Cosmology:
Lecture #2
Big Bang Theory
of the hot Universe

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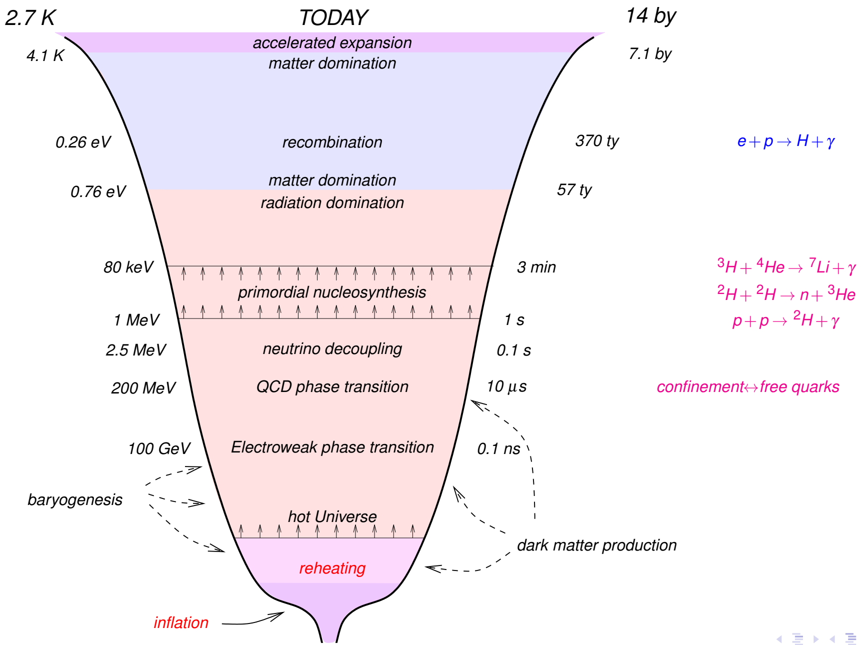
Outline


2.7 K
TODAY
14 by
accelerated expansion
4.1 K
matter domination
7.1 by
0.26 eV
recombination
370 ty
$e+p\to H+\gamma$
matter domination
0.76 eV
57 ty
radiation domination
80 keV
3 min
$^3H+{}^4He\to{}^7Li+\gamma$
primordial nucleosynthesis
$^2H+{}^2H\to n+{}^3He$
1 MeV
1 s
$p+p\to{}^2H+\gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD phase transition
10 µs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Outline

Einstein equations

$T_{\mu\nu}$: macroscopic description $\frac{1}{2}\int d^4x\sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu}$

$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - g_{\mu\nu}p$ ideal liquid with $\rho(t)$ and $p(t)$

in the comoving frame $u^0 = 1$, $\mathbf{u} = 0$ (almost) always works

$$T_\mu^\nu = diag(\rho,\, -p)$$

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j\ ,$$

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R\ :\quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

$$(00):\quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\varkappa}{a^2}$$

Friedmann equation (00) : $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{\varkappa}{a^2}$

$$\nabla_\mu T^{\mu 0} = 0 \quad \longrightarrow \quad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

the equation of state

$$p = p(\rho)$$

many-component liquid,
in case of thermal equilibrium

other equations

$$-3d(\ln a) = \frac{d\rho}{p+\rho} = d(\ln s)$$

entropy of cosmic primordial plasma is conserved in a comoving frame

$$sa^3 = \text{const}$$

Examples of cosmological solutions

$$\varkappa = 0 \qquad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho$$

dust: $p = 0$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^3}\,, \quad a(t) = const \cdot (t - t_s)^{2/3}\,, \quad \rho(t) = \frac{\text{const}}{(t - t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{2}{3t}\,, \quad \rho = \frac{3}{8\pi G} H^2 = \frac{1}{6\pi G}\frac{1}{t^2}$$

the Universe is too young

$$t_0 = \frac{2}{3H_0} = 0.9 \times 10^{10}\ \text{yr} \quad (h = 0.7)$$

Cosmological (particle) horizon $l_H(t)$

distance covered by photons emitted at $t = 0$

the size of causally-connected region — the size of the visible part of the Universe

in conformal coordinates: $ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$
coordinate size of the horizon equals $\eta(t) = \int d\eta$

$$l_H(t) = a(t)\eta(t) = a(t)\int_0^t \frac{dt'}{a(t')}$$

dust

$$l_H(t) = 3t = \frac{2}{H(t)}\,, \qquad l_{H,0} = 2.6 \times 10^{28}\text{cm} \quad (h = 0.7)$$

Examples of cosmological solutions

radiation: $p = \frac{1}{3}\rho$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^4}\,, \quad a(t) = const \cdot (t - t_s)^{1/2}\,, \quad \rho(t) = \frac{\text{const}}{(t - t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{1}{2t}\,, \quad \rho = \frac{3}{8\pi G}H^2 = \frac{3}{32\pi G}\frac{1}{t^2}$$

$$l_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} = 2t = \frac{1}{H(t)}\,.$$

In case of thermal equilibrium $T = \text{const}/a$

$$\rho_b = \frac{\pi^2}{30}g_b T^4\,, \quad \rho_f = \frac{7}{8}\frac{\pi^2}{30}g_f T^4$$

$$\rho = \frac{\pi^2}{30}g_* T^4\,, \quad g_* = \sum_b g_b + \frac{7}{8}\sum_f g_f = g_*(T)$$

