Two muon and heavy quark pair production in two-photon collisions at LHC

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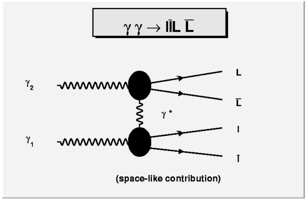
Outline

1. Introduction (past results)
2. Analytical Formulaes for the production of two fermion pairs, two scalar pairs or mixted fermion and scalar pairs
3. Very low angle Monte Carlo and application at PLC, ILC/FCC-ee and LHC
4. Summary-Outlook

Two fermions pair production ($\gamma\gamma \to l\bar{l}L\bar{L}$) : a bit of history (starting in $\simeq$ 1970) and motivation for PLC, ILC/FCC and LHC.

- Pseudo Pair Configurations (peripheral diagrams)



- Total cross section computation
  Two identical lepton pair production at infinite energy in $\gamma\gamma$ center of mass : L.N. Lipatov et al (1969),$\cdots$
  Two identical pion pair production : Chen et al. (1970)
- Total and differential cross section.
  Different pairs produced - main logarithmic approximation- gamma polarisation : V. G. Serbo et al. (1970 - 1985-1998 - $\cdots$)
- Factorisation Formulas
  cf. Kessler and C. Carimalo thesis (1974)
  Provide also powerful tools to calculate Helicity Amplitudes (Use of Helicity Coupling, $\cdots$)
- Motivation Today
  -Reference process for luminosity measurement at PLC
  - Can be a noise for low angle detector at ILC/FCC
  - Can be a background source to rare processes
  $\Rightarrow$ Only a realistic Monte-Carlo can give a correct answer (at low and high angle).
  $\Rightarrow$ have analytical formulas to test the validity of the Monte Carlo on some distributions

Our goal : obtain analytical formulas without mass approximation in order to have the accuracy of old formulas

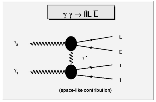
We use the Factorisation Formula : including all diagrams where the exchanged photon is space-like : Cf Kessler, Carimalo, $\cdots$ ($\gamma\gamma$ group of Collège de France)

$$\sigma = \int_{u_{min}}^{u_{max}} \int_{u'_{min}}^{u'_{max}} \int_{t_{min}}^{t_{max}} \frac{d\sigma}{dtdudu'} du\, du'\, dt$$

$$\frac{d\sigma}{dtdudu'} = \frac{uu'}{8\pi^3 s^2 t^2} \Big((1 + ch^2\theta)\sigma_T \sigma'_T + sh^2\theta(\sigma_T\sigma'_L + \sigma_L\sigma'_T) + ch^2\theta\sigma_L\sigma'_L \Big)$$

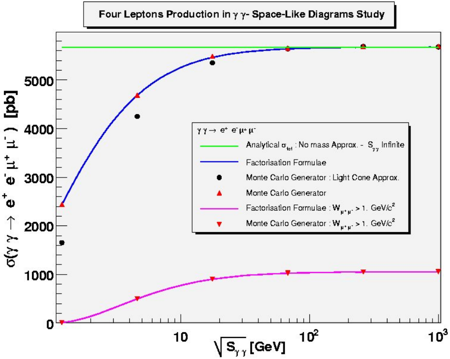
$$\sigma_T^{\gamma\gamma^\star \to l^+ l^-} = \frac{4\pi\alpha^2\beta u}{(u+t)^2} \left(\beta^2 - 2 + 2\frac{2t}{u} - \frac{t^2}{u^2} + \frac{3 - \beta^4 + 2t^2/u^2}{2\beta} L \right)$$

$$\sigma_L^{\gamma\gamma^\star \to l^+ l^-} = \frac{16\pi\alpha^2\beta t}{(u+t)^2} \left(1 - \frac{1-\beta^2}{2\beta} L \right)$$



$$\beta = \sqrt{1 - \frac{4m^2}{u}}$$

$$L = \ln\left(\frac{1+\beta}{1-\beta}\right)$$



- Blue line : Factorization Formula without cuts
- Pink line : Factorization Formula with cuts on muons
- Other results explain later in talk

