2nd Lecture
When (some) QCD matters

- Isospin and $SU(3)$ flavor
  Measuring $\alpha$, $SU(3)$

- The heavy quark limit
  Heavy quark symmetry, OPE, exclusive / inclusive decays

- Semileptonic and radiative $b$ decays
  $b \rightarrow s \gamma$, etc.

- SCET and nonleptonic decays — skip, but include slides
  $B$ decays to charm, $\Lambda_b$ decay
  charmless $B$ decays, different approaches
• How to learn about high energy physics from low energy hadronic processes?

• QCD coupling is scale dependent, $\alpha_s(m_B) \sim 0.2$

\[
\alpha_s(\mu) = \frac{\alpha_s(\Lambda)}{1 + \frac{\alpha_s}{2\pi} \beta_0 \ln \frac{\mu}{\Lambda}}, \quad \beta_0 = 11 - \frac{2}{3} n_f > 0
\]

Nobel prize in 2004:
Politzer, Wilczek, Gross
Interplay of electroweak and strong interactions

- How to learn about high energy physics from low energy hadronic processes?

- QCD coupling is scale dependent, $\alpha_s(m_B) \sim 0.2$

$$\alpha_s(\mu) = \frac{\alpha_s(\Lambda)}{1 + \frac{\alpha_s}{2\pi} \beta_0 \ln \frac{\mu}{\Lambda}}, \quad \beta_0 = 11 - \frac{2}{3} n_f > 0$$

High energy (short distance): perturbation theory is useful

Low energy (long distance): QCD becomes nonperturbative $\Rightarrow$ It is usually very hard, if not impossible, to make precise calculations

- Solutions: New symmetries in some limits: effective theories (heavy quark, chiral)
  Certain processes are determined by short-distance physics
  Lattice QCD (bite the bullet — limited cases)

- Incalculable nonperturbative hadronic effects sometimes limit sensitivity

ZL — p.2/1
Want to learn about electroweak physics, but hadronic physics is nonperturbative

Model independent continuum approaches:

• (1) Symmetries of QCD (exact or approximate)

E.g.: $\sin 2\beta$ from $B \rightarrow J/\psi K_S$: amplitude not calculable

Solution: $CP$ symmetry of QCD ($\theta_{QCD}$ can be neglected)

$$\langle \psi K_S | H | B^0 \rangle = -\langle \psi K_S | H | B^0 \rangle \times [1 + O(\alpha_s \lambda^2)]$$

• (2) Effective field theories (separation of scales)

E.g.: $|V_{cb}|$ and $|V_{ub}|$ from semileptonic $B$ decays

Solution: Heavy quark expansions

$$\Gamma = |V_{cb}|^2 \times \text{(known factors)} \times [1 + O(\Lambda_{QCD}^2/m_b^2)]$$
Many relevant scales: \( B \rightarrow X_s \gamma \)

- **Separate physics at:** \((m_{t,W} \sim 100 \text{ GeV}) \gg (m_b \sim 5 \text{ GeV}) \gg (\Lambda \sim 0.5 \text{ GeV})\)

Inclusive decay:
\[ X_s = K^*, K^{(*)}\pi, K^{(*)}\pi\pi, \text{ etc.} \]

Diagrams with many gluons are crucial, resumming certain subset of them affects rate at factor-of-two level

Rate in SM calculated to \(<10\%\), using several effective theories, renormalization group, operator product expansion... one of the most involved SM analyses

- **Solution:** Short distance dominated (some issues discussed later)
Some caveats

• Lot at stake: theoretical tools for semileptonic and rare decays are the same
  – Measurements of CKM elements
  – Better understanding of hadronic physics improves sensitivity to new physics

• For today’s talk: [strong interaction] model independent
  ≡ theor. uncertainty suppressed by small parameters
  ... so theorists argue about $O(1) \times (\text{small numbers})$ instead of $O(1)$ effects

• Most of the progress have come from expanding in powers of $\Lambda/m_Q, \alpha_s(m_Q)$
  ... a priori not known whether $\Lambda \sim 200\text{ MeV}$ or $\sim 2\text{ GeV}$ ($f_\pi, m_\rho, m_K^2/m_s$)
  ... need experimental guidance to see how well the theory works
To avoid...

The SM shows impressive consistency — separate what’s “proven” / “hoped”

Only robust deviations from model independent theory are likely to be interesting

(2σ: 50 theory papers  3σ: 200 theory papers  5σ: strong sign of an effect)
Isospin and $SU(3)$ flavor
Extracting $\alpha$ from $B \rightarrow \pi\pi$

- Until $\sim 1997$ the hope was to determine $\alpha$ simply from:
  \[
  \frac{\Gamma(B^0(t) \rightarrow \pi^+\pi^-) - \Gamma(B^0(t) \rightarrow \pi^+\pi^-)}{\Gamma(B^0(t) \rightarrow \pi^+\pi^-) + \Gamma(B^0(t) \rightarrow \pi^+\pi^-)} = S \sin(\Delta m t) - C \cos(\Delta m t)
  \]
  \[\arg \lambda_{\pi^+\pi^-} = (B\text{-mix} = 2\beta) + (\frac{\bar{A}}{A} = 2\gamma + \ldots) \Rightarrow \text{measures } \sin 2\alpha \text{ if amplitudes with one weak phase dominated} \text{ — relied on expectation that } P/T = \text{small}\]

- Models in which the dominant NP effect is the modification of the $B^0 - \bar{B}^0$ mixing amplitude have $\gamma = \pi - \beta - \alpha$, so reducing the uncertainty of $\alpha$ effectively improves the determination of $\gamma$ and the bound on NP
\( B \rightarrow \pi\pi \) — isospin analysis

- Isospin started with \((p, n)\) symmetry, broken by \((m_d - m_u)/\Lambda_{QCD}\)

- \((u, d)\): \(I\)-spin doublet
  - \((\pi\pi)_{\ell=0} \rightarrow I_f = 0 \quad \text{or} \quad I_f = 2\)
  - \(1 \times 1\)  \(\Delta I = \frac{1}{2}\)  \(\Delta I = \frac{3}{2}\)

- Other quarks and gluons: \(I = 0\)

- \(\gamma, Z\): mixtures of \(I = 0, 1\)

- \(I = 0\) final state forbidden by Bose symmetry

- Hamiltonian has two parts:
  - \(\Delta I = \frac{1}{2} \Rightarrow I_f = 0\)
  - \(\Delta I = \frac{3}{2} \Rightarrow I_f = 2\)

- Note: \(\gamma\) and \(Z\) penguins violate isospin and yield some (small) uncertainties

Experimentally, need all (tagged) rates + time dependent \(B \rightarrow \pi^+\pi^-\) asymmetry

- Three rates \(\bar{B}^0 \rightarrow \pi^+\pi^-\), \(\bar{B}^0 \rightarrow \pi^0\pi^0\), \(B^- \rightarrow \pi^0\pi^-\) determine magnitudes and relative strong phase of two amplitudes; similarly for \(B^0\) and \(B^+\) decay

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ZL — p.2/7
Isospin analysis (cont.)