Examples of cosmological solutions

vacuum: $T_{\mu\nu} = \rho_{vac}\eta_{\mu\nu}$ $p = -\rho$

$$S_G = -\frac{1}{16\pi G}\int R\sqrt{-g}d^4x\,, \quad S_\Lambda = -\Lambda\int\sqrt{-g}d^4x\,.$$

$$a = \text{const}\cdot e^{H_{\rm dS}t}\,, \quad H_{\rm dS} = \sqrt{\frac{8\pi}{3}G\rho_{vac}}$$

de Sitter space: space-time of constant curvature

$$ds^2 = dt^2 - e^{2H_{dS}t}d\mathbf{x}^2$$

$\ddot{a} > 0$, no initial singularity

$ds^2 = dt^2 - e^{2H_{dS}t} d\mathbf{x}^2$

no cosmological horizon: $l_{\mathrm{H}}(t) = e^{H_{dS}t} \int_{-\infty}^{t} dt' e^{-H_{dS}t'} = \infty$

de Sitter (events) horizon ($\mathbf{x} = 0$, t):

from which distance $l(t)$ one can detect light emitted at t?

in conformal coordinates: $ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$

coordinate size: $\eta(t \to \infty) - \eta(t) = \int_t^{\infty} \frac{dt'}{a(t')}$

physical size: $l_{dS} = a(t) \int_t^{\infty} \frac{dt'}{a(t')} = \frac{1}{H_{dS}}$

observer will never be informed what happens at distances larger than $l_{dS} = H_{dS}^{-1}$

$H_0 > H_{dS}$

Outline

Friedmann equation for the present Universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\left(\rho_{\rm M} + \rho_{rad} + \rho_\Lambda + \rho_{\rm curv}\right)$$

$$\frac{8\pi}{3} G\rho_{\rm curv} = -\frac{\varkappa}{a^2}\,, \quad \rho_c \equiv \frac{3}{8\pi G} H_0^2$$

$$\rho_c = \rho_{{\rm M},0} + \rho_{rad,0} + \rho_{\Lambda,0} = \rho_c = 0.52 \cdot 10^{-5} \frac{\rm GeV}{\rm cm^3}\,, \qquad \text{for } \; h = 0.7$$

$$\Omega_X \equiv \frac{\rho_{X,0}}{\rho_c}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho_c \left[\Omega_{\rm M}\left(\frac{a_0}{a}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_{curv}\left(\frac{a_0}{a}\right)^2\right]$$

Homogeneous and isotropic 3d manifolds

$$dl^2 = d\rho^2 + r^2(\rho)(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$r(\rho) = \begin{cases} R\sin(\rho/R)\,, & \text{3-sphere} \\ \rho\,, & \text{3-plane} \\ R\sinh(\rho/R)\,, & \text{3-hyperboloid} \end{cases}$$

ρ is a geodesic distance; $S = 4\pi r^2(\rho)$; $\Delta\theta = \frac{l}{r(\rho)}$

$$d\rho^2 = \frac{dr^2}{\cosh^2\frac{\rho}{R}} = \frac{dr^2}{1+\frac{r^2}{R^2}} \qquad d\rho^2 = \frac{dr^2}{\cos^2\frac{\rho}{R}} = \frac{dr^2}{1-\frac{r^2}{R^2}}$$

$$dl^2 = \frac{dr^2}{1-\varkappa\frac{r^2}{R^2}} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

Brightness–redshift dependence in the Universe

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + \sinh^2\chi\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

coordinate distance $\chi = \int_{t_i}^{t_0} \frac{dt}{a(t)}$ $\qquad z(t) = \frac{a_0}{a(t)} - 1$

$$\chi(z) = \int_0^z \frac{dz'}{a_0 H_0} \frac{1}{\sqrt{\Omega_M (z'+1)^3 + \Omega_\Lambda + \Omega_{curv}(z'+1)^2}}$$

$$a_0^2 H_0^2 \Omega_{curv} = 1 \,, \quad \Omega_M + \Omega_\Lambda + \Omega_{curv} = 1$$

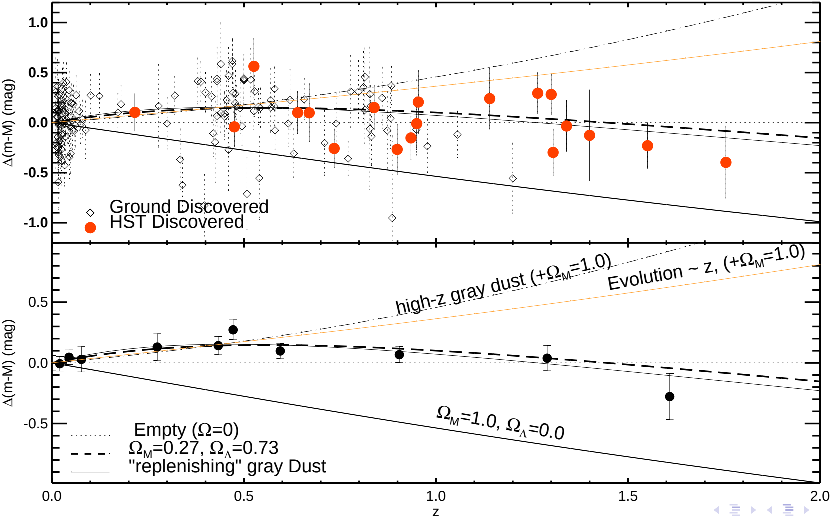
$$S(z) = 4\pi r^2(z) \,, \qquad r(z) = a_0 \sinh\chi(z)$$

