Quantum Field Theory and the Electroweak Standard Model

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Outline

1. Introduction

2. Elements of Quantum Field Theory

3. Construction of the Electroweak Standard Model Lagrangian

4. Phenomenology of the Electroweak Standard Model

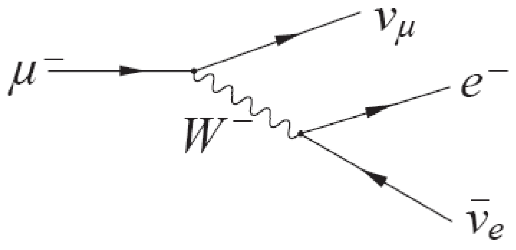
5. Concluding remarks

Phenomenology of the Electroweak Standard Model

The Fermi constant G_F is measured with high precision from muon life time

$$G_F = 1.166\,378\,7(6) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Since the muon mass $m_\mu \ll M_W$ one can neglect the W-boson mass in the propagator and immediately get the following relation



$$\frac{g_2^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$$

As we have seen the W boson mass is obtained in SM due to the Higgs mechanism and proportional to the Higgs vacuum expectation value v

$$M_W^2 = \frac{1}{4} g_2^2 v^2$$

From these two relations we obtain

$$v = \frac{1}{\sqrt{\sqrt{2} G_F}} = 246.22\ \mathrm{GeV}$$

At this point one can see the power of gauge invariance principle, g_2 is the same gauge coupling

The Higgs field expectation value v is determined by the Fermi constant G_F introduced long before the Higgs mechanism appeared!

$$M_W^2 = \frac{1}{4} g_2^2 v^2 \qquad g_2 s_W = e \qquad M_W = M_Z c_W$$

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi \alpha_{em}}{\sqrt{2} G_F} \equiv A_0^2$$

$\alpha_{em} = e^2/4\pi$ is the electromagnetic fine structure constant. The low energy value follows mainly from the electron anomalous magnetic measurements

$$\alpha_{em} = (137.035\,999\,074(44))^{-1}$$

One gets A_0 very precisely from low energy measurements

$$A_0 = 37.2804 \text{ GeV}$$

From the other hand one gets A_0 from measured values for the masses of W and Z bosons

$$\begin{array}{rclll} M_W & = & 80.385 & \pm 0.015 & \text{GeV} \\ M_Z & = & 91.1876 & \pm 0.0021 & \text{GeV} \end{array} \quad \Longrightarrow \quad A_0 = 37.95 \text{ GeV}$$

Values are close. The difference is about 1.5%.

CC and NC interactions of SM fermions, as we know already, have the following structure

$$L_{CC} = \frac{g_2}{2\sqrt{2}} \sum_{ij} V_{ij} \bar{u}_i \gamma_\mu (1-\gamma_5) d_j = \frac{e}{2\sqrt{2} s_W} \sum_{ij} V_{ij} \bar{u}_i \gamma_\mu (1-\gamma_5) d_j,$$

$$L_{NC} = e \sum_f Q_f \bar{f} \gamma_\mu f + \frac{e}{4 s_W c_W} \sum_f \bar{f} \gamma_\mu (v_f - a_f \gamma_5) f Z^\mu,$$

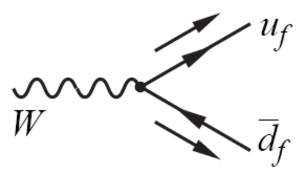
where Vij is the CKM matrix element, i, j = 1, 2, 3 - number of fermion generation

$$v_{u_i} = 1 - \tfrac{8}{3} s_W^2, \quad a_{u_i} = 1; \quad v_{d_i} = -1 + \tfrac{4}{3} s_W^2, \quad a_{d_i} = -1$$

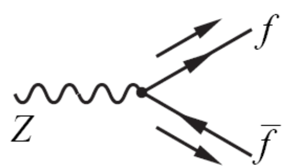
$$v_\ell = -1 + 4 s_W^2, \quad a_\ell = -1; \quad v_\nu = 1, \quad a_\nu = 1.$$

$$v_f = 2T_3^f - 4Q_f s_W^2, \quad a_f = 2T_3^f$$

The Feynman rules following from L_{CC} and L_{NC} allow to get tree level formulas for the W and Z boson widths



$$\Gamma(W \to u_f \bar{d}_f) = |V_{ij}|^2 N_c \frac{\alpha}{12 s_W^2} M_W,$$



$$\Gamma(Z \to f \bar{f}) = N_c \frac{\alpha M_Z}{12 \sin^2(2\theta_W)} [v_f^2 + a_f^2]$$

N_c = 3 for quarks, and N_c = 1 for leptons

Since CC for all fermions have the same (V – A) structure one can very easily obtain branching fractions for W decay modes

$$\sum_q \mathrm{Br}(W \to q\bar{q}) = 2N_c \cdot \frac{1}{9} = \frac{2}{3}$$
$$\sum_\ell \mathrm{Br}(W \to \ell\nu) = 3 \cdot \frac{1}{9} = \frac{1}{3}$$

Measured Br(W → ℓν) = (10.80±0.09)% is in a reasonable agreement with simple tree level result 1/9 = 11%

QCD corrections to Br(W → qq) improved the agreement

The decay width of the Z-boson to neutrinos, the invisible decay mode, allows to measure the number of light ($m_\nu < M_Z/2$) neutrinos

$$\Gamma^Z_{inv} = \Gamma^Z_{tot} - \Gamma^Z_{had} - \Gamma^Z_{\ell+\ell-}$$

$$\Gamma^Z_{tot} = 2.4952 \pm 0.0023 \text{ GeV}$$ is measured from the shape of the Z-boson resonance

$$\Gamma^Z_{\ell+\ell-} = 83.984 \pm 0.086 \text{ MeV}$$
$$\Gamma^Z_{had} = 1744.4 \pm 2.0 \text{ MeV}$$

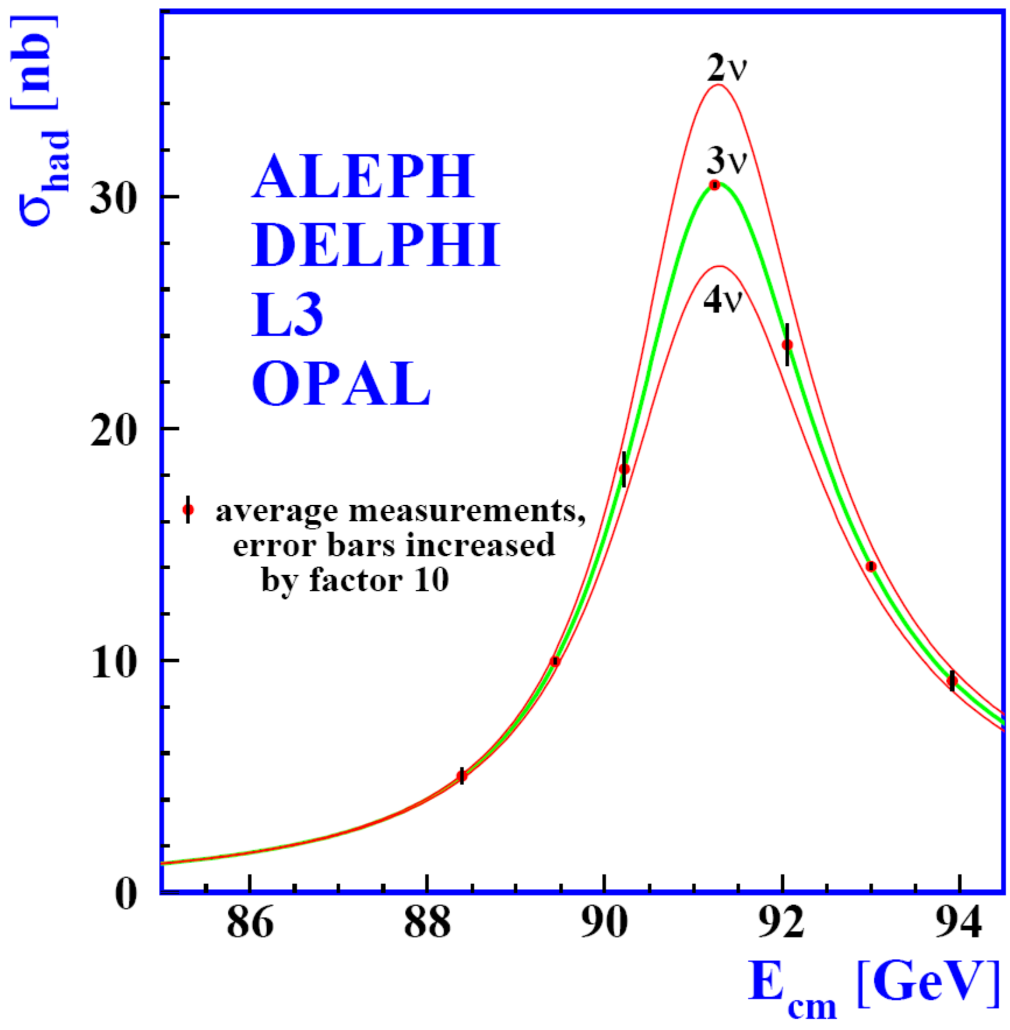
→ $$\Gamma^Z_{inv} = 499.0 \pm 1.5 \text{ MeV}$$