Analytic formula for two fermion Pair production in $\gamma\gamma$ at infinite energy

We obtain finally (green line in the right figure) :

$$\sigma = \frac{4\alpha^4}{9\pi m m'} \left\{ \frac{19}{16} \left[2\left(\frac{1}{u} - u\right) \ln(u) - \left(\frac{1}{u} + u\right) \left(2 + \ln^2(u)\right) \right] + \left[\frac{25}{4} + \frac{19}{32} \left(\frac{1}{u} - u\right)^2 \right] Poly(u) \right\}$$

where $Poly(u) = Poly(1/u) = \Lambda_3(u) - \Lambda_3(-u)$, $\Lambda_n(z) = \int_0^z \frac{\ln^{n-1}|t|}{1+t}\, dt$ (Kummer function)

cf PLB B718-2012-577

When the two masses are very different ($m \gg m'$), we obtain :

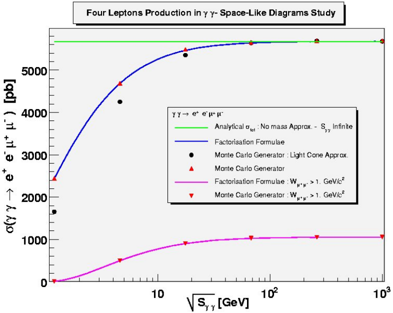
$$\sigma \simeq \frac{28\alpha^4}{27 m^2 \pi} \left(\ln^2(u^2) - \frac{103}{21} \ln(u^2) + \frac{485}{63} \right)$$

in agreement with Serbo et al. computation.
$ee\mu\mu$ Serbo et al production agrees with the exact expression within a relative accuracy of 10^{-5}.

When masses are equal ($m = m'$) we get :

$$\sigma = \frac{\alpha^4}{m^2 \pi} \left(\frac{175}{36} \zeta(3) - \frac{19}{18} \right)$$

in agreement also with the well-known formula for identical pair production.



Analytic formulae for $\gamma\gamma \to \pi^+\pi^- l^+ l^-$ cross section at Infinite Energy

Using the same method : for charged pions depicted as scalar point-like particles in QED we obtain ($u = \frac{m_l}{m_\pi}$) (red line in the right figure) (as to compare to finite size model including a "M. POPPE" pion form factor) :

$$\sigma = \frac{\alpha^4}{72\pi m_\pi m_l}\left[-2\left(\frac{19}{u}+5u\right)\ln(u) + \left(\frac{19}{u}-5u\right)\left(2+\ln^2(u)\right) + \left(\frac{5u^2}{2}+27-\frac{19}{2u^2}\right) Poly(u)\right]$$

When the two masses are very different ($m_\pi \gg m_e$), we obtain :

$$\sigma \simeq \frac{16\alpha^4}{27\pi m_\pi^2}\left[\ln^2\left(\frac{m_e}{m_\pi}\right) - \frac{8}{3}\ln\left(\frac{m_e}{m_\pi}\right) + \frac{163}{72}\right]$$

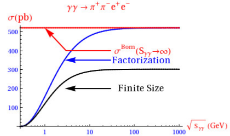
in agreement with Serbo et al. computation.

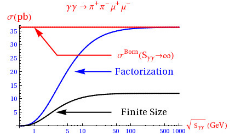
$\pi^+\pi^- e^+ e^-$ ($\pi^+\pi^-\mu^+\mu^-$)Serbo et al production, agrees with the exact expression within a relative accuracy of 5 10^{-6} (9 10^{-2}).