- Isospin symmetry implies that 6 amplitudes form two triangles with common base

\[
\frac{A^{-+}}{\sqrt{2}} + A^{00} = A^{+0}, \quad \frac{\bar{A}^{-+}}{\sqrt{2}} + \bar{A}^{00} = \bar{A}^{-0}
\]

\[|A^{+0}| = |\bar{A}^{-0}|\]

\[
A^{-+} \equiv A(B^0 \rightarrow \pi^+ \pi^-) \quad \bar{A}^{-+} \equiv A(\bar{B}^0 \rightarrow \pi^+ \pi^-)
\]

\[
A^{00} \equiv A(B^0 \rightarrow \pi^0 \pi^0) \quad \bar{A}^{00} \equiv A(\bar{B}^0 \rightarrow \pi^0 \pi^0)
\]

\[
A^{+0} \equiv A(B^+ \rightarrow \pi^+ \pi^0) \quad \bar{A}^{-0} \equiv A(B^- \rightarrow \pi^- \pi^0)
\]

- \(B \rightarrow \rho\pi\): 4 isospin amplitudes \(\Rightarrow\) pentagon relations (not used)

Dalitz plot analysis allows considering \(\pi^+ \pi^- \pi^0\) final state only

\[2\delta = \text{difference between } \arg \lambda_{\pi^+ \pi^-} \text{ and } 2\alpha\]
$B \rightarrow \rho\rho$: the best $\alpha$ at present

- $\rho\rho$ is mixture of $CP$ even/odd (as all $VV$ modes); data: $CP$ = even dominates
  Isospin analysis applies for each $L$, or in transversity basis for each $\sigma (= 0, \parallel, \perp)$

- Small rate: $B(B \rightarrow \rho^0\rho^0) = (0.73 \pm 0.28) \times 10^{-6} \Rightarrow$ small penguin pollution

\[
\frac{B(B \rightarrow \pi^0\pi^0)}{B(B \rightarrow \pi^+\pi^-)} \approx 0.35 \quad \text{vs.} \quad \frac{B(B \rightarrow \rho^0\rho^0)}{B(B \rightarrow \rho^+\rho^-)} \approx 0.03
\]

- Ultimately, more complicated than $\pi\pi$, $I = 1$ possible due to finite $\Gamma_\rho$, giving $O(\Gamma_\rho^2/m_\rho^2)$ effects [can be constrained]

$B \rightarrow \rho\rho$ isospin analysis: $\alpha = (90 \pm 5)^\circ$

- Also $B \rightarrow \rho\pi$ Dalitz plot analysis

- $\rho\rho$ mode dominates $\alpha$ determination for now, may change at a super $B$ factory

\[ZL -- p.2/9\]
Recall: the \( B \rightarrow K\pi \) puzzle

- Have we seen new physics in CPV?
  \[
  A_{K^+\pi^-} = -0.098 \pm 0.012 \quad (P + T)
  \]
  \[
  A_{K^+\pi^0} = 0.050 \pm 0.025 \quad (P + T + C + A + P_{ew})
  \]

  What’s the reason for large difference?
  \[
  A_{K^+\pi^0} - A_{K^+\pi^-} = 0.148 \pm 0.028
  \]

- SCET / factorization predicts: \( \arg(C/T) = \mathcal{O}(\Lambda_{QCD}/m_b) \) and \( A + P_{ew} \) small

  This makes it hard to understand above data:
  - \( P \) and \( T \): nonzero relative strong and weak phases to give \( A_{K^+\pi^-} \)
  - \( T \) and \( C \): same weak phase and predicted to have small relative strong phase

- Huge fluctuations? Breakdown of \( 1/m \) exp.? Missing something subtle? BSM?

\[\text{[Belle, Nature 452, 332 (2008)]}\]
Flavor \(SU(3)\) — a timely example

- First observation of \(B_s\) CPV: \(A_{\text{CP}}(B^0_s \to K^-\pi^+) = 0.27 \pm 0.04\) \[LHCb, \text{arXiv:1304.6173}\]

- Compare: \(B^0_d \to K^+\pi^-\) (\(\bar{b} \to \bar{s}q\bar{q}\)) vs. \(B^0_s \to K^-\pi^+\) (\(\bar{b} \to d\bar{q}\bar{q}\))

Can use \(U\)-spin \((d \leftrightarrow s)\) subgroup of \(SU(3)\): \((B_d, B_s), \mathcal{H}, (\pi^+, K^+), (\pi^-, K^-)\)

final state \(U = 0, 1 \Rightarrow \) two reduced matrix elements

\[
A(B^0_d \to K^+\pi^-) = V^*_{cb} V_{cs} (P_c - P_t + T_{c\bar{c}s}) + V^*_{ub} V_{us} (T_{u\bar{u}s} + P_u - P_t) \equiv P + T
\]

\[
A(B^0_s \to K^-\pi^+) = V^*_{cb} V_{cd} (P_c - P_t + T_{c\bar{c}s}) + V^*_{ub} V_{ud} (T_{u\bar{u}s} + P_u - P_t) = -\lambda P + \lambda^{-1} T
\]