In SM $\Gamma(Z \to f\bar{f}) = N_c \frac{\alpha M_Z}{12\sin^2(2\theta_W)}[v_f^2 + a_f^2]$ ⟹ $\Gamma^Z_{inv} = \Gamma^Z_{\nu\bar{\nu}} = N_\nu \cdot \frac{\alpha M_Z}{12\sin^2(2\theta_W)}(1+1)$

⇓

$$2.9840 \pm 0.0082$$

Confirmation of 3 fermion generations assumed in the SM and observed in nature



σ_had [nb]
ALEPH
DELPHI
L3
OPAL
2ν
3ν
4ν
average measurements,
error bars increased
by factor 10
0
10
20
30
86
88
90
92
94
E_cm [GeV]

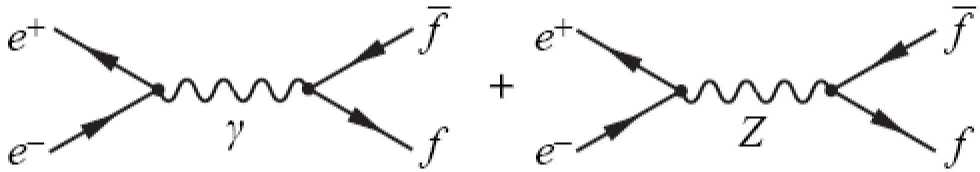
Another way to make this test

$$\frac{\Gamma^Z_{inv}}{\Gamma^Z_{e^+e^-}} = \frac{2N_\nu}{1+(1-4s^2_W)^2}$$

The experimental value 5.942 ± 0.016

$N_\nu = 3$ gives for the ratio about **5.970** in an agreement with the measured value ($s_W^2 = 0.2324$)

An important part of information about EW fermionic interactions and couplings comes from e+e− annihilation to fermion-antifermion pairs



$$\frac{d\sigma}{d\cos\theta} = \frac{2\pi\alpha^2}{4s} N_C \{(1+\cos^2\theta)\cdot \cdot\left[Q_f^2 - 2\chi_1 v_e v_f Q_f + \chi_2(a_e^2+v_e^2)(a_f^2+v_f^2)\right] + +2\cos\theta\left[-2\chi_1 a_e a_f Q_f + 4\chi_2 a_e a_f v_e v_f\right]\}$$

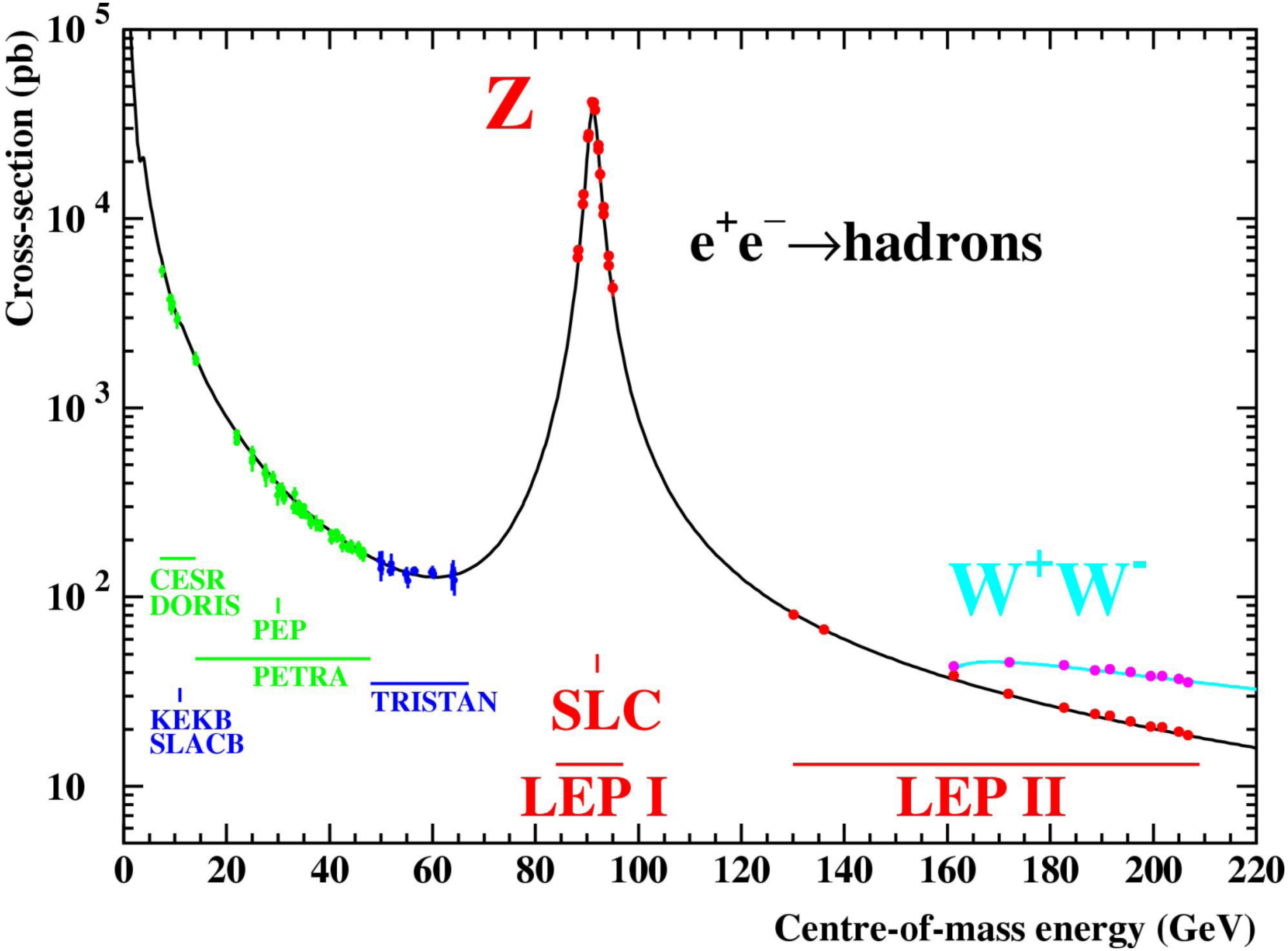
$$\chi_1 = \frac{1}{16 s_W^2 c_W^2}\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2},$$

$$\chi_2 = \frac{1}{256 s_W^2 c_W^2}\frac{s^2}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}.$$

In the region much below Z-boson pole one can neglect Z-boson exchange diagram and well known QED formula is restored

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} Q_f^2 N_C (1+\cos^2\theta), \quad \sigma = \frac{4\pi\alpha^2}{3} Q^2 N_C$$

N_c = 3 for quarks, and N_c = 1 for leptons



Cross-section (pb)
10^5
10^4
10^3
10^2
10
Z
$e^+e^- \rightarrow$ hadrons
CESR
DORIS
PEP
PETRA
KEKB
SLACB
TRISTAN
SLC
LEP I
LEP II
W^+W^-
0 20 40 60 80 100 120 140 160 180 200 220
Centre-of-mass energy (GeV)

