BEYOND THE STANDARD MODEL

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OUTLINE

LEC. I.
- THE HIERARCHY PROBLEM – NEED TO GO BSM
- BRIEF INTRO TO SUSY

LEC. II.
- THE MSSM
- VARIATIONS BEYOND THE MSSM

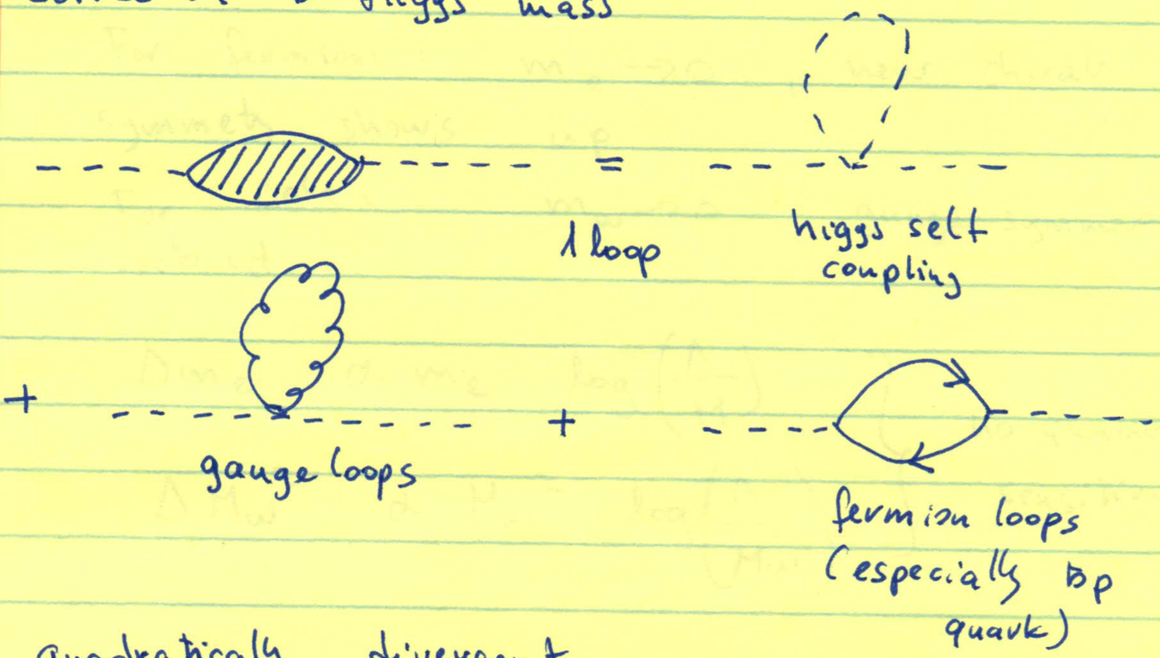
LEC. III.
- WARPED EXTRA DIMENSIONS
- COMPOSITE HIGGS

LEC. IV.
- LITTLE HIGGS

HIERARCHY PROBLEM

Traditional presentation

correction to Higgs mass



quadratically divergent

$$\propto \int \frac{d^4k}{(k^2-m^2)} \frac{1}{(2\pi)^4} \quad \propto \Lambda^2$$

Λ ← cut-off scale

$$\delta m^2 = \frac{\Lambda^2}{32\pi^2}\Big(\underset{\substack{\uparrow \\ \text{self-coupling} \\ (800\,\text{GeV})^2}}{6\lambda} + \underbrace{\tfrac{1}{4}(9g^2+3g'^2)}_{\substack{\uparrow \\ (600\,\text{GeV})^2}} \underset{\substack{\uparrow \\ -(1.5\,\text{TeV})^2}}{-6y_t^2} \Big)$$

If $\Lambda \gg 10\,\text{TeV}$ (for example $\Lambda \sim M_{Pl}$)

$\delta m_H^2 \gg m_H^2$ → hierarchy problem.

Higgs mass sensitive to ANY scale of new physics

<u>NOTE:</u> this problem is specific to elementary scalars.

