Lecture 2
QCD perturbation theory at fixed order: $\sigma(e^+e^- \to \text{hadrons})$

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Today

- ☛ Solution of the RGE
  - ☞ running coupling
  - ☞ running masses
- ☛ Consequences of renormalization of QCD
- ☛ Electron-positron annihilation to hadrons
  - ☞ LO
  - ☞ NLO
  - ☞ choosing the scale

Running coupling

- ☛ we introduce the running coupling $\alpha_s(Q^2)$ implicitly:

$$t = \int_{\alpha_s}^{\alpha_s(Q^2)} \frac{\mathrm{d}x}{\beta(x)}, \qquad \alpha_s \equiv \alpha_s(\mu^2)$$

- ☛ d/dt :

$$1 = \frac{1}{\beta\left(\alpha_s(Q^2)\right)} \frac{\partial \alpha_s(Q^2)}{\partial\, t}$$

- ☛ d/dα :

$$0 = -\frac{1}{\beta(\alpha_s)} + \frac{1}{\beta\left(\alpha_s(Q^2)\right)} \frac{\partial \alpha_s(Q^2)}{\partial \alpha_s}$$

➔ $$\frac{\partial \alpha_s(Q^2)}{\partial\, t} = \beta(\alpha_s) \frac{\partial \alpha_s(Q^2)}{\partial \alpha_s}$$

Same equation for $\alpha_s(Q^2)$ as RGE for $R(e^t, \alpha_s)$

Running coupling

☛ if $\mu^2 = Q^2$ ➔ $e^t = 1$, $R(1, \alpha_s(Q^2))$ is a solution of the RGE

➔ the scale-dependence in R enters only through $\alpha_s(Q^2)$, and we can predict the scale dependence of R by solving

$$t = \int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \frac{\mathrm{d}x}{\beta(x)} \quad \text{or} \quad \frac{\partial \alpha_s}{\partial t} = \beta(\alpha_s)$$

Let's solve it perturbatively!

(we analyse the validity of PT later)

Beta function in perturbation theory

- in PT

$$\beta(\alpha_s) = -\alpha_s \sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{4\pi}\right)^{n+1}$$

- known coefficients (hard computations!):

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_R N_f, \qquad \beta_1 = \frac{34}{3} C_A^2 - 4 C_F T_R N_f - \frac{20}{3} C_A T_R N_f$$

$$\beta_2 = \frac{2857}{2} - \frac{5033}{18} N_f + \frac{325}{54} N_f^2, \qquad \beta_3 = 29243 - 6946.3 N_f + 405.9 N_f^2 + 1.5 N_f^3$$

- another convention:

$$\beta(\alpha_s) = -b_0 \alpha_s^2 \left[1 + \sum_{n=1}^{\infty} b_n \, \alpha_s^n\right]$$

Beta function in perturbation theory

☛ if $\alpha_s(Q^2)$ is small, we can truncate the series, at leading order (LO):

$$\frac{\partial \alpha_s}{\partial t} = -b_0 \alpha_s^2 \qquad b_0 = \frac{\beta_0}{4\pi}$$

➔

$$-\left[\frac{1}{\alpha_s(Q^2)} - \frac{1}{\alpha_s(\mu^2)}\right] = -b_0 t$$

Solution if both $\alpha_s(Q^2)$ and $\alpha_s(\mu^2)$ are small:

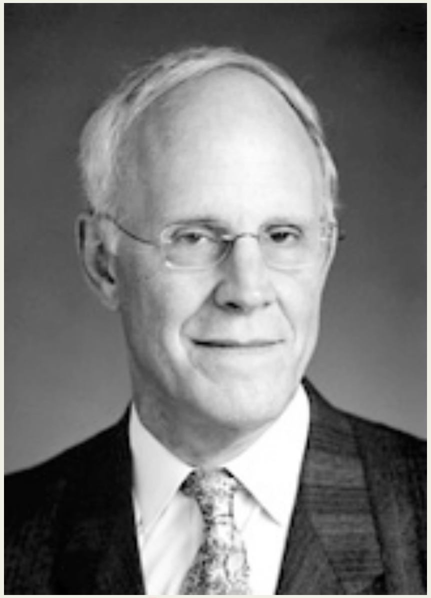
$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b_0 t\, \alpha_s(\mu^2)}$$