In the region close to the Z pole the photon exchange part is small

$$A_{FB} \equiv \frac{N_F - N_B}{N_F + N_B}$$

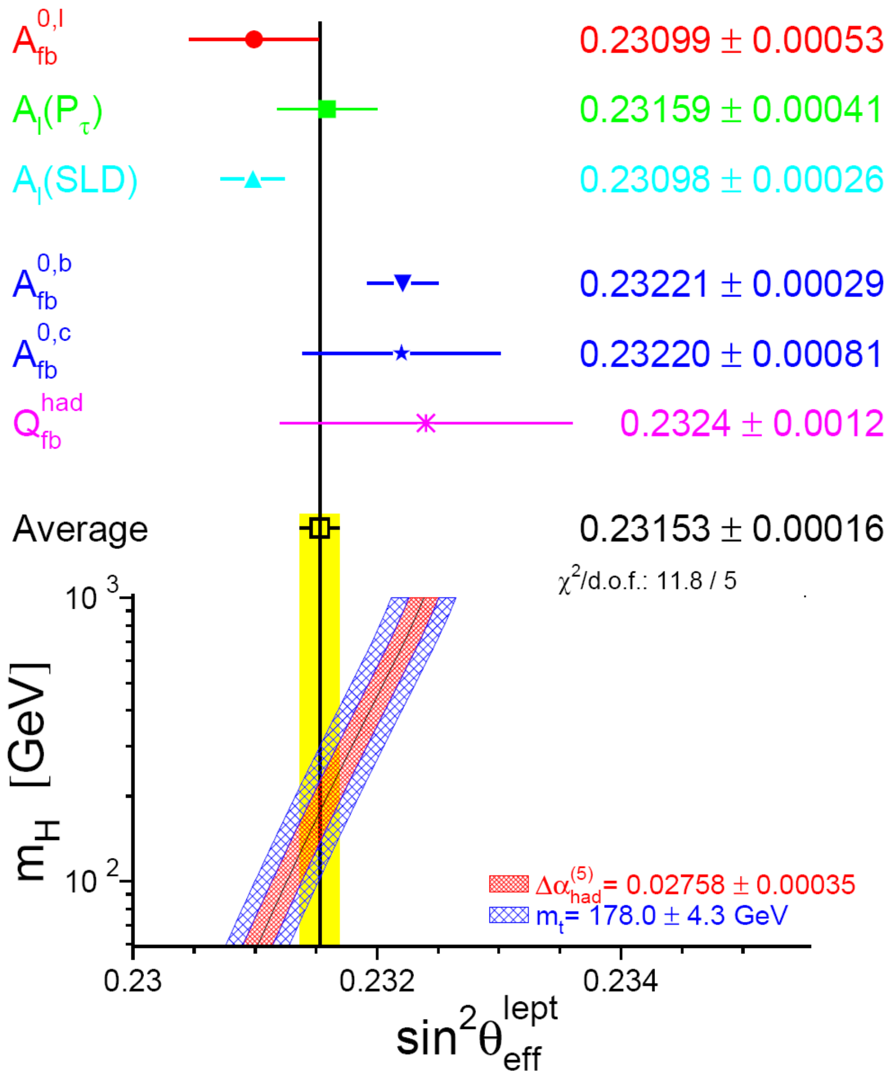
$$N_F = \int_0^1 d(\cos\theta) \frac{d\sigma}{d\cos\theta}, \quad N_B = \int_{-1}^0 d(\cos\theta) \frac{d\sigma}{d\cos\theta}$$

Asymmetries for different fermions allow to extract the coefficients a_f and v_f

$$A_{FB} = \frac{3}{2} A_e \cdot A_f, \quad A_{e,f} = \frac{2 a_{e,f} v_{e,f}}{a_{e,f}^2 + v_{e,f}^2}$$

$(a_f^2 + v_f^2)$ - from partial decay width of Z

The best way to measure s_W^2 $\quad \sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} \equiv \frac{1}{4}\left(1 - \frac{v_l}{a_l}\right)$



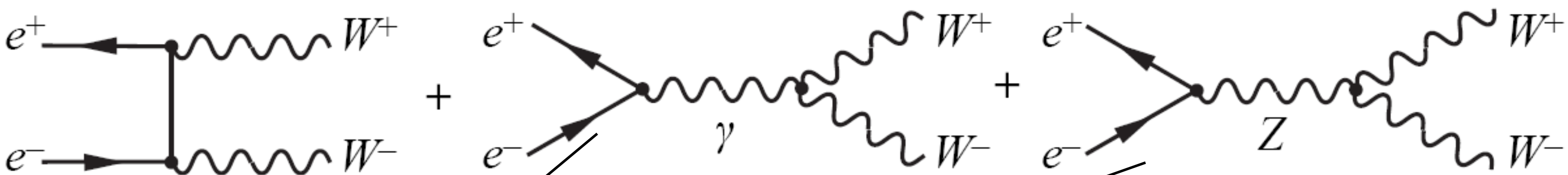
$A_{fb}^{0,l}$
0.23099 ± 0.00053
$A_l(P_\tau)$
0.23159 ± 0.00041
A_l(SLD)
0.23098 ± 0.00026
$A_{fb}^{0,b}$
0.23221 ± 0.00029
$A_{fb}^{0,c}$
0.23220 ± 0.00081
Q_{fb}^{had}
0.2324 ± 0.0012
Average
0.23153 ± 0.00016
χ^2/d.o.f.: 11.8 / 5
10^3
10^2
m_H [GeV]
$\Delta\alpha_{had}^{(5)}$ = 0.02758 ± 0.00035
m_t = 178.0 ± 4.3 GeV
0.23
0.232
0.234
$\sin^2\theta_{eff}^{lept}$

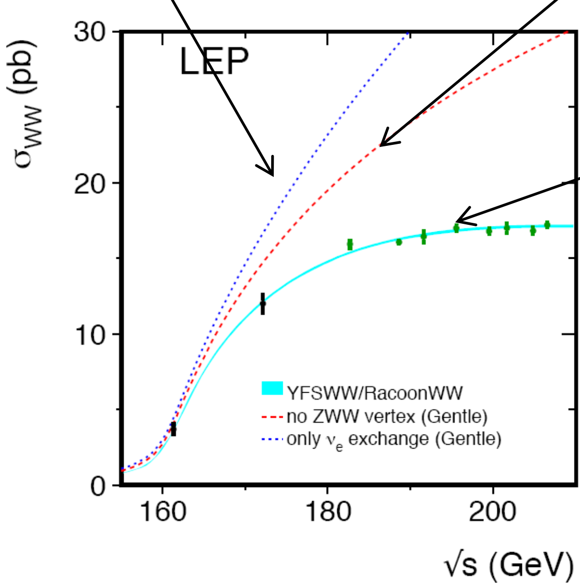
Well known example demonstrating correctness of the Yang-Mills interaction of gauge bosons is W-boson pair production. Triple gauge boson vertex WWγ and WWZ have been tested at LEP2 ($e^+e^- \to W^+W^-$) and at the Tevatron ($q\bar{q} \to W^+W^-,\ q\bar{q}' \to W\gamma,\ q\bar{q}' \to WZ$).

The triple vertex of Yang-Mills interaction

$$\Gamma^{WW\gamma/Z}_{m_1m_2m_3}(p_1p_2p_3) = g_{\gamma,Z}\left[(p_1-p_2)_{m_3}g_{m_1m_2} + (p_3-p_1)_{m_2}g_{m_1m_3} + (p_2-p_3)_{m_1}g_{m_2m_3}\right]$$

$$g_\gamma = e,\ g_Z = g_2 c_W = e\frac{c_W}{s_W}$$



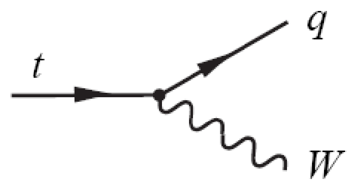


Three SM diagrams

The quartic gauge couplings WWγγ, WWγZ, WWZZ have not been tested yet. This is challenging task for the LHC. It will require high luminosity regime at a linear collider

In the SM there are no $1 \to 2$ decays of fermions to the real Z-boson due to absence of FCNC.

The top quark is heavy enough to decay to W-boson



$$V_{tb} \sim 1 \gg V_{ts}, V_{td}$$

In SM top decays to W-boson and b-quark practically with 100% probability

If one neglects the b-quark mass

$$\Gamma_{top} = \frac{G_F M_t^3}{8\pi\sqrt{2}} \left(1 - \frac{M_W^2}{M_t^2}\right)^2 \left(1 + 2\frac{M_W^2}{M_t^2}\right)$$

$$\Gamma(t \to bW)_{LO} \simeq 1.53 \text{ GeV}, \quad \Gamma(t \to bW)_{correc} = 1.42 \text{ GeV} \qquad \tau = 1/\Gamma$$

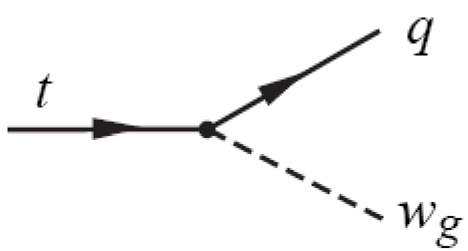
Top decays ($\tau_t \sim 5 \times 10^{-25}\ sec$) much faster than a typical time-scale for a formation of the strong bound states ($\tau_{QCD} \sim 3 \times 10^{-24}\ sec$). The top-quark decays before hadronization.

No top hadrons

In the limit $M_{top} >> M_W$ one can use the EW equivalence theorem to estimate to top width.

