2nd Lecture
When (some) QCD matters

- Isospin and $SU(3)$ flavor

  Measuing α, $SU(3)$

- The heavy quark limit

  Heavy quark symmetry, OPE, exclusive / inclusive decays

- Semileptonic and radiative b decays

  $b \to s\gamma$, etc.

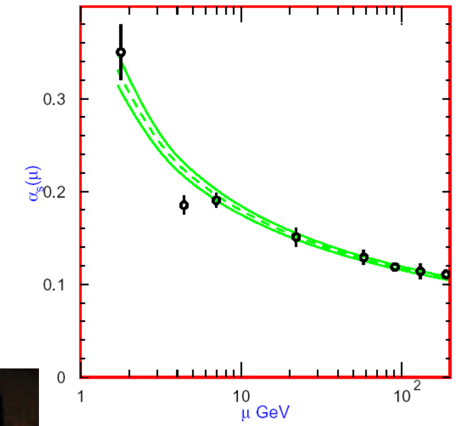
- SCET and nonleptonic decays — skip, but include slides

  B decays to charm, Λ_b decay

  charmless B decays, different approaches

Interplay of electroweak and strong interactions

- How to learn about high energy physics from low energy hadronic processes?
- QCD coupling is scale dependent, $\alpha_s(m_B) \sim 0.2$



$$\alpha_s(\mu) = \frac{\alpha_s(\Lambda)}{1 + \frac{\alpha_s}{2\pi}\beta_0 \ln\frac{\mu}{\Lambda}}, \qquad \beta_0 = 11 - \frac{2}{3}n_f > 0$$

Nobel prize in 2004:

Politzer, Wilczek, Gross

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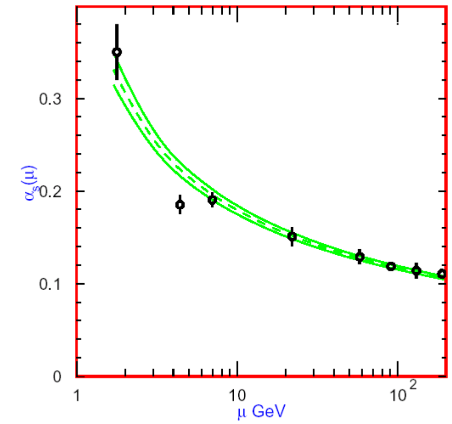


$$\alpha_s(\mu) = \frac{\alpha_s(\Lambda)}{1 + \frac{\alpha_s}{2\pi}\,\beta_0 \ln\frac{\mu}{\Lambda}}\,, \qquad \beta_0 = 11 - \frac{2}{3}n_f > 0$$

High energy (short distance): perturbation theory is useful

Low energy (long distance): QCD becomes nonperturbative $\Rightarrow$ It is usually very hard, if not impossible, to make precise calculations

- Solutions: New symmetries in some limits: effective theories (heavy quark, chiral)
  Certain processes are determined by short-distance physics
  Lattice QCD (bite the bullet — limited cases)
- Incalculable nonperturbative hadronic effects sometimes limit sensitivity

Disentangling weak and strong interactions

- Want to learn about electroweak physics, but hadronic physics is nonperturbative

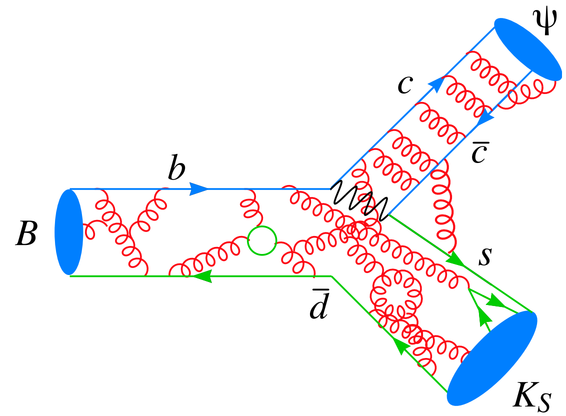
  Model independent continuum approaches:

- (1) Symmetries of QCD (exact or approximate)

  E.g.: $\sin 2\beta$ from $B \to J/\psi K_S$: amplitude not calculable

  Solution: CP symmetry of QCD ($\theta_{\rm QCD}$ can be neglected)

$$\langle \psi K_S | \mathcal{H} | B^0 \rangle = -\langle \psi K_S | \mathcal{H} | \overline{B}^0 \rangle \times [1 + \mathcal{O}(\alpha_s \lambda^2)]$$

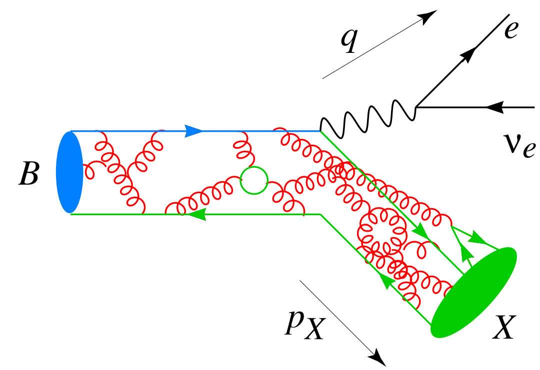


- (2) Effective field theories (separation of scales)

  E.g.: $|V_{cb}|$ and $|V_{ub}|$ from semileptonic B decays

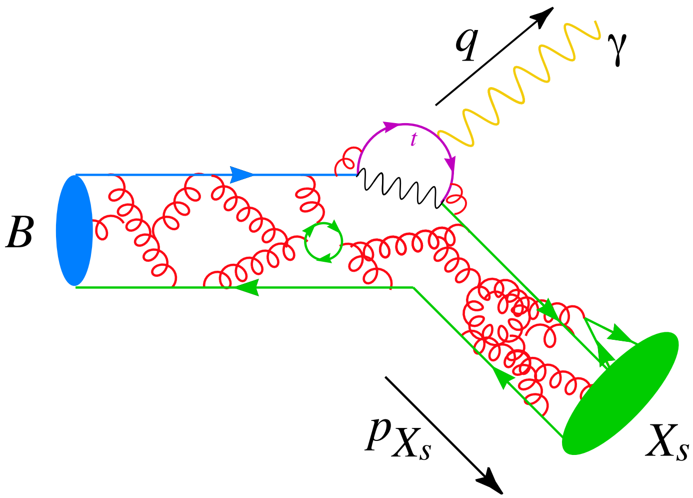
  Solution: Heavy quark expansions

$$\Gamma = |V_{cb}|^2 \times (\text{known factors}) \times [1 + \mathcal{O}(\Lambda_{\rm QCD}^2 / m_b^2)]$$



Many relevant scales: $B \to X_s\gamma$

- Separate physics at: $(m_{t,W} \sim 100\,\text{GeV}) \gg (m_b \sim 5\,\text{GeV}) \gg (\Lambda \sim 0.5\,\text{GeV})$



Inclusive decay:
$X_s = K^*,\ K^{(*)}\pi,\ K^{(*)}\pi\pi$, etc.

Diagrams with many gluons are crucial, resumming certain subset of them affects rate at factor-of-two level

Rate in SM calculated to $< 10\%$, using several effective theories, renormalization group, operator product expansion... one of the most involved SM analyses

- Solution: Short distance dominated (some issues discussed later)

Some caveats

- Lot at stake: theoretical tools for semileptonic and rare decays are the same
  - Measurements of CKM elements
  - Better understanding of hadronic physics improves sensitivity to new physics

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- For today's talk: [strong interaction] model independent

  $\equiv$ theor. uncertainty suppressed by small parameters

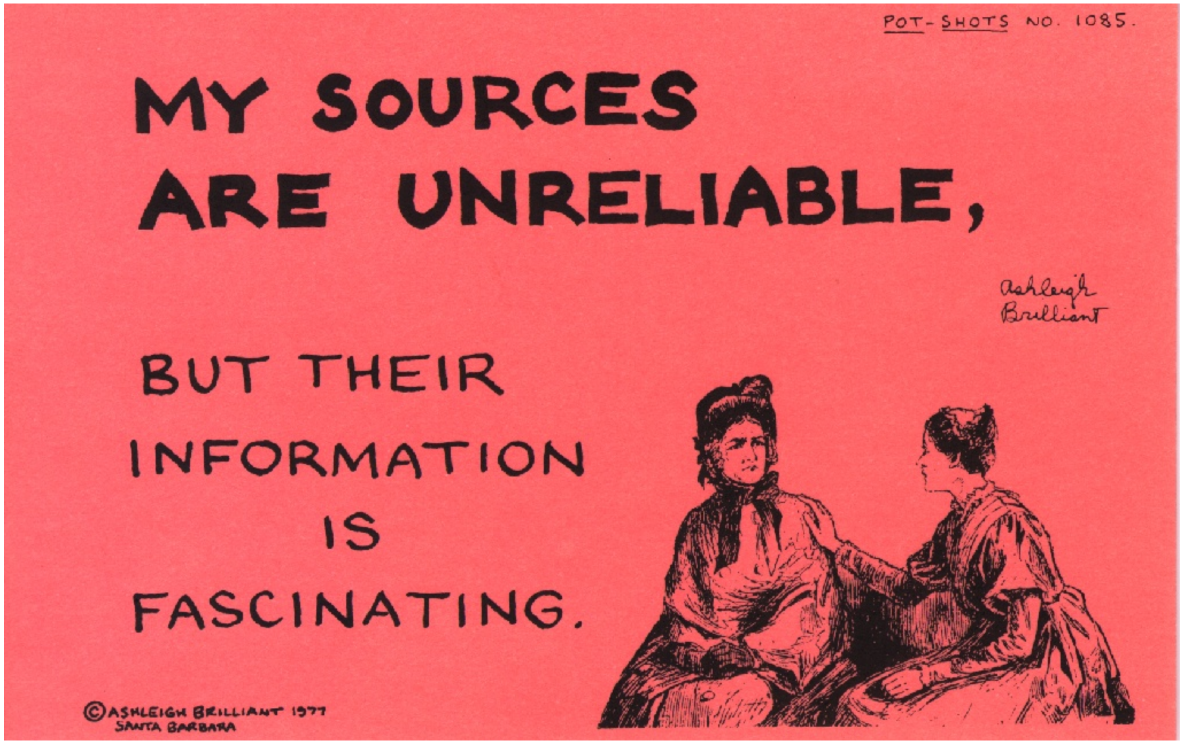
... so theorists argue about $\mathcal{O}(1)\times$(small numbers) instead of $\mathcal{O}(1)$ effects

- Most of the progress have come from expanding in powers of $\Lambda/m_Q,\, \alpha_s(m_Q)$

... a priori not known whether $\Lambda \sim 200\,\mathrm{MeV}$ or $\sim 2\,\mathrm{GeV}$ $(f_\pi, m_\rho, m_K^2/m_s)$

... need experimental guidance to see how well the theory works

To avoid...



The SM shows impressive consistency — separate what's "proven" / "hoped"

Only robust deviations from model independent theory are likely to be interesting

(2σ: 50 theory papers 3σ: 200 theory papers 5σ: strong sign of an effect)

Isospin and $SU(3)$ flavor

Extracting α from $B \to \pi\pi$

- Until $\sim$1997 the hope was to determine α simply from:

$$\frac{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) - \Gamma(B^0(t) \to \pi^+\pi^-)}{\Gamma(\overline{B}^0(t) \to \pi^+\pi^-) + \Gamma(B^0(t) \to \pi^+\pi^-)} = S\sin(\Delta m\, t) - C\cos(\Delta m\, t)$$

$\arg\lambda_{\pi^+\pi^-} = (B\text{-mix} = 2\beta) + (\overline{A}/A = 2\gamma + \ldots) \Rightarrow$ measures $\sin 2\alpha$ if amplitudes with one weak phase dominated — relied on expectation that $P/T =$ small

$\mathcal{B}(B \to K\pi) > \mathcal{B}(B \to \pi\pi) \Rightarrow$ comparable amplitudes with different weak & strong phases (roughly $|P/T| \gtrsim 0.3$)

- Models in which the dominant NP effect is the modification of the $B^0 - \overline{B}^0$ mixing amplitude have $\gamma = \pi - \beta - \alpha$, so reducing the uncertainty of α effectively improves the determination of γ and the bound on NP

$B \to \pi\pi$ — isospin analysis

- Isospin started with (p, n) symmetry, broken by $(m_d - m_u)/\Lambda_{\rm QCD}$
- $(u,\, d)$: I-spin doublet

  other quarks and gluons: $I = 0$

  $\gamma,\, Z$: mixtures of $I = 0, 1$

  $(\pi\pi)_{\ell=0} \;\to\; I_f = 0$ or $I_f = 2$

  (1×1) $\quad (\Delta I = \frac{1}{2})$ $\quad (\Delta I = \frac{3}{2})$

  $I = 0$ final state forbidden by Bose symmetry

- Hamiltonian has two parts: $\Delta I = \frac{1}{2} \;\Rightarrow\; I_f = 0$

  $\Delta I = \frac{3}{2} \;\Rightarrow\; I_f = 2$ ... only two amplitudes

  Note: γ and Z penguins violate isospin and yield some (small) uncertainties

  Experimentally, need all (tagged) rates $+$ time dependent $B \to \pi^+\pi^-$ asymmetry

- Three rates $\overline{B}{}^0 \to \pi^+\pi^-$, $\overline{B}{}^0 \to \pi^0\pi^0$, $B^- \to \pi^0\pi^-$ determine magnitudes and relative strong phase of two amplitudes; similarly for B^0 and B^+ decay

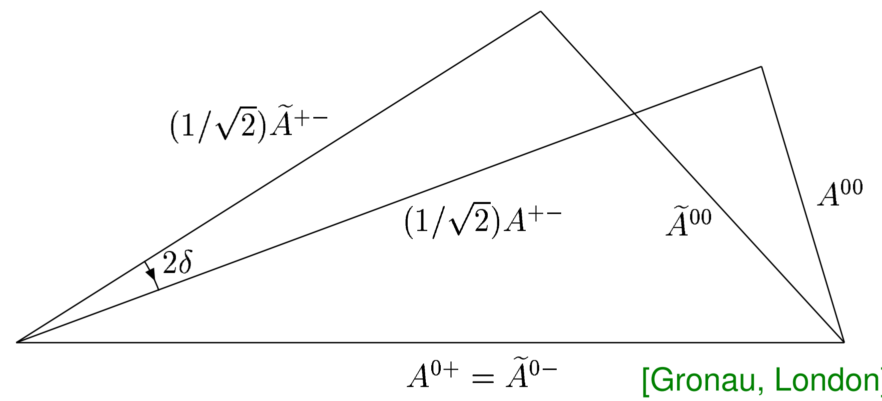
Isospin analysis (cont.)

