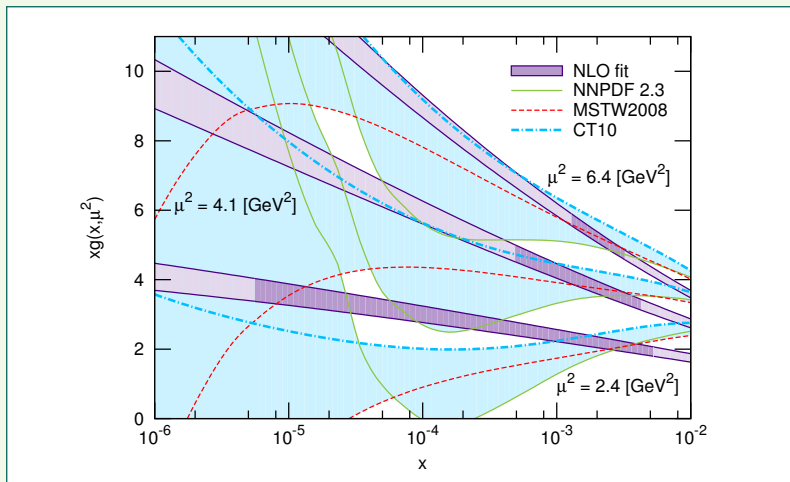


- Expect bigger relativistic corrections for $\psi(2S)$ than J/ψ
 - Some relativistic corrections accounted for in Γ_{ee}
 - Further corrections not accounted for

Gluon

- Scale dependence from k_T^2 integral and HERA electroproduction data
- Fitted gluons below global partons for higher $x \sim 10^{-2}$
- J/ψ data diminish the huge uncertainty on global gluons at low scale & small- x

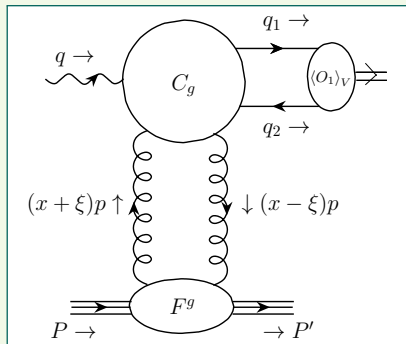
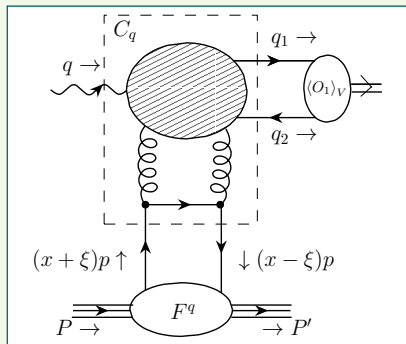




- LHCb data provides support for fit down to $x \sim 10^{-6}$
- 14 TeV data will probe even lower x

Towards full NLO Matrix Element Collinear factorisation + NRQCD framework

Quark & Gluon Contribution

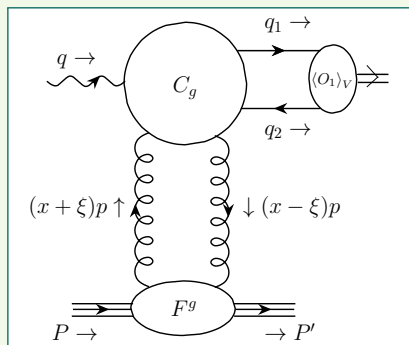


[Fig:Ivanov et al. 2004]

- Collinear factorisation (omitting μ_F and t): [Ji 1997, Collins et al. 1998]

$$A \propto \int_{-1}^1 dx \left[C_g \left(\frac{x}{x_B}, \frac{\xi}{x_B} \right) F^g(x, \xi) + C_q \left(\frac{x}{x_B}, \frac{\xi}{x_B} \right) F^q(x, \xi) \right]$$

- Set-up identical to [Ivanov et al. 2004]
 - Computed NLO matrix element for photoproduction using dispersive approach



- Our aim: check this result with standard integral reduction
 - $\langle O_1 \rangle_V$ - NRQCD long distance matrix element
 - Heavy quarks may have relative momenta, $r^\mu = q_1^\mu - q_2^\mu$
- $F^g(x, \xi, t, \mu_F)$ - Generalised Parton Distribution (GPD)
 - Related $\mathcal{O}(\xi)$ to PDFs via Shuvaev Transform [Shuvaev et. al 1999]
 - Full dependence on momentum fraction ξ and skew x