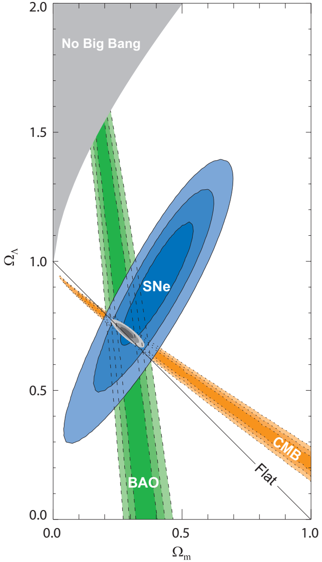
detector: $N_\gamma \propto S^{-1}$, $\omega = \omega_i/(1+z)$, $dt_0 = (1+z)dt_i$
hence the brightness (energy flux measured by a detector) is

$$J = \frac{L}{(1+z)^2 S(z)} \equiv \frac{L}{4\pi r_{ph}^2} \,, \quad r_{ph} = (1+z)\cdot r(z)$$

Brightness–redshift dependence: SNe Ia

$$\Delta(m-M) = 5\log\frac{r_{ph}}{r_{ph}(\Omega_c = 0.8, \Omega_M = 0.2)}$$




No Big Bang
SNe
BAO
CMB
Flat
Ω_Λ
Ω_m
0.0
0.5
1.0
1.5
2.0

Last scattering: $\gamma e \to \gamma e$

$$\sigma_{\mathrm{T}} = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2} \approx 0.67 \cdot 10^{-24}\,\mathrm{cm}^2\,, \qquad \tau_\gamma = \frac{1}{\sigma_{\mathrm{T}} \cdot n_e(T)}$$

last scattering: $\tau_\gamma(T_f) \simeq H^{-1}(T_f) \simeq t_f$

$$T_f = 0.26\ \mathrm{eV}\,, \quad z = 1100\,, \quad t_f = 370\,000\ \mathrm{yr}$$

for general processes one should solve kinetic equations

$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \int (production - destruction)$$

Boltzmann equation in a comoving volume: $\frac{d}{dt}\left(n\,a^3\right) = a^3 \int \dots$

Recombination: horizon

matter domination: $l_{H,r} = 2H_r^{-1}$

$$H_r^2 = \frac{8\pi}{3} G\rho_M(t_r) = \frac{8\pi}{3} G\rho_{M,0}\left(\frac{a_0}{a_r}\right)^3 = \frac{8\pi}{3} G\rho_c\Omega_{M,0}(1+z_r)^3 \, .$$

at recombination: $l_{H,r} = \frac{2}{H_0\sqrt{\Omega_M}} \frac{1}{(1+z_r)^{3/2}}$

today: $l_{H,r}(t_0) = l_{H,r} \times \frac{a_0}{a_r} = \frac{2}{H_0\sqrt{\Omega_M}} \frac{1}{\sqrt{1+z_r}}$

$$\frac{l_{H_0}}{l_{H,r}(t_0)} \sim \sqrt{1+z_r} \simeq 30$$

Recombination: angle

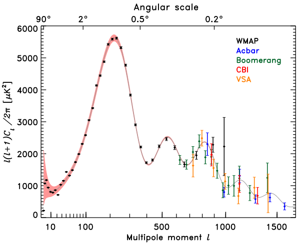
$$\chi_r = \int_{t_r}^{t_0} \frac{dt}{a(t)} , \quad \Delta\theta_r = \frac{l_{H,r}}{r_a(z_r)}$$

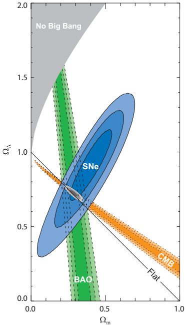
$$r_a(z_r) = (1+z_r)^{-1} \cdot a_0 \cdot \sinh\chi_r$$

$$\Delta\theta_r = \frac{1}{\sqrt{z_r+1}} , \quad \Omega_{curv} = \Omega_\Lambda = 0 .$$

$$\Delta\theta_r = \frac{1}{\sqrt{z_r+1}} \frac{2\sqrt{\Omega_{curv}/\Omega_M}}{\sinh\left(2\sqrt{\Omega_{curv}/\Omega_M} I\right)} .$$