According to the EW equivalence theorem amplitudes with external W and Z bosons are dominated by the longitudinal polarisation of the bosons $\left(e_L^{W,Z} \sim p^0/M_{W,Z}\right)$

But the longitudinal W,Z components in the SM appear from "eated" Goldstone bosons w_g, z_g



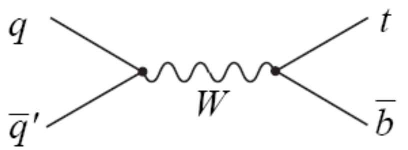
with the Yukawa vertex $M_t/(v\sqrt{2})$

$$\Gamma = \frac{2}{32\pi}\left(\frac{M_t}{v}\right)^2 \cdot M_t = \frac{G_F M_t^3}{8\pi\sqrt{2}}$$

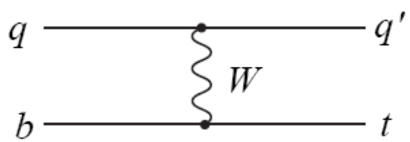
gives exactly the leading behavior

$$\Gamma_{top} = \frac{G_F M_t^3}{8\pi\sqrt{2}}\left(1 - \frac{M_W^2}{M_t^2}\right)^2\left(1 + 2\frac{M_W^2}{M_t^2}\right)$$

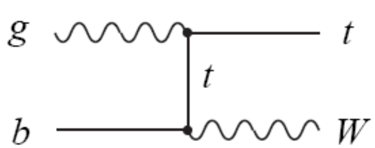
The electroweak single top quark production is another confirmation of the EW fermion structure of the SM



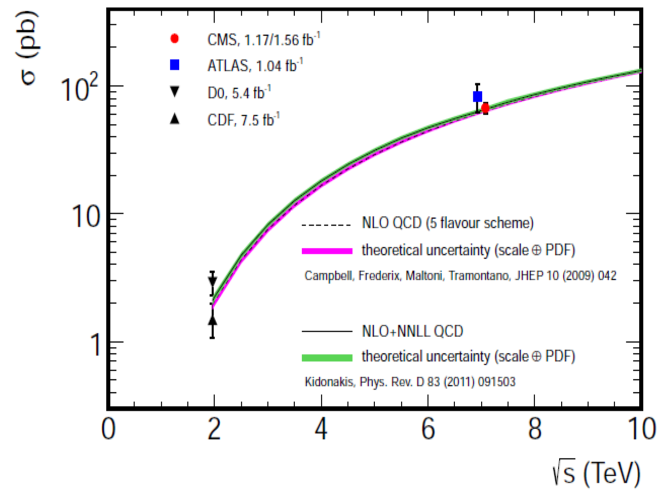
s-channel, $\quad Q_W^2 > 0,$



t-channel, $\quad Q_W^2 < 0,$



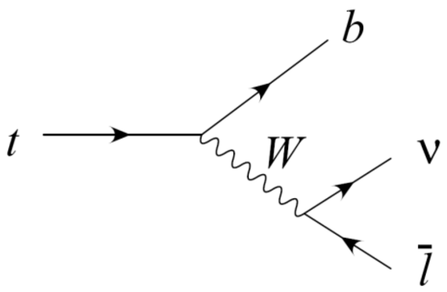
$+W$-associated, $\quad Q_W^2 \approx M_W^2.$



Reasonable agreement with SM including pQCD corrections

Spin correlations in single top

V-A vertex structure in SM



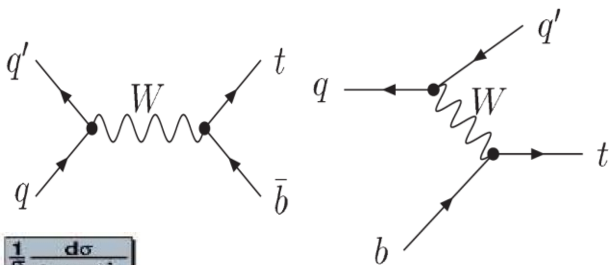
$$d\Gamma \sim |\mathcal{M}|^2 \sim (t + ms) \cdot \ell b \cdot \nu$$

where in the top-quark rest frame, the spin four-vector $s = (0, \hat{s})$ is a unity $\hat{s}$ vector that defines the spin quantization axis of the top quark. In the top quark rest frame:

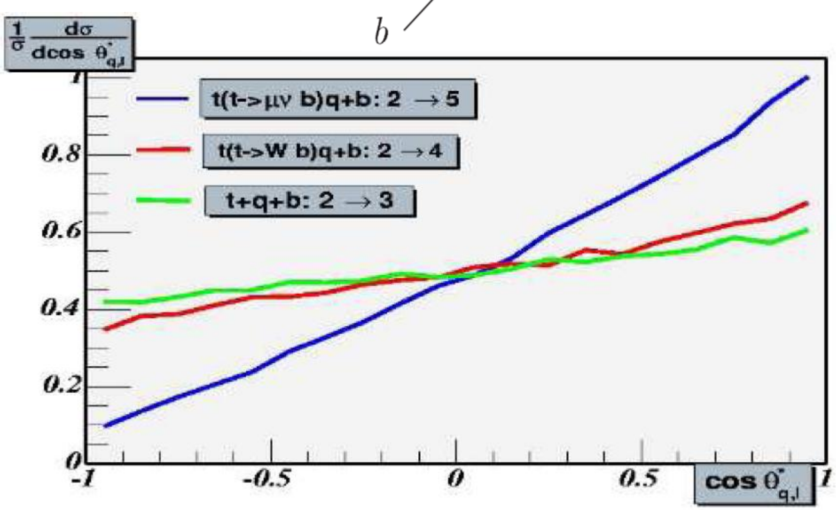
$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} = \frac{1}{2}(1 + \cos\theta_\ell)$$

Hence the charged lepton tends to point along the direction of top spin

Single top production as top decay back in time



Down-type component of weak isospin doublet - d-quark in production plays a role of charged lepton in decay



t-channel production

Best spin correlation variable - the angle between the lepton from W-decay and momentum of outgoing light jet in the top-quark rest frame. Polarization

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta^*_{ql}} = \frac{1 + P\cos\theta^*_{ql}}{2} \qquad P_{top} \approx 90\,\%$$

Electroweak SM beyond the leading order

in many cases a high accuracy of experimental measurements requires the SM computations beyond the leading order

Divergences in computing corrections

Hard: UV divergences

(renormalization)

Soft: IR/collinear divergences

(cancellation due to Kinoshita-Lee-Noenberg theorem)

Introduction to renormalization. QED as an example

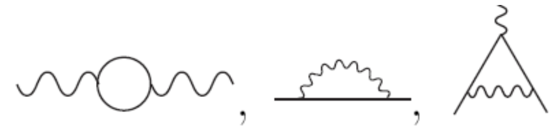
In the SM dimensions of all coupling constants are zero. This has an important sequences making the theory renormalizable. In renormalizable only few diagrams are UV divergent

$$\omega = 4 - L_\gamma - \frac{3}{2} L_e$$ index depends only on a number of external lines

All the UV divergences may be incorporated into few constants such as coupling constants, masses, and field normalization constants.

The generating functional integral

$$Z[J,\eta,\bar\eta] \;=\; \int D(\bar\Psi\Psi A)\exp\Big(i\int d^4x\bar\Psi(i\not{D}-m)\Psi + ieA + J_\mu A^\mu$$
$$+\,\bar\eta\Psi + \bar\Psi\eta - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2\xi}\int d^4x(\partial_\mu A^\mu)^2\Big)$$

There are only three divergent graphs in QED: 

Dyson-Schwinger equation for the photon propagator. The equation is a sequences of the invariance of the measure of functional integral with respect to the shift $A_\mu(x)\to A_\mu(x)+\varepsilon_\mu(x)$

$$D^{-1}_{\alpha\beta}(k) = (D_0)^{-1}_{\alpha\beta} + \Pi_{\alpha\beta}$$

trancated 1 particle irreducible vertex function Γ_μ(p1, p2, k)

At one loop level $\Pi_{\alpha\beta}(k)$ is given by the following (divergent) Feynman integral

$$(-ie)^2\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\frac{\not{p}+m}{p^2-m^2+i0}\gamma_\alpha\frac{(\not{p}-\not{k})+m}{(p-k)^2-m^2+i0}\gamma_\beta\right]$$

Dimensional regularization

$$d^4p \to d^Dp(\mu^2)^{2-D/2}$$

$\Pi_{\alpha\beta}$ has the following sructure

$$\Pi_{\alpha\beta}(k) = \left(g_{\alpha\beta}k^2 - k_\alpha k_\beta\right)\Pi(k^2)$$

Because of Ward identity

$$k^\mu\Gamma_\mu(p_1,p_2,k) = S^{-1}(p_1) - S^{-1}(p_2) \qquad k_\alpha\gamma^\alpha = (\not p) - (\not p - \not k) = (\not p - m) - [(\not p - \not k) - m]$$

$$\int \frac{d^Dp}{(2\pi)^D}\mathrm{Tr}\left[\left(\frac{\not p + m}{p^2 - m^2 + i0} - \frac{(\not p - \not k) + m}{(p-k)^2 - m^2 + i0}\right)\gamma_\beta\right] = 0$$

Therefore the dressed photon propagator

$$D_{\alpha\beta}(k) = -\frac{i}{k^2}\left[\frac{1}{1+\Pi_\gamma(k^2,\varepsilon,\mu^2)}\left(g_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}\right) + \xi\frac{k_\alpha k_\beta}{k^2}\right]$$

For Π = 0 one gets the free photon propagator

$$D^0_{\mu\nu}(k) = -i\frac{1}{k^2+i0}\left[g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]$$

Correct normalization of the kinetic term by rescaling $A_\mu(x)$ field

$$A_\mu(x) \to \frac{1}{\sqrt{Z_3}} A_\mu, \text{ where } Z_3^{(a)} = (1 + \Pi_\gamma(0,\varepsilon))^{-1}$$

Direct computation of 1-loop integral with well known Feynman technics

$$\Pi_\gamma(k^2, \varepsilon, \mu^2) = \frac{\alpha}{3\pi\varepsilon} + \Pi_{finite}$$