NRQCD (HVM Formation)

- Effective field theory for production of heavy quarkonium [Bodwin et al. 1995]

$$\sigma_V = \sigma_{q\bar{q}} \cdot \langle O \rangle_V$$

- Relativistic corrections systematically computed by expanding matrix elements in powers of v :

$$\mathcal{M}[J/\psi] \propto (\mathcal{A}_\rho + \mathcal{B}_{\rho\sigma} r^\sigma + \mathcal{C}_{\rho\sigma\tau} r^\sigma r^\tau + \dots) \epsilon_{J/\psi}^\rho$$

$\mathcal{A}, \mathcal{B}, \mathcal{C}$ - matrix elements $\epsilon_{J/\psi}^\rho$ - J/ψ polarization

- We will compute to leading order in relative quark velocity v , for J/ψ :

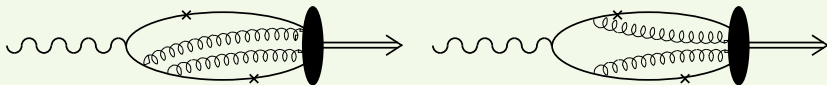
$$\mathcal{M}[J/\psi] = \left(\frac{\langle O_1 \rangle_{J/\psi}}{2N_c m_C} \right)^{\frac{1}{2}} \mathcal{A}_\rho \epsilon_{J/\psi}^\rho$$

- Compute $\Gamma_{ee} \propto \langle O_1 \rangle_{J/\psi}$
 - Extract $\langle O_1 \rangle_{J/\psi}$ from measurement of Γ_{ee}
 - Justification for using Γ_{ee} in k_T factorisation model

Relativistic Effects: J/ψ Formation

Hoodbhoy Study [Hoodbhoy 1997]

- Accounts for Fermi motion of the $c\bar{c}$ pair
 - Work in J/ψ rest frame using Coulomb gauge
 - Expand in powers of heavy quark relative velocity upto $\mathcal{O}(v^2)$
 - Necessary to include extra gluon fields to maintain gauge invariance
 - Procedure accounts for largest contribution from Fermi motion



Result

- Correction factor due to Fermi motion ≈ 0.94 (for cross-section)

NLO Matrix Element

Linearly dependent momenta

- GPDs: $(x + \xi)p^\mu \propto (x - \xi)p^\mu$
- Non-relativistic limit: $q_1^\mu = q_2^\mu$

$$I^{\mu_1 \dots \mu_m} = \int [dk] \frac{k_1^{\mu_1} \dots k_m^{\mu_m}}{N_0 \dots N_n}$$

1. Tensor (Integral) Decomposition

- Perform Sudakov decomposition:

$$q^\mu = (q \cdot n)p^\mu + (q \cdot p)n^\mu + q_T^\mu$$

$$p^2 = n^2 = p \cdot q_T = n \cdot q_T = 0 \text{ and } p \cdot n = 1$$

- Kinematics: all q_T^μ vanish
- Choose basis constructed from just p^μ , n^ν and $\eta^{\mu\nu}$

$$I^{\mu\nu} = \eta^{\mu\nu} I_{00} + p^\mu p^\nu I_{11} + p^\mu n^\nu I_{12} + n^\mu p^\nu I_{21} + n^\mu n^\nu I_{22}$$

2. Integral Reduction

- Linearly dependent momenta \Rightarrow relation between propagators N_i
- Example:
 - Suppose $\sum_{i=0}^3 a_i N_i = 1$, $a_i \in \mathbb{R} \setminus \{0\}$

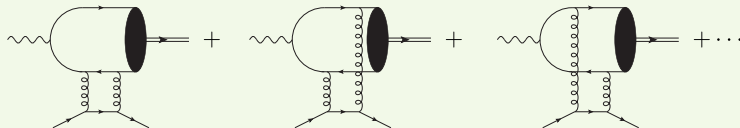
$$\frac{N_2}{N_0 N_1} = \frac{1}{a_2} \frac{1}{N_0 N_1} - \frac{a_0}{a_2} \frac{1}{N_1} - \frac{a_1}{a_2} \frac{1}{N_0}$$