- LHCb quotes the \(SU(3)\) relation:

\[
\Delta \equiv \frac{A_{\text{CP}}(B_d \to K^+\pi^-)}{A_{\text{CP}}(B_s \to K^-\pi^+)} + \frac{\mathcal{B}(B_s \to K^-\pi^+)}{\mathcal{B}(B_d \to K^+\pi^-)} \frac{\tau_d}{\tau_s} = -0.02 \pm 0.05 \pm 0.04
\]

- Looks obscure — where does this come from?
Flavor $SU(3)$ vs factorization at $\mu \sim m_b$

- Saw yesterday: $\Gamma_{CP}(i \to f) \equiv \Gamma(i \to f) - \Gamma(\bar{i} \to \bar{f}) = 4A_1A_2 \sin(\phi_1 - \phi_2) \sin(\delta_1 - \delta_2)$

Define: $\tilde{\Delta} \equiv \frac{\Gamma_{CP}(B_d \to K^+\pi^-) + \Gamma_{CP}(B_s \to K^-\pi^+)}{\Gamma_{CP}(B_d \to K^+\pi^-) - \Gamma_{CP}(B_s \to K^-\pi^+)} = 0.01 \pm 0.11$

In fact $\tilde{\Delta} = 0 +$ typical size of $SU(3)$ breaking, whereas $\Delta$ depends also on $|P/T|$

- Using factorization $\tilde{\Delta} \ll 1$ iff: $F_{B \to \pi} f_K \approx F_{B_s \to K} f_\pi$

Need $B_s \to K$ form factor from LQCD (extract $|V_{ub}|$ at LHCb from $B_s \to K^+ \mu^- \nu$?)

Similar $SU(3)$ relations: $B_{s0} \to K^+K^- \leftrightarrow B_{d0} \to \pi^+\pi^-$

$B_{s0} \to \pi^+\pi^- \leftrightarrow B_{d0} \to K^+K^-$

- Which relation works well will help answer what’s at play [Grossman, ZL, Robinson, to appear]
Heavy quark symmetry
• $Q\bar{Q}$: positronium-type bound state, perturbative in the $m_Q \gg \Lambda_{QCD}$ limit

• $Q\bar{q}$: wave function of the light degrees of freedom
  (“brown muck”) insensitive to spin and flavor of $Q$

$B$ meson is a lot more complicated than just a $b\bar{q}$ pair

In the $m_Q \gg \Lambda_{QCD}$ limit, the heavy quark acts as a static color source with fixed four-velocity $v^\mu$

$\Rightarrow SU(2n)$ heavy quark spin-flavor symmetry at fixed $v^\mu$

• Similar to atomic physics: $(m_e \ll m_N)$

  1. Flavor symmetry $\sim$ isotopes have similar chemistry [$\Psi_e$ independent of $m_N$]

  2. Spin symmetry $\sim$ hyperfine levels almost degenerate [$\vec{s}_e - \vec{s}_N$ interaction $\rightarrow 0$]

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ZL — p.2/13
Spectroscopy of heavy-light mesons

- In \( m_Q \gg \Lambda_{QCD} \) limit, spin of the heavy quark is a good quantum number, and so is the spin of the light d.o.f., since \( \vec{J} = \vec{s}_Q + \vec{s}_l \) and

  \[
  \text{angular momentum conservation: } [\vec{J}, \mathcal{H}] = 0 \]

  \[
  \text{heavy quark symmetry: } [\vec{s}_Q, \mathcal{H}] = 0 \Rightarrow [\vec{s}_l, \mathcal{H}] = 0
  \]

- For a given \( s_l \), two degenerate states:

  \[
  J_\pm = s_l \pm \frac{1}{2}
  \]

  \[
  \Rightarrow \Delta_i = \mathcal{O}(\Lambda_{QCD}) \text{ — same in } B \text{ and } D \text{ sector}
  \]

  Doublets are split by order \( \Lambda_{QCD}^2/m_Q \), e.g.:

  \[
  m_{D^*} - m_D \simeq 140 \text{ MeV}
  \]

  \[
  m_{B^*} - m_B \simeq 45 \text{ MeV}
  \]
Aside: a puzzle

• Vector–pseudoscalar mass splitting is $\propto 1/m_Q \Rightarrow m_V^2 - m_P^2 = \text{const.}$

Experimentally:

$\begin{align*}
    m_{B^*}^2 - m_B^2 &= 0.49 \text{ GeV}^2 \\
    m_{D^*}^2 - m_D^2 &= 0.54 \text{ GeV}^2
\end{align*}$

$\begin{align*}
    m_{B_s^*}^2 - m_{B_s}^2 &= 0.50 \text{ GeV}^2 \\
    m_{D_s^*}^2 - m_{D_s}^2 &= 0.58 \text{ GeV}^2
\end{align*}$
Aside: a puzzle

- Vector–pseudoscalar mass splitting is $\propto 1/m_Q \Rightarrow m_V^2 - m_P^2 = \text{const.}$

Experimentally:

- $m_{B^*}^2 - m_B^2 = 0.49 \text{ GeV}^2$
- $m_{D^*}^2 - m_D^2 = 0.54 \text{ GeV}^2$
- $m_{K^*}^2 - m_K^2 = 0.55 \text{ GeV}^2$
- $m_{\rho}^2 - m_{\pi}^2 = 0.57 \text{ GeV}^2$
- $m_{B_{s}^*}^2 - m_{B_s}^2 = 0.50 \text{ GeV}^2$
- $m_{D_{s}^*}^2 - m_{D_s}^2 = 0.58 \text{ GeV}^2$

- The HQS argument relies on $m_Q \gg \Lambda_{QCD}$, so something more has to go on...

- It is not only important to test how a theory works, but also how it breaks down!