- Isospin symmetry implies that 6 amplitudes form two triangles with common base

$$\frac{A^{+-}}{\sqrt{2}} + A^{00} = A^{+0}, \qquad \frac{\bar{A}^{+-}}{\sqrt{2}} + \bar{A}^{00} = \bar{A}^{-0}$$

$$|A^{+0}| = |\bar{A}^{-0}|$$

$$A^{+-} \equiv A(B^0 \to \pi^+\pi^-) \qquad \bar{A}^{+-} \equiv A(\bar{B}^0 \to \pi^+\pi^-)$$
$$A^{00} \equiv A(B^0 \to \pi^0\pi^0) \qquad \bar{A}^{00} \equiv A(\bar{B}^0 \to \pi^0\pi^0)$$
$$A^{+0} \equiv A(B^+ \to \pi^+\pi^0) \qquad \bar{A}^{-0} \equiv A(B^- \to \pi^-\pi^0)$$

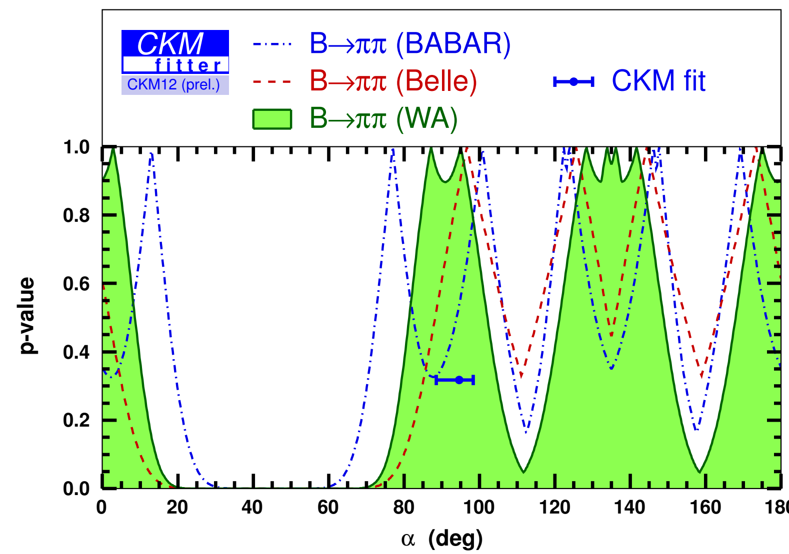


[Gronau, London]

2δ = difference between $\arg \lambda_{\pi^+\pi^-}$ and 2α

- $B \to \rho\pi$: 4 isospin amplitudes $\Rightarrow$ pentagon relations (not used)

Dalitz plot analysis allows considering $\pi^+\pi^-\pi^0$ final state only



$B \to \rho\rho$: the best α at present

- $\rho\rho$ is mixture of CP even/odd (as all VV modes); data: $CP =$ even dominates
  Isospin analysis applies for each L, or in transversity basis for each σ $(= 0, \|, \perp)$

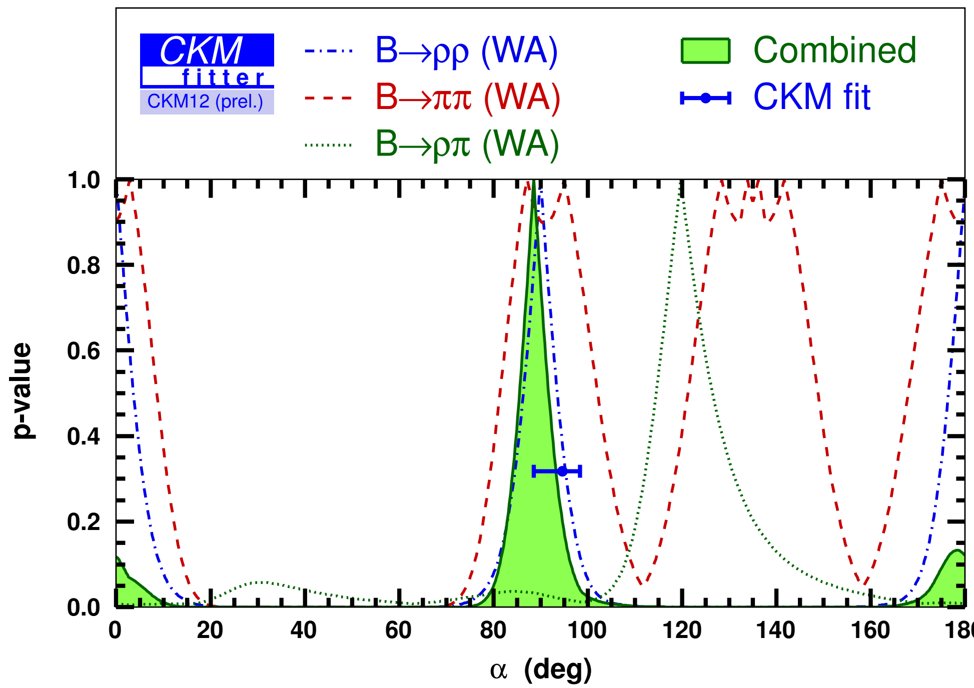
- Small rate: $\mathcal{B}(B \to \rho^0\rho^0) = (0.73 \pm 0.28) \times 10^{-6}$ $\Rightarrow$ small penguin pollution
  $\frac{\mathcal{B}(B\to\pi^0\pi^0)}{\mathcal{B}(B\to\pi^+\pi^0)} \approx 0.35$ vs. $\frac{\mathcal{B}(B\to\rho^0\rho^0)}{\mathcal{B}(B\to\rho^+\rho^0)} \approx 0.03$

- Ultimately, more complicated than $\pi\pi$, $I = 1$ possible due to finite Γ_ρ, giving $\mathcal{O}(\Gamma_\rho^2/m_\rho^2)$ effects [can be constrained]

  $B \to \rho\rho$ isospin analysis: $\alpha = (90 \pm 5)^\circ$

- Also $B \to \rho\pi$ Dalitz plot analysis

- $\rho\rho$ mode dominates α determination for now, may change at a super B factory



Recall: the $B \to K\pi$ puzzle

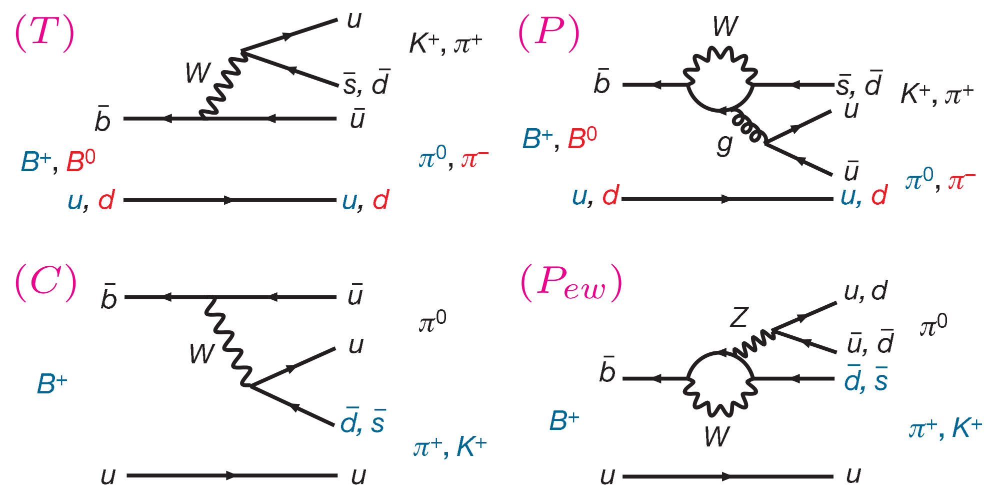
- Have we seen new physics in CPV?

$$A_{K^+\pi^-} = -0.098 \pm 0.012 \quad (P+T)$$

$$A_{K^+\pi^0} = 0.050 \pm 0.025 \quad (P+T+C+A+P_{ew})$$

What's the reason for large difference?

$$A_{K^+\pi^0} - A_{K^+\pi^-} = 0.148 \pm 0.028$$



(Annihilation not shown)

[Belle, Nature 452, 332 (2008)]

- SCET / factorization predicts: $\arg(C/T) = \mathcal{O}(\Lambda_{QCD}/m_b)$ and $A + P_{ew}$ small

This makes it hard to understand above data:

– P and T: nonzero relative strong and weak phases to give $A_{K^+\pi^-}$

– T and C: same weak phase and predicted to have small relative strong phase

- Huge fluctuations? Breakdown of $1/m$ exp.? Missing something subtle? BSM?

Flavor $SU(3)$ — a timely example

- First observation of B_s CPV: $A_{CP}(B_s^0 \to K^-\pi^+) = 0.27 \pm 0.04$ [LHCb, arXiv:1304.6173]
- Compare: $B_d^0 \to K^+\pi^-$ $(\bar b \to \bar s q \bar q)$ vs. $B_s^0 \to K^-\pi^+$ $(\bar b \to \bar d q \bar q)$

  Can use U-spin $(d \leftrightarrow s)$ subgroup of $SU(3)$: (B_d, B_s), $\mathcal{H}$, (π^+, K^+), (π^-, K^-)
  final state $U = 0, 1 \Rightarrow$ two reduced matrix elements

$$A(B_d^0 \to K^+\pi^-) = V_{cb}^* V_{cs}\,(P_c - P_t + T_{c\bar c s}) + V_{ub}^* V_{us}\,(T_{u\bar u s} + P_u - P_t) \equiv P + T$$

$$A(B_s^0 \to K^-\pi^+) = V_{cb}^* V_{cd}\,(P_c - P_t + T_{c\bar c s}) + V_{ub}^* V_{ud}\,(T_{u\bar u s} + P_u - P_t) = -\lambda\, P + \lambda^{-1}\, T$$

- LHCb quotes the $SU(3)$ relation:

$$\Delta \equiv \frac{\mathcal{A}_{\rm CP}(B_d \to K^+\pi^-)}{\mathcal{A}_{\rm CP}(B_s \to K^-\pi^+)} + \frac{\mathcal{B}(B_s \to K^-\pi^+)}{\mathcal{B}(B_d \to K^+\pi^-)}\,\frac{\tau_d}{\tau_s} = -0.02 \pm 0.05 \pm 0.04$$

- Looks obscure — where does this come from?

Flavor $SU(3)$ vs factorization at $\mu \sim m_b$

- Saw yesterday: $\Gamma_{\rm CP}(i \to f) \equiv \Gamma(i \to f) - \Gamma(\bar{i} \to \bar{f}) = 4 A_1 A_2 \,\sin(\phi_1 - \phi_2)\, \sin(\delta_1 - \delta_2)$

  Define: $$\widetilde{\Delta} \equiv \frac{\Gamma_{\rm CP}(B_d \to K^+\pi^-) + \Gamma_{\rm CP}(B_s \to K^-\pi^+)}{\Gamma_{\rm CP}(B_d \to K^+\pi^-) - \Gamma_{\rm CP}(B_s \to K^-\pi^+)} = 0.01 \pm 0.11$$

  In fact $\widetilde{\Delta} = 0 +$ typical size of $SU(3)$ breaking, whereas Δ depends also on $|P/T|$

- Using factorization $\widetilde{\Delta} \ll 1$ iff: $F_{B\to\pi}\, f_K \approx F_{B_s\to K}\, f_\pi$

  Need $B_s \to K$ form factor from LQCD (extract $|V_{ub}|$ at LHCb from $B_s \to K^+\mu^-\nu$?)