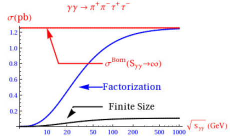
If we make $m_\pi \simeq m_\mu$ and $m_\pi \ll m_\tau$ we get :

$$\sigma \simeq \frac{7\alpha^4(5\zeta(3)+2)}{36\pi m_\pi^2}\left[1 + \frac{62-7\zeta(3)}{7(5\zeta(3)+2)}\left(1-\frac{m_\mu}{m_\pi}\right)\right]$$

$$\sigma \simeq \frac{252\alpha^4}{243\pi m_\tau^2}\left[\ln^2\left(\frac{m_\tau}{m_\pi}\right) + \frac{20}{21}\ln\left(\frac{m_\tau}{m_\pi}\right) + \frac{247}{126}\right]$$





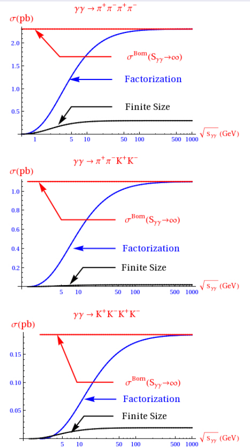


Analytic formulae for $\gamma\gamma \to \pi^+\pi^- K^+ K^-$ cross section at Infinite Energy

In QED scalar approximation, we obtain also (red line in the right figure) :

$$\sigma = \frac{5\alpha^4}{144\pi m_\pi m_K}\left[2\left(u-\frac{1}{u}\right)\ln(u)+\left(\frac{1}{u}+u\right)\left(2+\ln^2(u)\right)+\left(\frac{4}{5}-\frac{1}{2}\left(\frac{1}{u}-u\right)^2\right)Poly(u)\right]$$

where $\quad Poly(u) = Poly(1/u)$ (already defined), $\quad$ and $u = \dfrac{m_\pi}{m_K}$



If masses are equal (i.e. put $m_K \to m_\pi$) we get :

$$\sigma^{\mathrm{Born}}_{\gamma\gamma\to\pi^+\pi^-\pi^+\pi^-} = \frac{\alpha^4}{144\pi m_\pi^2}\left(7\zeta(3)+10\right)$$

in agreement with H. Cheng and T. T. Wu.

If we put $u = \frac{m'}{m} \ll 1$ we get :

$$\sigma \simeq \frac{36\alpha^4}{243\pi m^2}\left[\ln^2(u)-\frac{7}{6}\ln(u)+\frac{77}{36}\right]$$

Inclusive cross section in $\gamma\gamma$ collisions at infinite energy :$\pi^+\pi^-K^+K^-$, $\pi^+\pi^-e^+e^-$, $\pi^+\pi^-\mu^+\mu^-$

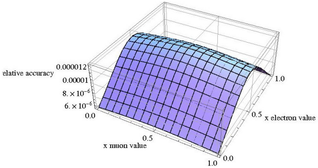
Using the impact factor method the energy distribution of particles moving along the momentum of one photon is given by E.A. Kuraev, A. Schiller and V. G. Serbo

$$\frac{d\sigma_a}{dx_+ dx'_+} = \frac{4\alpha^4}{\pi}\int_0^\infty f\left(t, X, x_+\right) f'\left(t, X', x'_+\right)\, dt$$

$$f\left(t, X, x_+\right) = \begin{cases} \frac{x_+ x_-}{t} F_s(X) & \text{(scalar)} \\ \frac{-2x_+ x_-}{t} F_s(X) + \frac{X}{t}\ln\frac{1+X}{1-X} & \text{(lepton)} \end{cases}$$

$$F_s(X) = -1 + \frac{1}{2}\left(X + \frac{1}{X}\right)\ln\frac{1+X}{1-X}$$

where $x_\pm$, is the incoming photon energy fraction carried by the pos. (neg.) particle.



After integration without any mass approximation we have for scalars and leptons $\Rightarrow$

$$\frac{d\sigma_a^{\gamma\gamma\to s^+s^-s'^+s'^-}}{dx_+ dx'_+} = 18 x_+ x_- x'_+ x'_- \times \sigma^{\gamma\gamma\to s^+s^-s'^+s'^-}$$

$$\frac{d\sigma_a^{\gamma\gamma\to s^+s^-l'^+l'^-}}{dx_+} = 3 x_+ x_- \sigma^{\gamma\gamma\to s^+s^-l'^+l'^-}$$

$$\simeq \frac{4\alpha^4}{9\pi m_\pi^2} x_+ x_- \left[\ln^2 u^2 - \frac{16}{3}\ln u^2 + \frac{163}{18}\right]$$

($u = \frac{m_e}{m_\pi} \ll 1$), in agreement with Serbo et al. computation.

for example : $\pi^+\pi^-e^+e^-$ $(ee\mu\mu)$ Serbo et al production agrees with the exact expression with a relative accuracy lower than 5 10^{-6} (2 10^{-5}) ! .