[An approximation should work when it the expansion parameter is small, and fail when it’s $O(1)$]
Successes in charm spectrum

- $D_1$ narrow width:
  - $S$-wave $D_1 \rightarrow D^* \pi$ allowed by angular momentum conservation, but forbidden in the $m_Q \rightarrow \infty$ limit by heavy quark spin symmetry

- Mass splittings of orbitally excited states is small:
  - $m_{D_2^*} - m_{D_1} = 37$ MeV $\ll m_{D^*} - m_D$ vanishes in the quark model, since it arise from $\langle \vec{s}_Q \cdot \vec{s}_\bar{q} \delta^3(\vec{r}) \rangle$

Spectroscopy of D mesons

- $L = 0$
  - $J^P = 0^-, 1^-, 0^*$
- $L = 1$
  - $J^P = 1^+, 1^+, 2^*$

(hep-ex/9908009)
Semileptonic and rare $B$ decays

- $|V_{ub}|$ is the dominant uncertainty of the side of the UT opposite to $\beta$
- $|V_{ub}|$ is crucial for comparing tree-dominated and loop-mediated processes
- Error of $|V_{cb}|$ is a large part of the uncertainty in the $\epsilon_K$ constraint, and in $K \rightarrow \pi \nu \bar{\nu}$ when it’s measured

Rare $b \rightarrow s \gamma$, $s \ell^+ \ell^-$, and $s \nu \bar{\nu}$ decays are sensitive probes of the Standard Model
Exclusive $B \rightarrow D(\ast)\ell\bar{\nu}$ decay

- In the $m_{b,c} \gg \Lambda_{QCD}$ limit, configuration of brown muck only depends on the four-velocity of the heavy quark, but not on its mass and spin.

- On a time scale $\ll \Lambda_{QCD}^{-1}$ weak current changes $b \rightarrow c$.

  i.e.: $\vec{p}_b \rightarrow \vec{p}_c$ and possibly $\vec{s}_Q$ flips.

In $m_{b,c} \gg \Lambda_{QCD}$ limit brown muck only feels $v_b \rightarrow v_c$.

Form factors independent of Dirac structure of weak current $\Rightarrow$ all form factors related to a single function of $w = v \cdot v'$, the Isgur-Wise function, $\xi(w)$.

Contains all nonperturbative low-energy hadronic physics.

- $\xi(1) = 1$, because at “zero recoil” configuration of brown muck not changed at all.
$B \rightarrow D^{(*)}\ell\bar{\nu}$ form factors

- Lorentz invariance $\Rightarrow$ 6 form factors
  \[
  \langle D(v')|V_{\nu}|B(v)\rangle = \sqrt{m_Bm_D} \left[ h_+ (v + v')_{\nu} + h_- (v - v')_{\nu} \right]
  \]
  \[
  \langle D^*(v')|V_{\nu}|B(v)\rangle = i\sqrt{m_Bm_{D^*}} \ h_V \ \epsilon_{\nu\alpha\beta\gamma} \epsilon^{*\alpha} v'^{\beta} v^{\gamma}
  \]
  \[
  \langle D(v')|A_{\nu}|B(v)\rangle = 0
  \]
  \[
  \langle D^*(v')|A_{\nu}|B(v)\rangle = \sqrt{m_Bm_{D^*}} \left[ h_{A_1} (w + 1) \epsilon_{\nu}^{*} - h_{A_2} (\epsilon^{*} \cdot v)_{\nu} - h_{A_3} (\epsilon^{*} \cdot v')_{\nu} \right]
  \]
  \[V_{\nu} = \bar{c}\gamma_{\nu} b, \quad A_{\nu} = \bar{c}\gamma_{\nu}\gamma_5 b, \quad w \equiv v \cdot v' = \frac{m_B^2 + m_D^2 - q^2}{2m_Bm_D}, \quad \text{and} \quad h_i = h_i(w, \mu)\]

- In $m_Q \gg \Lambda_{QCD}$ limit, up to corrections suppressed by $\alpha_s$ and $\Lambda_{QCD}/m_{c,b}$
  \[
  h_- = h_{A_2} = 0, \quad h_+ = h_V = h_{A_1} = h_{A_3} = \xi(w)
  \]
  The $\alpha_s$ corrections are calculable
  $\Lambda_{QCD}/m_{c,b}$ corrections is where model dependence enters

\[ZL \quad p.2/18\]
\[ |V_{cb}| \text{ from } B \rightarrow D^{(*)} \ell \bar{\nu} \]

- Extract \( |V_{cb}| \) from \( w \equiv v \cdot v' = (m_B^2 + m_D^2 - q^2)/(2m_Bm_D) \rightarrow 1 \) limit of the rate
  \[
  \frac{d\Gamma(B \rightarrow D^{(*)} \ell \bar{\nu})}{dw} = (...) (w^2 - 1)^{3/2(1/2)} \left| V_{cb} \right|^2 \mathcal{F}^2_{(*)}(w)
  \]
  Isgur-Wise function + ...

\[
\mathcal{F}(1) = 1_{\text{Isgur-Wise}} + 0.02 \frac{\alpha_s, \alpha_s^2}{m_{c,b}} + \ldots
\]

\[
\mathcal{F}^*_{(*)}(1) = 1_{\text{Isgur-Wise}} - 0.04 \frac{\alpha_s, \alpha_s^2}{m_{c,b}} + \frac{0_{\text{Luke}}}{m_{c,b}} + \frac{\alpha_s, \alpha_s^2}{m_{c,b}^2} + \ldots
\]

- Lattice QCD: \( \mathcal{F}^*_{(*)}(1) = 0.921 \pm 0.024, \; \mathcal{F}(1) = 1.074 \pm 0.024 \) [arXiv:0808.2519, hep-lat/0409116]

- Need constraints on shape to fit [Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert]

- Need some understanding of decays to higher mass \( X_c \) states (backgrounds)

- Data: \( |V_{cb} \mathcal{F}^*_{(*)}(1)| = (35.75 \pm 0.42) \times 10^{-3}, \; |V_{cb} \mathcal{F}(1)| = (42.3 \pm 1.5) \times 10^{-3} \) [HFAG]
  [note: \( \chi^2/\text{dof} = 39.6/21 \; (56.9/21), \; \text{CL} = 0.8\% \; (4E-5))]

ZL — p.2/19
Heavy quark expansion
Physics at $r \sim L$ is complicated
Depends on the details of the charge distribution
The multipole expansion

Physics at $r \gg L$ is much simpler

Charge distribution characterized by total charge, $q$

Details suppressed by powers of $L/r$, and can be parameterized in terms of $p_i, Q_{ij}, \ldots$

Simplifications occur due to separating physics at different distance scales

Complicated charge distribution can be replaced by a point source with additional interactions (multipoles) — underlying idea of effective theories
The multipole expansion (cont.)