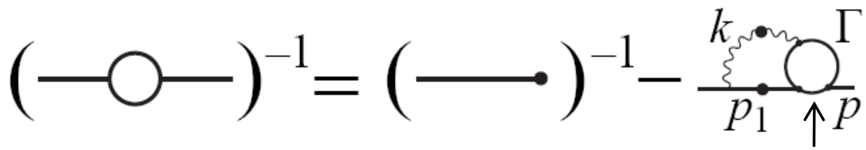
$$\varepsilon = (4 - D)/2$$

⇓

$$Z_3^{-1} = 1 + \frac{\alpha}{3\pi\varepsilon}$$

Dyson-Schwinger equation for the dressed fermion propagator

$$S^{-1}(p) = S_0^{-1}(p) - \Sigma(p)$$



the same trancated vertex function Γ_μ(p1, p2, k)

In the second order of perturbation theory $\Sigma^{(2)}$(p)

$$-i\Sigma^{(2)}(p) = (-ie)^2 \int \frac{d^D k}{(2\pi)^D} (\mu^2)^{2-D/2} \cdot \gamma^\mu D_{\mu\nu}(k) \frac{\not{p} - \not{k} + m}{(p-k)^2 - m^2} \gamma_\nu$$

Direct computation gives the following answer:

$$\Sigma_2 = \frac{\alpha}{8\pi}(4m - \not{p})\frac{2}{\varepsilon} + \Sigma_{finite}$$

Generic structure of Σ(p) and fermion propagator:

$$\Sigma(p) = \not{p} f_1(p^2) - m f_2(p^2)$$

$$S(p) = \frac{1}{\not{p}(1 - f_1(p^2)) - m(1 - f_2(p^2))} = -\frac{1}{1 - f_1(p^2)} \frac{1}{\not{p} - m\frac{1-f_2(p^2)}{1-f_1(p^2)}}$$

The physics mass:

$$m_{phys} = m\frac{1 - f_2(m^2_{phys})}{1 - f_1(m^2_{phys})}$$

The fermion propagator has the following form close to physics mass

$$S(p) = \frac{Z_2(\varepsilon, \mu)}{\hat{p} - m_{ph}(\varepsilon, \mu)}$$

$$Z_2 = (1 - f_1)^{-1} \qquad m_{ph} = m\frac{Z_2}{Z_m}$$

$$Z_m = (1 - f_2)^{-1}$$

↓

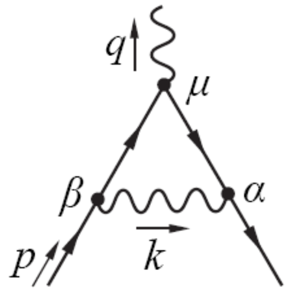
At one loop:

$$\Sigma_2 = \frac{\alpha}{8\pi}(4m - \not{p})\frac{2}{\varepsilon} + \Sigma_{finite}$$

$$Z_2^{-1} = 1 + \frac{\alpha}{4\pi\varepsilon} + O(\alpha)$$

$$Z_m^{-1} = 1 + \frac{\alpha}{\pi\varepsilon} + O(\alpha)$$

The remaining divergent QED diagram is the vertex function correction



$$
\begin{aligned}
ie\Gamma^{(2)}_{\mu}(p,q) \;=\;& (-ie)^3(\mu^2)^{2-D/2}\int \frac{d^Dk}{(2\pi)^D}\cdot \\
& \cdot\gamma^{\alpha}(i)\frac{\not{p}-\not{q}-\not{k}+m}{(p-q-k)^2-m^2+i0} \\
& \gamma_{\mu}(i)\frac{\not{p}-\not{q}+m}{(p-q)^2-m^2+i0}\cdot\gamma_{\beta}\frac{-i}{k^2+i0}g^{\alpha\beta}
\end{aligned}
$$

In order to compute the divergent part one can compute the diagram in the limit $q \to 0$

$$\Gamma^{(2)}_{\mu}(p,0) = \gamma_{\mu}\left[\frac{\alpha}{4\pi}\frac{1}{\varepsilon} + O(\alpha)\right]$$

Therefore the vertex function including 1-loop correction may be written in the form

$$-ie\Gamma_{\mu} = -ieZ_1\gamma_{\mu} \qquad Z_1 = 1 - \frac{\alpha}{4\pi}\frac{1}{\varepsilon} + O(\alpha)$$

Therefore $Z_1 = Z_2$ at 1-loop level. But this is correct to all orders of perturbation theory due to the Ward identity

$$k^{\mu}\Gamma_{\mu}(p_1,p_2,k) = S^{-1}(p_1) - S^{-1}(p_2) \qquad k \to 0 \qquad \Gamma_{\mu}(p,0) = \partial_{\mu}S^{-1}(p)$$

Let us rewrite our initial or bare (before renormalization) QED Lagrangian

$$L = -\frac{1}{4} F^0_{\mu\nu} F^{0\mu\nu} - \bar{\Psi}_0 (i \not{D} - m_0) \Psi_0 \qquad F^0_{\mu\nu} = \partial_\mu A^0_\nu - \partial_\nu A^0_\mu,\ D_{\mu 0} = \partial_\mu - i e_0 A^0_\mu$$

in the following way:

$$L = -\frac{1}{4} F^{ph}_{\mu\nu} F^{ph\mu\nu} - \bar{\Psi}_{ph} (i \not{D}_{ph} - m_{ph}) \Psi_{ph} + \Delta L$$

$$\begin{aligned} \Delta L \ &= \ -(Z_3 - 1)\tfrac{1}{4} F^{ph}_{\mu\nu} F^{ph\mu\nu} + (Z_2 - 1) \bar{\Psi}_{ph} (i \not{\partial}) \Psi_{ph} - \\ & -(Z_m - 1) m_{ph} \bar{\Psi}_{ph} \Psi_{ph} - (Z_1 - 1) e_{ph} \bar{\Psi}_{ph} (\not{A}_{ph}) \Psi_{ph} \end{aligned}$$

$$A^{ph}_\mu = Z_3^{-1/2} A^0_\mu,\ \Psi_{ph} = Z_2^{-1/2} \Psi,\ m_{ph} = (Z_2/Z_m) m_0$$

$$e_0 = Z_1 Z_2^{-1} Z_3^{-1/2} (\mu)^{D/2-2} e_{ph},\ D^{ph}_\mu = \partial_\mu - i e_{ph} A^{ph}_\mu$$

The terms in the Lagrangian ΔL are called counter-terms

When one computes some effects using the Lagrangian L+ΔL all UV divergences are cancelled out order by order in perturbation theory by contributions of the counter-terms. Number of the counter-terms is finite!

$$e_0 = Z_1 Z_2^{-1} Z_3^{-1/2} (\mu)^{D/2-2} e_{ph}(\mu)$$

dimension of the charge

$Z_1 = Z_2$ due to the Ward identity ⇒ $e_0 = Z_3^{-1/2} (\mu)^{D/2-2} e_{ph}(\mu)$

Note that e_0 does not depend on μ

For the coupling constant $\alpha = \frac{e^2}{4\pi}$ $D = 4 - 2\varepsilon$

$$\alpha_0 = Z_3^{-1} (\mu^2)^{-\varepsilon} \alpha_{ph}(\mu)$$

Taking the derivative $\mu \frac{\partial}{\partial \mu}$: $\mu \frac{\partial \alpha}{\partial \mu} = \frac{2\alpha^2}{3\pi} \equiv \beta(\alpha)$

The equation is a particular example of the renormalization group equation which we do not discuss in these short lecture course

At one-loop level the β–function in QED $\beta(\alpha) = \frac{b_0}{\pi} \alpha^2, \quad b_0 = \frac{2}{3}$

The equation for the coupling constant α can be easily solved

$$\alpha(\mu) = \frac{\alpha(\mu_0)}{1 - \frac{\alpha(\mu_0)}{3\pi}\ln(\mu/\mu_0)^2}$$