  Similar $SU(3)$ relations: $B_s^0 \to K^+K^- \longleftrightarrow B_d^0 \to \pi^+\pi^-$

  $$B_s^0 \to \pi^+\pi^- \longleftrightarrow B_d^0 \to K^+K^-$$

- Which relation works well will help answer what's at play [Grossman, ZL, Robinson, to appear]

Heavy quark symmetry

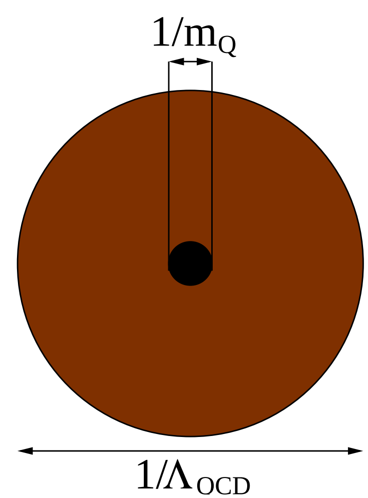
Heavy quark symmetry

- $Q\,\overline{Q}$: positronium-type bound state, perturbative in the $m_Q \gg \Lambda_{\rm QCD}$ limit
- $Q\,\bar{q}$: wave function of the light degrees of freedom
  ("brown muck") insensitive to spin and flavor of Q

  B meson is a lot more complicated than just a $b\,\bar{q}$ pair

  In the $m_Q \gg \Lambda_{\rm QCD}$ limit, the heavy quark acts as a static color source with fixed four-velocity v^μ

  $\Rightarrow$ $SU(2n)$ heavy quark spin-flavor symmetry at fixed v^μ



- Similar to atomic physics: $(m_e \ll m_N)$

1. Flavor symmetry $\sim$ isotopes have similar chemistry [Ψ_e independent of m_N]
2. Spin symmetry $\sim$ hyperfine levels almost degenerate [$\vec{s}_e - \vec{s}_N$ interaction $\to 0$]

Spectroscopy of heavy-light mesons

- In $m_Q \gg \Lambda_{\rm QCD}$ limit, spin of the heavy quark is a good quantum number, and so is the spin of the light d.o.f., since $\vec{J} = \vec{s}_Q + \vec{s}_l$ and

$$\left.\begin{array}{r} \text{angular momentum conservation: } [\vec{J}, \mathcal{H}] = 0 \\ \text{heavy quark symmetry: } [\vec{s}_Q, \mathcal{H}] = 0 \end{array}\right\} \Rightarrow [\vec{s}_l, \mathcal{H}] = 0$$

- For a given s_l, two degenerate states:

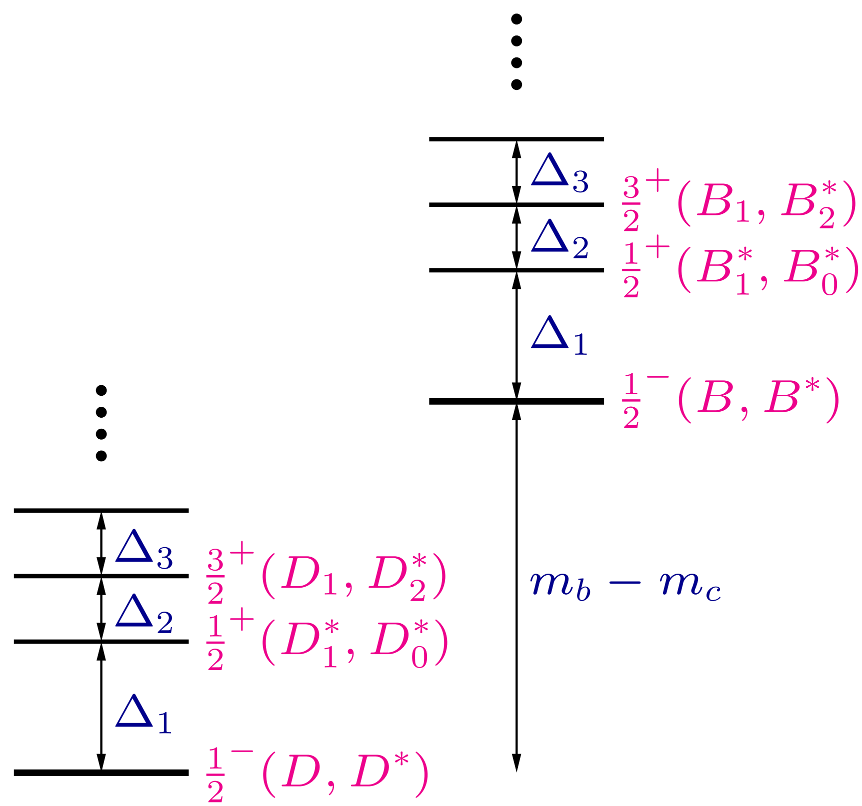
$$J_\pm = s_l \pm \tfrac{1}{2}$$

$\Rightarrow \Delta_i = \mathcal{O}(\Lambda_{\rm QCD})$ — same in B and D sector

Doublets are split by order $\Lambda_{\rm QCD}^2/m_Q$, e.g.:

$m_{D^*} - m_D \simeq 140$ MeV

$m_{B^*} - m_B \simeq 45$ MeV



Aside: a puzzle

- Vector–pseudoscalar mass splitting is $\propto 1/m_Q \Rightarrow m_V^2 - m_P^2 =$ const.

Experimentally:

$$m_{B^*}^2 - m_B^2 = 0.49\,\mathrm{GeV}^2 \qquad m_{B_s^*}^2 - m_{B_s}^2 = 0.50\,\mathrm{GeV}^2$$

$$m_{D^*}^2 - m_D^2 = 0.54\,\mathrm{GeV}^2 \qquad m_{D_s^*}^2 - m_{D_s}^2 = 0.58\,\mathrm{GeV}^2$$

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$$m_{K^*}^2 - m_K^2 = 0.55\,\text{GeV}^2$$

$$m_\rho^2 - m_\pi^2 = 0.57\,\text{GeV}^2$$

- The HQS argument relies on $m_Q \gg \Lambda_{\text{QCD}}$, so something more has to go on...

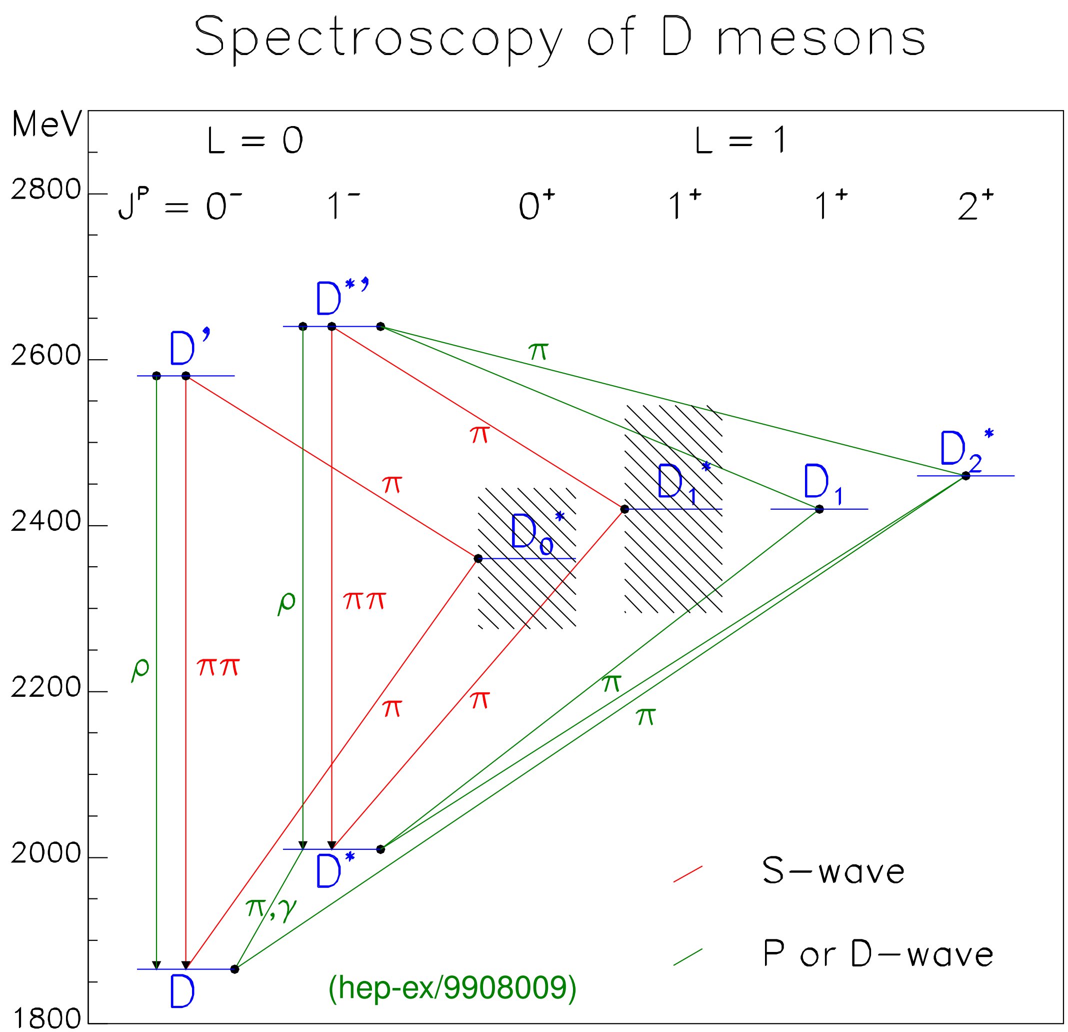
- It is not only important to test how a theory works, but also how it breaks down!

[An approximation should work when it the expansion parameter is small, and fail when it's $\mathcal{O}(1)$]

Successes in charm spectrum

- D_1 narrow width:
  S-wave $D_1 \to D^* \pi$ allowed by angular momentum conservation, but forbidden in the $m_Q \to \infty$ limit by heavy quark spin symmetry

- Mass splittings of orbitally excited states is small:
  $$m_{D_2^*} - m_{D_1} = 37\,\mathrm{MeV} \ll m_{D^*} - m_D$$
  vanishes in the quark model, since it arise from $\langle \vec{s}_Q \cdot \vec{s}_{\bar{q}}\, \delta^3(\vec{r}) \rangle$



CERN

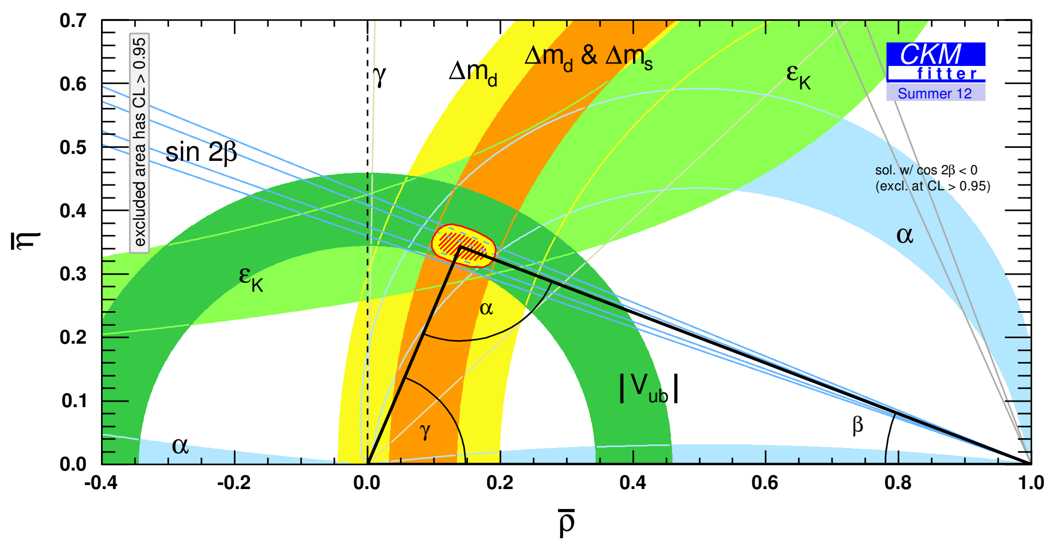
Berkeley Lab

Semileptonic and rare B decays

- $|V_{ub}|$ is the dominant uncertainty of the side of the UT opposite to β

  $|V_{ub}|$ is crucial for comparing tree-dominated and loop-mediated processes

- Error of $|V_{cb}|$ is a large part of the uncertainty in the ϵ_K constraint, and in $K \to \pi\nu\bar{\nu}$ when it's measured



Rare $b \to s\gamma$, $s\,\ell^+\ell^-$, and $s\,\nu\bar{\nu}$ decays are sensitive probes of the Standard Model

Exclusive $B \to D^{(*)}\ell\bar{\nu}$ decay

- In the $m_{b,c} \gg \Lambda_{\rm QCD}$ limit, configuration of brown muck only depends on the four-velocity of the heavy quark, but not on its mass and spin

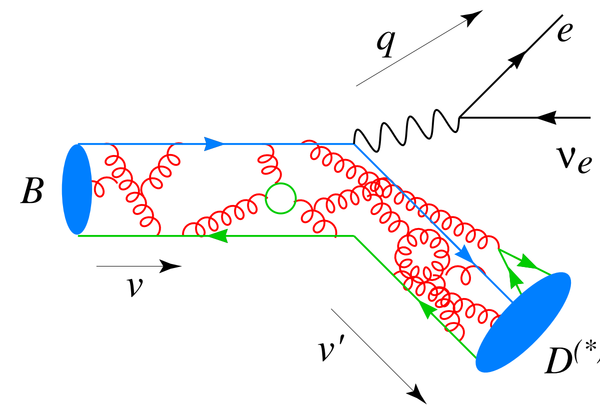
- On a time scale $\ll \Lambda_{\rm QCD}^{-1}$ weak current changes $b \to c$
  i.e.: $\vec{p}_b \to \vec{p}_c$ and possibly $\vec{s}_Q$ flips

  In $m_{b,c} \gg \Lambda_{\rm QCD}$ limit brown muck only feels $v_b \to v_c$

  Form factors independent of Dirac structure of weak current $\Rightarrow$ all form factors related to a single function of $w = v \cdot v'$, the Isgur-Wise function, $\xi(w)$

  $\Uparrow$

  Contains all nonperturbative low-energy hadronic physics



- $\xi(1) = 1$, because at "zero recoil" configuration of brown muck not changed at all

$B \to D^{(*)}\ell\bar{\nu}$ form factors

- Lorentz invariance $\Rightarrow$ 6 form factors

$$\langle D(v')|V_\nu|B(v)\rangle = \sqrt{m_B m_D}\,\left[h_+\,(v+v')_\nu + h_-\,(v-v')_\nu\right]$$

$$\langle D^*(v')|V_\nu|B(v)\rangle = i\sqrt{m_B m_{D^*}}\,h_V\,\epsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha}v'^\beta v^\gamma$$

$$\langle D(v')|A_\nu|B(v)\rangle = 0$$

$$\langle D^*(v')|A_\nu|B(v)\rangle = \sqrt{m_B m_{D^*}}\,\left[h_{A_1}\,(w+1)\epsilon^*_\nu - h_{A_2}\,(\epsilon^*\cdot v)v_\nu - h_{A_3}\,(\epsilon^*\cdot v)v'_\nu\right]$$

$V_\nu = \bar{c}\gamma_\nu b$, $A_\nu = \bar{c}\gamma_\nu\gamma_5 b$, $w \equiv v\cdot v' = \dfrac{m_B^2 + m_D^2 - q^2}{2m_B m_D}$, and $h_i = h_i(w,\mu)$