- Potential:
  \[ V(x) = q \frac{1}{r} + p_i \frac{x_i}{r^3} + \frac{1}{2} Q_{ij} \frac{x_i x_j}{r^5} + \ldots \]

Short distance quantities: \[ q = \int \rho(x) \, d^3x, \quad p_i = \int x_i \rho(x) \, d^3x, \quad \text{etc.} \]

Long distance quantities: \[ \langle \frac{1}{r} \rangle, \quad \langle \frac{x_i}{r^3} \rangle, \quad \langle \frac{x_i x_j}{r^5} \rangle, \quad \text{etc.} \]

- Higher multipoles: new interactions from “integrating out” short distance physics

- Useful tool independent of the fact whether we know the underlying theory or not

- Any theory at momentum \( p \ll M \) can be described by an effective Hamiltonian

  \[ H_{\text{eff}} = H_0 + \sum_i \frac{C_i}{M^{n_i}} O_i \]

  \( M \to \infty \) limit + corrections with well-defined power counting

  \( H_0 \) may have more symmetries than full theory at nonzero \( p/M \)

  Can work to higher orders in \( p/M \); can sum logs of \( p/M \)

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ZL — p.2/21
Inclusive heavy hadron decays

- Sum over hadronic final states, subject to constraints determined by short distance physics

Decay: short distance (calculable)

Hadronization: long distance (nonperturbative), but probability to hadronize is unity; sum over details

- Optical theorem + operator product expansion (OPE) + heavy quark symmetry

\[ \nu \nu \mu q \quad \text{and} \quad v q \]

\[ \quad p = m_b v - q + k \]

\[ \quad p_b = m_b v + k \]

\[ \sum \text{field theoretic version of multipole expansion} \]

Can think of the OPE as expansion of forward scattering amplitude in \( k \sim \Lambda_{\text{QCD}} \)

ZL — p.2/22
Consider semileptonic $b \to u$ decay: 

$$O_{bu} = -\frac{4G_F}{\sqrt{2}} V_{ub} \left( \bar{u} \gamma^\mu P_L b \right) \left( \bar{\ell} \gamma_\mu P_L \nu \right)$$

Decay rate: 

$$\Gamma(B \to X_u \ell \bar{\nu}) \sim \sum_{X_c} \int d[PS] \left| \langle X_u \ell \bar{\nu} | O_{bu} | B \rangle \right|^2$$

Factor to: $B \to X_u W^*$ and $W^* \to \ell \bar{\nu}$, concentrate on hadronic part

$$W^{\mu\nu} \sim \sum_{X_c} \delta^4(p_B - q - p_X) \left| \langle B | J^{\mu\dagger}_{bu} | X_u \rangle \langle X_u | J^\nu_{bu} | B \rangle \right|^2 = \text{Im} \, T^{\mu\nu}$$

(optical theorem)

$$T^{\mu\nu} = i \int dx \, e^{-i q \cdot x} \langle B | T \{ J^{\mu\dagger}_{bu}(x) \, J^\nu_{bu}(0) \} | B \rangle$$

Operators: $\bar{b} b \to$ free quark decay, $\langle \bar{b} D^2 b \rangle$, $\langle \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b \rangle \sim m_{B^*}^2 - m_B^2$, etc.

$$d\Gamma = \left( \begin{array}{c} b \text{ quark decay} \end{array} \right) \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \ldots + \alpha_s(\ldots) + \frac{\alpha_s^2(\ldots)}{m_b^2} + \ldots \right\}$$

As for $e^+ e^- \to$ hadrons, question is when perturbative calculation can be trusted.
Classic application: inclusive $|V_{cb}|$

- Want to determine $|V_{cb}|$ from $B \rightarrow X_c \ell \bar{\nu}$:

$$
\Gamma(B \rightarrow X_c \ell \bar{\nu}) = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} (4.7 \text{ GeV})^5 (0.534) \times \\
\left[ 1 - 0.22\left( \frac{\Lambda_1 S}{500 \text{ MeV}} \right) - 0.011\left( \frac{\Lambda_1 S}{500 \text{ MeV}} \right)^2 - 0.052\left( \frac{\lambda_1}{(500 \text{ MeV})^2} \right) - 0.071\left( \frac{\lambda_2}{(500 \text{ MeV})^2} \right) \\
- 0.006\left( \frac{\lambda_1 \Lambda_1 S}{(500 \text{ MeV})^3} \right) + 0.011\left( \frac{\lambda_2 \Lambda_1 S}{(500 \text{ MeV})^3} \right) - 0.006\left( \frac{\rho_1}{(500 \text{ MeV})^3} \right) + 0.008\left( \frac{\rho_2}{(500 \text{ MeV})^3} \right) \\
+ 0.011\left( \frac{T_1}{(500 \text{ MeV})^3} \right) + 0.002\left( \frac{T_2}{(500 \text{ MeV})^3} \right) - 0.017\left( \frac{T_3}{(500 \text{ MeV})^3} \right) - 0.008\left( \frac{T_4}{(500 \text{ MeV})^3} \right) \\
+ 0.096\epsilon - 0.030\epsilon_{\text{BLM}}^2 + 0.015\epsilon \left( \frac{\Lambda_1 S}{500 \text{ MeV}} \right) + \ldots \right]
$$

Corrections: $O(\Lambda/m): \sim 20\%$, $O(\Lambda^2/m^2): \sim 5\%$, $O(\Lambda^3/m^3): \sim 1 - 2\%$, $O(\alpha_s): \sim 10\%$, Unknown terms: $< 1 - 2\%$

Matrix elements extracted from shape variables — good fit to lots of data

- Error of $|V_{cb}| \sim 2\%$ — a precision field; uncomfortable $\sim 2\sigma$ tension with exclusive

ZL — p.2/24
The data...