Provide a "Pseudo pair" Monte-Carlo Generator for $\gamma\gamma \to 4$ *particles*

- Very low angle Space Phase :
  Perform a special 4 particles space phase for generating particles at very low angle.
- Helicity Amplitude Computation :
  First method : Impact Factor approximation (very fast).
  Gives us the dominant term at low angle and high energy (black points in the figure) .

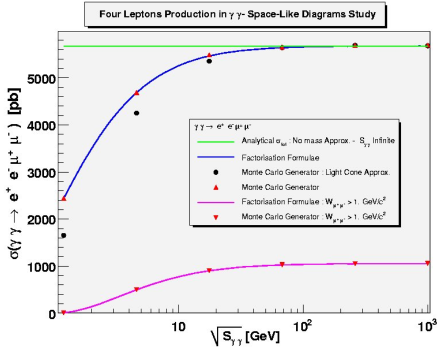
$$|M|^2 \sim L_{++} R_{--}$$

$$L_{++} = 4k_+^2 \left[\left\{ (1-\bar{x})\, P_{l/\gamma}(x) + l \leftrightarrow \bar{l} \right\} + 2\frac{\vec{l}_\perp . \vec{\bar{l}}_\perp}{ab} \left(x(1-\bar{x}) + \bar{x}(1-x) \right) - 2\frac{m^2}{ab} \right]$$

$$P_{l/\gamma}(x) = \frac{2m^2 x}{a^2} + \frac{x^2 + (1-x)^2}{a}$$

where $P_{l/\gamma}(x)$ is the lepton spectrum distribution, $l_+, l_-, \vec{l}_\perp$ are the lepton light cone variables, $x = 1 - \frac{l_+}{k_+}$ is the fraction of the photon energy taken by the quasi-real lepton l, $a = k_+ l_-$, $b = k_+ \bar{l}_-$ and $k_+ = \sqrt{S_{\gamma\gamma}}$

- Second method : no approximation (take time). (red triangles in the figure) in perfect agreement with the numerical integration of the factorization formulae (blue and pink (invariant mass cut)) lines.



Study mechanisms of pion pair production at ILC in $\gamma\gamma \to \pi^+\pi^- l^+ l^-$

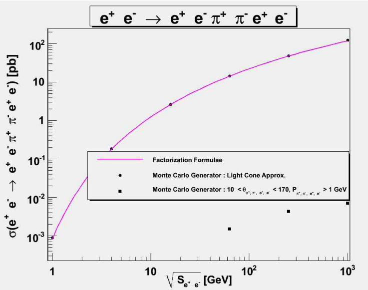
Lepton pair of $\gamma\gamma \to \pi^+\pi^- e^+ e^-$ $(\mu^+\mu^-)$ processes can be used for tagging pion pair

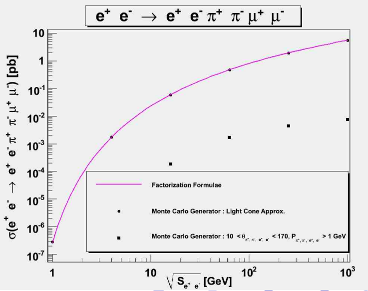
Cross Section Computation :

$$\sigma = \int_{z_{min}}^{z_{max}} dz\, 2z \int_{\frac{z^2}{z_{max}}}^{z_{max}} \frac{dy}{y}\, f_{\gamma/e}(y) f_{\gamma/e}\left(\frac{z^2}{y}\right)$$

$$\times\, \sigma^{\gamma\gamma \to \pi^+\pi^- l^+ l^-}\left(z\sqrt{S_{e^+e^-}}\right), \quad y = \frac{E_\gamma}{E_{beam}}$$