Asymptotic freedom

$$\alpha_s(Q^2) \lim_{Q^2 \to \infty} \simeq \frac{1}{b_0 t} \longrightarrow 0$$

Most important property of QCD ➔

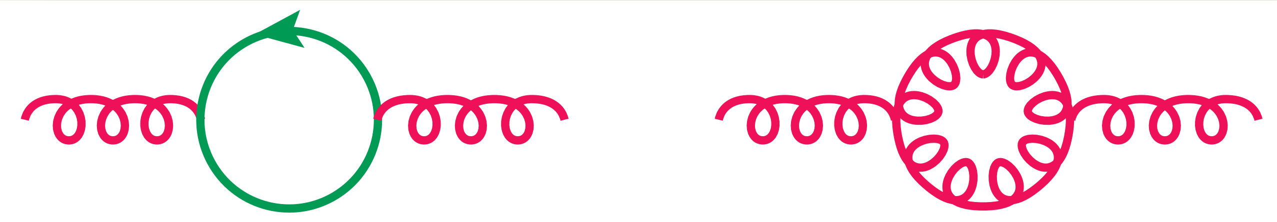
Nobel prize 2004



David J. Gross, David H. Politzer, Frank Wilczek

Asymptotic freedom

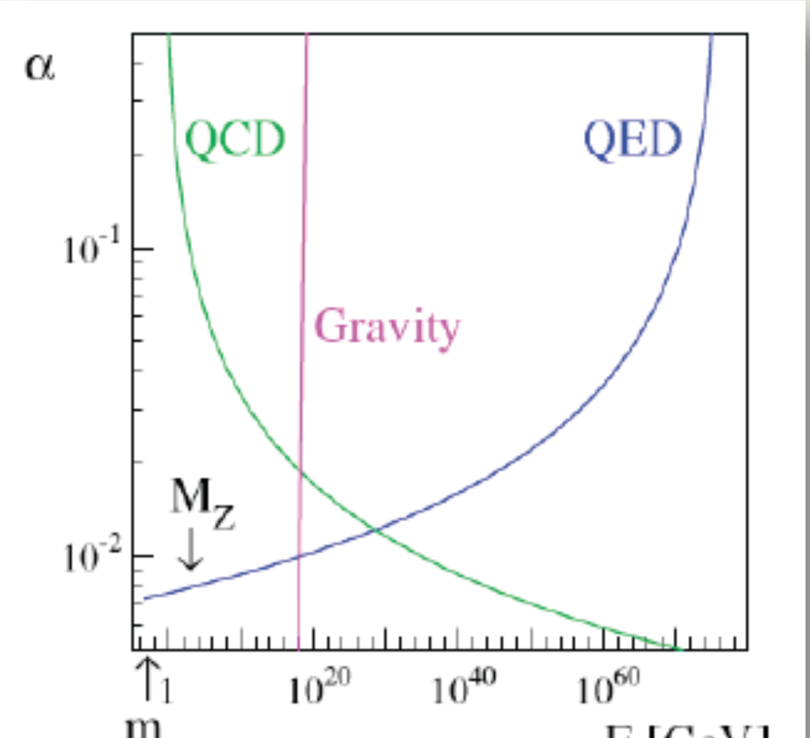
- justifies the use of PT
- sign of b_0 is crucial
- in background field gauge 2 graphs contribute:



  - quark loop negative: $-4T_R N_f/3$
  - gluon loop positive: $11C_A/3$

Asymptotic freedom

- ☛ Gluon self interaction makes QCD perfect in PT
- ☛ in QED $b_0 < 0$, hence coupling increases at high energies, but remains perturbative up to the Planck scale



Fabio Maltoni

Asymptotic freedom

- ☛ gives rationale to pQCD,
  but we shall see that LO is not enough
- ☛ can compute also at NLO:

$$\left[\alpha_s^2(1+b_1\alpha_s)\right]^{-1}\frac{\partial\alpha_s}{\partial t} = -b_0 \qquad b_1 = \frac{\beta_1}{4\pi\beta_0}$$

$\alpha_s(Q^2)$ is given implicitly by

$$\frac{1}{\alpha_s(Q^2)} - \frac{1}{\alpha_s(\mu^2)} + b_1 \ln\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} - b_1 \ln\frac{1+b_1\alpha_s(Q^2)}{1+b_1\alpha_s(\mu^2)} = bt$$

can be solved numerically

The running coupling resums logs

- ☛ if $R = R_0 + R_1\alpha_s + O(\alpha_s^2)$, use $(1+x)^{-1} = \Sigma_j(-x)^j$:

$$R(1, \alpha_s(Q^2)) = R_0 + R_1\alpha_s(\mu^2)\sum_{j=0}^{\infty}\left[-\alpha_s(\mu^2)b_0 t\right]^j$$

- ☛ $R_2\alpha_s^2$ gives logs with one power less in each term
- ☛ for $\alpha_s(Q^2)$ we need to measure $\alpha_s(\mu^2)$ at some scale
- ☛ different choices for μ give (in PT) subleading differences in $\alpha_s(Q^2)$ that can be significant numerically → choose the reference carefully

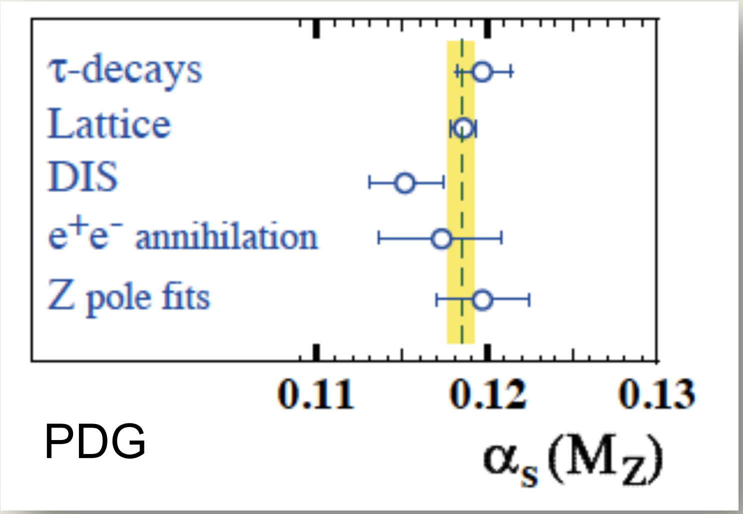
Λ_{QCD}

- Another approach to solving the RGE by introducing a Λ reference scale:

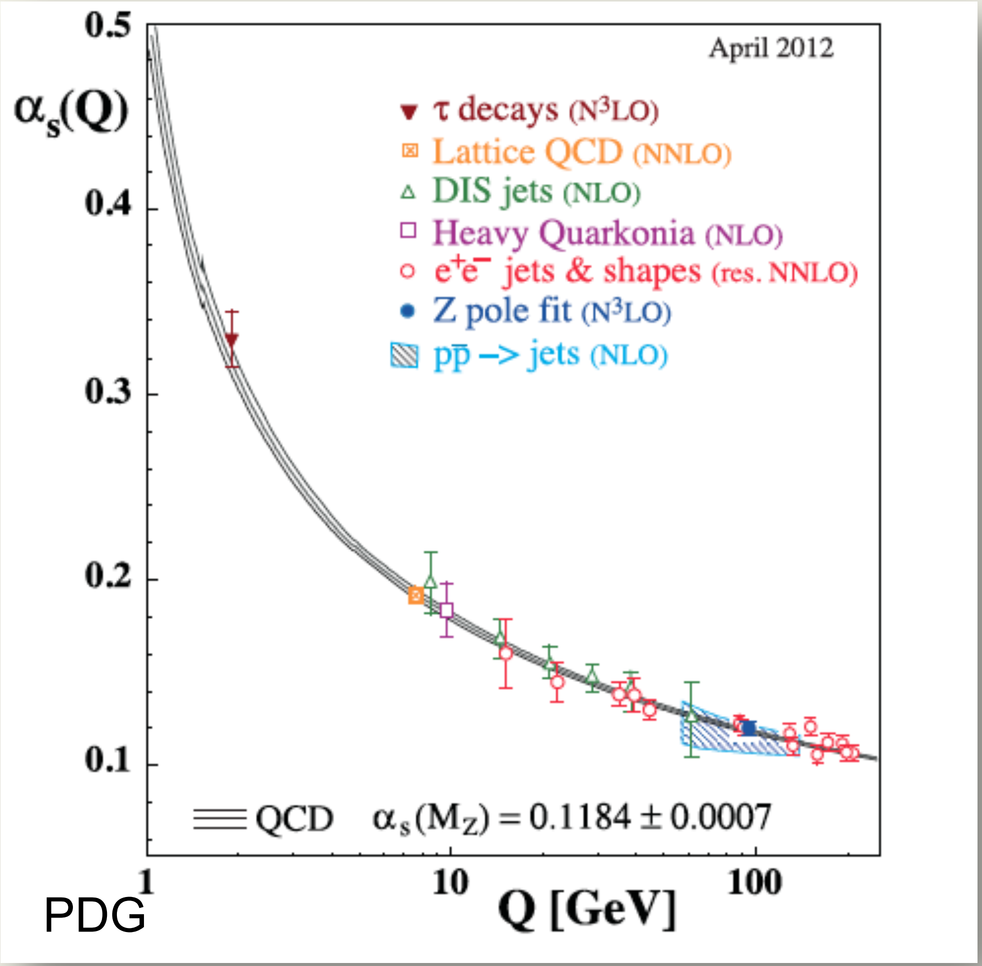
$$\ln \frac{Q^2}{\Lambda^2} = \int_{\alpha_s(Q^2)}^{\infty} \frac{\mathrm{d}x}{\beta(x)}$$

- Λ indicates the scale at which $\alpha_s(Q^2)$ gets strong
- LO: $$\alpha_s(Q^2) = \frac{1}{b_0 t}, \quad t = \ln \frac{Q^2}{\Lambda^2}$$
- NLO: $$\alpha_s(Q^2) = \frac{1}{b_0 t}\left(1 - \frac{b_1}{b_0^2}\frac{\ln t}{t}\right)$$

Running coupling in nature



Measurements at different scales lead to consistent values when evolved to the same reference scale



What about quark masses?

- ☛ one flavour with renormalized mass *m*: yet another mass scale

$$\left(\mu^2\frac{\partial}{\partial\mu^2}+\beta(\alpha_s)\frac{\partial}{\partial\alpha_s}-\gamma_m(\alpha_s)m\frac{\partial}{\partial m}\right)R\left(\frac{Q^2}{\mu^2},\alpha_s,\frac{m}{Q}\right)=0$$

γ_m is the mass anomalous dimension, PT:

$$\gamma_m=c_0\alpha_s(Q^2)\left[1+c_1\alpha_s(Q^2)+\dots\right]\qquad c_0=\frac{1}{\pi}$$

- ☛ *R* is dimensionless

$$c_1=\frac{303-10N_f}{72\pi}$$

➔ $$\left(Q^2\frac{\partial}{\partial Q^2}+\mu^2\frac{\partial}{\partial\mu^2}+m^2\frac{\partial}{\partial m^2}\right)R\left(\frac{Q^2}{\mu^2},\alpha_s,\frac{m}{Q}\right)=0$$

$$\left(Q^2\frac{\partial}{\partial Q^2}-\beta(\alpha_s)\frac{\partial}{\partial\alpha_s}+\left(\frac{1}{2}+\gamma(\alpha_s)\right)m\frac{\partial}{\partial m}\right)R\left(\frac{Q^2}{\mu^2},\alpha_s,\frac{m}{Q}\right)=0$$

Running quark mass

- ☛ to solve the RGE, introduce the running quark mass $m(Q^2)$

$$R\left(\frac{Q^2}{\mu^2}, \alpha_s, \frac{m}{Q}\right) \to R\left(1, \alpha_s(Q^2), \frac{m(Q^2)}{Q}\right)$$

- ☛ expand around $m(Q^2) = 0$:

$$= R\left(\frac{Q^2}{\mu^2}, \alpha_s, 0\right) + \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{m(Q^2)}{Q}\right)^n R^{(n)}\left(\frac{Q^2}{\mu^2}, \alpha_s, 0\right)$$

- ☛ derivative terms are suppressed by Q^{-n} at high Q^2
  - ☞ dropping the quark masses is justified
  - ☞ only IR-safe observables (?) can be computed

Running quark mass

- ☛ all non-trivial scale dependence of *R* can be included in the running of mass and coupling, for mass:

$$Q^2 \frac{\partial m}{\partial Q^2} = -\gamma_m(\alpha_s) m(Q^2)$$

- ☛ solution (check):

$$m(Q^2) = m(\mu^2) \exp\left[-\int_{\mu^2}^{Q^2} \frac{\mathrm{d}Q^2}{Q^2} \gamma_m\Big(\alpha_s(Q^2)\Big)\right]$$