• Reasonably good fits

No evidence for deviations from quark-hadron duality

[BaBar, arXiv:0908.0415, similar results from Belle]
The challenge of $|V_{ub}|$ measurements

- Side opposite to $\beta$; precision crucial to be sensitive to NP in $\sin 2\beta$ via mixing

- Inclusive: rate known to $\sim 5\%$; cuts to remove $B \to X_c\ell\bar{\nu}$ introduce small parameters that complicate expansions

Nonperturbative $b$ distribution function (“shape function”) determines tails (e.g., shifts endpoint $\frac{1}{2}m_b \to \frac{1}{2}m_B$) ⇒ related to $B \to X_s\gamma$ photon spectrum

- Exclusive:

$$\frac{d\Gamma(B^0 \to \pi^+\ell\bar{\nu})}{dq^2} = \frac{G_F^2|\vec{p}_\pi|^3}{24\pi^3} |V_{ub}|^2 f_+(q^2)^2$$

Tools: Lattice QCD, under control at large $q^2$ (small $|\vec{p}_\pi|$)
Dispersion rel: constrain shape using few $f_+(q^2)$ values

- Many challenging open questions, active areas to date

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$ZL - p.2/26$
Shape function: lepton endpoint vs. $B \rightarrow X_s \gamma$

$b$ quark decay spectrum

with a model for $b$ quark PDF
Shape function: lepton endpoint vs. $B \rightarrow X_s \gamma$

$b$ quark decay spectrum

with a model for $b$ quark PDF
Shape function: lepton endpoint vs. $B \rightarrow X_s \gamma$

$b$ quark decay spectrum

with a model for $b$ quark PDF
Shape function: lepton endpoint vs. $B \rightarrow X_s \gamma$

$b$ quark decay spectrum

with a model for $b$ quark PDF

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[CLEO, 2001]
Shape function: lepton endpoint vs. $B \rightarrow X_s \gamma$

- $b$ quark decay spectrum

with a model for $b$ quark PDF

- Both of these spectra determined at lowest order by the $b$ quark PDF in $B$ meson
- Lots of work toward extending beyond leading order; some open issues remain

[CLEO, 2001]
Inclusive $B \to X_s \gamma$ calculations

- Two-body decay at lowest order: $O_7 = \bar{m}_b \bar{s} \sigma_{\mu \nu} e F^{\mu \nu} P_R b$

One of the (if not “the”) most elaborate SM calculations (constrains many models)

- NNLO practically completed
  
  [Misiak et al., hep-ph/0609232]

$\mathcal{O}(10^4)$ diagrams, 4-loop running, 3-loop matching and matrix elements

- SM prediction: $\mathcal{B}(B \to X_s \gamma)|_{E_\gamma > 1.6 \text{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$

  Measurement: $(3.43 \pm 0.22) \times 10^{-4}$
Regions of $B \to X_s \gamma$ phase space

- Important both for $|V_{ub}|$ and constraining NP

- $m_B - 2E_\gamma \lesssim 2$ GeV, and $< 1$ GeV at the peak

  Three cases:
  1) $\Lambda_{QCD} \sim m_B - 2E_\gamma \ll m_B$
  2) $\Lambda_{QCD} \ll m_B - 2E_\gamma \ll m_B$
  3) $\Lambda_{QCD} \ll m_B - 2E_\gamma \sim m_B$

  Neither 1) nor 2) is fully appropriate

  [Sometimes called: 1) SCET and 2) MSOPE regions]

- Reducing $E^\text{cut}_\gamma$ to $\sim 1.7$ GeV is probably not optimal / practical

- $B \to X_u \ell \bar{\nu}$ is more complicated: hadronic physics depends not on one ($E_\gamma$) but two variables (best choice: $p^{\pm}_X = E_X \mp |\vec{p}_X|$ — “jettyness” of hadronic final state)

- Existing approaches based on theory in one region, expect future improvements
\( B \rightarrow X_s \gamma \) and the 2HDM

- In Type-II 2HDM (as in the MSSM) the \( H^\pm \) contribution always enhances the rate.

  In SUSY cancellations can occur, strong bounds remain, depend on many param’s.

- Curves show ±1σ bands.

  Solid: \( B(B \rightarrow X_s \gamma) \) for \( \tan \beta = 2 \) in Type-II 2HDM.

  Dashed: SM prediction.

  Dotted: experimental data.

\[ 10^4 \times B(B \rightarrow X_s \gamma) \text{ vs. } M_{H^\pm} \text{ [GeV]} \]

[Misiak et al., hep-ph/0609232]

ZL — p.2/30
If all else fails: “Grinstein-type double ratios”

- Continuum theory may be competitive using HQS + chiral symmetry suppression

\[
\frac{f_B}{f_{B_s}} \times \frac{f_{D_s}}{f_D} \quad \text{— lattice: double ratio } = 1 \text{ within few } \%
\]

\[
\frac{f(B \to \rho \ell \bar{\nu})}{f(B \to K^* \ell \bar{\nu})} \times \frac{f(D \to K^* \ell \bar{\nu})}{f(D \to \rho \ell \bar{\nu})} \quad \text{or } q^2 \text{ spectra — accessible soon?}
\]

Numerous variations in the literature

\[
\frac{\mathcal{B}(B \to \ell \bar{\nu})}{\mathcal{B}(B \to \ell^+ \ell^-)} \times \frac{\mathcal{B}(D_s \to \ell \bar{\nu})}{\mathcal{B}(D \to \ell \bar{\nu})} \quad \text{— very clean... by } \sim 2020?\]

\[
\frac{\mathcal{B}(B_u \to \ell \bar{\nu})}{\mathcal{B}(B_d \to \mu^+ \mu^-)} \quad \text{— uses only isospin... around 2025?}
\]

The theoretically cleanest $|V_{ub}|$ I know... Need lots of LHCb and Belle II data...

[A high precision $\mathcal{B}(B_d \to \mu^+ \mu^-)$ measurement — run/upgrade LHCb forever...]

ZL — p.2/31
New physics in $V_{ub}$?

- Inclusive & exclusive $V_{ub}$ determinations in tension: $[(4.4 \pm 0.3) \text{ vs. } (3.4 \pm 0.3)] \times 10^{-3}$

CKM fit in the SM favors smaller values from exclusive decays

- A right-handed current

$$\epsilon_R (\bar{u} \gamma^\mu P_R b) (\bar{\ell} \gamma^\mu P_L \nu)$$

affects inclusive $B \to X_u \ell \bar{\nu}$ rate $\propto \epsilon_R^2$

It affects the exclusive rates $\propto \epsilon_R$

- NP at 10 – 20% of the SM may still contribute to semileptonic decays as well!

