Lecture 3
QCD perturbation theory at fixed order: event shapes and jets

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Today

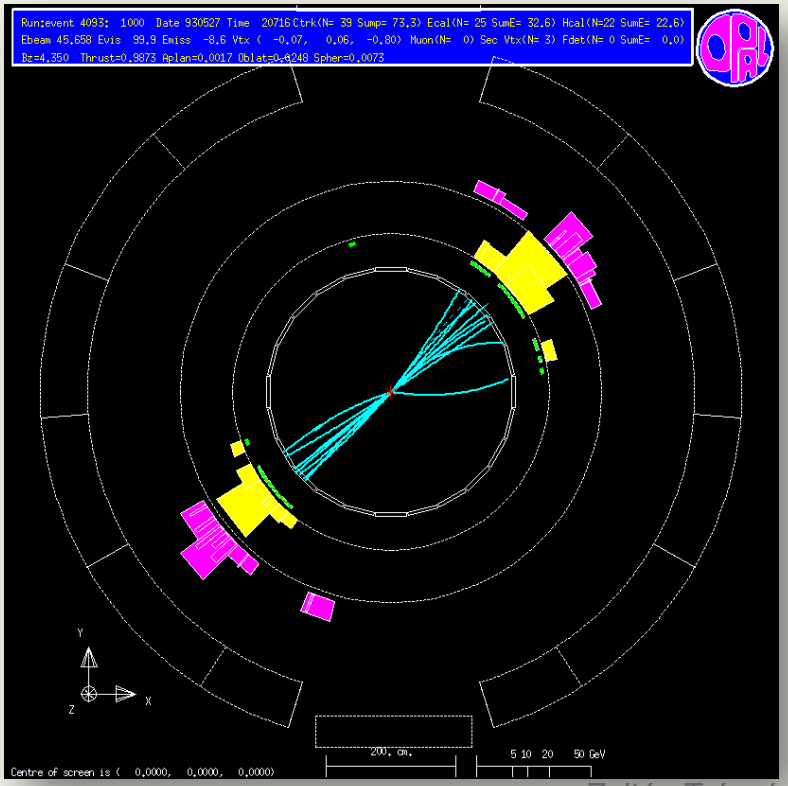
- ☛ IR safety
  - ☞ event shapes
  - ☞ jet algorithms
- ☛ A general method for computing QCD jet cross sections

Jet cross sections

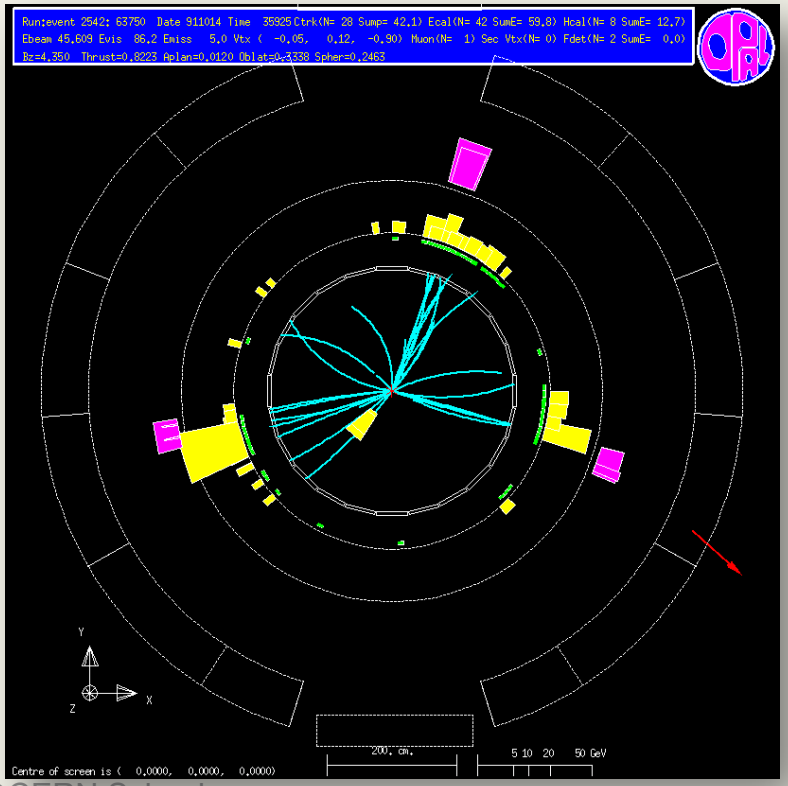
Are there less inclusive IR safe observables?