Running coupling constant

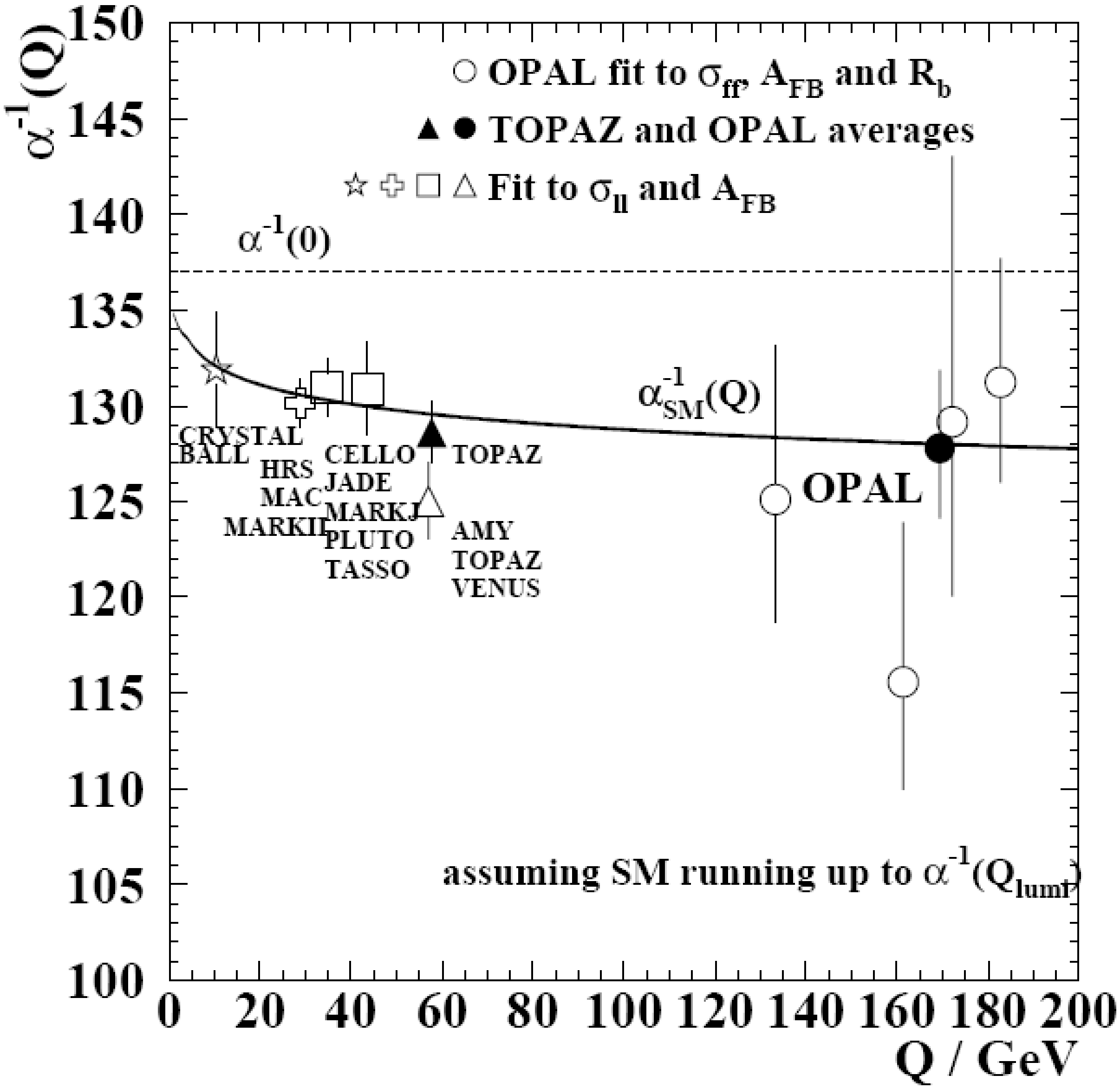
$\alpha = 1/137$ being measured at very small scale in Thompson scattering increases with the scale growing and becomes $\alpha(M_Z) \approx 1/129$ at the Z-mass. This fact was confirmed by LEP experiments.

This means the charged particle-antiparticle virtual pairs screen the bare charge at small μ^2 or at large distances

If the scale μ increases to very large values the well known Landau pole approaches where the perturbation picture in QED brakes down

$$\frac{b_0}{\pi}\ln(\mu/\mu_0)^2 = 1$$

Note that in QCD the β-function is negative leading to anti-screening effect, the α_s becomes smaller with increasing of the momuntum scale (momentum transfer) or decreasing distances ("asymptotic freedom")



α⁻¹(Q)
150
145
140
135
130
125
120
115
110
105
100
OPAL fit to σff, AFB and Rb
TOPAZ and OPAL averages
Fit to σll and AFB
α⁻¹(0)
α⁻¹SM(Q)
CRYSTAL BALL
HRS
MAC
MARKII
CELLO
JADE
MARKJ
PLUTO
TASSO
TOPAZ
AMY
TOPAZ
VENUS
OPAL
assuming SM running up to α⁻¹(Qlumi)
0 20 40 60 80 100 120 140 160 180 200
Q / GeV

All terms of the SM Lagrangian have dimension 4, and all the coupling constants are dimensionless. So, the SM is the renormalizable theory in the same manner as QED.

The perturbation theory expansion EW parameters α/π with $\alpha_{em} \sim 1/129$ and $\alpha_{weak} \sim 1/30$ are very small

Naively - the EW higher order corrections are not that important

However, the experimental accuracies are in some cases so high, that even 1-loop EW corrections might not be sufficient

$$\begin{array}{lclcllr} M_Z & = & 91.1875 & \pm & 0.0021 & \text{GeV} & 0.002\% \\ \Gamma_Z & = & 2.4952 & \pm & 0.0023 & \text{GeV} & 0.09\% \\ M_W & = & 80.385 & \pm & 0.015 & \text{GeV} & 0.02\% \\ M_{top} & = & 173.2 & \pm & 0.9 & \text{GeV} & 0.52\% \end{array}$$

$$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2\theta_W}$$

$$\rho_\ell = 1.0050 \pm 0.0010$$

$$\sin^2\theta_{eff}^{lept} = 0.23153 \pm 0.00016$$

Most important corretions:

Resummation of large logs - $\log(M_{top}^2/m_e^2) \approx 24.2$;

Corrections proportional to M_{top}^2/M_W^2 coming from longitudinal modes

CDF ($\int L dt$ = 2.2 fb^{-1})

Electron and Muon

M_W = 80387 ± 19 MeV

Dzero ($\int L dt$ = 5.2 fb^{-1})