$$I = \int_0^1 \frac{dy}{\sqrt{1+\frac{\Omega_\Lambda}{\Omega_M}y^6}}$$



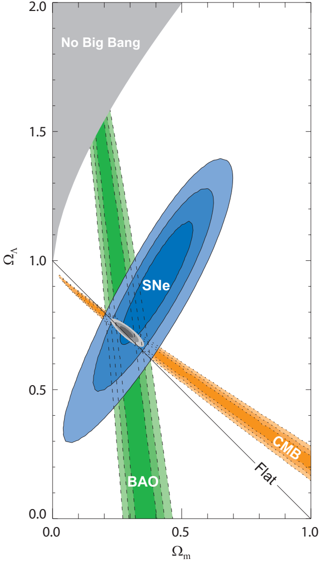


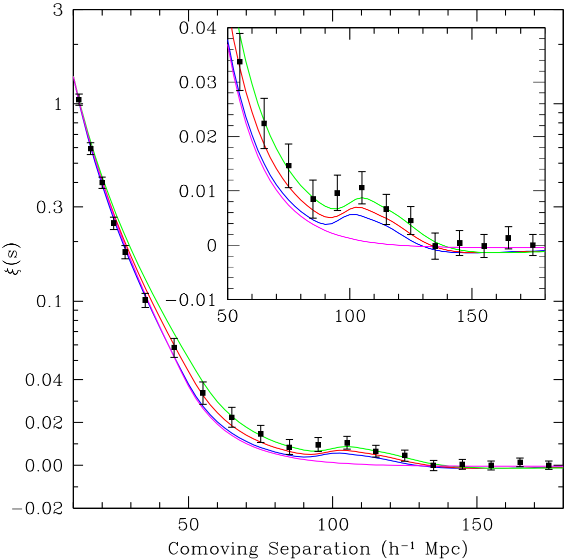
Acoustic oscillations in relativistic plasma: What matters is the sound horizon:

$$l_{s,r} = l_{H,r} \cdot v_s \approx l_{H,r}/\sqrt{3}$$

Then $\Delta\theta_{r,s} =$

$$\frac{1}{\sqrt{3}} \frac{1}{\sqrt{1+z}} \times \frac{180^\circ}{\pi} \simeq 1^\circ$$





$$110/0.7\,\mathrm{Mpc} \simeq l_{H,r}(t_0) \times \sqrt{v_s^2} \simeq l_{H_0}/\sqrt{3}/\sqrt{1+z_r}$$

Outline

Neutrino freeze-out

$T > m_e$ $\qquad$ $e^+e^- \leftrightarrow \nu\bar{\nu}$, $e\nu \leftrightarrow e\nu$

$$\sigma_\nu \sim G_F^2 E^2$$

neutrino interaction rate

$$\tau_\nu = \frac{1}{\langle \sigma_\nu n v \rangle} \sim \frac{1}{G_F^2 T^5} \qquad H^2 = \frac{8\pi}{3 M_{Pl}^2} \frac{\pi^2}{30} g_* T^4 \equiv \frac{T^4}{M_{Pl}^{*2}}$$

$$\tau_\nu(T) \sim H^{-1}(T) = \frac{M_{Pl}^*}{T^2}$$

$$T_{\nu,f} \sim \left(\frac{1}{G_F^2 M_{Pl}^*} \right)^{1/3} \sim 2 \div 3 \text{ MeV}$$

Neutron decoupling $p+e \longleftrightarrow n+\nu_e$

typical energy scales

$$T \gtrsim \Delta m = 1.3\text{ MeV}, \qquad T \gtrsim m_e = 0.5\text{ MeV}$$

neutron interaction rate

$$\tau_{n\leftrightarrow p} = \frac{1}{\Gamma_{n\leftrightarrow p}} = \frac{1}{C_n G_F^2 T^5}$$

neutron decoupling $\Gamma_{n\leftrightarrow p}(T) \sim H(T) = T^2/M_{Pl}^*$

$$T_n = \frac{1}{\left(C_n M_{Pl}^* G_F^2\right)^{1/3}} \approx 0.8\text{ MeV}$$

$$g_* = 2 + \frac{7}{8}\cdot 4 + \frac{7}{8}\cdot 2\cdot N_\nu \qquad t = \frac{1}{2H(T_n)} = \frac{M_{Pl}^*}{2T_n^2} \approx 1\text{ s}$$

Neutron density at decoupling

$$n_n = g_n \left(\frac{m_n T}{2\pi}\right)^{3/2} e^{\frac{\mu_n - m_n}{T}} \qquad \mu_n + \mu_\nu = \mu_p + \mu_e$$

$$\frac{n_n}{n_p} = e^{-\frac{m_n - m_p}{T}} e^{\frac{\mu_n - \mu_p}{T}}$$

for relativistic e^+ and e^-

$$n_{e^-} - n_{e^+} \sim \mu_e T^2 \longrightarrow \frac{\mu_e}{T} \sim \frac{n_{e^-} - n_{e^+}}{T^3} = \frac{n_p}{T^3} \sim \eta_B \equiv \frac{n_p}{n_\gamma} = 6 \times 10^{-10}$$

$$\frac{n_n}{n_p} = e^{-\frac{m_n - m_p}{T_n}} \equiv e^{-\frac{\Delta m}{T_n}} \approx \frac{1}{5} e^{-\frac{\mu_\nu}{T_n}}$$

$p+n \longrightarrow \mathrm{D}+\gamma$ Saha equation

$$n_n = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_n - m_n}{T}}\,, \qquad n_p = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_p - m_p}{T}}\,,$$

Chemical equilibrium for nuclei: $\mu_A = \mu_p \cdot Z + \mu_n \cdot (A-Z)$

$$X_A = \frac{A n_A}{n_\mathrm{B}}\,, \qquad X_A = X_p^Z X_n^{A-Z} 2^{-A} g_A A^{5/2} \eta_\mathrm{B}^{A-1} \left(\frac{2.5\,T}{m_p}\right)^{\frac{3}{2}(A-1)} \mathrm{e}^{\frac{\Delta A}{T}}$$

Temperature of BBN T_{NS}: $X_\mathrm{D} \sim 1$

$\Delta_\mathrm{D} = 2.23$ MeV $t_{NS} \approx 3\,\mathrm{min}$

$$X_\mathrm{D}(T_{NS}) \sim \eta_\mathrm{B} \left(\frac{2.5\,T_{NS}}{m_p}\right)^{3/2} \mathrm{e}^{\frac{\Delta_\mathrm{D}}{T_{NS}}} \sim 1 \longrightarrow T_{NS} \approx 80\ \mathrm{keV}$$