- Retains initial basis of propagators
- Reduces N -point integrals to $(N - 1)$ -point
- Finally: Apply Laporta Algorithm, all integrals reduced to masters

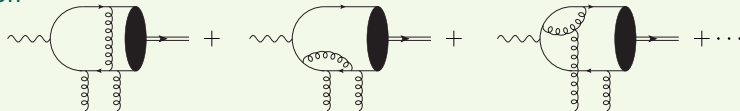
3. Master Integrals

- Known at 1-loop
 - Analytic: ['t Hooft, Veltman 1979] [Ellis et al. 2004]
 - Many numerical implementations: QCDLoop, LoopTools, ...

Quark



Gluon



Diagrams	Tree	1-Loop
Quark	0	6
Gluon	6	81

- Quark NLO complete!
 - Agrees with Ivanov et al.
- Gluon NLO underway

Conclusion

- In k_T factorisation:
 - Updated MNRT gluon fit to include recent LHCb data
[[Martin et al. 2008](#)]
 - Provided predictions for J/ψ , $\psi(2S)$ ultraperipheral production
 $\sqrt{s} = 7, 8 \text{ \& } 14 \text{ TeV}$
 - Not a complete NLO analysis (main kinematic effects included)
 - Considerable scale uncertainty remains
 - Can not directly identify extracted gluon with e.g. $\overline{\text{MS}}$ partons
- In Collinear factorisation:
 - Work under way to include NLO gluon diagrams
 - Better estimation of scale & check of scale variation
 - Clear scheme choice ($\overline{\text{MS}}$)
 - Can use same techniques to compute electroproduction

Extra

Scale Uncertainty

Observations from MNRT Study

- Varying μ from $\mu/2$ to 2μ in Sudakov factor (T) gives small effect
 - Inclusion of T at all only slightly alters the behaviour of the gluon
 - Reason: T mostly contributes in k_T^2 integral where derivative is taken
- Varying μ in α_s causes strong scale dependence
- Varying scale in both α_s and T simultaneously from $\mu/2$ to 2μ gives large effect $\pm 20\%$
 - Even including scale variation still obtain better gluon certainty than global partons!
 - Most scale variation is absorbed into the normalisation of the gluon
 - x -behaviour reasonably stable

GPDs

- Off-forward $P \neq P'$ generalisation of parton distribution functions
- Nonperturbative contribution factored into density matrix

$$\begin{aligned} M_{\alpha\beta}^q &= \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle P' | \bar{\psi}_\beta \left(\frac{\lambda}{2} n \right) \mathcal{P}\{\} \psi_\alpha \left(-\frac{\lambda}{2} n \right) | P \rangle \\ &= \frac{1}{2} F^q(x, \xi) \not{x}_{\alpha\beta} + \dots \end{aligned}$$

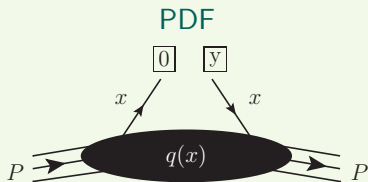
- Omitted higher-twist, terms relevant for polarized case

Universal

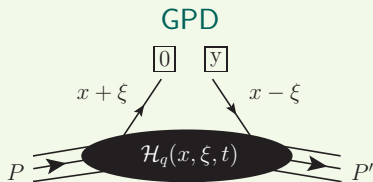
- Same distribution in other off-forward processes (eg... DVCS)

GPDs vs PDFs

- Unpolarized, neglecting nucleon helicity flip: $F^q = \sqrt{1 - \xi^2} \mathcal{H}_q$



$$\langle P | \bar{\psi}_q(y) \mathcal{P} \{ \} \psi_q(0) | P \rangle$$



$$\langle P' | \bar{\psi}_q(y) \mathcal{P} \{ \} \psi_q(0) | P \rangle$$

$$\mathcal{H}_q(x, 0) = q(x)$$

$$x > 0$$

$$\mathcal{H}_q(x, 0) = -\bar{q}(-x)$$

$$x < 0$$

Shuvaev Transform

- In small- x and ξ limit GPDs are related to PDFs via Shuvaev transform [Shuvaev et. al 1999]