- ☛ change dQ^2 to dα_s

$$= m(\mu^2) \exp\left[-\int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \mathrm{d}\alpha_s \frac{\gamma_m(\alpha_s)}{\beta(\alpha_s)}\right]$$

Running quark mass in PT

☛ at LO

$$-\frac{\gamma_m(\alpha_s)}{\beta(\alpha_s)} = \frac{c_0}{b_0 \alpha_s}$$

$$m(Q^2) = \underbrace{m(\mu^2)\left[\alpha_s(\mu^2)\right]^{-\frac{c_0}{b_0}}}_{\equiv \overline{m}} \left[\alpha_s(Q^2)\right]^{\frac{c_0}{b_0}}$$

☛ running quark mass vanishes at high Q^2 with running coupling

☛ effect of mass in *R* is suppressed by
- ☞ its physical dimension
- ☞ and anomalous dimension

Use tools

So far we have computed almost everything explicitly. This luxury is mostly over.

(I'll try, but the journey will be 'tour de Mont Blanc'.)

I'll have to present results without justification, but you can check those using freely available computer programs. Some useful links: (FeynCalc)

Tracer.m http://library.wolfram.com/infocenter/MathSource/2987/

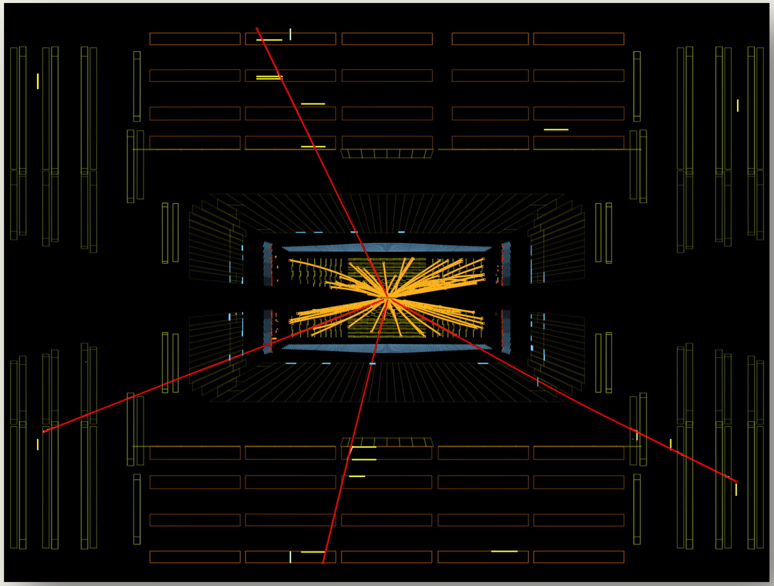
MadGraph http://madgraph.phys.ucl.ac.be/

Calchep http://theory.sinp.msu.ru/~pukhov/calchep.html

Comphep http://comphep.sinp.msu.ru/

What's the use of fixed-order predictions?

- we collect collision events with something interesting in the final state
- counting event rates
  we measure xsections
- we compare measured xsections to predictions
- parton-hadron duality
  ➔ need predictions at parton level



event with four hard muons in the CMS detector

Cross section: partons in the final state

☛ the formula:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \mathcal{N} \int \mathrm{d}\phi_m(p_1,\dots,p_m;Q)\,\frac{1}{S_{\{m\}}}\,|\mathcal{M}_m(p_1,\dots,p_m)|^2 \times \delta\big(O - O_m(p_1,\dots,p_m)\big)$$

- ☞ $\mathcal{N}$ contains non-QCD factors, e.g. flux = 1/2*s*
- ☞ dΦ_m is phase space of *m* particles
- ☞ $S_{\{m\}}$ is symmetry factor
- ☞ $|\mathcal{M}_m|^2$ is squared matrix element – the hard part
- ☞ *O* is the observable

Colour-state formalism

- ☛ basis in colour and helicity space of *m* partons: $|c_1,\ldots,c_m\rangle \otimes |s_1,\ldots,s_m\rangle$
- ☛ $|\mathcal{A}_m\rangle$ is a state vector in this space
- ☛ scattering amplitude for producing *m* partons of colour $(c_1,\ldots,c_m)$, spin $(s_1,\ldots,s_m)$ and momentum $(p_1,\ldots,p_m)$:

$$\mathcal{A}_m^{\{c_i\},\{s_i\}}\big(\{p_i\}\big) \equiv \langle c_1 \ldots c_m| \otimes \langle s_1 \ldots s_m|\mathcal{A}_m\big(\{p_i\}\big)\rangle$$

➔

$$\sum_{\text{colour}}\sum_{\text{helicity}} \left|\mathcal{A}_m^{\{c_i\},\{s_i\}}\big(\{p_i\}\big)\right|^2 = \langle \mathcal{A}_m\big(\{p_i\}\big)|\mathcal{A}_m\big(\{p_i\}\big)\rangle$$

Loop-expansion

☛ PT expansion of $|\mathcal{A}_m\rangle$ in space-time with $d = 4 - 2\epsilon$ dimensions

$$|\mathcal{A}_m\rangle = \left(\frac{\alpha_{\rm s}^{(0)}\,\mu^{2\epsilon}}{4\pi}\right)^{\frac{q}{2}}\left[|\mathcal{A}_m^{(0)}\rangle + \left(\frac{\alpha_{\rm s}^{(0)}\,\mu^{2\epsilon}}{4\pi}\right)|\mathcal{A}_m^{(1)}\rangle + \mathcal{O}\left(\left(\alpha_{\rm s}^{(0)}\right)\right)^2\right]$$