Helium density (chemical equilibrium)

$$X_A = X_p^Z X_n^{A-Z} 2^{-A} g_A A^{5/2} \eta_B^{A-1} \left(\frac{2.5T}{m_p}\right)^{\frac{3}{2}(A-1)} e^{\frac{\Delta A}{T}}$$

$X_{^4\mathrm{He}}$ @ $T = T_{NS}$? $\Delta_{^4\mathrm{He}} = 28.3$ MeV

$$X_{^4\mathrm{He}} = X_p^2 X_n^2 \cdot 8\eta_B^3 \left(\frac{2.5T}{m_p}\right)^{9/2} e^{\frac{\Delta_{^4\mathrm{He}}}{T}} \longrightarrow 10^{128}$$

Let $X_{^4\mathrm{He}} \sim 1$ $(2p, 2n)$, $n_n < n_p$ $X_p \sim 1$

$$X_n = X_{^4\mathrm{He}}^{1/2} \eta_B^{-3/2} \left(\frac{2.5T}{m_p}\right)^{-9/4} e^{-\frac{\Delta_{^4\mathrm{He}}}{2T}}$$

Light element densities (chemical equilibrium)

$$X_A = \left[\eta_{\rm B} \cdot \left(\frac{2.5T}{m_p}\right)^{3/2}\right]^{\frac{3}{2}Z - \frac{1}{2}A - 1} \mathrm{e}^{\frac{\Delta_A - \Delta_{^4\mathrm{He}}(A-Z)/2}{T}} \simeq 10^{7.4(A+2-3Z)}\, \mathrm{e}^{(A-Z)\frac{\Delta_A/(A-Z) - \Delta_{^4\mathrm{He}}/2}{T}}$$

Z	Nucleus	Δ_A	Δ_A/A	$\Delta_A/(A-Z)$	X_A
1	^{2}H $\equiv$ D	2.23	1.11	2.23	10^{-79}
	^{3}H $\equiv$ T	8.48	2.83	4.24	10^{-118}
2	^{3}He	7.72	2.57	7.72	10^{-51}
	^{4}He $\equiv \alpha$	28.30	7.75	14.15	1
3	^{6}Li	31.99	5.33	10.66	10^{-78}
	^{7}Li	39.24	5.61	9.81	10^{-116}
4	^{7}Be	37.60	5.37	12.53	10^{-55}
5	^{8}B	37.73	4.71	12.58	10^{-69}
6	^{12}C	92.2	7.68	15.37	10^{19}

Helium abundance (NO chemical equilibrium)

Neutrons remain mostly in helium

$$n_{^4\text{He}}(T_{NS}) = \frac{1}{2} n_n(T_{NS}) \,,$$

neutron-to-proton ratio $\tau_n \approx 880$ s

$$\frac{n_n(T_{NS})}{n_p(T_{NS})} \approx \frac{1}{5} \cdot \mathrm{e}^{-\frac{t_{NS}}{\tau_n}} \cdot \mathrm{e}^{-\frac{\mu_\nu}{T_n}} \approx \frac{1}{7} \,,$$

$$Y_p \equiv X_{^4\text{He}} = \frac{m_{^4\text{He}} \cdot n_{^4\text{He}}(T_{NS})}{m_p\left(n_p(T_{NS}) + n_n(T_{NS})\right)} = \frac{2}{\frac{n_p(T_{NS})}{n_n(T_{NS})} + 1} \approx 25\%$$

from observations of relic helium abundance:

$$\Delta N_{\nu,eff} \leq 1 \,, \qquad \left|\frac{\mu_\nu}{T_n}\right| \lesssim 0.01$$

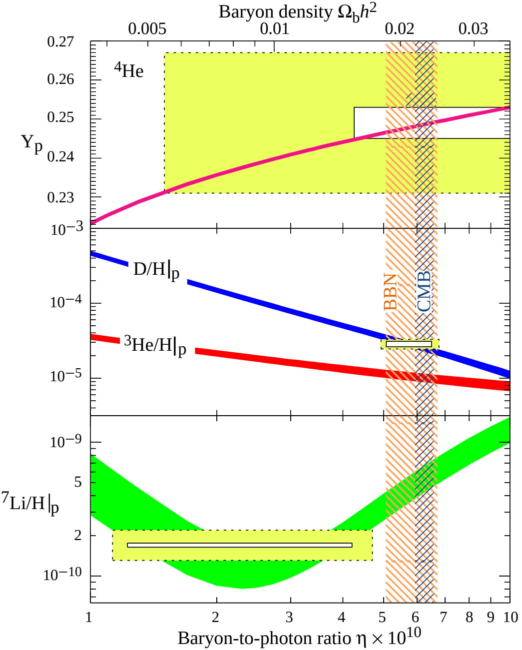
Main nuclear reactions

1. $p(n,\gamma)$D — deuterium production, BBN starts.
2. D$(p,\gamma)^3$He, D(D,n)^{3}He, D(D,p)T, ^{3}He(n,p)T — intermediate stage.
3. T(D,n)^{4}He, ^{3}He(D,p)^{4}He — production of ^{4}He.
4. T$(\alpha,\gamma)^7$Li, ^{3}He$(\alpha,\gamma)^7$Be, ^{7}Be$(n,p)^7$Li — production of the heaviest baryonic relics.
5. ^{7}Li$(p,\alpha)^4$He — ^{7}Li burning.