- In $m_Q \gg \Lambda_{\rm QCD}$ limit, up to corrections suppressed by α_s and $\Lambda_{\rm QCD}/m_{c,b}$

$$h_- = h_{A_2} = 0\,, \qquad h_+ = h_V = h_{A_1} = h_{A_3} = \xi(w)$$

$\uparrow$ Isgur-Wise function

The α_s corrections are calculable

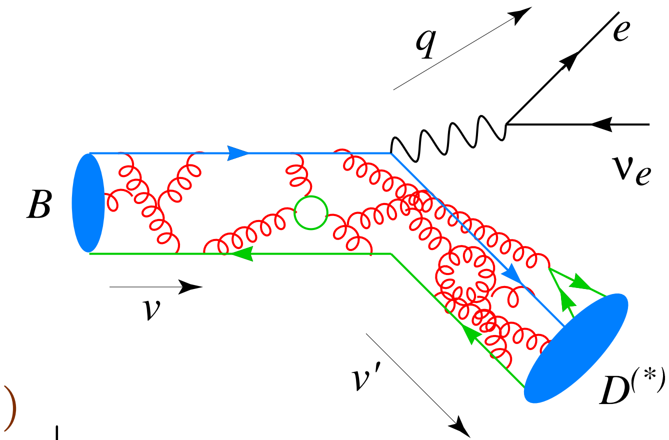
$\Lambda_{\rm QCD}/m_{c,b}$ corrections is where model dependence enters

$|V_{cb}|$ from $B \to D^{(*)}\ell\bar{\nu}$

- Extract $|V_{cb}|$ from $w \equiv v \cdot v' = (m_B^2 + m_D^2 - q^2)/(2 m_B m_D) \to 1$ limit of the rate

$$\frac{d\Gamma(B \to D^{(*)}\ell\bar{\nu})}{dw} = (\dots)\,(w^2-1)^{3/2(1/2)}\,|V_{cb}|^2\,\mathcal{F}^2_{(*)}(w)$$

$\nwarrow w \equiv v \cdot v'$ Isgur-Wise function $+ \dots \nearrow$



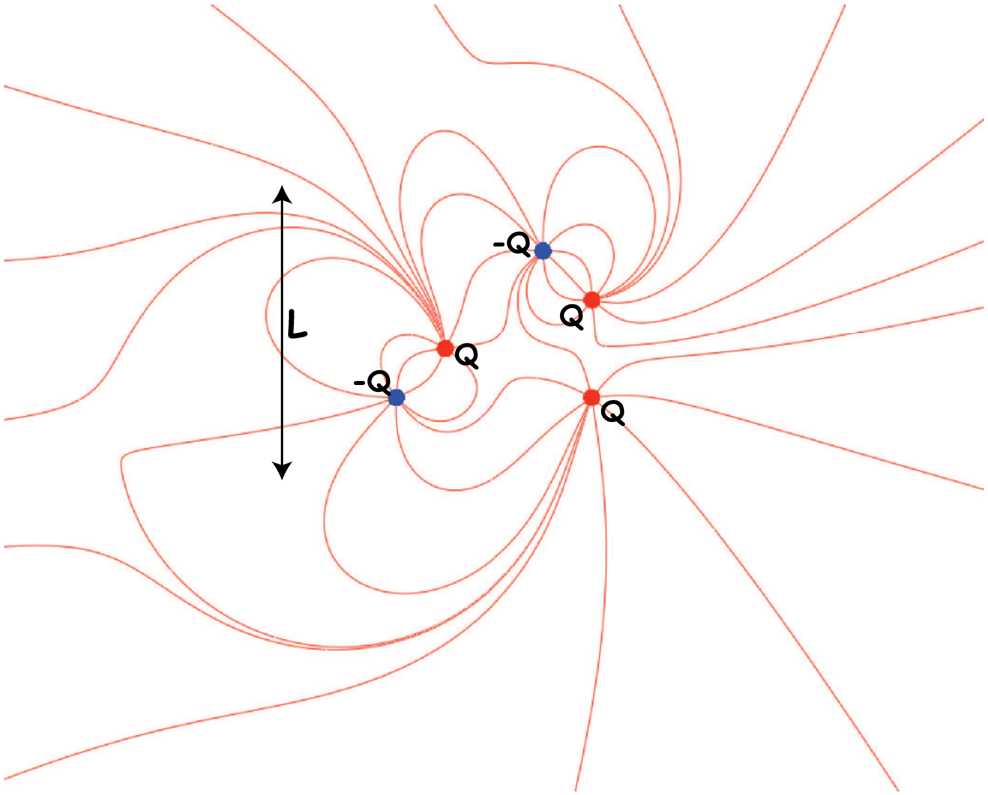
$$\mathcal{F}(1) = 1_{\rm Isgur-Wise} + 0.02_{\alpha_s,\alpha_s^2} + \frac{\text{(lattice or models)}}{m_{c,b}} + \dots$$

$$\mathcal{F}_*(1) = 1_{\rm Isgur-Wise} - 0.04_{\alpha_s,\alpha_s^2} + \frac{0_{\rm Luke}}{m_{c,b}} + \frac{\text{(lattice or models)}}{m_{c,b}^2} + \dots$$

- Lattice QCD: $\mathcal{F}_*(1) = 0.921 \pm 0.024,\ \ \mathcal{F}(1) = 1.074 \pm 0.024$ [arXiv:0808.2519, hep-lat/0409116]
- Need constraints on shape to fit [Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert]
- Need some understanding of decays to higher mass X_c states (backgrounds)
- Data: $|V_{cb}\,\mathcal{F}_*(1)| = (35.75 \pm 0.42) \times 10^{-3},\ \ |V_{cb}\,\mathcal{F}(1)| = (42.3 \pm 1.5) \times 10^{-3}$ [HFAG]
  [note: χ^2/dof $= 39.6/21\ (56.9/21)$, CL $= 0.8\%\ (4\text{E}-5)$]

Heavy quark expansion

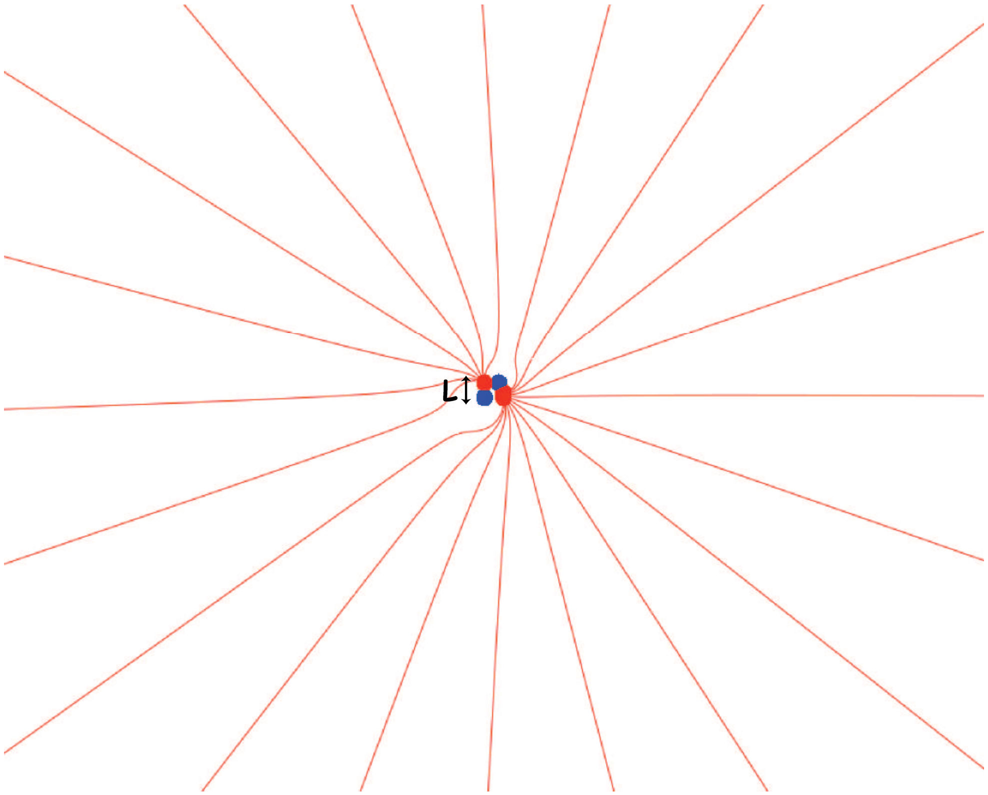
The multipole expansion



Physics at $r \sim L$ is complicated

Depends on the details of the charge distribution

The multipole expansion



Physics at $r \gg L$ is much simpler

Charge distribution characterized by total charge, q

Details suppressed by powers of L/r, and can be parameterized in terms of $p_i,\ Q_{ij},\ \ldots$

Simplifications occur due to separating physics at different distance scales

- Complicated charge distribution can be replaced by a point source with additional interactions (multipoles) — underlying idea of effective theories

The multipole expansion (cont.)

- Potential: $$V(x) = q\,\frac{1}{r} + p_i\,\frac{x_i}{r^3} + \frac{1}{2}\,Q_{ij}\,\frac{x_i x_j}{r^5} + \dots$$

  Short distance quantities: $q = \int \rho(x)\, \mathrm{d}^3x, \quad p_i = \int x_i\, \rho(x)\, \mathrm{d}^3x,$ etc.

  Long distance quantities: $\left\langle \frac{1}{r} \right\rangle, \quad \left\langle \frac{x_i}{r^3} \right\rangle, \quad \left\langle \frac{x_i x_j}{r^5} \right\rangle,$ etc.

- Higher multipoles: new interactions from "integrating out" short distance physics
- Useful tool independent of the fact whether we know the underlying theory or not

---

- Any theory at momentum $p \ll M$ can be described by an effective Hamiltonian

$$H_{\text{eff}} = H_0 + \sum_i \frac{C_i}{M^{n_i}}\, O_i$$

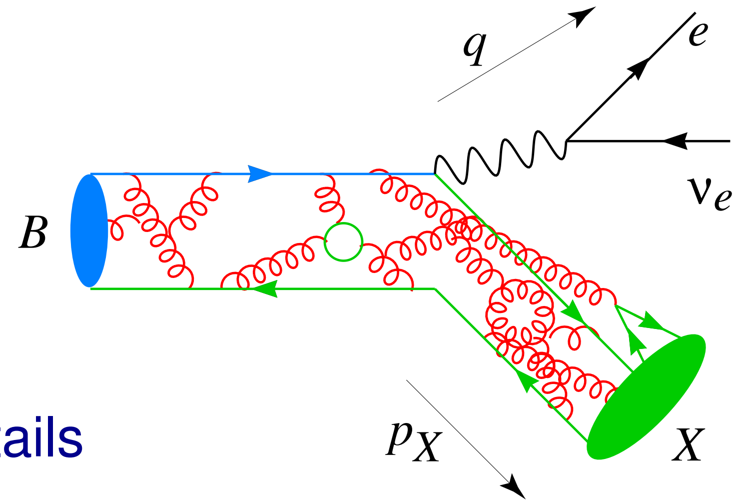
$M \to \infty$ limit $+$ corrections with well-defined power counting
H_0 may have more symmetries than full theory at nonzero p/M
Can work to higher orders in p/M; can sum logs of p/M

Inclusive heavy hadron decays

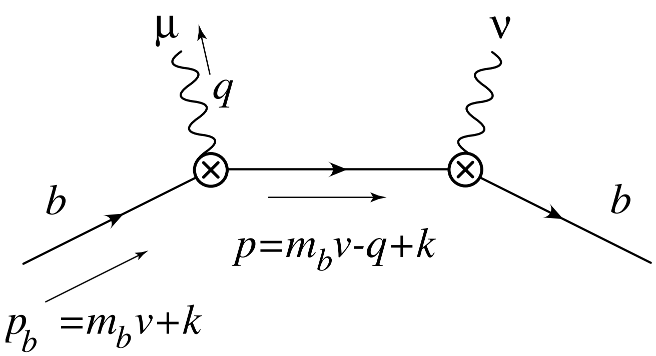
- Sum over hadronic final states, subject to constraints determined by short distance physics

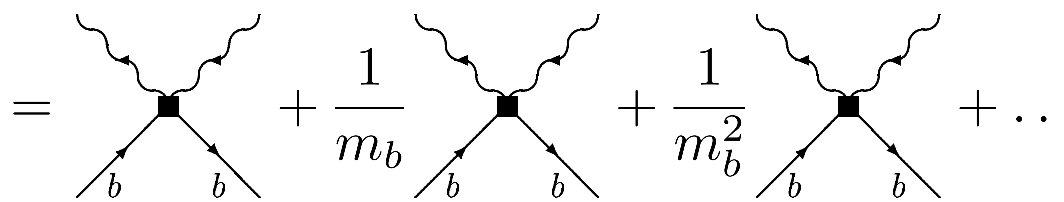
  Decay: short distance (calculable)

  Hadronization: long distance (nonperturbative), but probability to hadronize is unity; sum over details



- Optical theorem + operator product expansion (OPE) + heavy quark symmetry





$\sim$ field theoretic version of multipole expansion

Can think of the OPE as expansion of forward scattering amplitude in $k \sim \Lambda_{\rm QCD}$

Operator product expansion

- Consider semileptonic $b \to u$ decay: $O_{bu} = -\frac{4G_F}{\sqrt{2}} V_{ub} \underbrace{(\overline{u}\,\gamma^\mu P_L\, b)}_{J^\mu_{bu}} \underbrace{(\overline{\ell}\,\gamma_\mu P_L\, \nu)}_{J_{\ell\nu}}$

  Decay rate: $\Gamma(B \to X_u \ell \bar\nu) \sim \sum_{X_c} \int \mathrm{d}[\mathrm{PS}]\, \big|\langle X_u \ell \bar\nu | O_{bu} | B\rangle\big|^2$