Outline

- Anomalous dimensions of Gegenbauer moments, G_N of $\mathcal{H}(x, \xi)$, are equal to anomalous dimensions of conventional Mellin moments, $M_N = \int_0^1 x^N \mathcal{H}(x, 0) dx$
- Polynomiality: $G_N = \sum_{n=0}^N c_n^N \xi^{2n}$, allows all Gegenbauer moments to be determined $\mathcal{O}(\xi^2)$ from conventional PDFs ($c_0^N = M_N$)¹

Regge-based Assumption

- No singularities in the right half N -plane of diagonal small- x input distributions [Martin et al. 2009]

¹reduces to $\mathcal{O}(\xi)$ at NLO

Skewing & Real Part

- We use 'maximal skewing' limit ($\xi = x$), pure power PDF ($xg \sim x^{-\lambda}$) in transform which gives skewing factor

$$R_g = \frac{2^{2\lambda+3} \Gamma(\lambda + \frac{5}{2})}{\sqrt{\pi} \Gamma(\lambda + 4)}$$

- $\mathcal{O}(20 - 30\%)$ error on cross section compared to full transform
[Harland-Lang 2013]
- $\mathcal{O}(10 - 15\%)$ on gluon parameters

Real Part

- Real contribution included via dispersion relation assuming in the low x region $A \propto x^{-\lambda} + (-x)^{-\lambda}$ $\frac{\text{Re}A}{\text{Im}A} \simeq \frac{\pi}{2} \lambda$

IR Part and Scale Choices

IR Part

- For $k_T < Q_0$ assume linear behaviour of gluon at small k_T^2

$$xg(x, k_T^2) \sqrt{T(k_T^2, \mu^2)} = xg(x, Q_0^2) \sqrt{T(Q_0^2, \mu_{\text{IR}}^2)} k_T^2 / Q_0^2$$

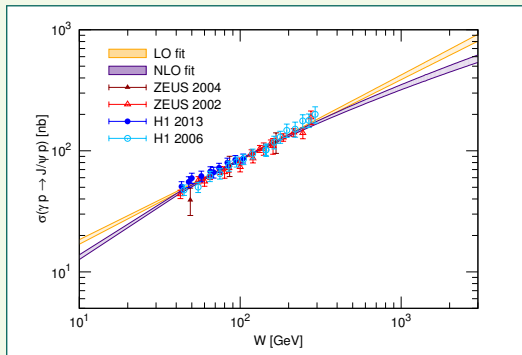
- Gives IR Part:

$$\ln \left(\frac{\bar{Q}^2 + Q_0^2}{\bar{Q}^2} \right) \frac{\alpha_s(\mu_{\text{IR}}^2)}{\bar{Q}^2 Q_0^2} xg(x, Q_0^2) \sqrt{T(Q_0^2, \mu_{\text{IR}}^2)}$$

Scale Choice

- Scale choice ambiguity remains (is extracted gluon e.g. $\overline{\text{MS}}$?)
- Choose $\mu^2 = \max(k_T^2, \bar{Q}^2)$ and $\mu_{\text{IR}}^2 = \max(Q_0^2, \bar{Q}^2)$
- Scale in IR Part matches lowest scale in integral
- Electroproduction typically contributes at higher scale
- $Q_0^2 = 1 \text{ GeV}^2$ (fit relatively insensitive to this)

HERA ($\gamma^* p \rightarrow J/\psi + p$)



Parameters

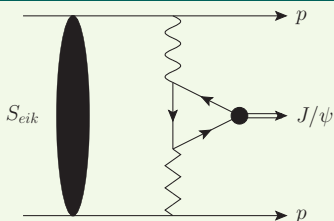
	LO	'NLO'
N	1.21	0.30
a	0.05	-0.11
b	0.08	-0.20
$\chi_{d.o.f}^2$	0.5	0.6

- Update to MNRT fit
[Martin et al. 2008]

- Probes gluon for $10^{-4} \lesssim x \lesssim 10^{-2}$
- **Note:** Electroproduction data included in fit (not shown here)

Survival factors

- For $pp \rightarrow p + J/\psi + p$ non-negligible interactions between spectator quarks
- Can populate rapidity gap
- Event not selected