One has to compare reaction rates to the expansion rate

$$H(T_{NS} = 80\ \text{keV}) = 4 \times 10^{-3}\ \text{s}^{-1}$$

to obtain nonequilibrium concentrations



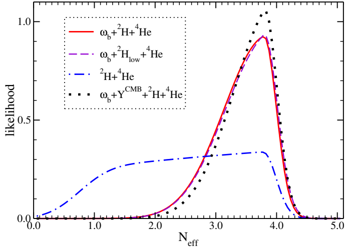
Measurement of $\eta_B = n_B/n_\gamma$ at $T \sim 1$ MeV

Lack of Lithium. . . Exotics needed?

$$Y_p = 0.2581 \pm 0.025\,,$$

$$D/H\big|_p = (2.87 \pm 0.21) \times 10^{-5}$$

1103.1261



similar results from other recent studies including structure formation

1001.4440, 1001.5218, 1202.2889

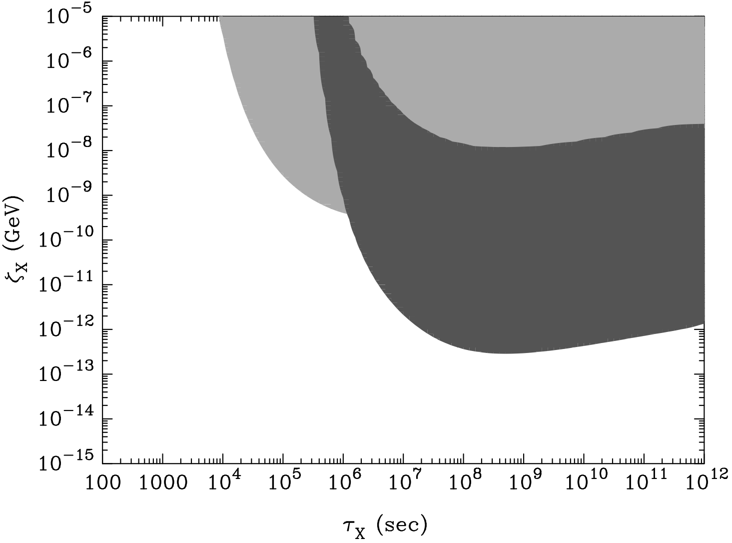
$N_{\nu,eff} < 4.2$ @ 95%CL

$N_{\nu,eff} < 4.0$ from D/H 1205.3785

BBN limits on Unstable relics $\zeta_X = M_X n_X / n_\gamma$

astro-ph/0605255

$X \to \gamma + x$



Outline

Dark Matter Properties $p = 0$

(If) particles:

1. stable on cosmological time-scale
2. nonrelativistic long before RD/MD-transition (either Cold or Warm, $v_{RD/MD} \lesssim 10^{-3}$)
3. (almost) collisionless
4. (almost) electrically neutral

If were in thermal equilibrium: $M_X \gtrsim 1$ keV

If not: for bosons

$\lambda = 2\pi/(M_X v_X)$, in a galaxy $v_X \sim 0.5 \cdot 10^{-3} \longrightarrow M_X \gtrsim 3 \cdot 10^{-22}$ eV

for fermions

Pauli blocking: $M_X \gtrsim 750$ eV

$$f(\mathbf{p}, \mathbf{x}) = \frac{\rho_X(\mathbf{x})}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi} M_X v_X\right)^3} \cdot \mathrm{e}^{-\frac{\mathbf{p}^2}{2M_X^2 v_X^2}} \Bigg|_{\mathbf{p}=0} \leq \frac{g_X}{(2\pi)^3}$$

Dark Matter Candidates

- WIMPs (neutralino, . . .)
- sterile neutrinos
- gravitino
- axion
- Heavy relics
- (Topological) defects
- Massive Astrophysical Compact Halo Objects
- Primordial black hole remnants

Weakly Interacting Massive Particles

Assumptions:

1. no $X - \bar{X}$ asymmetry $\qquad n_X = n_{\bar{X}}$
2. @ $T < M_X$ in thermal equilibrium with plasma

$$n_X = n_{\bar{X}} = g_X \left(\frac{M_X T}{2\pi} \right)^{3/2} e^{-M_X/T}$$

$$X\bar{X} \longrightarrow \text{light particles}$$

freeze-out temperature T_f $\qquad M_{Pl}^* = M_{Pl}/1.66\sqrt{g_*}$

$$\frac{1}{n_X} \frac{1}{\langle \sigma_{ann} v \rangle} = H^{-1}(T_f) \longrightarrow T_f = \frac{M_X}{\ln\left(\frac{g_X M_X M_{Pl}^* \sigma_0}{(2\pi)^{3/2}} \right)} .$$

Bethe formula: $\qquad$ annihilation in s-wave: $\sigma_{ann} = \frac{\sigma_0}{v}$

Weakly Interacting Massive Particles (WIMPs)

density after freeze-out: $n_X(T_f) = \frac{T_f^2}{M_{Pl}^* \sigma_0}$

present density: $n_X(T_0) = \left(\frac{a(T_f)}{a(T_0)}\right)^3 n_X(T_f) = \left(\frac{s_0}{s(T_f)}\right) n_X(T_f) \propto \frac{1}{T_f} \propto \frac{1}{M_X}$