☛ μ is dimensional regularization scale to keep coupling dimensionless in *d* dim

☛ $|\mathcal{A}_m^{(1)}\rangle$ is divergent in *d* = 4, the singularities appear as $\frac{1}{\epsilon^2},\ \frac{1}{\epsilon}$ poles with both UV and IR origin

Renormalization of $|\mathcal{A}_m\rangle$

- ☛ UV poles can be removed by multiplicative redefinition of the fields and parameters in the Lagrangian, systematically order by order in PT – a hard task even at one loop, known up to four loops: A.Chetyrkin, **hep-ph/0405193**
- ☛ for scattering amplitudes renormalization at one-loop can be achieved by the substitution

$$\alpha_{\rm s}^{(0)}\mu^{2\epsilon} \to \alpha_{\rm s}(\mu_{\rm R}^2)\mu_{\rm R}^{2\epsilon}S_\epsilon^{-1}\left[1-\frac{\alpha_{\rm s}(\mu_{\rm R}^2)}{4\pi}\frac{\beta_0}{\epsilon}\right]$$

Renormalization of $|\mathcal{A}_m\rangle$

$$\left[1-\frac{\alpha_s(\mu_R^2)}{4\pi}\,\frac{\beta_0}{\epsilon}\right]^{\frac{q}{2}} = 1-\frac{q}{2}\,\frac{\alpha_s(\mu_R^2)}{4\pi}\,\frac{\beta_0}{\epsilon}+O(\alpha_s^2)$$

➔

$$|\mathcal{M}_m^{(0)}\rangle = \left(\frac{\alpha_s(\mu_R^2)\mu_R^{2\epsilon}}{4\pi}\,S_\epsilon^{-1}\right)^{\frac{q}{2}} |\mathcal{A}_m^{(0)}\rangle \qquad q\in\mathbb{N},$$

$$|\mathcal{M}_m^{(1)}\rangle = \left(\frac{\alpha_s(\mu_R^2)\mu_R^{2\epsilon}}{4\pi}\,S_\epsilon^{-1}\right)^{\frac{q}{2}} \frac{\alpha_s(\mu_R^2)}{4\pi}\,S_\epsilon^{-1}$$

$$\times\left(\mu_R^{2\epsilon}|\mathcal{A}_m^{(1)}\rangle - \frac{q}{2}\,\frac{\beta_0}{\epsilon}\,S_\epsilon|\mathcal{A}_m^{(0)}\rangle\right)$$

☛ $|\mathcal{M}_m^{(0)}\rangle$ and $|\mathcal{M}_m^{(1)}\rangle$ are renormalized,

$$S_\epsilon = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}$$

Renormalization of $|\mathcal{A}_m\rangle$

- the renormalized amplitude up to one-loop accuracy, $\big|\mathcal{M}_m\big\rangle = \big|\mathcal{M}_m^{(0)}\big\rangle + \big|\mathcal{M}_m^{(1)}\big\rangle$
  finite in the UV, but still contains IR poles
- use dimensional regularization to regulate IR poles: *d* > 4, epsilon negative
- for IR safe observables these IR poles vanish and we can set *d* = 4, so

$$\left[\left(\frac{\alpha_s\mu_R^{2\epsilon}}{4\pi}S_\epsilon^{-1}\right)^{\frac{q}{2}}\left(\frac{\alpha_s}{4\pi}S_\epsilon^{-1}\right)^{i}\right] \to \left(\frac{\alpha_s}{4\pi}\right)^{i+\frac{q}{2}}$$

- with UV finite, IR regularized $|\mathcal{M}_m|^2 \to \sigma$

pQCD with partons in the final state: $e^+e^- \to$ hadrons

- consider our old friend the hadronic *R* ratio:

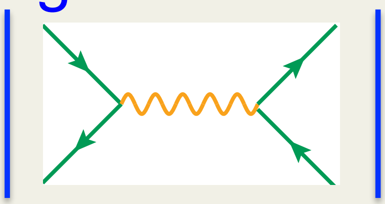
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \approx \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- $2 \to 2$ scattering has one free kinematical parameter, the *θ* scattering angle
- the differential cross section for $e^+e^- \to f\bar{f}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{\pi\alpha^2}{2s}\left\{(1+\cos^2\theta)\left[Q_f^2 + (A_e^2+V_e^2)(A_f^2+V_f^2)\frac{\kappa^2 s^2}{(s-M_Z)^2+\Gamma_Z^2 M_Z^2} + \ldots\right.\right.$$

+ terms that vanish at $s=M_Z$, or after integration

- below the *Z* pole $\sigma_0 = \frac{4\pi\alpha^2}{3s}Q_f^2$



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pQCD with partons in the final state: $e^+e^- \to$ hadrons

- consider our old friend the hadronic *R* ratio:

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- $2 \to 2$ scattering has one free kinematical parameter, the θ scattering angle
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+ terms that vanish at $s=M_Z$, or after integration

- on the *Z* pole $\sigma_0 = \frac{4\pi\alpha^2}{3s}\left[Q_f^2 + (A_e^2+V_e^2)(A_f^2+V_f^2)\kappa^2\frac{s}{\Gamma_Z^2}\right]$

R ratio at LO

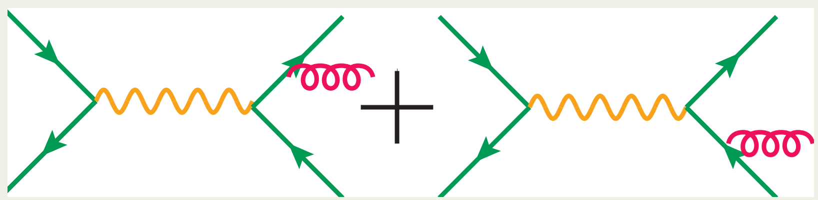
- LO: the hadronic cross section is obtained by counting the possible final states:

$$R = 3\sum_q e_q^2 \equiv R_0 \qquad\qquad R = 3\frac{\sum_q \left(A_q^2 + V_q^2\right)}{\left(A_\mu^2 + V_\mu^2\right)}$$