  Factor to: $B \to X_u W^*$ and $W^* \to \ell\bar\nu$, concentrate on hadronic part

$$W^{\mu\nu} \sim \sum_{X_c} \delta^4(p_B - q - p_X)\, \big|\langle B | J^{\mu\dagger}_{bu} | X_u\rangle\, \langle X_u | J^\nu_{bu} | B\rangle\big|^2 = \mathrm{Im}\, T^{\mu\nu}$$

(optical theorem) $\quad T^{\mu\nu} = i \int \mathrm{d}x\, e^{-iq\cdot x}\, \langle B|\, T\big\{J^{\mu\dagger}_{bu}(x)\, J^\nu_{bu}(0)\big\}\, |B\rangle$

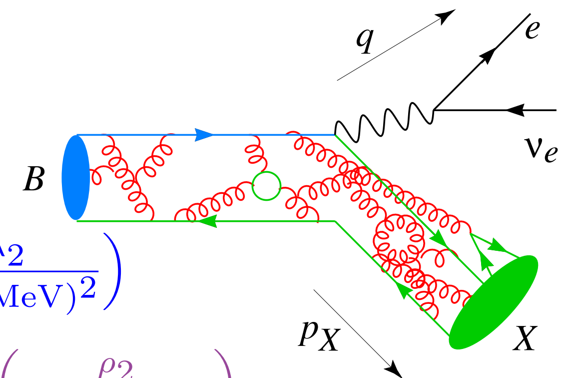
- Operators: $\bar b\, b \to$ free quark decay, $\langle \bar b D^2 b\rangle$, $\langle \bar b \sigma_{\mu\nu} G^{\mu\nu} b\rangle \sim m^2_{B^*} - m^2_B$, etc.

$$\mathrm{d}\Gamma = \begin{pmatrix} b\ \text{quark} \\ \text{decay} \end{pmatrix} \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \ldots + \alpha_s(\ldots) + \alpha_s^2(\ldots) + \ldots \right\}$$

- As for $e^+e^- \to$ hadrons, question is when perturbative calculation can be trusted

Classic application: inclusive $|V_{cb}|$

- Want to determine $|V_{cb}|$ from $B \to X_c \ell \bar\nu$:



$$\Gamma(B \to X_c \ell \bar\nu) = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} (4.7\,\mathrm{GeV})^5\,(0.534) \times$$

$$\Big[1 - 0.22\left(\frac{\Lambda_{1S}}{500\,\mathrm{MeV}}\right) - 0.011\left(\frac{\Lambda_{1S}}{500\,\mathrm{MeV}}\right)^2 - 0.052\left(\frac{\lambda_1}{(500\,\mathrm{MeV})^2}\right) - 0.071\left(\frac{\lambda_2}{(500\,\mathrm{MeV})^2}\right)$$

$$- 0.006\left(\frac{\lambda_1\Lambda_{1S}}{(500\,\mathrm{MeV})^3}\right) + 0.011\left(\frac{\lambda_2\Lambda_{1S}}{(500\,\mathrm{MeV})^3}\right) - 0.006\left(\frac{\rho_1}{(500\,\mathrm{MeV})^3}\right) + 0.008\left(\frac{\rho_2}{(500\,\mathrm{MeV})^3}\right)$$

$$+ 0.011\left(\frac{T_1}{(500\,\mathrm{MeV})^3}\right) + 0.002\left(\frac{T_2}{(500\,\mathrm{MeV})^3}\right) - 0.017\left(\frac{T_3}{(500\,\mathrm{MeV})^3}\right) - 0.008\left(\frac{T_4}{(500\,\mathrm{MeV})^3}\right)$$

$$+ 0.096\epsilon - 0.030\epsilon^2_{\mathrm{BLM}} + 0.015\epsilon\left(\frac{\Lambda_{1S}}{500\,\mathrm{MeV}}\right) + \ldots\Big]$$

Corrections: $\mathcal{O}(\Lambda/m)$: $\sim 20\%$, $\mathcal{O}(\Lambda^2/m^2)$: $\sim 5\%$, $\mathcal{O}(\Lambda^3/m^3)$: $\sim 1-2\%$, $\mathcal{O}(\alpha_s)$: $\sim 10\%$, Unknown terms: $< 1-2\%$

Matrix elements extracted from shape variables — good fit to lots of data

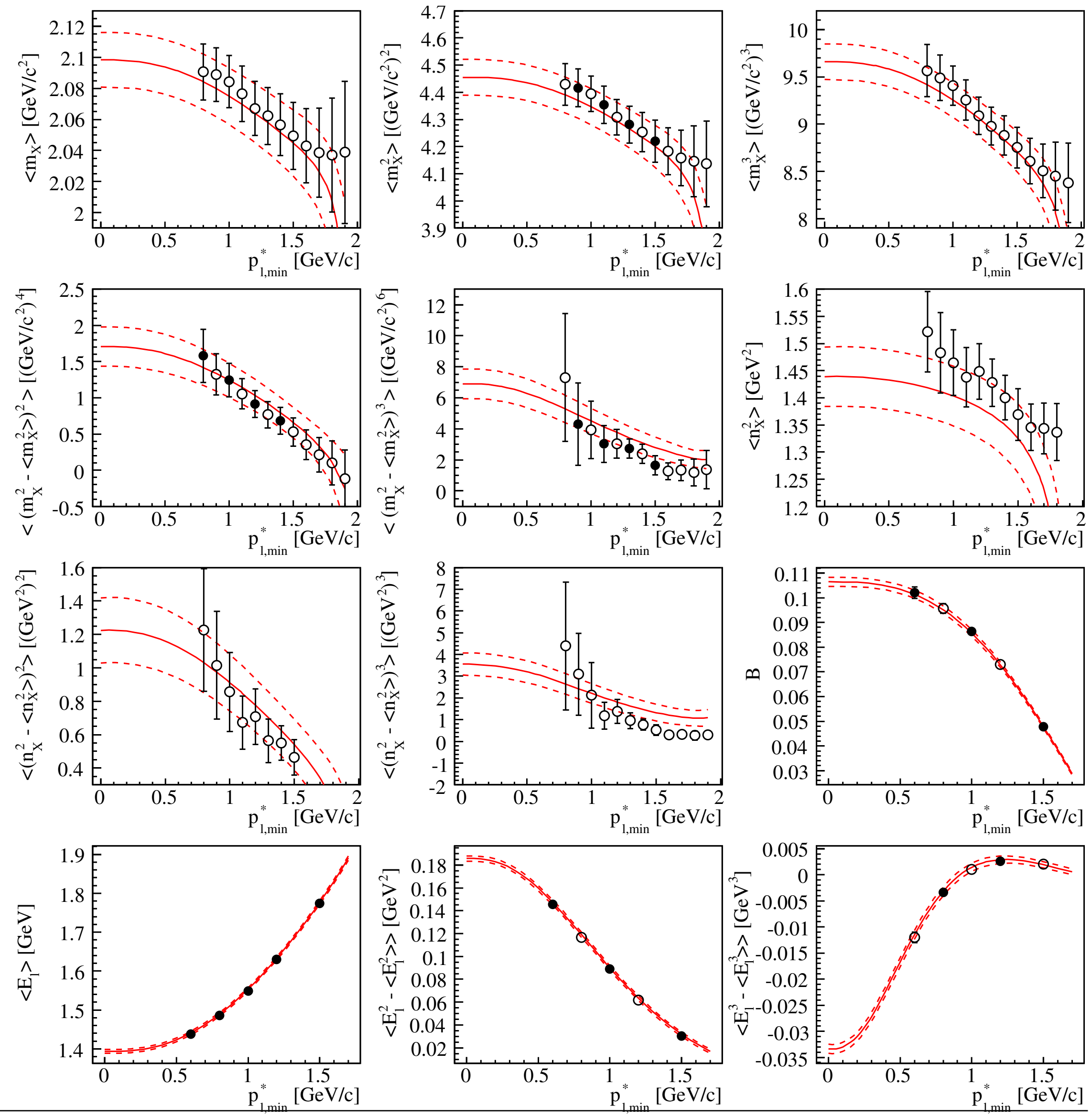
- Error of $|V_{cb}| \sim 2\%$ — a precision field; uncomfortable $\sim 2\sigma$ tension with exclusive

The data...

- Reasonably good fits

  No evidence for deviations from quark-hadron duality

[BaBar, arXiv:0908.0415, similar results from Belle]

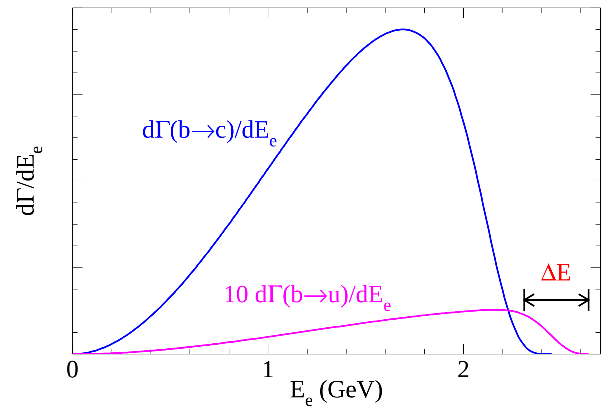


The challenge of $|V_{ub}|$ measurements

- Side opposite to β; precision crucial to be sensitive to NP in $\sin 2\beta$ via mixing

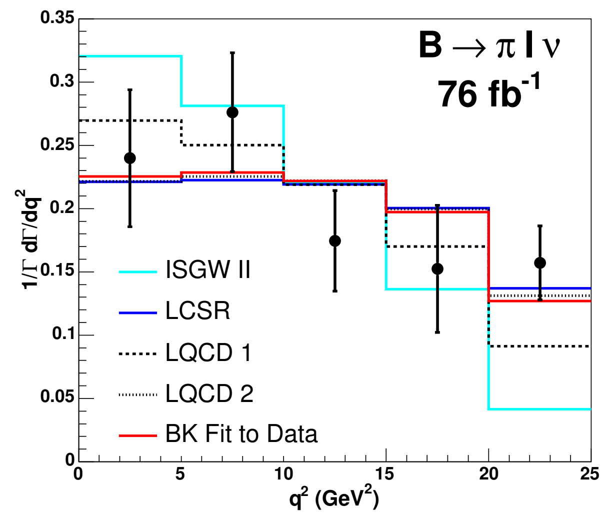
- Inclusive: rate known to $\sim 5\%$; cuts to remove $B \to X_c \ell \bar{\nu}$ introduce small parameters that complicate expansions

  Nonperturbative b distribution function ("shape function") determines tails (e.g., shifts endpoint $\frac{1}{2} m_b \to \frac{1}{2} m_B$)
  $\Rightarrow$ related to $B \to X_s \gamma$ photon spectrum



- Exclusive: $$\frac{d\Gamma(\overline{B}^0 \to \pi^+ \ell \bar{\nu})}{dq^2} = \frac{G_F^2 |\vec{p}_\pi|^3}{24\pi^3} |V_{ub}|^2 \, |f_+(q^2)|^2$$

  Tools: Lattice QCD, under control at large q^2 (small $|\vec{p}_\pi|$)
  Dispersion rel: constrain shape using few $f_+(q^2)$ values

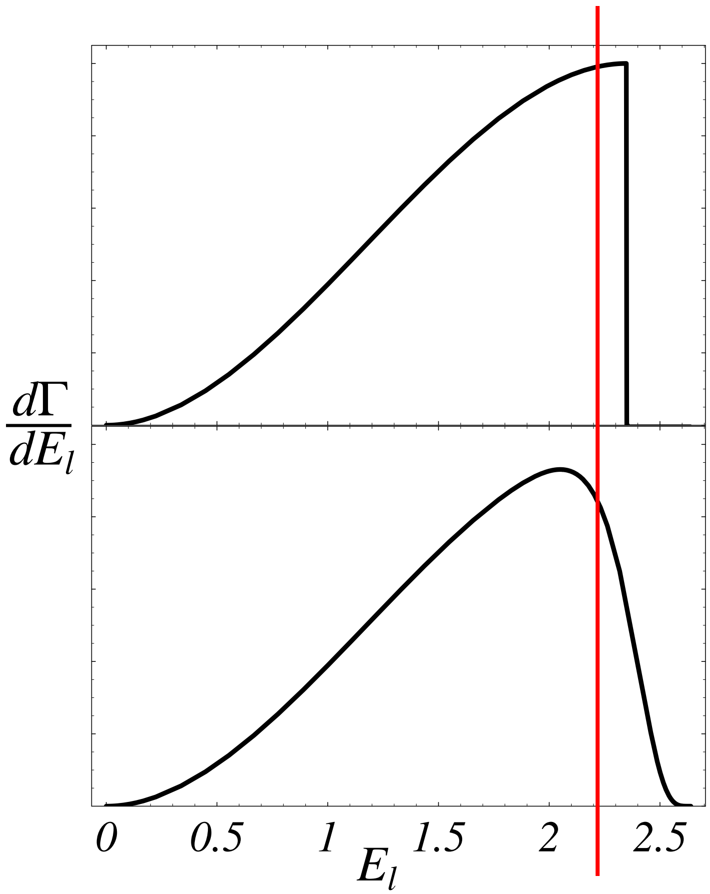


- Many challenging open questions, active areas to date

Shape function: lepton endpoint vs. $B \to X_s \gamma$

b quark decay spectrum

with a model for b quark PDF



Shape function: lepton endpoint vs. $B \to X_s\gamma$

b quark decay spectrum

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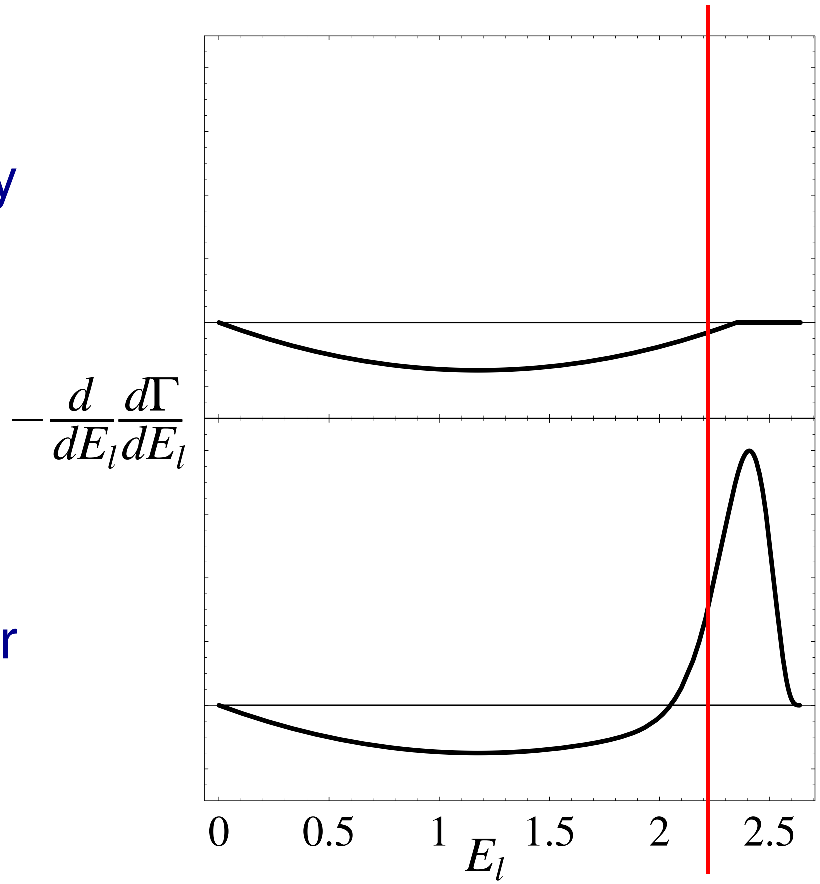