$X + \bar{X}$ contribution to critical density:

$$\Omega_X = 2\frac{M_X n_X(T_0)}{\rho_c} = 7.6 \frac{s_0 \ln\left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}}\right)}{\rho_c \sigma_0 M_{Pl} \sqrt{g_*(T_f)}}$$

$$= 0.1 \cdot \left(\frac{(10\ \text{TeV})^{-2}}{\sigma_0}\right) \frac{0.3}{\sqrt{g_*(T_f)}} \ln\left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}}\right) \cdot \frac{1}{2h^2}$$

natural dark matter: $\sigma_0 \sim 0.01 \times \sigma_{weak}$

naturaly “light” $\sigma_0 \lesssim \frac{4\pi}{M_X^2} \longrightarrow M_X \lesssim 100$ TeV

WIMPs are mostly welcome

- Do not need new physical scale (and interaction?)
- Can search for WIMPs in collision experiments (LHC):

$$X + \bar{X} \leftrightarrow \text{SM} + \text{SM'} + \dots$$

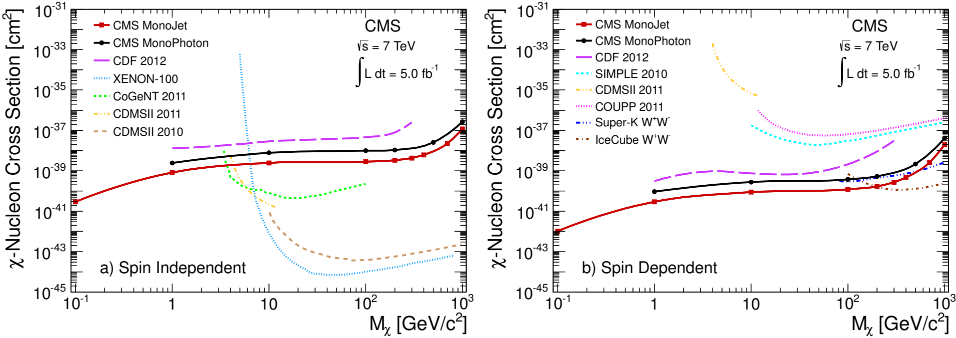
- Can search for WIMPs in cosmic rays: products of WIMPs annihilation (in Galactic center, dwarf galaxies, Sun)

$$X + \bar{X} \to p\bar{p}\,,\;\; e^+e^-\,,\;\; \nu, \gamma, \dots$$

- Direct searches for Galactic Dark Matter ($v \sim 10^{-3}$)

$$X + \text{nuclei} \to X + \text{nuclei} + \Delta E$$

Recent results of (in)direct searches @ 7 TeV

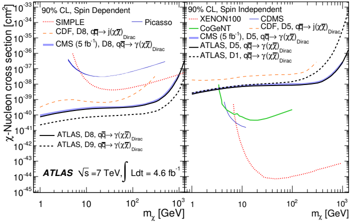


for WIMPs 1206.5663

Logic: no light superpartners, $M_{SUSY} > 500\,\text{GeV}$

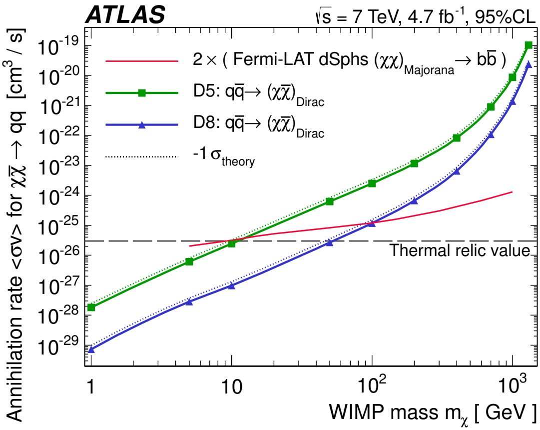
$D1\ (scalar): \quad \frac{m_q}{M_*^3}\bar{\chi}\chi\bar{q}q \qquad D8\ (axial): \quad \frac{1}{M_*^2}\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$

$D5\ (vector): \frac{1}{M_*^2}\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q \quad D9\ (tensor): \frac{1}{M_*^2}\bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma_{\mu\nu}q$

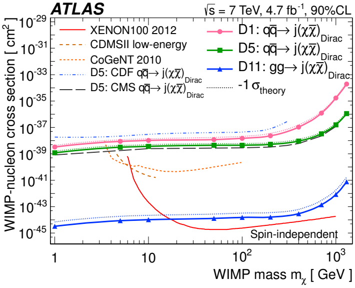


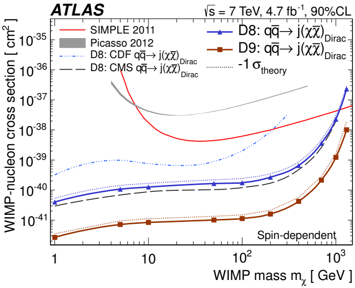
1209.4625

Recent results of (in)direct searches @ 7 TeV



1210.4491v1





Decoupling of relativistic specia (DM?)

Thermal equilibrium is forbidden:

$T_d \gg M_X$, and then $n_X/s = \text{const}$

$$\Omega_{3/2} = \frac{m_X \cdot n_{X,0}}{\rho_c} = \frac{m_X \cdot s_0}{\rho_c}\frac{n_{X,0}}{s_0} = 0.2\frac{M_X}{100\text{ eV}}\left(\frac{g_X}{2}\right)\cdot\left(\frac{100}{g_*(T_d)}\right)\cdot\frac{1}{2h^2}$$

- If fermions: limit from Pauli-blocking
- Generally: too hot at Equality:
  from structure formation we need at $T_{Eq} \sim 1$ eV, $v_{DM} \lesssim 10^{-3}$

Other Dark Matter candidates are not in equilibrium!