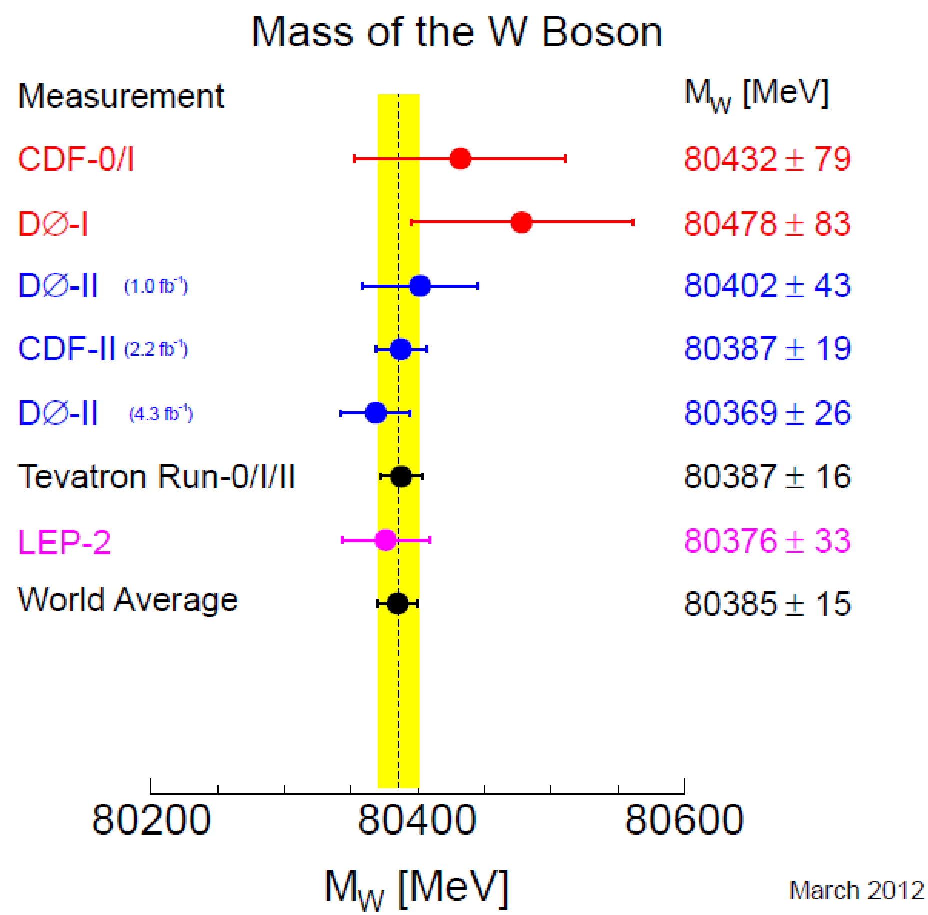
Electron only

M_W = 80369 ± 26 MeV

difficult analysis

Calibration / alignment

Understanding of recoil

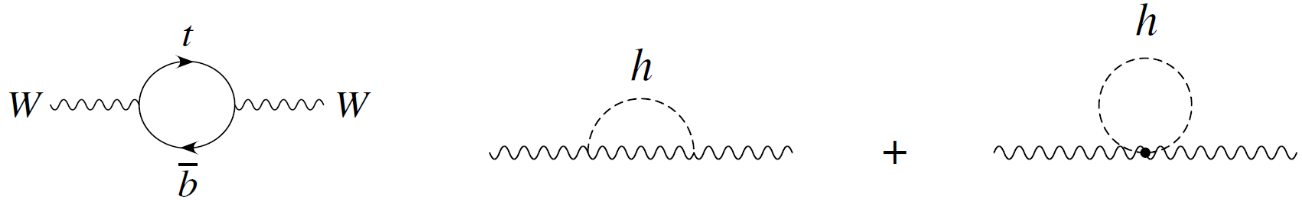


Combination : M_W = 80385 ± 15 MeV

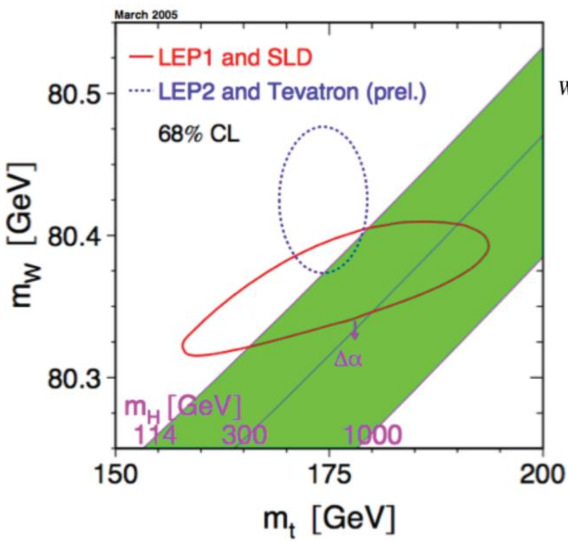
0.02%

M_W is a function of M_{top} and M_H in SM

$$(1-\frac{m_W^2}{m_Z^2})\frac{m_W^2}{m_Z^2} = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2}\frac{1}{1-\Delta r_W}$$



$$(\Delta r)_{\text{top}} \approx -\frac{3G_F m_t^2}{8\sqrt{2}\pi^2}\frac{1}{t_W^2} \qquad (\Delta r)_{\text{Higgs}} \approx \frac{11 G_F M_Z^2 c_W^2}{24\sqrt{2}\pi^2}\ln\frac{m_h^2}{M_Z^2}$$



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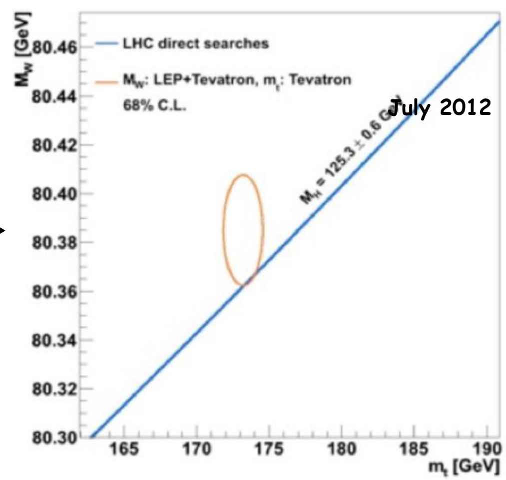


Top mass most precisely measured quark mass !

$$m_t^{\text{comb}} = 173.18 \pm 0.56\,(\text{stat}) \pm 0.75\,(\text{syst})\ \text{GeV} = 173.18 \pm 0.94\ \text{GeV}$$

LHC: m_t=173.3±0.5(stat.)±1.3(sist.) GeV

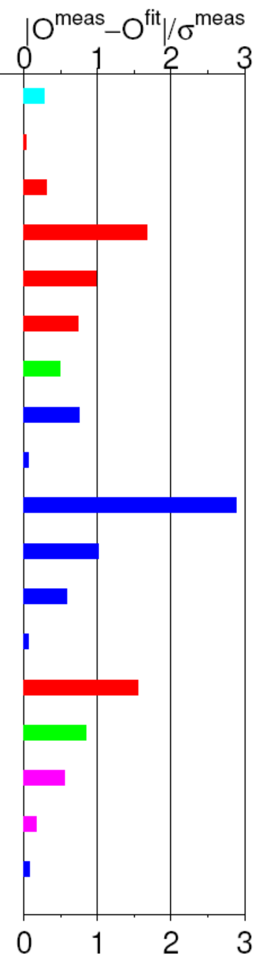
The top quark mass has been determined indirectly from the analysis of loop corrections before direct observation

$$m_t = 178 \pm 8\ {}^{+17}_{-20}\ \text{GeV}$$

Loop corrections lead to the fact that SM parameters (coupling constants, masses, widths) are the running parameters, and **they are nontrivial functions of each other.**

Summary of comparisons of the EW precision measurements at LEP1, LEP2, SLD, and the Tevatron and a global parameter fit

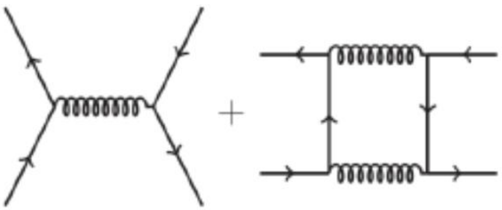
	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26

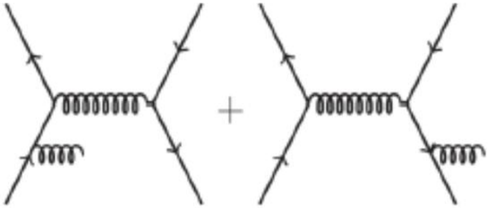


March 2012

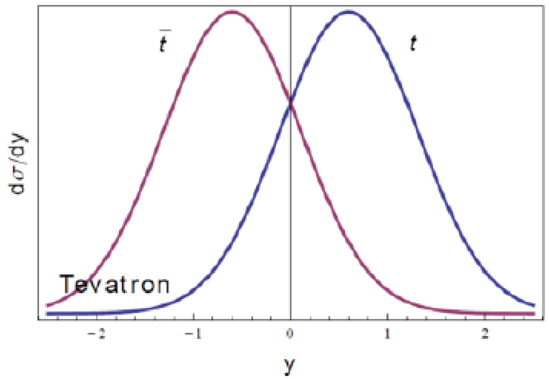
$$\sin^2\theta^{lept}_{eff} \equiv \frac{1}{4}\left(1-\frac{v_l}{a_l}\right)$$

tT forward-backward and charge assymetries





Tevatron



$$A_{\rm rest}(t\bar{t}) = \frac{N_t(\Delta y > 0) - N_t(\Delta y < 0)}{N(\Delta y > 0) + N_(\Delta < 0)}$$

$$A_{\rm rest}^{\rm theory} = 0.07 \pm 0.006$$ Kuehn, Rodrigo

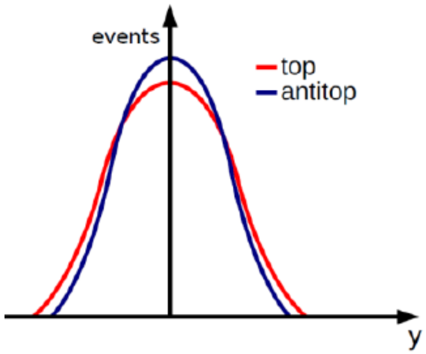
$$A_{\rm rest} = 0.15 \pm 0.05 \quad \rm CDF$$

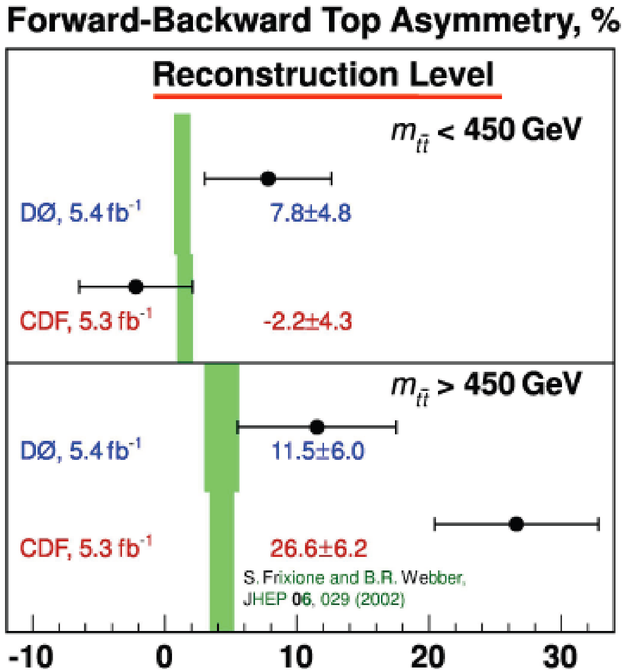
$$A_{\rm rest} = 0.196 \pm 0.065, \quad \rm D0$$