- Potential background for detectors at very low angle never taken into account (Need more studies with our Monte Carlo)
- Strong dependence of the visible cross section with angular and energy cuts applied
- $\sigma^{ILC}_{Visible} \simeq 0.1 - 10$ fb (black square in the right figure)
- $N_{evt} \simeq 10^2 - 10^4$ if $L(ILC) = 1\ \mathrm{ab}^{-1}$
- Need realistic simulation for bgk rejection (pion decay in flight background to $\gamma\gamma \to \pi^+\pi^-\mu^+\mu^-$ $, \cdots)$





- Pseudorapidity Distribution (η)

Figure (right-top) :

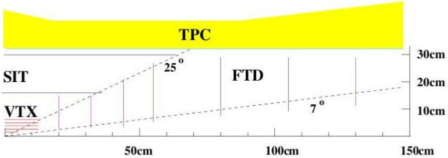
- Pion : $|\eta_{\mathrm{MAX.}}| \simeq 4$, but can be seen in FTD/VTX (blue line)
- Electron $|\eta_{\mathrm{MAX.}}| \simeq 6.5$, mostly in bean pipe but can be seen in FTD/VTX (red line)

Figure (right-midle) :

- Muon : $|\eta_{\mathrm{MAX.}}| \simeq 4$, but can be seen in FTD/VTX(red line)

Figure (right-bottom) :

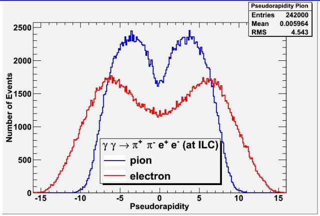
- $\gamma\gamma \rightarrow \pi^+\pi^-$ (blue line)$u\bar{u}$ (blue line), photon exchanged beetween $\pi^+\pi^-$ pair and $u\bar{u}$ pair. Products of $u\bar{u}$ fragmentation (green line)

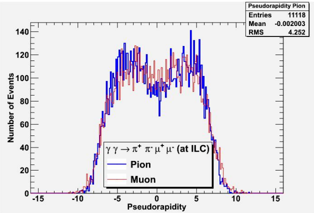


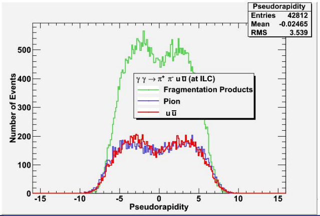
Vertex Detector (VTX) $-1.9 \lesssim \eta \lesssim 1.9$

Forward Tracker (FTD) $-3 \lesssim \eta \lesssim 3$

$\Rightarrow$ Many particles are produced in the beam pipe, but a significant fraction can be seen at low angle.







Study mechanisms of pion pair production at LHC in $\gamma\gamma \rightarrow \pi^+\pi^- l^+ l^-$

- Cross Section Computation :

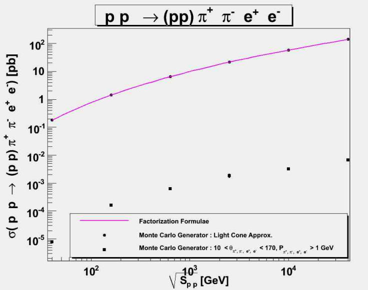
$$\sigma = \int_{z_{min}}^{z_{max}} dz 2z \int_{\frac{z^2}{z_{max}}}^{z_{max}} \frac{dy}{y} f_{\gamma/p}(y) f_{\gamma/p}(\frac{z^2}{y})$$

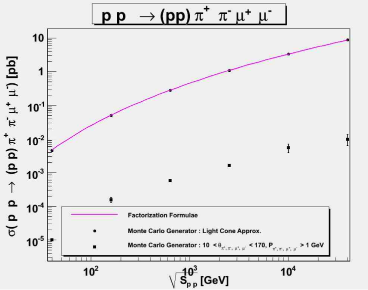
$$\sigma^{\gamma\gamma \rightarrow \pi^+\pi^- l^+ l^-}(W_{\gamma\gamma})$$

$$f_{\gamma/p}(y, \mu^2) = f_{\gamma(el)/p}(y) + f_{\gamma(inel)/p,Q^2}(y)$$

$y = \frac{E_\gamma}{E_{beam}}$ and Q^2 is the resolution scale at which the proton is probed.