- WIMPs (neutralino, ...) $\Longleftarrow$ thermal ! $\Longrightarrow$ Singlet scalar field:

$$\mathscr{L} = \mathscr{L}_{SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{m_0^2}{2} S^2 - \lambda S^2 H^\dagger H + ...$$

Invisible decay $H \to SS$ if kinematically allowed, missing energy

direct searches for dark matter

- sterile neutrinos $\Longleftarrow$ Price: sensitive to mass and couplings! not seesaw neutrino!
- axion $\Longleftarrow$ Price: sensitive to mass and (=couplings) and history!
- gravitino $\Longleftarrow$ Price: sensitive to mass, couplings and reheating temperature !!! yet it is natural LSP if $\Lambda_{SUSY} \lesssim 10^{10}$ GeV
- Heavy relics $\Longleftarrow$ Price: sensitive to mass and untestable
- Asymmetric WIMPS, $n_X \neq n_{\bar{X}}$ $\Longleftarrow$ No cosmic ray signals, but trapped in stars
  Why asymmetric? But other matter, baryons, is asymmetric...
- Why non-thermal DM? $\Longleftarrow$ But major processes we know (recombination and nucleosymthesis) were out-of-equilibrium...

Outline

Electroweak sphalerons: $B-L$

$$\partial^\mu j_\mu{}^{\mathrm{B}} = 3\frac{g^2}{16\pi^2} V^{a\,\mu\nu}\tilde{V}^a{}_{\mu\nu}\,,$$

$$\partial^\mu j_\mu{}^{\mathrm{L}n} = \frac{g^2}{16\pi^2} V^{a\,\mu\nu}\tilde{V}^a{}_{\mu\nu}\,,\quad n = 1,2,3\,,$$

$V^a{}_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g\varepsilon^{abc}V^b_\mu V^c_\nu$ refer to $SU(2)_{\mathrm{W}}$, $\tilde{V}^a{}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}V^{a\,\lambda\rho}$
Anomaly: only left fermions couple to fields V^a_μ.
For nontrivial gauge fields in vacuum or plasma

$$\Delta B = B(t_f) - B(t_i) = \int_{t_i}^{t_f} dt \int d^3\mathbf{x}\,\partial^\mu j^{\mathrm{B}}_\mu = 3\int_{t_i}^{t_f} d^4x\,\frac{g^2}{16\pi^2} V^{a\,\mu\nu}\tilde{V}^a{}_{\mu\nu}\,,$$

Strong fields are needed: $V^a{}_{\mu\nu} \propto \frac{1}{g}$, (integral is natural number!). Energies of such configurations $\propto \frac{1}{g^2}$.

$$\Delta B = 3\,\Delta L_e = 3\,\Delta L_\mu = 3\,\Delta L_\tau$$

At temperatures 100 GeV $\lesssim T \lesssim 10^{12}$ GeV only 3 linear combinations survive, e.g.

$$B-L\,,\quad L_e - L_\mu\,,\quad L_e - L_\tau$$

where $$L \equiv L_e + L_\mu + L_\tau$$

Baryogenesis

Sakharov conditions of successful baryogenesis

- B-violation $(\Delta B \neq 0)\ XY \dots \to X'Y' \dots B$
- C- & CP-violation $(\Delta C \neq 0,\ \Delta CP \neq 0)\ \bar{X}\bar{Y} \dots \to \bar{X}'\bar{Y}' \dots \bar{B}$
- processes above are out of equilibrium $X'Y' \dots B \to XY \dots$

At 100 GeV$\lesssim T \lesssim 10^{12}$ GeV nonperturbative processes (EW-sphalerons) violate B, L_α, so that only three charges are conserved out of four, e.g.

$$B - L\,, \quad L_e - L_\mu\,, \quad L_e - L_\tau$$

and $B = \alpha \times (B - L)$, $L = (\alpha - 1) \times (B - L)$

Leptogenesis: Baryogenesis from lepton asymmetry of the Universe ... due to sterile neutrinos

Why $\Omega_B \sim \Omega_{DM}$? antropic principle?

Lepton asymmetry from sterile neutrino decays

Most general renormalizable lagrangian with Majorana neutrinos N_I, $I,\alpha = 1,2,3$.

$$\mathscr{L}_{SM} + \overline{N}_I i\partial\!\!\!/ N_I - y_{I\alpha}\bar{L}_\alpha \widetilde{H} N_I - \frac{M_I}{2}\overline{N}_I^c N_I + \text{h.c.}$$

where $\widetilde{H}_i = \varepsilon_{ij}H_j^*$, $i,j = 1,2$; complex Yukawas, Majorana mass: $\Delta L \neq 0$ lepton number violating processes ($N = N^c$!):

$$N_I \to h l_\alpha \;, \quad N_I \to h\bar{l}_\alpha \;,$$
$$h l_\alpha \to h\bar{l}_\beta$$

- neutrino oscillations are explained
- BAU via leptogenesis (decays for $M_I > 10^9$ GeV or oscillations for light neutrinos, even $M_I \gtrsim 100\,\text{MeV}$
- dark matter with $M_I \sim$ 1-100 keV