KMR Model

$$S^2 = \langle S^2(b_t) \rangle = \frac{\int \sum_i |\mathcal{M}_i(s, b_t^2)|^2 \exp[-\Omega_i(s, b_t^2)] d^2b_t}{\int \sum_i |\mathcal{M}_i(s, b_t^2)|^2 d^2b_t}$$

- \mathcal{M}_i - process dependent matrix elements
- b_t - impact parameter, Ω_i - 'universal' proton opacities

[Khoze et al. 2002] [Khoze et al. 2013]

KMR Model

- Fitted to diffractive pp and $p\bar{p}$ data:
 - σ_{tot} - Total cross section ($\sigma_{\text{el}} + \sigma_{\text{inel}}$)
 - $d\sigma/dt$ - Elastic cross section
 - $\sigma_{\text{lowM}}^{\text{D}}$ - Low mass dissociation ($pp \rightarrow N^* + p$)
 - $d\sigma/d(\Delta\eta)$ - High mass dissociation
- Data from:
 - CERN ISR 1975–1980
 - CERN SPS 1982–1993
 - TEVATRON (CDF, DØ) 1990–2012
 - TOTEM 2011–2013
 - ATLAS 2012
- Two-channel eikonal model with one ‘effective pomeron’
- Proton wave function written as superposition of two diffractive Good-Walker eigenstates $|p\rangle = \sum_i a_i |\phi_i\rangle$ with $i = 1, 2$

KMR Model (II)

- Use an opacity matrix Ω_{ik} corresponding to one-pomeron-exchange between states ϕ_i and ϕ_k
- Observables in terms of GW eigenstates depend on this opacity e.g.

$$\sigma_{\text{inel}} = \int d^2b_t \sum_{i,k} |a_i|^2 |a_k|^2 (1 - \exp[-\Omega_{ik}(b_t)])$$

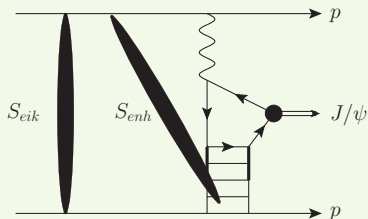
- Each GW eigenstate $|\phi_i\rangle$ independently parametrised by a form factor

$$F_i(t) = \exp \left[-(b_i(c_i - t))^{d_i} + (b_i c_i)^{d_i} \right]$$

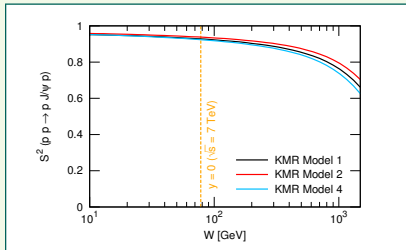
- 3 parameters per eigenstate + 1 relative weighting
- 'Effective' pomeron has energy dependent coupling to eigenstates
- 6 pomeron trajectory parameters: intercept (Δ), slope (α') and couplings (gives $b_0 = 4.9$, $\alpha' = 0.06$ for b slope)

KMR Model (III)

- Survival factors reasonably certain ($\mathcal{O}(5\%)$ difference between KMR models)
- Less certain for high rapidity



- Include this possibility using method of KMR [Ryskin et al. 2009]
- Find small effect from including S_{enh}



- Possibility of ‘enhanced rescattering’
- Interaction between spectator quarks and parton in ladder

Photon Flux

$$\frac{dn}{dk} = \frac{\alpha}{\pi k} \int_0^\infty dq_T^2 \frac{q_T^2 F_p^2(q_T^2)}{(t_{\min} + q_T^2)^2}$$

- k - photon energy
- q_T - photon trans. momentum
- t_{\min} - kinematic q^2 cut-off

- Proton form factor:

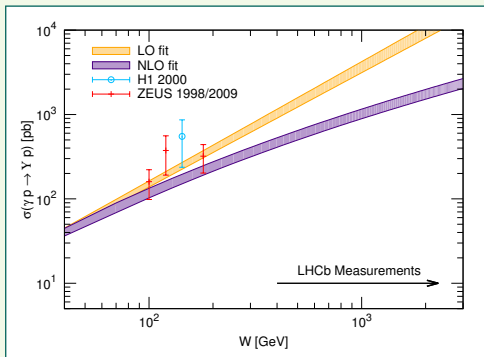
$$F_p(q_T^2) = \left(1 + \frac{t_{\min} + q_T^2}{0.71 \text{ GeV}^2}\right)^{-2}, \quad t_{\min} \approx \frac{(x_\gamma m_p)^2}{1 - x_\gamma}$$