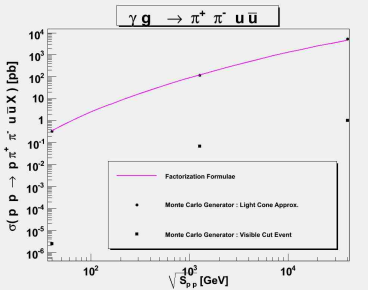
- Photon content of proton used :
  -elastic contribution (Bernd A. Kniehl)
  -inelastic comtribution (M. Glück et al),
- Few events but clear signature
  $\sigma^{LHC}_{Visible} \simeq 0.1 - 1\ \mathrm{fb}$ (depending strongly on CUTS)
  if $L(LHC) = 100\ \mathrm{fb}^{-1} \rightarrow \mathrm{N_{evt}} \simeq 10 - 100$
  (need realistic simulation for bgk rejection $\Rightarrow$ conclusion)
- Use Roman pot to tag proton(s) ?

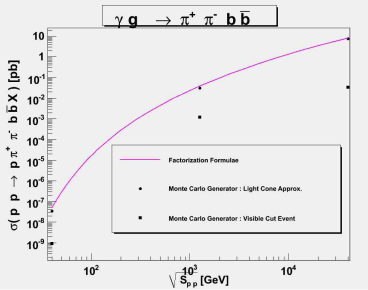




Study mechanisms of $\gamma g \to \pi^+\pi^- Q\bar{Q}$ production in LHC

- Photon exchanged beetween scalar pion pair and (heavy) quark pair
  Use only Born approximation (very crude approximation)
- PDF :
  Gluon content of the proton used : CTEQ6
  Photon content of the proton used : elastic (Bernd A. Kniehl) and inelastic (M. Glück et al).
- Pion pair and $Q\bar{Q}$ pair inside detector
  $\sigma^{LHC\,:\gamma g\to\,\pi^+\pi^-u\bar{u}}_{Visible} \simeq 100 \text{ fb} \;-\; 1000 \text{ fb}$
  $\sigma^{LHC\,:\gamma g\to\,\pi^+\pi^-b\bar{b}}_{Visible} \simeq 1 \text{ fb} \;-\; 10 \text{ fb}$
- Need realistic LHC estimation of background (simulation, pile-up $,\cdots$) to see if we can extract the signal
- Remember : pseudo pair configuration can give some “strange” events
  pion pair and $Q\bar{Q}$ are back to back in γg center of mass and are produced at low angle $\Rightarrow$ After boost some pion pair are not visible and $Q\bar{Q}$ pair is visible from one side of the detector $\Rightarrow$ strange events





- Cross Section Computation :

$$\sigma = \int d\omega_1 d\omega_2 L_{\gamma\gamma}(\omega_1, \omega_2)\, \sigma_{\gamma\gamma}(\omega_1, \omega_2)$$

$$L_{\gamma\gamma}(\omega_1, \omega_2) = \int_{R_1^{\mathrm{Cut.}}} d^2 b_1\; n(\omega_1, b_1) \int_{R_2^{Cut.}} d^2 b_2\; n(\omega_2, b_2)\, \Theta\left(\left|\vec{b}_1 - \vec{b}_2\right| - (R_1 + R_2)\right)$$

- Photon content of ion (cf G. Baur, J. Nystrand et al):

  -$\Theta(..) \Rightarrow$ No frontal Collission

  - $R_1^{Cut.} = R_2^{Cut.} = 0 \Rightarrow$ pair production inside ion allowed,

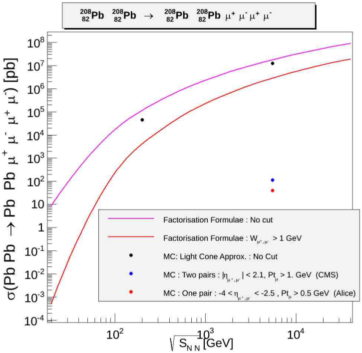
  - $n\left(\omega_{1(2)}, b_{1(2)}\right) = \frac{Z^2\alpha}{\pi^2} \times$
  $\frac{1}{b^2\omega}\left|\int_0^\infty J_1(v)\, F\left(-\frac{u^2+v^2}{b^2}\right)\frac{v^2 dv}{u^2+v^2}\right|^2 u = \omega b/(\gamma)$,
  available if $\omega < \omega_{\mathrm{max.}} \sim \frac{\gamma}{R} \sim 100$ GeV for $\mathrm{F(Q^2)}$ we use the simple monopole form factor with $\Lambda = 80$ MeV