For fermions: $m_e \to 0$, new chiral symmetry shows up

For GB's: $m_W \to 0$ gauge symmetry restored

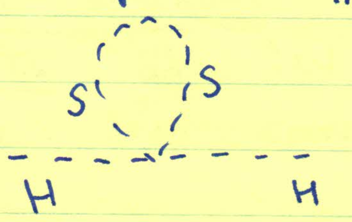
$$\left.\begin{array}{l} \Delta m_e \propto m_e \log\left(\frac{\Lambda}{M}\right) \\ \Delta M_W^2 \propto M_W^2 \log\left(\frac{\Lambda}{M_W}\right) \end{array}\right\} \text{ no quadratic sensitivity!}$$

<u>Isn't this dependent on what kind of regularization</u> I use (e.g. in dim reg no quadratic divergences, $1/\epsilon$ poles $\sim$ log div's).

<u>No!</u> any new physics coupled to Higgs will introduce it

<u>Example:</u> a new phsics at scale m_S,

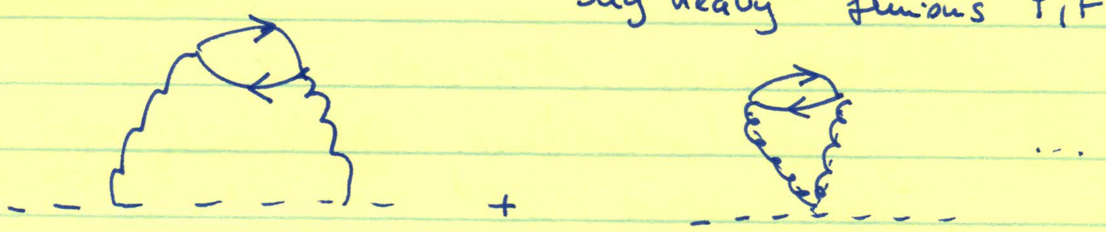
for example $\lambda_S |H|^2 |S|^2$



$$\delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{UV}^2 - 2m_S^2 \log\frac{\Lambda_{UV}}{m_S} + \text{finite}\right]$$

even if $\Lambda_{UV} \to 0$, log divergent & finite contributions $\propto m_S^2$ ...

Even if NOT directly coupled to Higgs, but coupled to some SM fields.
Say heavy fermions $F, \bar{F}$



2 loop: $\delta m_H^2 \propto \left(\frac{g^2}{16\pi^2}\right)^2 \left[a \Lambda_{UV}^2 + 48\, m_F^2 \log \frac{\Lambda_{UV}}{m_F} + \dots \right]$

Real formulation of hierarchy problem:

m_H sensitive to any high scale in theory, if indirectly coupled to SM

Other words: Higgs mass relevant operator, relative importance grows toward IR.
Only relevant operator in SM!

Usual solution: there needs to be new physics at TeV scale that eliminates large loop corrections above TeV scale!

Possibilities:

- Relate elementary scalar to fermions via SUSY. Chiral sym. of fermions ensures together with SUSY cancellation of div's.
- Relate to elementary gauge field (gauge-Higgs unification)
- There is no higgs boson, just a condensate dynamically generated (technicolor, higgsless)
- There is a Higgs, but it is not elementary. At ~TeV start feeling large form factors, that suppress corrections (~ "Higgs dissolves...") Composite higgs, warped extra dim's, RS
- The higgs is a pseudo-Goldstone boson, that gives some protection (usually 1-loop) from quadratic divergences. Still need to combine with some of the other mechanisms (little Higgs)
- The fundamental scale of all of new physics is actually 1TeV (large extra dimensions)

BRIEF INTRODUCTION TO SUPERSYMMETRY

Ordinary symmetry (internal symmetry):

$\varphi_i \to i\epsilon^a T^a_{ij}\varphi_j$ $\qquad$ T^a generator of symmetry, assumed to be a boson.

$\varphi \longleftrightarrow \varphi$

boson $\longleftrightarrow$ boson

fermion $\longleftrightarrow$ fermion

Once you have a symmetry, fields form representations (basic building blocks) of symmetry.

Supersymmetry is like ordinary symmetry, <u>except</u> generator <u>fermionic</u>.