- Need much larger Belle II data sets to probe this conclusively
Also related to $B \rightarrow X_s \ell^+ \ell^-$

- Complementary to $B \rightarrow X_s \gamma$, depends on:

$$O_7 = \bar{m}_b \bar{s} \sigma_{\mu\nu} e F^{\mu\nu} P_R b,$$
$$O_9 = e^2 (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \ell),$$
$$O_{10} = e^2 (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

- Theory most precise for $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$
  - NNLL perturbative calculations
  - Nonperturbative corrections to $q^2$ spectrum

- In small $q^2$ region experiments require additional $m_{X_s} \lesssim 2 \text{ GeV}$ cut to suppress $b \rightarrow c(\rightarrow s \ell^+ \nu) \ell^- \bar{\nu} \Rightarrow$ nonperturbative effects

- Theory same as for in inclusive $|V_{ub}|$ measurements (similar phase space cuts)
A_{FB} in \( B \rightarrow K^* \ell^+ \ell^- \)

- Noticed that zero of \( A_{FB} \) was insensitive to form factor models

Decay rate depends on several form factors, were assumed to be independent functions of \( q^2 = m_{\ell^+ \ell^-}^2 \)

Despite the spectrum being model dependent, zero-crossing looked insensitive to model used

- For \( q^2 \ll m_B^2 \), the \( K^* \) is highly boosted; can use soft-collinear effective theory (SCET) to write form factors as sum of two terms: soft form factor & hard scattering

First term obeys symmetry relations, unclear to what extent it dominates over 2nd

- If it does, then \( A_{FB} \) changes sign:

\[
C_9^{\text{eff}}(s_0) = \frac{-2m_Bm_b}{s_0} C_7^{\text{eff}} \times [1 + O(\alpha_s, \Lambda_{\text{QCD}}/m_b)]
\]

\[ZL ~ p.2/34\]
Substantial discovery potential in many modes

- Some of the theoretically cleanest modes ($\nu$, $\tau$, inclusive) only possible at $e^+e^-$
- Many modes first seen at Belle II or LHCb
- In some decay modes, even in 2025:
  \[(\text{Exp. bound})/\text{SM} \gtrsim 10^3\]
  (E.g.: $B_{(s)} \rightarrow \tau^+\tau^-$, $e^+e^-$
  lots of model building...)

<table>
<thead>
<tr>
<th>Observable</th>
<th>Approximate SM prediction</th>
<th>Present status</th>
<th>Uncertainty / number of events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Super-$B$ (50 ab$^{-1}$) LHCb (10 fb$^{-1}$)</td>
</tr>
<tr>
<td>$S_{\psi K}$</td>
<td>input</td>
<td>0.671 ± 0.024</td>
<td>0.005 0.1</td>
</tr>
<tr>
<td>$S_{s K}$</td>
<td>$S_{\psi K}$</td>
<td>0.44 ± 0.18</td>
<td>0.03 0.1</td>
</tr>
<tr>
<td>$S_{n K}$</td>
<td>$S_{\psi K}$</td>
<td>0.59 ± 0.07</td>
<td>0.02 not studied</td>
</tr>
<tr>
<td>$\alpha(\pi\pi, \rho\rho, \rho\pi)$</td>
<td>$\alpha$</td>
<td>(89 ± 4)$^\circ$</td>
<td>$2^\circ$ 4$^\circ$</td>
</tr>
<tr>
<td>$\gamma(DK^+)$</td>
<td>$\gamma$</td>
<td>(70$^{+37}_{-30})^\circ$</td>
<td>$2^\circ$ 3$^\circ$</td>
</tr>
<tr>
<td>$S_{K\gamma}$</td>
<td>few × 0.01</td>
<td>−0.16 ± 0.22</td>
<td>0.03 —</td>
</tr>
<tr>
<td>$S_{B_s \rightarrow \phi\gamma}$</td>
<td>few × 0.01</td>
<td>—</td>
<td>0.05 —</td>
</tr>
<tr>
<td>$\beta_s(B_s \rightarrow \psi\phi)$</td>
<td>$1^\circ$</td>
<td>(22$^{+10}_{-8})^\circ$</td>
<td>— 0.3$^\circ$</td>
</tr>
<tr>
<td>$\beta_s(B_s \rightarrow \phi\phi)$</td>
<td>$1^\circ$</td>
<td>—</td>
<td>— 1.5$^\circ$</td>
</tr>
<tr>
<td>$A_{S_L}$</td>
<td>$-5 \times 10^{-4}$</td>
<td>$-(5.8 \pm 3.4) \times 10^{-3}$</td>
<td>$10^{-3}$ 10$^{-3}$</td>
</tr>
<tr>
<td>$A_{S_L}$</td>
<td>$2 \times 10^{-5}$</td>
<td>$(1.6 \pm 8.5) \times 10^{-3}$</td>
<td>$10^{-3}$ 10$^{-3}$</td>
</tr>
<tr>
<td>$A_{\text{CP}}(b \rightarrow s\gamma)$</td>
<td>$&lt; 0.01$</td>
<td>$-0.012 \pm 0.028$</td>
<td>0.005 —</td>
</tr>
<tr>
<td>$</td>
<td>V_{tb}</td>
<td>$</td>
<td>input</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>$</td>
<td>input</td>
</tr>
<tr>
<td>$B \rightarrow X_s \gamma$</td>
<td>$3.2 \times 10^{-4}$</td>
<td>$(3.52 \pm 0.25) \times 10^{-4}$</td>
<td>4% —</td>
</tr>
<tr>
<td>$B \rightarrow \tau\nu$</td>
<td>$1 \times 10^{-4}$</td>
<td>$(1.73 \pm 0.35) \times 10^{-4}$</td>
<td>5% —</td>
</tr>
<tr>
<td>$B \rightarrow X_s \nu\bar{\nu}$</td>
<td>$3 \times 10^{-5}$</td>
<td>$&lt; 6.4 \times 10^{-4}$</td>
<td>— only $K\nu\bar{\nu}$</td>
</tr>
<tr>
<td>$B \rightarrow X_s \ell^+\ell^-$</td>
<td>$6 \times 10^{-6}$</td>
<td>$(4.5 \pm 1.0) \times 10^{-6}$</td>
<td>6% not studied</td>
</tr>
<tr>
<td>$B_{s} \rightarrow \tau^+\tau^-$</td>
<td>$1 \times 10^{-6}$</td>
<td>$&lt; \text{few }%$</td>
<td>— Y(5S) run?</td>
</tr>
<tr>
<td>$B \rightarrow X_s \tau^+\tau^-$</td>
<td>$5 \times 10^{-7}$</td>
<td>$&lt; \text{few }%$</td>
<td>— not studied</td>
</tr>
<tr>
<td>$B \rightarrow \mu\ell$</td>
<td>$4 \times 10^{-7}$</td>
<td>$&lt; 1.3 \times 10^{-6}$</td>
<td>6% —</td>
</tr>
<tr>
<td>$B \rightarrow \tau^+\tau^-$</td>
<td>$5 \times 10^{-8}$</td>
<td>$&lt; 4.1 \times 10^{-3}$</td>
<td>— $\mathcal{O}(10^{-4})$</td>
</tr>
<tr>
<td>$B_{s} \rightarrow \mu^+\mu^-$</td>
<td>$3 \times 10^{-9}$</td>
<td>$&lt; 5 \times 10^{-8}$</td>
<td>— &gt; 5$\sigma$ in SM</td>
</tr>
<tr>
<td>$B \rightarrow \mu^+\mu^-$</td>
<td>$1 \times 10^{-10}$</td>
<td>$&lt; 1.5 \times 10^{-8}$</td>
<td>— not studied</td>
</tr>
<tr>
<td>$B \rightarrow K^* \ell^+\ell^-$</td>
<td>$1 \times 10^{-6}$</td>
<td>$(1 \pm 0.1) \times 10^{-6}$</td>
<td>15k 36k</td>
</tr>
<tr>
<td>$B \rightarrow K\nu\bar{\nu}$</td>
<td>$4 \times 10^{-6}$</td>
<td>$&lt; 1.4 \times 10^{-5}$</td>
<td>20% —</td>
</tr>
</tbody>
</table>