Shape function: lepton endpoint vs. $B \to X_s \gamma$

b quark decay spectrum

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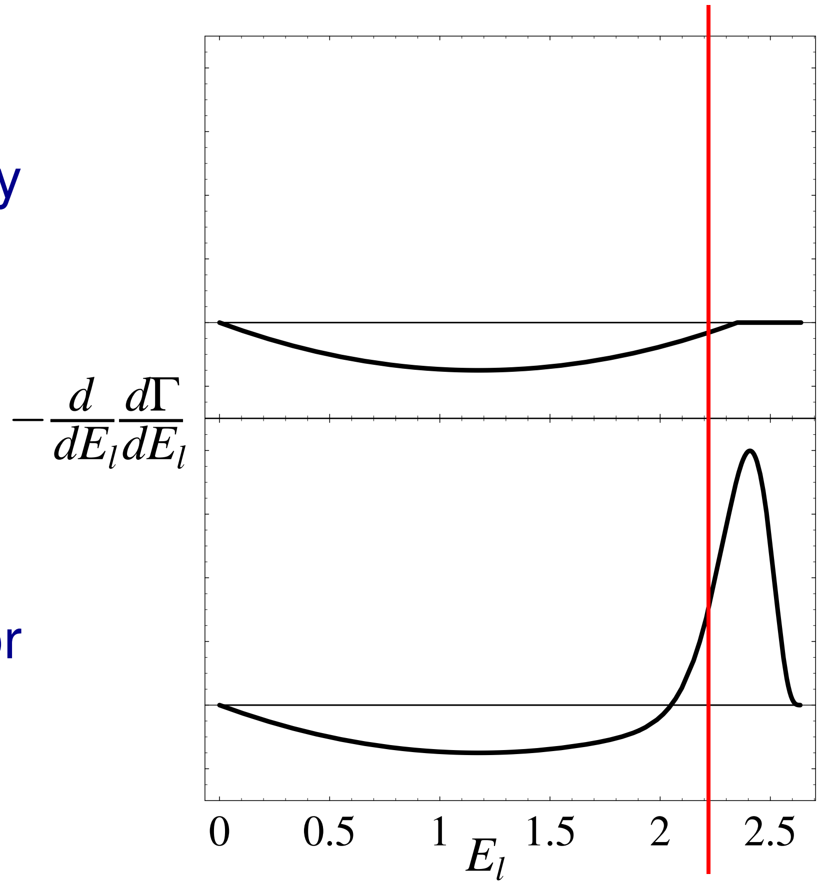
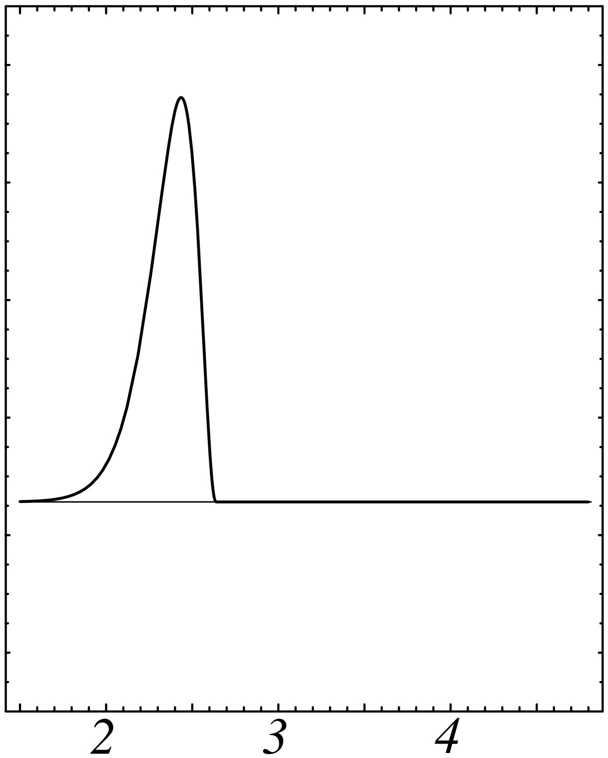


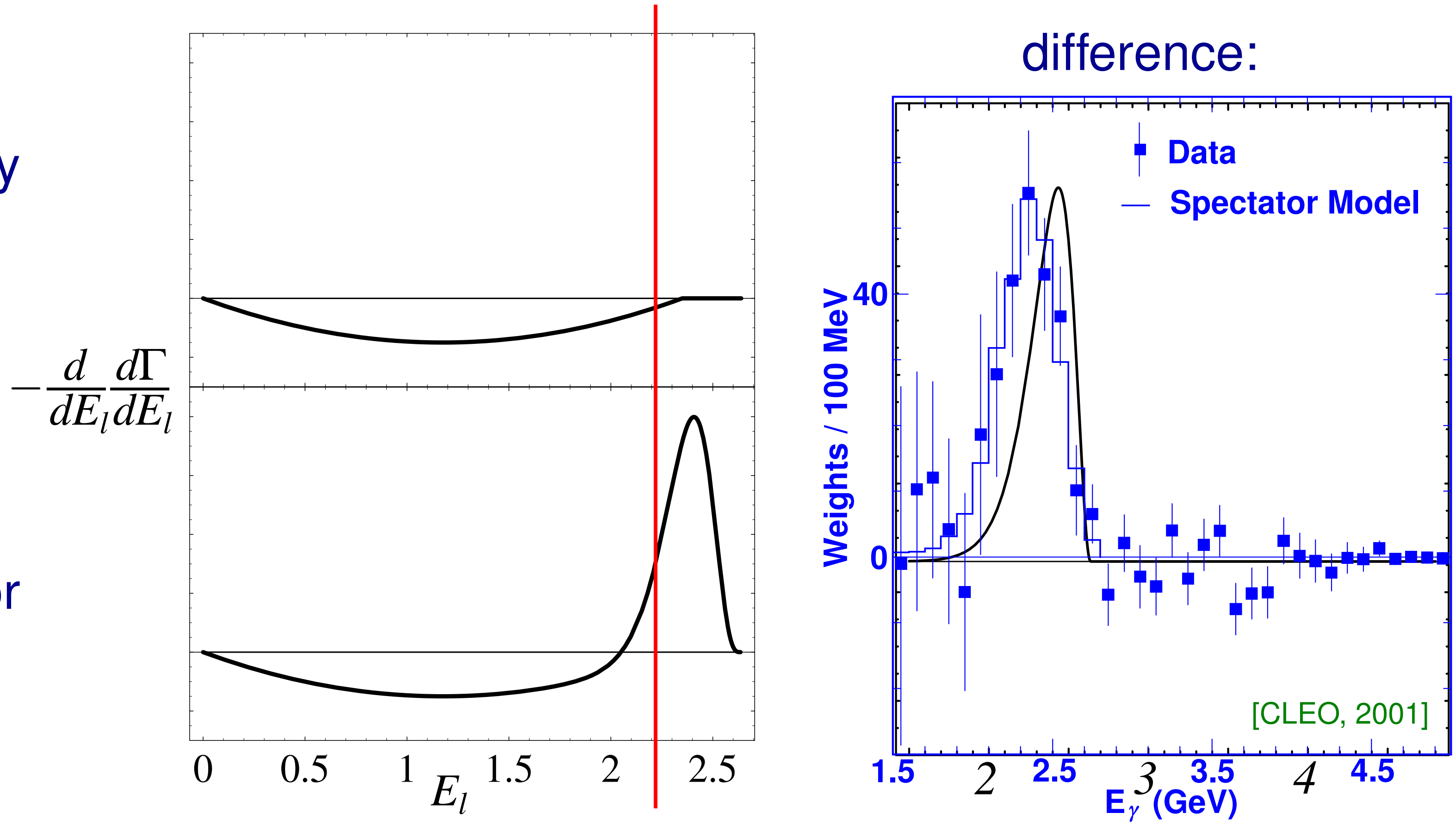
difference:



Shape function: lepton endpoint vs. $B \to X_s\gamma$

b quark decay spectrum

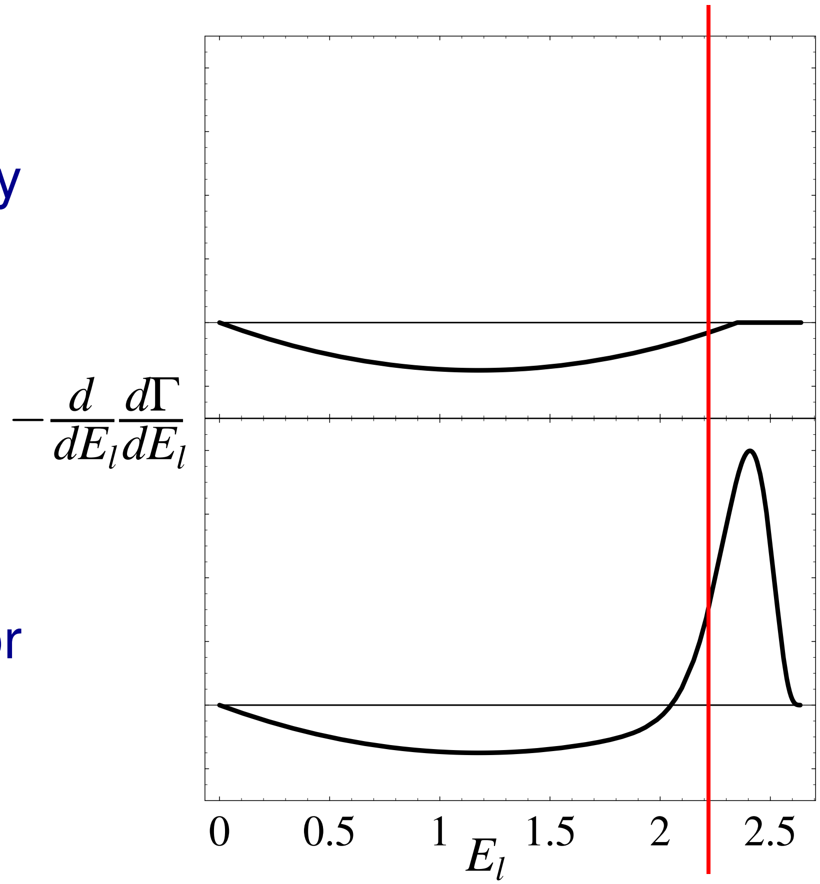
with a model for b quark PDF



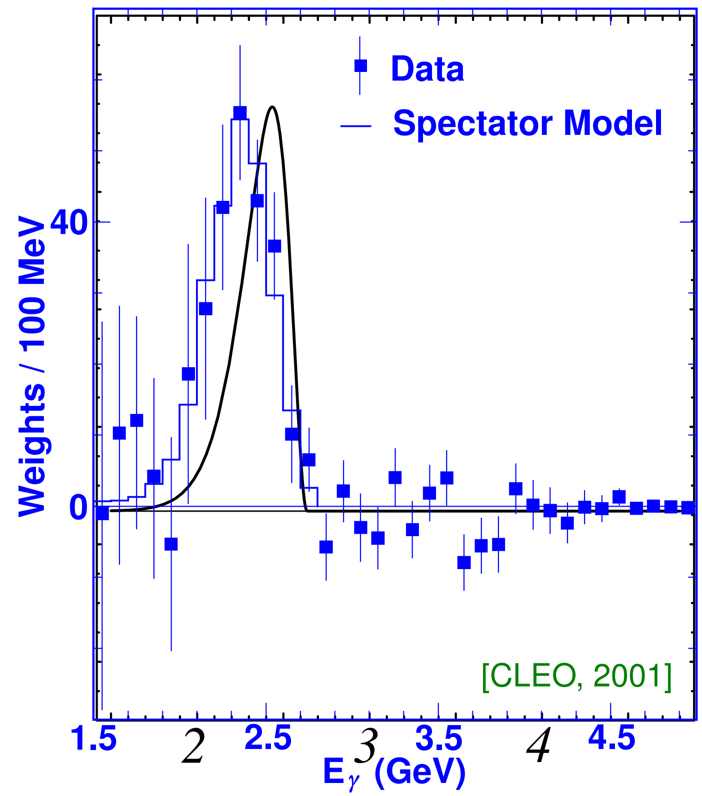
Shape function: lepton endpoint vs. $B \to X_s\gamma$

b quark decay spectrum

with a model for b quark PDF



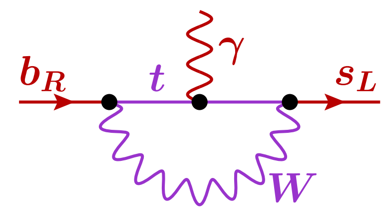
difference:



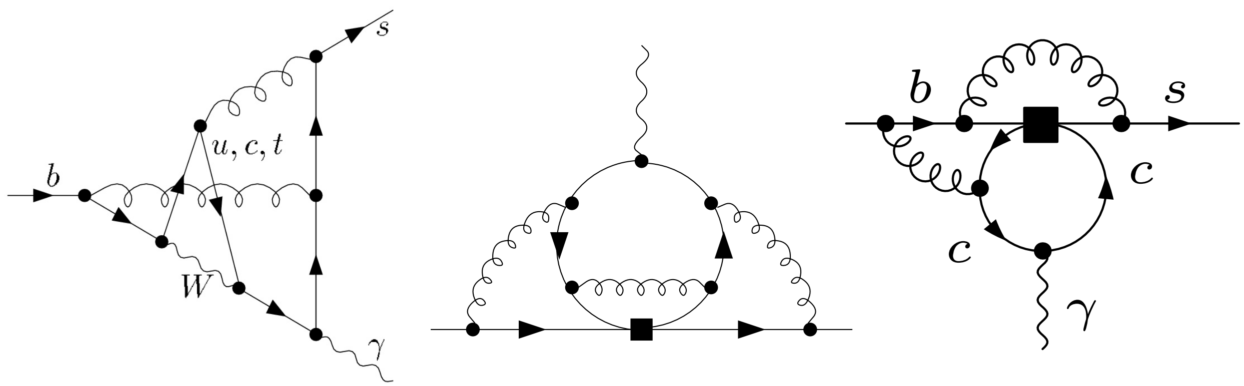
- Both of these spectra determined at lowest order by the b quark PDF in B meson
- Lots of work toward extending beyond leading order; some open issues remain

Inclusive $B \to X_s\gamma$ calculations

- Two-body decay at lowest order: $O_7 = \overline{m}_b \bar{s} \sigma_{\mu\nu} e F^{\mu\nu} P_R b$
  One of the (if not "the") most elaborate SM calculations (constrains many models)



- NNLO practically completed [Misiak et al., hep-ph/0609232]
  $\mathcal{O}(10^4)$ diagrams, 4-loop running, 3-loop matching and matrix elements



- SM prediction: $\mathcal{B}(B \to X_s\gamma)\big|_{E_\gamma > 1.6\mathrm{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$
  Measurement: $(3.43 \pm 0.22) \times 10^{-4}$

Regions of $B \to X_s\gamma$ phase space

- Important both for $|V_{ub}|$ and constraining NP

- $m_B - 2E_\gamma \lesssim 2\,\text{GeV}$, and $<1\,\text{GeV}$ at the peak

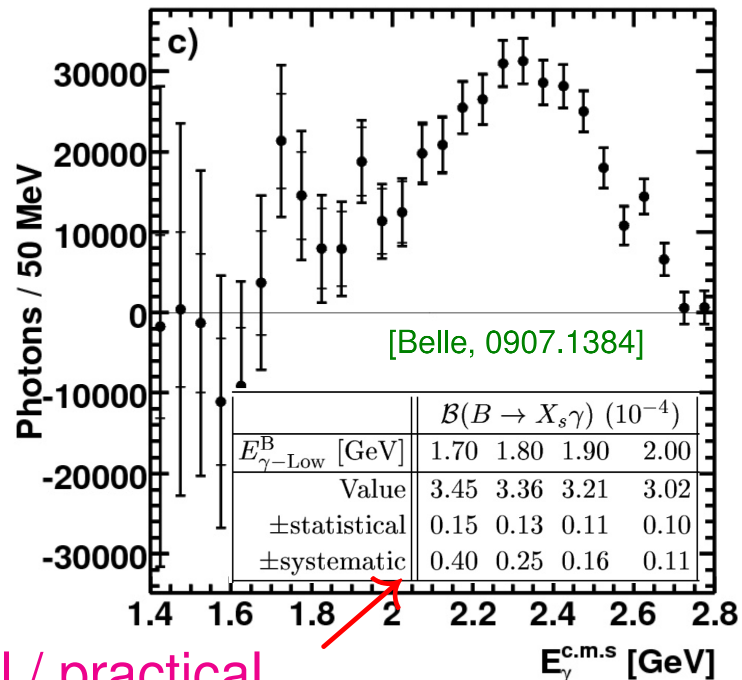
  Three cases: 1) $\Lambda_{\text{QCD}} \sim m_B - 2E_\gamma \ll m_B$

  2) $\Lambda_{\text{QCD}} \ll m_B - 2E_\gamma \ll m_B$

  3) $\Lambda_{\text{QCD}} \ll m_B - 2E_\gamma \sim m_B$

  Neither 1) nor 2) is fully appropriate

  [Sometimes called: 1) SCET and 2) MSOPE regions]



- Reducing E_γ^{cut} to $\sim 1.7\,\text{GeV}$ is probably not optimal / practical

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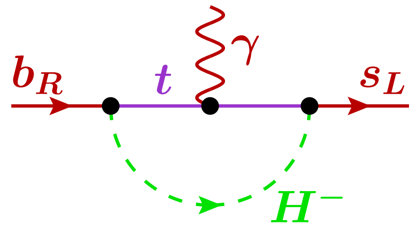
- $B \to X_u \ell\bar\nu$ is more complicated: hadronic physics depends not on one (E_γ) but two variables (best choice: $p_X^\pm = E_X \mp |\vec{p}_X|$ — “jettyness” of hadronic final state)

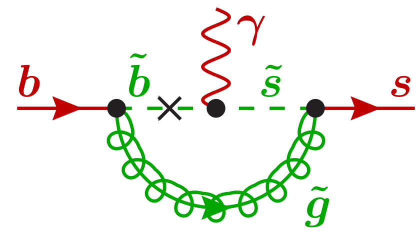
- Existing approaches based on theory in one region, expect future improvements

$B \to X_s\gamma$ and the 2HDM

- In Type-II 2HDM (as in the MSSM) the $H^\pm$ contribution always enhances the rate

  In SUSY cancellations can occur, strong bounds remain, depend on many param's





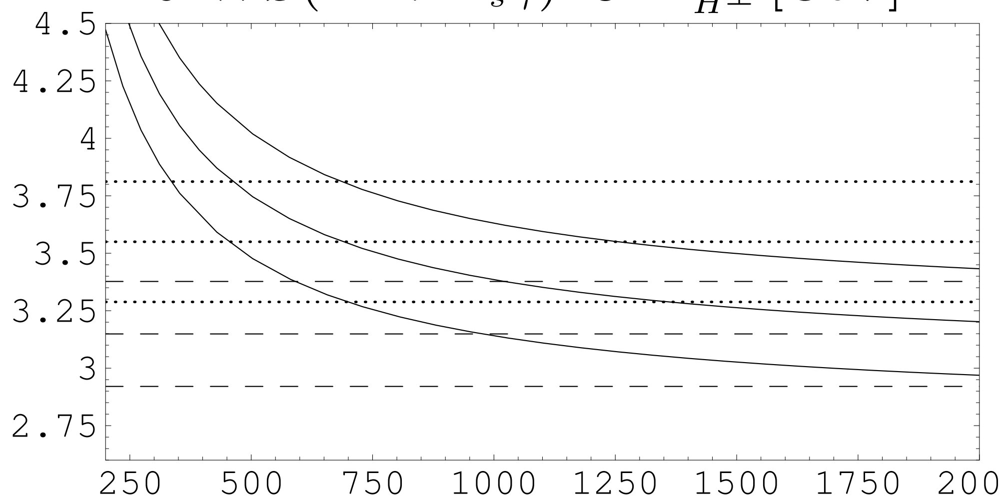
- Curves show $\pm 1\sigma$ bands

  Solid: $\mathcal{B}(B \to X_s\gamma)$ for $\tan\beta = 2$
  in Type-II 2HDM

  Dashed: SM prediction

  Dotted: experimental data

$10^4 \times \mathcal{B}(B \to X_s\gamma)$ vs. $M_{H^\pm}$ [GeV]



[Misiak et al., hep-ph/0609232]

If all else fails: “Grinstein-type double ratios”

- Continuum theory may be competitive using HQS + chiral symmetry suppression
- $\dfrac{f_B}{f_{B_s}} \times \dfrac{f_{D_s}}{f_D}$ — lattice: double ratio $= 1$ within few % [Grinstein '93]
- $\dfrac{f^{(B\to\rho\ell\bar\nu)}}{f^{(B\to K^*\ell^+\ell^-)}} \times \dfrac{f^{(D\to K^*\ell\bar\nu)}}{f^{(D\to\rho\ell\bar\nu)}}$ or q^2 spectra — accessible soon? [ZL, Wise; Grinstein, Pirjol]

  Numerous variations in the literature
- $\dfrac{\mathcal{B}(B\to\ell\bar\nu)}{\mathcal{B}(B_s\to\ell^+\ell^-)} \times \dfrac{\mathcal{B}(D_s\to\ell\bar\nu)}{\mathcal{B}(D\to\ell\bar\nu)}$ — very clean... by ~ 2020? [ZL, Ringberg '03]
- $\dfrac{\mathcal{B}(B_u\to\ell\bar\nu)}{\mathcal{B}(B_d\to\mu^+\mu^-)}$ — uses only isospin... around 2025? [Grinstein, CKM'06]

The theoretically cleanest $|V_{ub}|$ I know... Need lots of LHCb and Belle II data...

[A high precision $\mathcal{B}(B_d\to\mu^+\mu^-)$ measurement — run/upgrade LHCb forever...]

New physics in V_{ub}?

- Inclusive & exclusive V_{ub} determinations in tension: $[(4.4 \pm 0.3)$ vs. $(3.4 \pm 0.3)] \times 10^{-3}$
  CKM fit in the SM favors smaller values from exclusive decays

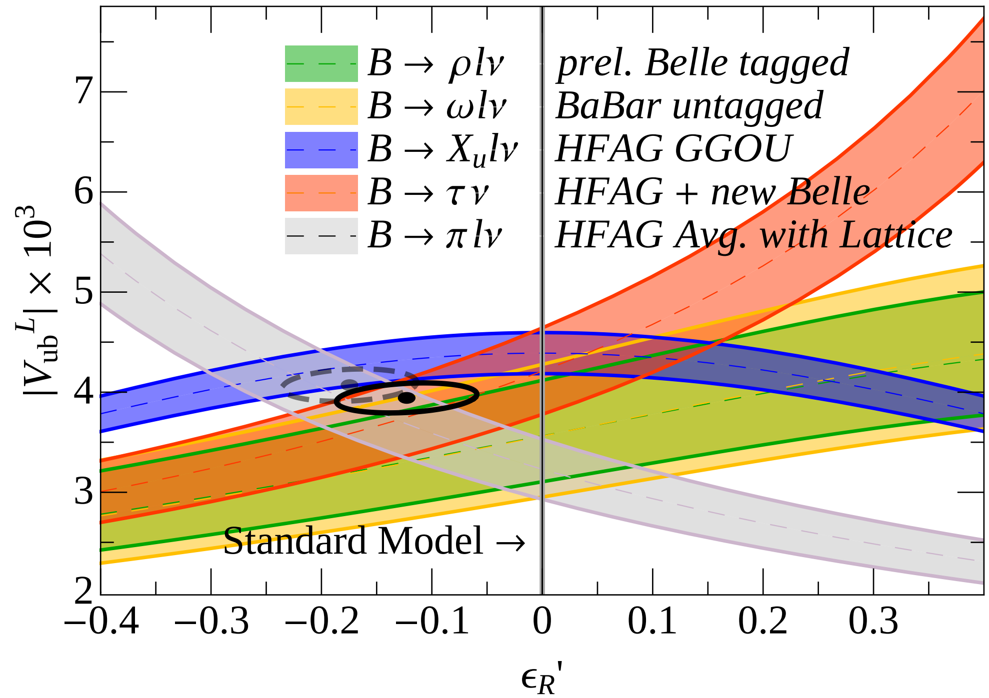
- A right-handed current

$$\epsilon_R\, (\overline{u}\, \gamma^\mu P_R\, b)\, (\overline{\ell}\, \gamma_\mu P_L\, \nu)$$

affects inclusive $B \to X_u \ell \bar{\nu}$ rate $\propto \epsilon_R^2$

It affects the exclusive rates $\propto \epsilon_R$

- NP at $10-20\,\%$ of the SM may still contribute to semileptonic decays as well!