- (Two) few visible events but clear signature

  - at $\sqrt{S_{NN}} = 5.5$ TeV $\Rightarrow \sigma_{\mathrm{Tot.}}^{\gamma\gamma\to 4\mu} \simeq 12\ \mu$b

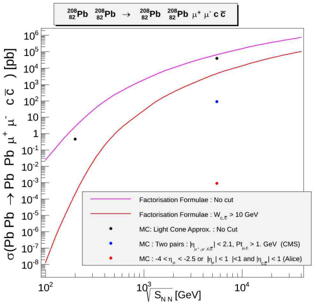
  - $\sigma_{\mathrm{Visible}} \simeq 100$ pb : if only one muon pair detected in Alice very forward region and $P_T > 0.5$ GeV

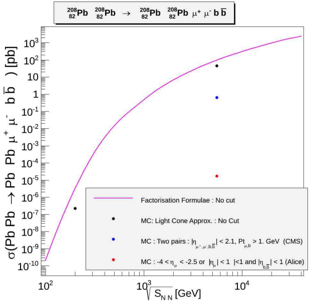
  - $\sigma_{\mathrm{Visible}} \simeq 40$ pb : if two muon pairs detected in CMS/ATLAS barrel region and $P_T > 1$ GeV



Heavy Ion Collisions : one muon pair and heavy quark pair production in $\gamma\gamma \to \ \mu^+\mu^- c\bar{c}\ (b\bar{b})$

- Heavy quark treatment :
  Use only Born Term approximation (very crude approximation)
- Photon content of ion (cf G. Baur, J. Nystrand et al):
  -$\Theta(..) \Rightarrow$ No frontal Collission
  - $R_1^{Cut.} = R_2^{Cut.} = R_A \Rightarrow$ No pair production inside ion allowed,
  - $n\left(\omega_{1(2)}, b_{1(2)}\right)$: for $\mathrm{F(Q^2)}$ we use the simple monopole form factor with $\Lambda = 80\ \mathrm{MeV}$
- (Two) few visible events but clear signature
  - at $\sqrt{S_{NN}} = 5.5\ \mathrm{TeV} \Rightarrow \sigma_{\mathrm{Tot.}}^{\gamma\gamma\to 2\mu 2c(2b)} \simeq 40\ (0.05)\ \mathrm{nb}$
  - $\sigma_{\mathrm{Visible}} < 0.1\ \mathrm{fb}$: very low if only muon $\in$ Alice very region (or Barrel) and heavy quark detected in Barrel
  - $\sigma_{\mathrm{Visible}} \simeq 91(0.7)\ \mathrm{pb}$: if one muon pair and one heavy quark pair detected in CMS/ATLAS barrel region $P_T > 1\ \mathrm{GeV}$





Summary

- The Monte-carlo for Pseudo Pair production at very low angle is ready (fully integrated in ROOT software).
- We provide analytical formulas without mass approximation for the total and inclusive cross section of several the processes $\gamma\gamma \to \pi^+\pi^- + \text{ 2 leptons}$, $\gamma\gamma \to Q\bar{Q} + \text{ 2 leptons}$, $\gamma\gamma \to \pi^+\pi^- K^+ K^-, \cdots$.
- We have to include realistic photon flux induced by proton and heavy ions

- Outlook
  - Need to continue the study of each process if we want the error on the number of events computed (validity range of Born approximation, contributions of other process with same final state, $\cdots$)
  - Include "realistic ILC and FCC photon fluxes" and simulate theses events at ILC and at FCC
  - $\cdots$