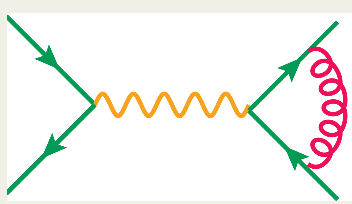
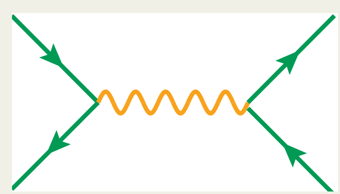
- with $q = u,d,s,c,b$ R =11/3 & R_Z =20.09
- measured value at LEP: R_Z =20.79±0.04
- the 3.5% difference is mainly due to QCD radiation effects: NLO corrections

R ratio at NLO

- NLO: two kind of corrections
  - real: $\langle \mathcal{M}_3^{(0)} | \mathcal{M}_3^{(0)} \rangle =$

    $\Big|$  $\Big|^2$

  - virtual: $\langle \mathcal{M}_2^{(1)} | \mathcal{M}_2^{(0)} \rangle + \langle \mathcal{M}_2^{(0)} | \mathcal{M}_2^{(1)} \rangle =$
    (loop)

    2 Re $\Big\langle$  $\Big|$  $\Big\rangle$

NLO: real gluon emission

- ☛ three-body phase space has 5 independent variables: 2 energies and 3 angles
- ☛ integrate over the angles & use $y_{ij} = 2p_i \cdot p_j/s$ scaled two-particle invariants, $y_{12}+y_{13}+y_{23}=1$
- ☛ real contribution to the total xsection:

$$\sigma^{\mathrm{R}} = \sigma_0 R_0 \int_0^1 \mathrm{d}y_{13} \int_0^1 \mathrm{d}y_{23}\, C_F \frac{\alpha_{\mathrm{s}}}{2\pi} \left(\frac{y_{23}}{y_{13}} + \frac{y_{13}}{y_{23}} + \frac{2y_{12}}{y_{13}y_{23}} \right) \Theta(1 - y_{13} - y_{23})$$

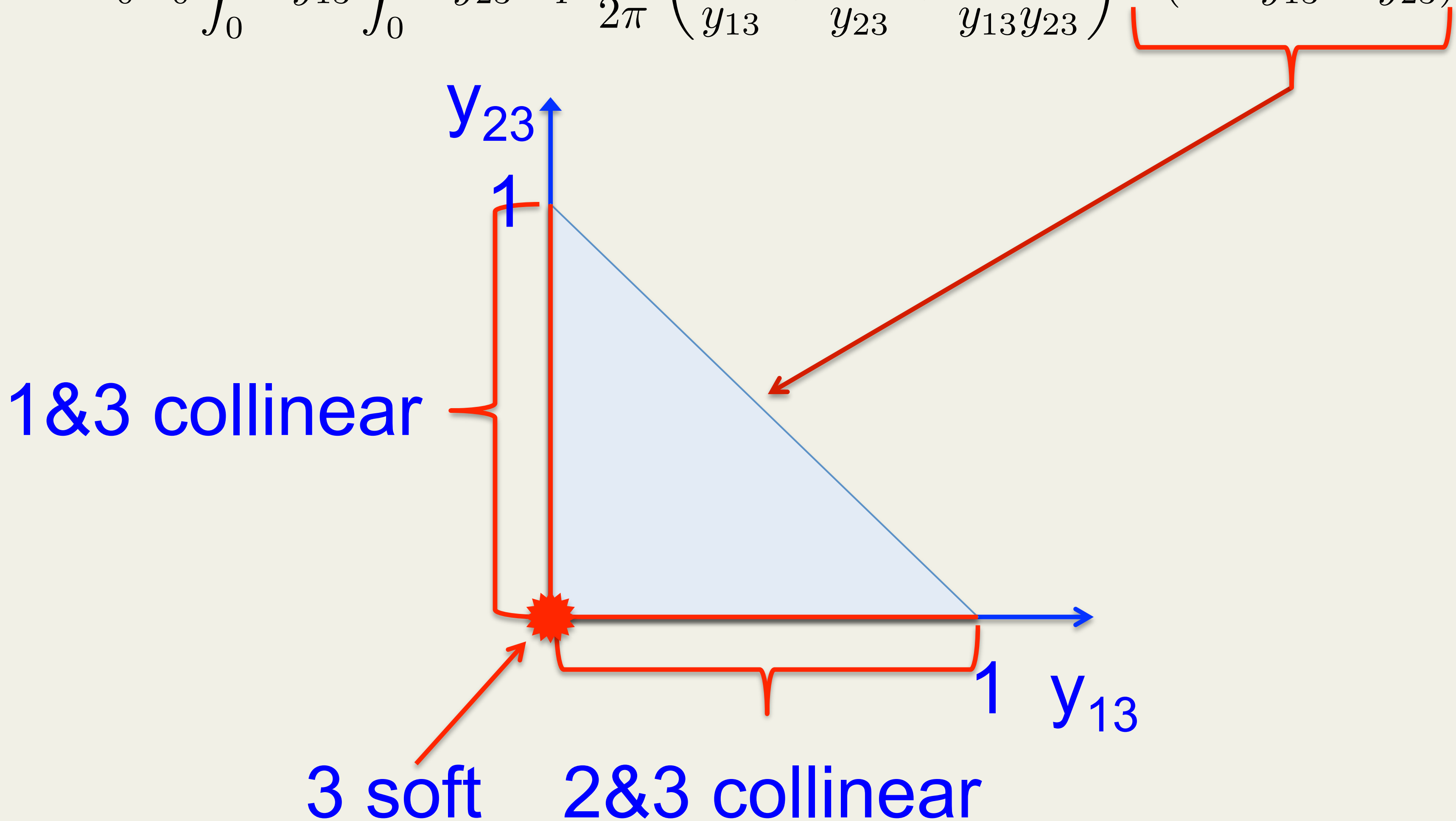
- ☛ Divergent along the boundaries at $y_{i3} = 0$:

$$y_{i3} s = 2E_i E_3 (1 - \cos\theta_{i3})$$

- ☛ Divergent when $E_3 \to 0$ (soft gluon), or $\theta_{i3} \to 0$ (collinear gluon)

Real corrections: the phase space

$$\sigma^{\mathrm{R}} = \sigma_0 R_0 \int_0^1 \mathrm{d}y_{13} \int_0^1 \mathrm{d}y_{23}\, C_F \frac{\alpha_{\mathrm{s}}}{2\pi} \left(\frac{y_{23}}{y_{13}} + \frac{y_{13}}{y_{23}} + \frac{2y_{12}}{y_{13}y_{23}} \right) \Theta(1 - y_{13} - y_{23})$$



NLO: real corrections in $d \neq 4$

☛ make sense of the real contribution use dimensional regularization

$$\sigma^{\mathrm{R}} = \sigma_0 R_0 H(\varepsilon) \int_0^1 \frac{\mathrm{d}y_{13}}{y_{13}^{\varepsilon}} \int_0^1 \frac{\mathrm{d}y_{23}}{y_{23}^{\varepsilon}} C_F \frac{\alpha_{\mathrm{s}}}{2\pi} \left[(1-\varepsilon)\left(\frac{y_{23}}{y_{13}} + \frac{y_{13}}{y_{23}} \right) + \frac{2y_{12}}{y_{13}y_{23}} - 2\varepsilon \right]$$

$$= \sigma_0 R_0 H(\varepsilon) C_F \frac{\alpha_{\mathrm{s}}}{2\pi} \left[\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + \frac{19}{2} - \pi^2 + \mathrm{O}(\varepsilon) \right] \qquad H(\varepsilon) = 1 + \mathrm{O}(\varepsilon)$$

☛ to be combined with virtual correction

$$\sigma^{\mathrm{V}} = \sigma_0 R_0 H(\varepsilon) C_F \frac{\alpha_{\mathrm{s}}}{2\pi} \left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \pi^2 + \mathrm{O}(\varepsilon) \right]$$

☛ sum of real and virtual contributions is finite in $d = 4$: (same for R_Z)

$$R = R_0 \left(1 + \frac{\alpha_{\mathrm{s}}}{\pi} + \mathrm{O}\left(\alpha_{\mathrm{s}}^2\right) \right)$$

Total hadronic cross section at O(α_s^3)

- total xsection is computed more easily using the optical theorem $\left(\sigma \propto \mathrm{Im}\, f(\gamma \to \gamma)\right)$

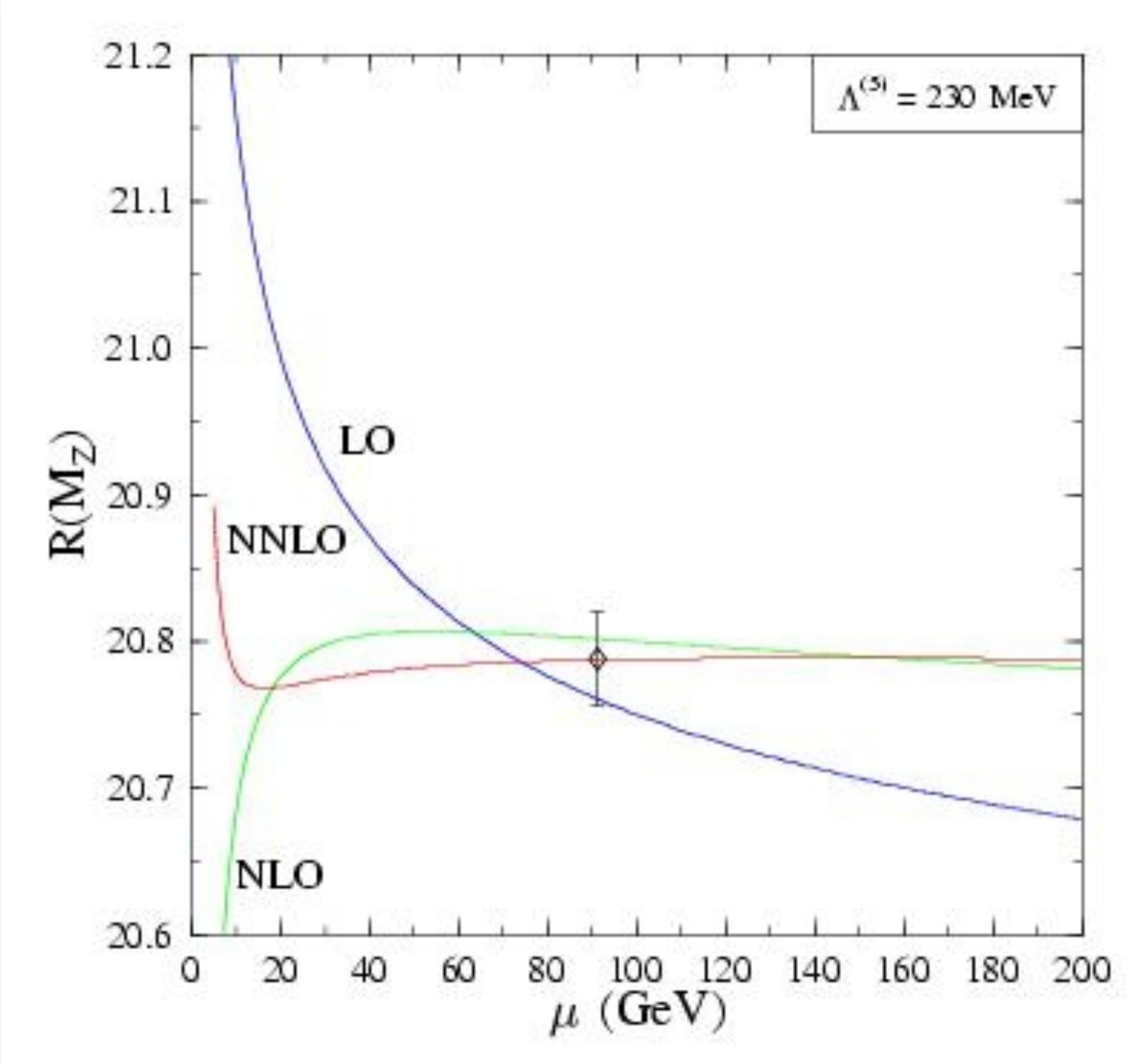
$$R = R_0 \left\{ 1 + c_1 \alpha_s(\mu^2) + \left[c_2 + c_1 b_0 \ln\frac{\mu^2}{Q^2} \right] \alpha_s(\mu^2)^2 + \right.$$