SUSY: fermions $\longleftrightarrow$ bosons

<u>History</u>: '60s very successful for classifying ~~[illegible]~~ hadrons based on SU(3) internal symmetry (Gell-Mann's SU(3))
Then people tried to enlarge this to SU(6) which was combination of $SU(3)_{GM} \times SU(2)_{spin}$.
Coleman-Mandula: this does <u>NOT</u> make sense. <u>C-M thm:</u> ~~[illegible]~~ no non-trivial

combination of internal symmetry with Lorentz symmetry (space-time symmetry).

<u>One exception</u> Haag-Lopuszanski-Sohnius

only non-trivial way of avoiding C-M theorem is via <u>graded</u> Lie algebras $\equiv$ generators fermionic. HLS theorem $\rightarrow$ SUSY algebra

\# of supercharges

$$\{Q^L_\alpha, \bar{Q}_{\dot{\alpha}M}\} = 2 P_\mu \sigma^\mu_{\alpha\dot{\alpha}} \delta^L_M$$

Lorentz index

Q : SUSY generator

fermionic $\rightarrow$ $\{\ \}$ instead of $[\ ,\ ]$

$$[P_\mu, Q^A_\alpha] = 0 \quad , \quad [Q^A_\alpha, M_{\mu\nu}] = -i(\sigma_{\mu\nu})_\alpha{}^\beta Q^A_\beta$$

$$\{Q^L_\alpha, Q^M_\beta\} = \varepsilon_{\alpha\beta} Z^{LM}$$

central charge.

<u>If theory supersymmetric</u>

1. \# of fermions = \# of bosons

$$(-1)^{N_F} |q\rangle = +|q\rangle \quad \text{boson}$$
$$\qquad\qquad -|q\rangle \quad \text{fermion}$$

fermion # op.

$$(-1)^{N_F} Q^A_\alpha |q\rangle = -Q^A_\alpha (-1)^{N_F} |q\rangle$$

$$\{(-1)^{N_F}, Q^A_\alpha\} = 0$$

over a representation

$$\mathrm{Tr}\left[(-1)^{N_F}\ \{Q_\alpha{}^A, \bar{Q}_{\dot\beta}{}^B\}\right] =$$

$$= \mathrm{Tr}\left[(-1)^{N_F}\left(Q_\alpha{}^A \bar{Q}_{\dot\beta}{}^B + \bar{Q}_{\dot\beta}{}^B Q_\alpha{}^A\right)\right] =$$

$$= \mathrm{Tr}\left[-Q_\alpha{}^A (-1)^{N_F} \bar{Q}_{\dot\beta}{}^B + (-1)^{N_F}\bar{Q}_{\dot\beta}{}^B Q_\alpha{}^A\right] = 0$$

Trace cyclic

in an irrep fixed P_μ

$$= \mathrm{Tr}\left[(-1)^{N_F}\ 2\sigma^\mu_{\alpha\dot\beta} P_\mu\right] \rightarrow \quad \mathrm{Tr}\,(-1)^{N_F} = 0$$

equal # of fermions & bosons.

Similarly

2. $\rightarrow$ masses must be equal

3. $\langle\psi|H|\psi\rangle \geq 0$ energy positive def. for SUSY

if SUSY unbroken $\langle|H|\rangle = 0$

~~Say // characterize~~

Can classify all SUSY representations

Will not go through that.

For basic case $N=1$ SUSY (means a single fermionic generator Q_α & its conjugate $\bar{Q}$)

representations: massless case

chiral superfield : 1 complex scalar
1 2 component Weyl spinor

vector superfield 1 ~~[illegible]~~ 2 component Weyl spinor

1 gauge field

ONLY $N=1$ reps that do not involve spin >1 fields

NOTE: if more SUSY charges ($N=2,4,\ldots$) no chiral reps. There smallest rep. "hypermultiplet" 1 Dirac fermion + 2 complex scalars ...

For SUSY extension of SM will only need $N=1$ chiral SF + vector SF.

Note: Chiral SF

1 complex scalar $\to$ 2 DOF

1 complex Weyl spinor $\to$ 4 DOF, but only 2 propagating (2 helicity states)

$4-2=2$

If you want to use fields & have SUSY off-shell as well need to add 2 dummy scalar DOF.