[F. Bernlochner, CKM 2012]

- Need much larger Belle II data sets to probe this conclusively

Also related to $B \to X_s \ell^+ \ell^-$

- Complementary to $B \to X_s\gamma$, depends on:

$$O_7 = \overline{m}_b\, \bar{s}\sigma_{\mu\nu} e F^{\mu\nu} P_R b,$$

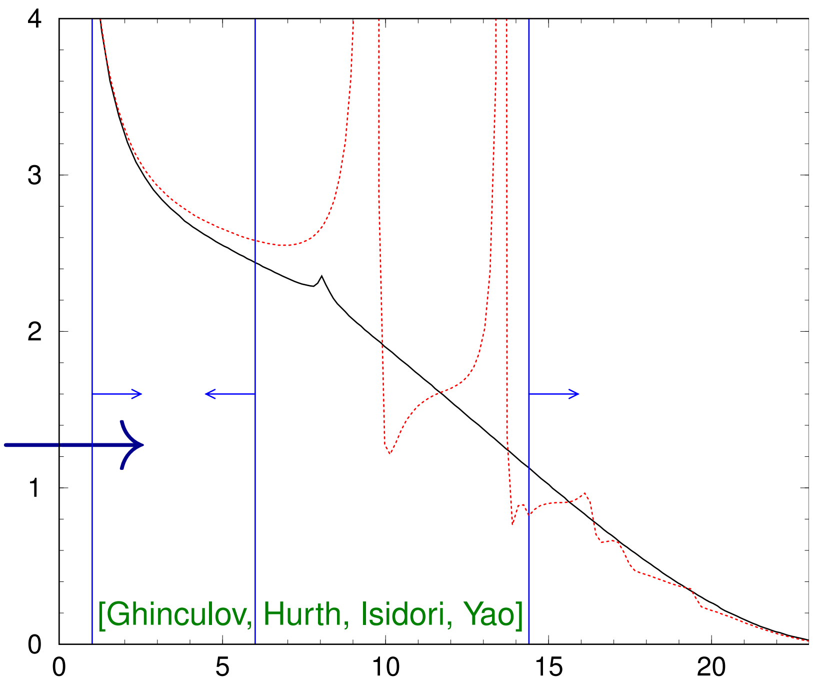
$$O_9 = e^2(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell),$$

$$O_{10} = e^2(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

Theory most precise for $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$ →

– NNLL perturbative calculations

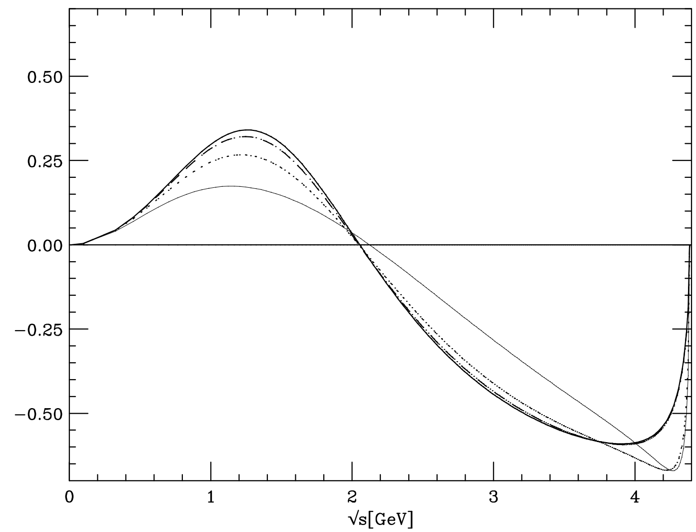
– Nonperturbative corrections to q^2 spectrum



- In small q^2 region experiments require additional $m_{X_s} \lesssim 2\,\mathrm{GeV}$ cut to suppress $b \to c(\to s\ell^+\nu)\ell^-\bar{\nu}$ $\Rightarrow$ nonperturbative effects [Ali & Hiller; Lee, ZL, Stewart, Tackmann]

- Theory same as for in inclusive $|V_{ub}|$ measurements (similar phase space cuts)

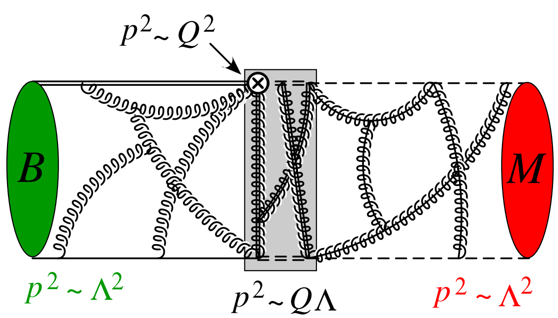
$A_{\mathbf{FB}}$ in $B \to K^*\ell^+\ell^-$



- Noticed that zero of $A_{\mathbf{FB}}$ was insensitive to form factor models [Burdman]

  Decay rate depends on several form factors, were assumed to be independent functions of $q^2 = m^2_{\ell^+\ell^-}$

  Despite the spectrum being model dependent, zero-crossing looked insensitive to model used



- For $q^2 \ll m_B^2$, the K^* is highly boosted; can use soft-collinear effective theory (SCET) to write form factors as sum of two terms: soft form factor & hard scattering

  First term obeys symmetry relations, unclear to what extent it dominates over 2nd

- If it does, then $A_{\mathbf{FB}}$ changes sign: $C_9^{\rm eff}(s_0) = -\dfrac{2m_B m_b}{s_0}\, C_7^{\rm eff} \times [\,1 + O(\alpha_s, \Lambda_{\rm QCD}/m_b)]$

Substantial discovery potential in many modes

- Some of the theoretically cleanest modes (ν, τ, inclusive) only possible at e^+e^-
- Many modes first seen at Belle II or LHCb
- In some decay modes, even in 2025:

  (Exp. bound)/SM $\gtrsim 10^3$

  (E.g.: $B_{(s)} \to \tau^+\tau^-$, e^+e^- lots of model building...)

[Grossman, ZL, Nir, 0904.4262, Prog. Theor. Phys. special issue commemorating the KM Nobel Prize]

Observable	Approximate SM prediction	Present status	Uncertainty / number of events Super-B (50 ab^{-1})	LHCb (10 fb^{-1})
$S_{\psi K}$	input	0.671 ± 0.024	0.005	0.01
$S_{\phi K}$	$S_{\psi K}$	0.44 ± 0.18	0.03	0.1
$S_{\eta' K}$	$S_{\psi K}$	0.59 ± 0.07	0.02	not studied
$\alpha(\pi\pi, \rho\rho, \rho\pi)$	α	$(89 \pm 4)^\circ$	2°	4°
$\gamma(DK)$	γ	$(70^{+27}_{-30})^\circ$	2°	3°
$S_{K^*\gamma}$	few $\times$ 0.01	-0.16 ± 0.22	0.03	—
$S_{B_s \to \phi\gamma}$	few $\times$ 0.01	—	—	0.05
$\beta_s(B_s \to \psi\phi)$	1°	$(22^{+10}_{-8})^\circ$	—	0.3°
$\beta_s(B_s \to \phi\phi)$	1°	—	—	1.5°
$A^d_{\rm SL}$	-5×10^{-4}	$-(5.8 \pm 3.4) \times 10^{-3}$	10^{-3}	10^{-3}
$A^s_{\rm SL}$	2×10^{-5}	$(1.6 \pm 8.5) \times 10^{-3}$	$\Upsilon(5S)$ run?	10^{-3}
$A_{CP}(b \to s\gamma)$	< 0.01	-0.012 ± 0.028	0.005	—
$\lvert V_{cb}\rvert$	input	$(41.2 \pm 1.1) \times 10^{-3}$	1%	—
$\lvert V_{ub}\rvert$	input	$(3.93 \pm 0.36) \times 10^{-3}$	4%	—
$B \to X_s\gamma$	3.2×10^{-4}	$(3.52 \pm 0.25) \times 10^{-4}$	4%	—
$B \to \tau\nu$	1×10^{-4}	$(1.73 \pm 0.35) \times 10^{-4}$	5%	—
$B \to X_s\nu\bar\nu$	3×10^{-5}	$< 6.4 \times 10^{-4}$	only $K\nu\bar\nu$?	—
$B \to X_s\ell^+\ell^-$	6×10^{-6}	$(4.5 \pm 1.0) \times 10^{-6}$	6%	not studied
$B_s \to \tau^+\tau^-$	1×10^{-6}	< few %	$\Upsilon(5S)$ run?	—
$B \to X_s\tau^+\tau^-$	5×10^{-7}	< few %	not studied	—
$B \to \mu\nu$	4×10^{-7}	$< 1.3 \times 10^{-6}$	6%	—
$B \to \tau^+\tau^-$	5×10^{-8}	$< 4.1 \times 10^{-3}$	$\mathcal{O}(10^{-4})$	—
$B_s \to \mu^+\mu^-$	3×10^{-9}	$< 5 \times 10^{-8}$	—	$> 5\sigma$ in SM
$B \to \mu^+\mu^-$	1×10^{-10}	$< 1.5 \times 10^{-8}$	$< 7 \times 10^{-9}$	not studied
$B \to K^*\ell^+\ell^-$	1×10^{-6}	$(1 \pm 0.1) \times 10^{-6}$	15k	36k
$B \to K\nu\bar\nu$	4×10^{-6}	$< 1.4 \times 10^{-5}$	20%	—

Look for "odd" things

- Cast a wide net — broad program is critical:

  $B \to (\gamma +)\,\text{invisible}$ [Belle, 1206.5948; BaBar, 1206.2543]

  $B \to X_s + \text{invisible}$

  $\Upsilon(1S) \to \text{invisible}$ [Belle, hep-ex/0611041; BaBar, 0908.2840]

  $\Upsilon(nS) \to \gamma + \text{invisible}$ [e.g., for $1S$ and $3S$: BaBar, 0808.0017, 1007.4646]

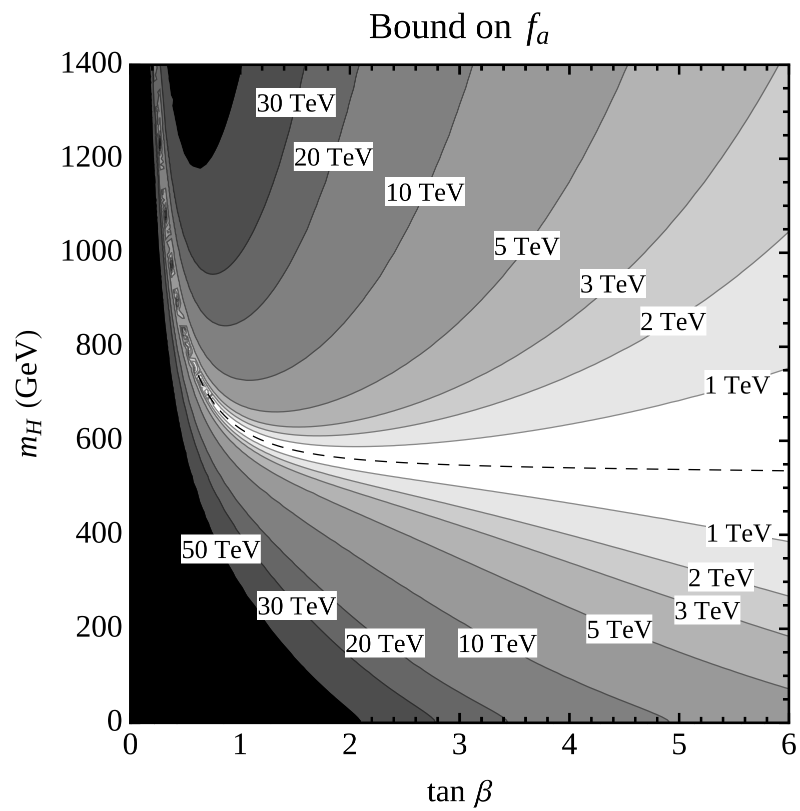
  $e^+e^- \to (\gamma +)\,\text{invisible}$

- Also include "invisible" replaced by a new resonance; may decay to $\ell^+\ell^-$, etc.
- τ and μ lepton flavor violation
- Searches for violations of conservation laws
- Obvious! most cited Belle paper: $X(3872)$, most cited BaBar paper: $D_{s0}^*(2317)$

Example: bump searches in $B \to K^{(*)}\ell^+\ell^-$

- Can probe certain DM models with B decays

  E.g., "axion portal": light ($\lesssim 1\,\mathrm{GeV}$) scalar particle coupling as $(m_\psi/f_a)\,\bar\psi\gamma_5\psi\,a$



[Freytsis, ZL, Thaler, arXiv:0911.5355]

- In most of parameter space best bound is from $B \to K\ell^+\ell^-$

My Belle II “best buy” list

- Key observables: (i) sensitive to different NP, (ii) measurements can improve by order of magnitude, (iii) not limited by hadronic uncertainties
  - Difference of CP asymmetries, $S_{\psi K_S} - S_{\phi K_S}$, $S_{\psi K_S} - S_{\eta' K_S}$, etc.
  - γ from CP asymmetries in tree-level decays vs. γ from $S_{\psi K_S}$ and $\Delta m_d/\Delta m_s$
  - Search for charged lepton flavor violation, $\tau \to \mu\gamma$, $\tau \to 3\mu$, and similar modes
  - Search for CP violation in $D^0 - \overline{D}^0$ mixing
  - CP asymmetry in semileptonic decay (dilepton asymmetry), $A_{\rm SL}$
  - CP asymmetry in the radiative decay, $S_{K_S\pi^0\gamma}$
  - Rare decay searches and refinements: $b \to s\nu\bar{\nu}$, $B \to \tau\bar{\nu}$, etc.
  - Improve magnitudes of CKM elements
- Complementary to LHCb
- Any one of these measurements has the potential to establish new physics

My LHCb "best buy" list

- LHCb will probe B_s sector at a level comparable to B_d
  - The CP asymmetry, $S_{B_s\to\psi\phi}$
  - Difference of CP asymmetries, $S_{B_s\to\psi\phi} - S_{B_s\to\phi\phi}$
  - $B_s \to \mu^+\mu^-$, search for $B_d \to \mu^+\mu^-$, other rare / forbidden decays
  - 10^{4-5} events in $B \to K^{(*)}\ell^+\ell^-$, $B_s \to \phi\gamma, \ldots$ — test Dirac structure, BSM op's
  - γ from $B \to DK$ and $B_s \to D_s K$
  - Search for charged lepton flavor violation, $\tau \to 3\mu$ and other modes if possible
  - Search for CP violation in $D^0 - \overline{D}^0$ mixing
- Very broad program, complementary to Belle II
- With large BSM discovery potential

Summary

- Lots of progress for $|V_{cb}|$ and $|V_{ub}|$, determinations from exclusive decays largely in the hands of lattice QCD, room for progress in continuum — tension is troubling

- Theoretical tools for rare decays are similar, so developments often simultaneous

- Theory progress in understanding nonleptonic decays; unfortunately the best understood cases are not the most interesting to learn about weak scale physics

- More work and data needed to understand the expansions

  Why some predictions work at $\lesssim 10\%$ level, while others receive $\sim 30\%$ corrections

  Clarify role of charming penguins, chirally enhanced terms, annihilation, etc.

- Active field, experimental data stimulated lots of theory developments, expect more work & progress as LHCb and Belle II provide challenges and opportunities

Read at your own risk...