$$\left. + \left[c_3 + \left(2c_2 b_0 + c_1 b_1 + c_1 b_0^2 \ln\frac{\mu^2}{Q^2} \right) \ln\frac{\mu^2}{Q^2} \right] \alpha_s(\mu^2)^3 + \mathrm{O}(\alpha_s^4) \right\}$$

$$c_1 = \frac{1}{\pi} \qquad c_2 = \frac{1.409}{\pi^2} \qquad c_3 = -\frac{12.85}{\pi^3}$$

- Satisfies the renormalization-group equation to order α_s^4

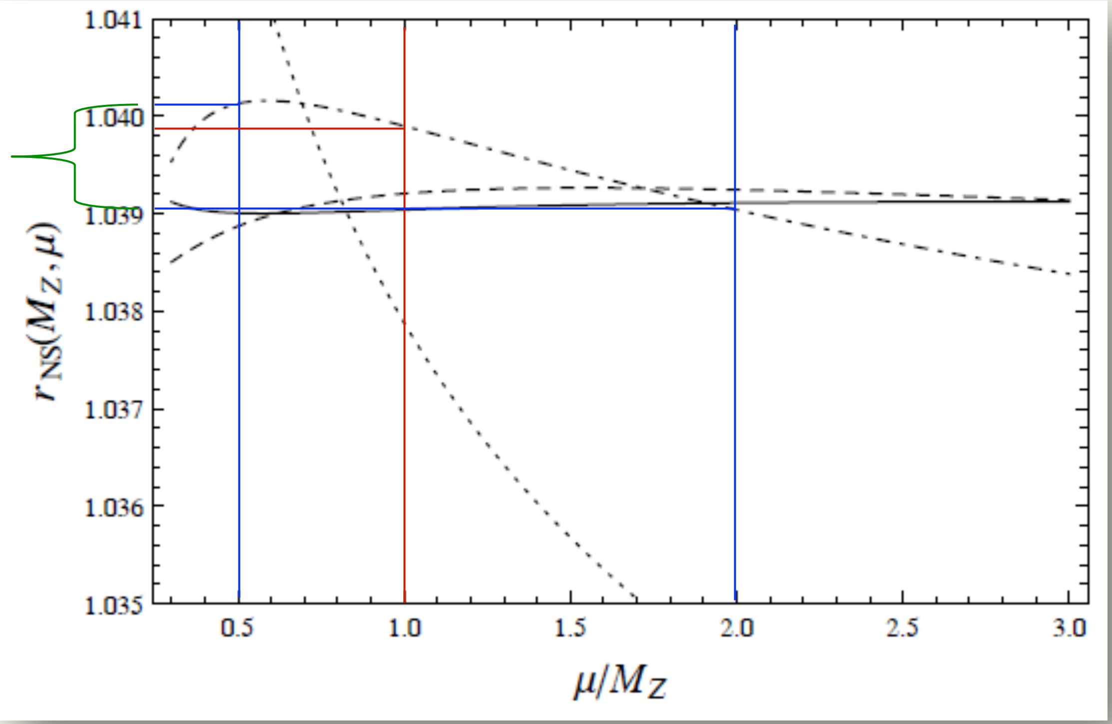
Total hadronic cross section at $O(\alpha_s^3)$



Total hadronic cross section at $O(\alpha_s^3)$

What should be the scale μ?

NLO band



(non-singlet contribution)

Choosing the scale

- There is no theorem that gives the proper scale choice and scale-interval to estimate the theoretical uncertainty at 95% confidence level

- There are several recommendations based on educated guesses, such as
  - principle of minimal sensitivity
  - BLM, choices
- At hadron colliders the case is worse as there is a second (factorization) scale and often several physical scales (e.g. particle masses)

Summary

- ☛ Solved the RGE and found asymptotic freedom
- ☛ Set our playground: pQCD with massless light quarks
- ☛ Showed that PT can only be fully consistent in an asymptotically free theory, like QCD
- ☛ Computed the QCD corrections to the total hadronic xsection in electron positron annihilation

How about more exclusive observables?