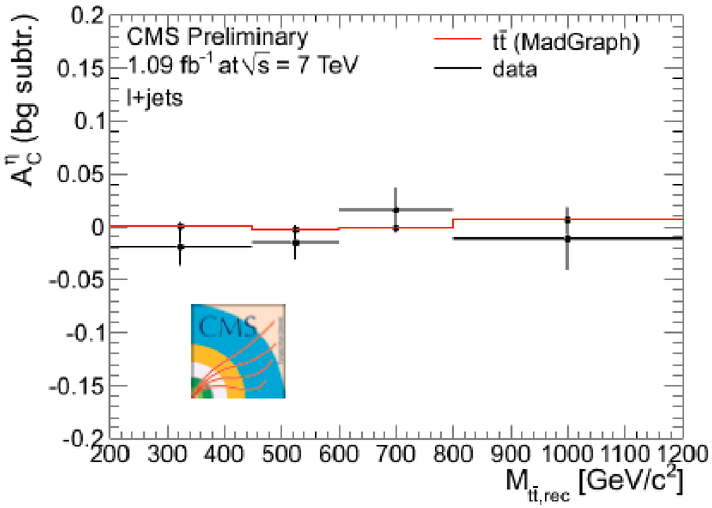
LHC

$$A_C = \frac{N(\Delta|y|>0) - N(\Delta|y|<0)}{N(\Delta|y|>0) + N(\Delta|y|<0)}$$

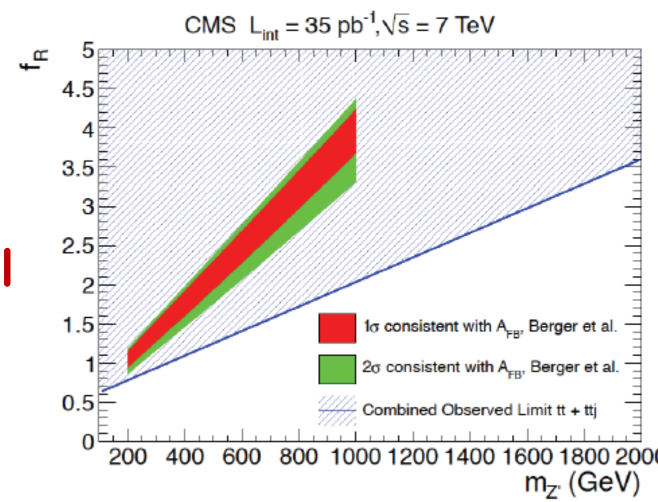
$$\Delta|y| = |y_t| - |y_{\bar{t}}|$$







LHC rulled out most interesting t-channel possibilities



Concluding remarks

1. Standard Model is the renormalizable anomaly free gauge quantum field theory with spontaneously broken electroweak symmetry. Remarkable agreement with many experimental measurements.

2. All SM leptons, quarks, gauge bosons, and, probably, the Higgs boson have been discovered

3. SM predicts the structure of all interactions: fermion-gauge, gauge self couplings, Higgs-gauge, Higgs-fermion, Higgs self couplings (but not all couplings were tested yet experimentally)

4. The EW SM has 17 parameters (from experiments)
   gauge-Higgs sector contains 4 parameters:
   g_1, g_2, μ^2, λ $\longrightarrow$ best measured α_{em}, G_F, M_Z (or α_{em}, s_W, M_W) plus M_H

   In addition, 6 quarks masses, 3 lepton masses,
   3 mixing angles and one phase of the CKM matrix

   plus α_{QCD} $\Rightarrow$ 18 SM parameters

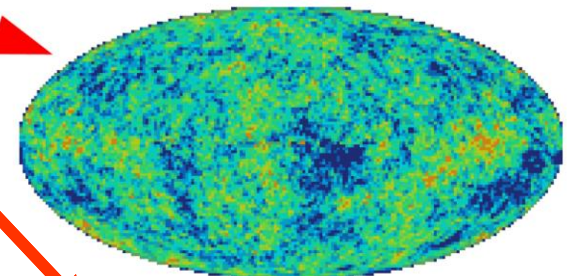
   (+ may be masses and mixing parameters from neutrino sector)

5. Facts which can not be explained in SM

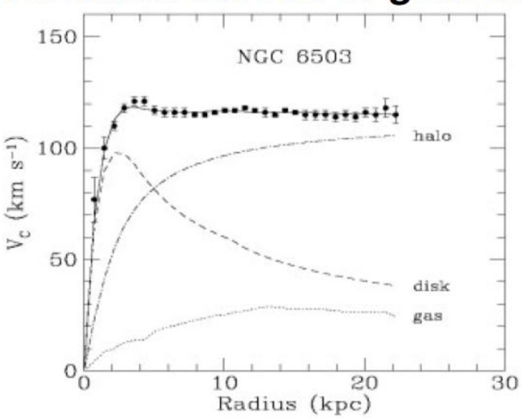
- EW symmetry is broken – photon is massless, W and Z are massive prticles
Fermions have very much different masses
(Mtop ≈ 172 GeV, Me ≈ 0.5 MeV, $\Delta M\nu \approx 10^{-3}$ eV)

- Dark Matter exists in the Universe

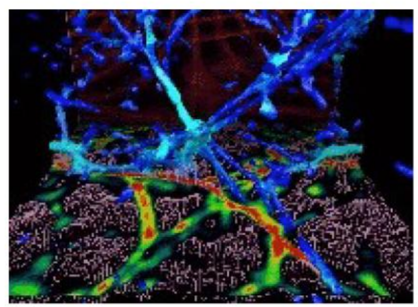
CMB



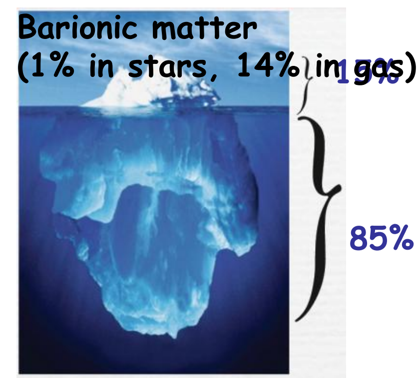
Rotation curves of galaxies



Large Scale Structure



Lensing



Dark unknown matter

- (g-2)μ (about 3.5 σ)
- Neutrino masses, mixing, oscillations

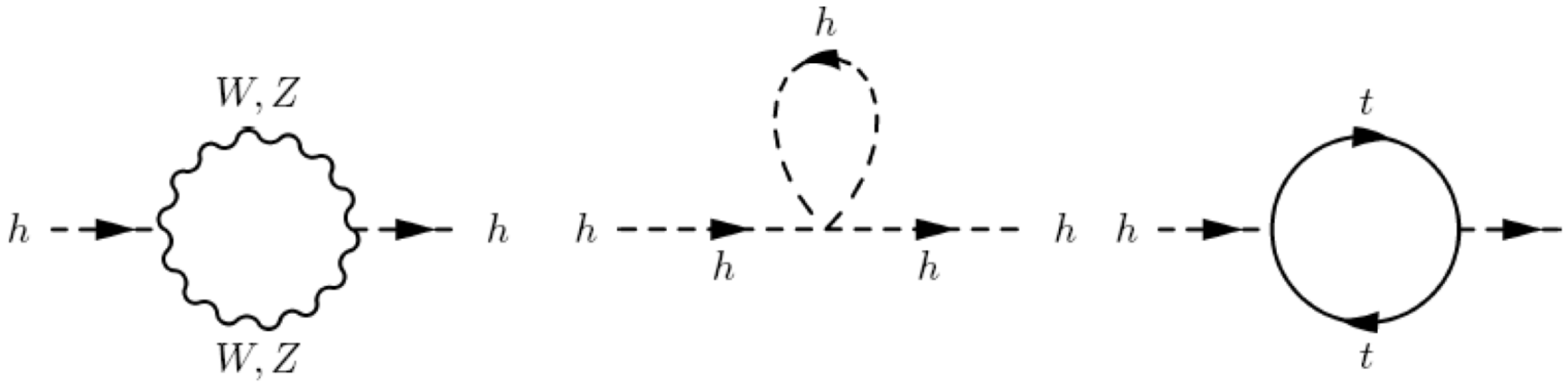
- Particle – antiparticle asymmetry in the Universe,
CP violation

baryon asymmetry: $\frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \sim 10^{-10}$

CKM phase – too small efect

6. The simplest Higgs mechanism SM is not stable with respect to quantum corrections (naturalness problem)

Loop corrections to the Higgs mass



$$\delta m_H^2 = \frac{3G_F}{4\sqrt{2}\pi^2}\left(2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2\right)\Lambda^2 \approx -(0.2\,\Lambda)^2$$

$\delta m_H < m_H$ $\Lambda < 1$ TeV

In SM there is no symmetry which protects a strong dependence of Higgs mass on a possible new scale

Something is needed in addition to SM…

7. In addition to mentioned problems (naturalness/hierarchy, dark matter content, CP violation) SM does not give answers to many questions

What is a generation? Why there are only 3 generations?

How quarks and leptons related to each other, what is a nature of quark-lepton analogy?

What is responsible for gauge symmetries, why charges are quantize? Are there additional gauge symmetries?

What is responsible for a formation of the Higgs potential?

To which accuracy the CPT symmetry is exact?

Why gravity is so weak comparing to other interactions?

………

“It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong”.

Richard P. Feynman