- Photon flux consistent with KMR model
 - Similar to equivalent photon approximation (EPA)
 - But: neglect terms \propto anomalous magnetic moment of the proton

Accuracy

- Neglected terms $\propto q_T^2$ have no singularity at $q_T^2 \rightarrow 0$
- Contributions from $q_T \sim 1/R_p$ are concentrated at small b_t , suppressed by large opacities

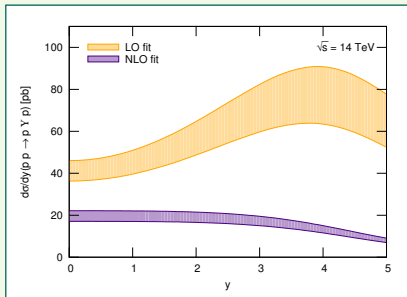
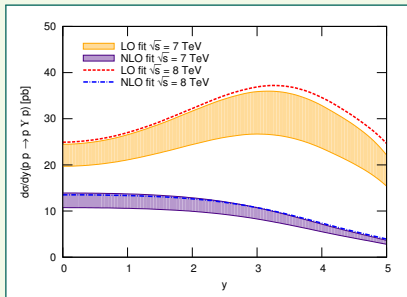
Υ Prediction ($\gamma p \rightarrow \Upsilon + p$)



- Very little data available for comparison
- Need high energy Υ to determine scale dependence
- Huge discrepancy between extrapolated LO and 'NLO' fits

- Υ scale $\bar{Q}^2 \approx 23 \text{ GeV}^2$ vs J/ψ scale $\bar{Q}^2 \approx 2.4 \text{ GeV}^2$
- Future: Use full DGLAP evolution
 - Approximate treatment within our gluon uncertainty for $x \gtrsim 10^{-4}$
 - DGLAP gives $\approx 30\%$ larger gluon for $x \sim 10^{-5}$

Υ Prediction ($pp \rightarrow p + \Upsilon + p$)



- 'NLO' gluon parametrisation grows $1/x$ and $\ln(\mu^2)$ less steep than $xg \propto x^{-\lambda}$
- Large discrepancy between LO and 'NLO' due to poor high energy constraints on scale behaviour
- Large real and skewing corrections when W_- is small
- But: W_- component only $\mathcal{O}(15 - 20\%)$ of $d\sigma(pp)/dy$

NRQCD

- To compute $\gamma p \rightarrow J/\psi + p$
 - First $\gamma p \rightarrow c\bar{c} + p$ with $c\bar{c}$ on shell
 - Project onto quarkonium state $v\bar{u} \rightarrow \dots \not{\epsilon}_{J/\psi}$
- Leading order in v : set $q_1^\mu = q_2^\mu$

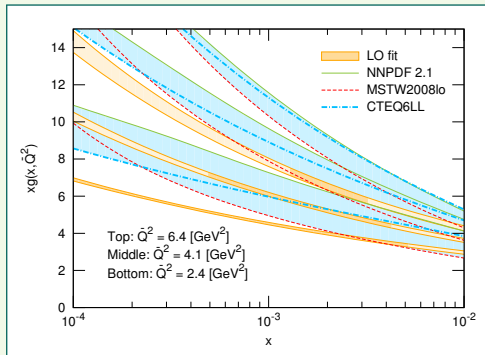
NRQCD Long Distance Matrix Element

- Can compute electronic decay rate

$$\Gamma[J/\psi \rightarrow e^+e^-] = \frac{2e_c^2\pi\alpha^2}{3} \frac{\langle O_1 \rangle_{J/\psi}}{m_c^2} \left(1 - \frac{8\alpha_s}{3\pi}\right)^2$$

- Experimentally measure Γ_{ee} to determine $\langle O_1 \rangle_{J/\psi}$
- Justification for using Γ_{ee} in k_T factorisation model

LO Gluon Fit



- Decreased uncertainty
- Steeply rising behaviour as for global partons
- Receive sizeable correction when including NLO effects (Like global partons)