Chiral SF:

	off-shell	on-shell
φ	2	2
ψ	4	2
F	2	0

~~vector SF~~

~~1 complex 2 component scalar~~ ~~$\to$ 2 DOF~~

~~1 gauge field~~ ~~$\to$ 3 DOF off-shell, 2 helicities on-shell~~

~~need 1 real scalar~~

vector SF

1 Weyl spinor — 4 DOF off-shell
2 on-shell

1 gauge boson — 3 comp's off-shell
2 comp's on-shell

need 1 dummy boson off-shell → real scalar D

	off-shell	on-shell
ψ	4	2
A_μ	3	2
D	1	0

To describe these use
SUPER SPACE.

SUSY transformation: a "fermionic space-time transformation"

Natural to introduce superspace coordinate θ
θ like a fermionic space-time coordinate.
Every field function of x, θ → superfield in superspace formulation.

By SUSY algebra effect of SUSY transf.

$$(x^\mu, \theta, \bar\theta) \longrightarrow \left(x^\mu + i\theta\sigma^\mu\bar\xi - i\xi\sigma^\mu\bar\theta,\ \theta+\xi,\ \bar\theta+\bar\xi\right)$$

superspace derivatives

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu$$

$$\bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta_{\dot\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

Generic superfield: $z = (x, \theta, \bar\theta)$

$$F(z) = F(x,\theta,\bar\theta) = f(x) + \theta\psi(x)$$
$$+\bar\theta\bar\chi(x) + \theta^2 m(x) + \bar\theta^2 n(x) + \theta\sigma^\mu\bar\theta v_\mu(x)$$
$$+\theta^2\bar\theta\bar\lambda + \bar\theta^2\theta\xi + \theta^2\bar\theta^2 d$$

BUT : not irreducible $\equiv$ some of these fields do not transform each other.

Imposing SUSY covariant conditions get

chiral SF $\bar D\phi = 0$

vector SF $V = V^+$

Chiral SF:

$\bar D_{\dot\alpha}\phi = 0$ check: $\bar D_{\dot\alpha}\overbrace{(x^\mu + i\theta\sigma^\mu\bar\theta)}^{\text{any function of this}} = 0$

$y = x + i\theta\sigma\bar{\theta}$

ch. SF:

$$\Phi(y,\theta) = \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y)$$

φ: complex scalar

ψ: Weyl spinor

F: complex scalar dummy field (aux. field)

Transformation of XSF:

$$\delta_\epsilon \varphi = \sqrt{2}\,\epsilon\psi$$

$$\delta_\epsilon \psi = \sqrt{2}\, i\, \sigma^\mu \bar{\epsilon}\, \partial_\mu \varphi + \sqrt{2}\,\epsilon F$$

$$\delta_\epsilon F = \sqrt{2}\, i\, \bar{\epsilon}\bar{\sigma}^\mu \partial_\mu \psi$$

transforms into a total derivative.

Highest component of XSF $\rightarrow$ SUSY inv.

$\rightarrow$ $\boxed{\int d^2\theta \;\; W(\phi) \;\; + h.c.}$ a candidate SUSY Lagrangian term!

$W(\phi) \rightarrow$ a "holomorphic function of XSF's"

$\int d^2\theta \rightarrow$ picks out highest component (ordinary fermionic integral)

Similarly: $\Phi^\dagger\Phi$ not a chiral SF,

can show $\Phi^\dagger\Phi|_{\theta^2\bar\theta^2} = FF^* + \frac{1}{4}\varphi^*\Box\varphi$

$+\frac{1}{4}\Box\varphi^*\varphi - \frac{1}{2}\partial_\mu\varphi^*\partial^\mu\varphi$

$+\frac{i}{2}\partial_\mu\bar\psi\,\bar\sigma^\mu\psi - \frac{i}{2}\bar\psi\,\bar\sigma\,\partial_\mu\psi$

Kinetic terms of scalars.

$$\mathcal{L} = \int d^4\theta\ \Phi^\dagger\Phi + \int d^2\theta\, W(\Phi) + h.c.$$

More general:

$K(\Phi^\dagger, \Phi)$ generic real function of $\Phi, \Phi^\dagger$

Kähler potential

$\rightarrow \int d^4\theta\ K(\Phi^\dagger, \Phi)$

In terms of components (after integrating out auxiliary field F)

$$\mathcal{L} = \partial_\mu\varphi_i^*\partial^\mu\varphi_i + i\bar\psi_i\,\bar\sigma^\mu\partial_\mu\psi_i$$

$$-\frac{\partial^2 W}{\partial\varphi_i\partial\varphi_j}\psi_i\psi_j - \sum_i\left|\frac{\partial W}{\partial\varphi_i}\right|^2$$

For gauge fields → vector superfields

ϕ carries U(1) charge

$$\phi \to e^{-i\Lambda}\phi$$

a symmetry. To promote to local sym. need real field

$$\phi^+\phi \to e^{-i(\Lambda-\Lambda^+)}$$

$$V \to V + i(\Lambda - \Lambda^+)$$

$\phi^+ e^V \phi$ will be gauge invariant!

Generic vector SF $V = V^+$ has lots of components, many can be eliminated via gauge fixing → Wess-Zumino gauge

$$V = -\underbrace{\theta\sigma^\mu\bar\theta V_\mu}_{\text{gauge field}} + i\theta^2\bar\theta\bar\lambda - i\bar\theta^2\theta\lambda + \frac{1}{2}\theta^2\bar\theta^2 D$$

gaugino

auxiliary field

Important: can make a chiral SF out of VSF that contains field strength

$$W_\alpha = -1/4\, \bar D\bar D D_\alpha V$$

$$W_\alpha = -i\lambda_\alpha(y) + \theta_\beta\left[\delta_\alpha{}^\beta D(y) - \frac{i}{2}(\sigma^\mu\bar\sigma^\nu)^\beta{}_\alpha F_{\mu\nu}\right] + \theta^2\sigma^\mu{}_{\alpha\dot\alpha}\,\partial_\mu\bar\lambda^{\dot\alpha}$$

$W_\alpha W^\alpha |_{\theta^2}$ → kinetic term of gauge field.

Final full interacting gauge invariant Lagrangian:

$$\mathcal{L}^{SUSY} = \int d^4\theta \quad \Phi_i^+ e^{gV} \Phi_i$$
$$+ \int d^2\theta \left(1/4\, W_\alpha W^\alpha + h.c. \right)$$
$$+ \int d^2\theta \left[W(\Phi) + h.c. \right]$$

<u>Example:</u> SUSY QED

supersymmetrized interaction of electron & positron.

Φ_+ : positron χSF

Φ_- : electron χSF $(e<0)$

$$\mathcal{L} = \int 1/4 \left(W_\alpha W^\alpha + \bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha} \right) d^2\theta$$
$$+ \int \left(\Phi_+^+ e^{eV} \Phi_+ + \Phi_-^+ e^{-eV} \Phi_- \right) d^4\theta$$
$$+ \int \left[m \{ \Phi_+ \Phi_- d^2\theta + m \Phi_+^+ \Phi_-^+ \} d^2\bar\theta \right]$$

In components:

$$\int \left(W_\alpha W^\alpha + h.c.\right) d^2\theta$$

$$\mathcal{L}_{SQED} = \left[\frac{1}{2} D^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i \lambda \sigma^\mu \partial_\mu \bar{\lambda}\right]$$

$$+ F_+^* F_+ + F_- F_-^* + |D_\mu \varphi_+|^2 + |D_\mu \varphi_-|^2$$

$$+ i \bar{\psi}_+ D_\mu \bar{\sigma}^\mu \psi_+ + i \bar{\psi}_- D_\mu \bar{\sigma}^\mu \psi_-$$

$$- \frac{ie}{\sqrt{2}} \left(\varphi_+ \bar{\psi}_+ \bar{\lambda} - \varphi_- \bar{\psi}_- \bar{\lambda}\right) + h.c.$$

$$+ \frac{e}{2} D \left[|\varphi_+|^2 - |\varphi_-|^2\right]$$

$$+ m \left[\varphi_+ F_- + \varphi_- F_+ - \psi_+ \psi_-\right] + h.c.$$

ψ_+, ψ_- $\rightarrow$ combine into Dirac electron

φ_+, φ_- $\rightarrow$ scalar electron (selectron)

2 complex scalars, L & R version

λ $\rightarrow$ gaugino (photino)

v_μ $\rightarrow$ photon