Look for “odd” things

- Cast a wide net — broad program is critical:
  
  \[ B \to (\gamma^+) \text{ invisible} \]  
  \[ B \to X_s + \text{ invisible} \]  
  \[ \Upsilon(1S) \to \text{ invisible} \]  
  \[ \Upsilon(nS) \to \gamma + \text{ invisible} \]  
  \[ e^+e^- \to (\gamma^+) \text{ invisible} \]  

- Also include “invisible” replaced by a new resonance; may decay to \( \ell^+\ell^- \), etc.

- \( \tau \) and \( \mu \) lepton flavor violation

- Searches for violations of conservation laws

- Obvious! most cited Belle paper: \( X(3872) \), most cited BaBar paper: \( D^*_0(2317) \)
Example: bump searches in $B \to K^{(*)}\ell^{+}\ell^{-}$

- Can probe certain DM models with $B$ decays
  E.g., “axion portal”: light ($\lesssim 1 \text{ GeV}$) scalar particle coupling as $(m_{\psi}/f_a) \bar{\psi}\gamma_5\psi a$

- In most of parameter space best bound is from $B \to K\ell^{+}\ell^{-}$

[Freytsis, ZL, Thaler, arXiv:0911.5355]
My Belle II “best buy” list

- Key observables: (i) sensitive to different NP, (ii) measurements can improve by order of magnitude, (iii) not limited by hadronic uncertainties
  - Difference of $CP$ asymmetries, $S_{\psi K_S} - S_{\phi K_S}$, $S_{\psi K_S} - S_{\eta' K_S}$, etc.
  - $\gamma$ from $CP$ asymmetries in tree-level decays vs. $\gamma$ from $S_{\psi K_S}$ and $\Delta m_d/\Delta m_s$
  - Search for charged lepton flavor violation, $\tau \to \mu \gamma$, $\tau \to 3\mu$, and similar modes
  - Search for $CP$ violation in $D^0 - \bar{D}^0$ mixing
  - $CP$ asymmetry in semileptonic decay (dilepton asymmetry), $A_{SL}$
  - $CP$ asymmetry in the radiative decay, $S_{K_S\pi^0\gamma}$
  - Rare decay searches and refinements: $b \to s\nu\bar{\nu}$, $B \to \tau\bar{\nu}$, etc.
  - Improve magnitudes of CKM elements
- Complementary to LHCb
- Any one of these measurements has the potential to establish new physics
My LHCb “best buy” list

- LHCb will probe $B_s$ sector at a level comparable to $B_d$
  - The $CP$ asymmetry, $S_{B_s \to \psi \phi}$
  - Difference of $CP$ asymmetries, $S_{B_s \to \psi \phi} - S_{B_s \to \phi \phi}$
  - $B_s \to \mu^+ \mu^-$, search for $B_d \to \mu^+ \mu^-$, other rare / forbidden decays
  - $10^{4-5}$ events in $B \to K^{(*)} \ell^+ \ell^-$, $B_s \to \phi \gamma$, ... — test Dirac structure, BSM op’s
  - $\gamma$ from $B \to DK$ and $B_s \to D_s K$
  - Search for charged lepton flavor violation, $\tau \to 3\mu$ and other modes if possible
  - Search for $CP$ violation in $D^0 - \bar{D}^0$ mixing

- Very broad program, complementary to Belle II
- With large BSM discovery potential
Summary

- Lots of progress for $|V_{cb}|$ and $|V_{ub}|$, determinations from exclusive decays largely in the hands of lattice QCD, room for progress in continuum — tension is troubling

- Theoretical tools for rare decays are similar, so developments often simultaneous

- Theory progress in understanding nonleptonic decays; unfortunately the best understood cases are not the most interesting to learn about weak scale physics

- More work and data needed to understand the expansions
  Why some predictions work at $\lesssim 10\%$ level, while others receive $\sim 30\%$ corrections
  Clarify role of charming penguins, chirally enhanced terms, annihilation, etc.

- Active field, experimental data stimulated lots of theory developments, expect more work & progress as LHCb and Belle II provide challenges and opportunities
Read at your own risk...