(typical final states have structure)

2 jets



3 jets

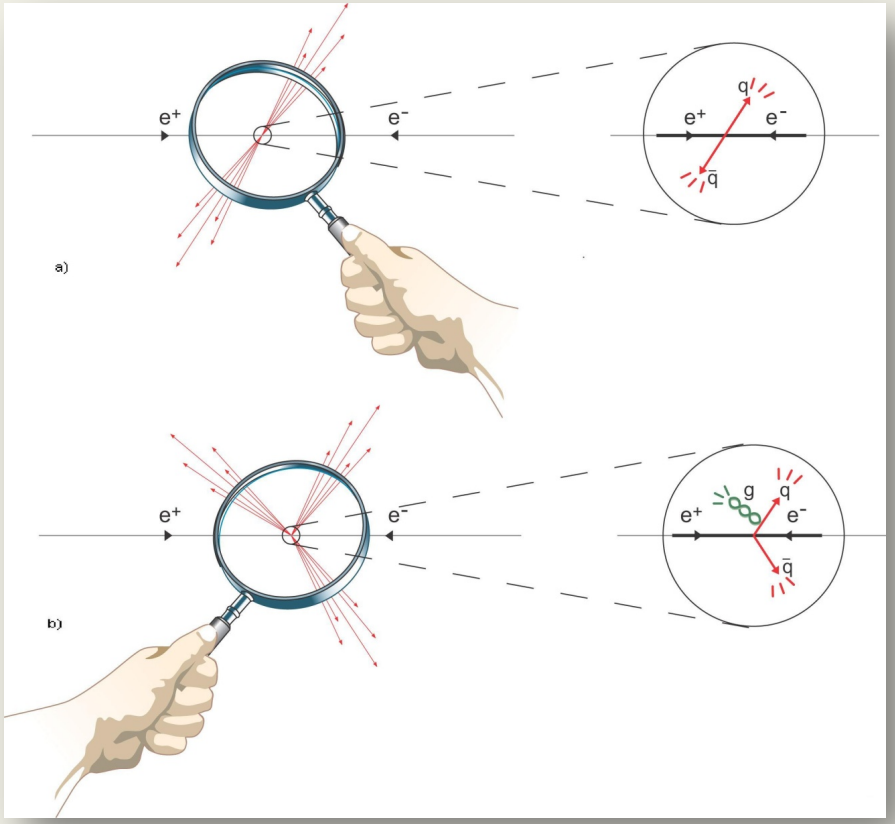


Parton-hadron duality

☛Production probability pattern:

2jets : 3jets : 4jets ~ $O(\alpha_s^0) : O(\alpha_s^1) : O(\alpha_s^2)$

➔ jets reflect the partonic structure



Jet cross sections: 2 jets

- average over event orientation $\Rightarrow$ $|\mathcal{M}_2|^2$ has no dependence on the parton momenta: $\sigma^{\mathrm{LO}} = |\mathcal{M}_2|^2 \int_0^1 \mathrm{d}y_{12}\, \delta(1-y_{12}) \underbrace{J_2(p_1,p_2)}_{\text{jet function}}$
- NLO corrections:

$$\mathrm{d}\sigma^{\mathrm{R}} = |\mathcal{M}_2|^2 S_\varepsilon \frac{\mathrm{d}y_{13}}{y_{13}^{\varepsilon}} \frac{\mathrm{d}y_{23}}{y_{23}^{\varepsilon}} C_F \frac{\alpha_s}{2\pi} \left[(1-\varepsilon)\left(\frac{y_{23}}{y_{13}} + \frac{y_{13}}{y_{23}} \right) + \frac{2y_{12}}{y_{13}y_{23}} - 2\varepsilon \right]$$

$$\times J_3(p_1,p_2,p_3)$$

$$\mathrm{d}\sigma^{\mathrm{V}} = |\mathcal{M}_2|^2 S_\varepsilon C_F \frac{\alpha_s}{2\pi} \left(\frac{\mu^2}{s} \right)^{\varepsilon} \left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \pi^2 + \mathrm{O}(\varepsilon) \right]$$

$$\times \mathrm{d}y_{12} \delta(1-y_{12}) J_2(p_1,p_2)$$

Subtraction method

- Cannot combine the integrands (as for σ_{tot})

$$\sigma^{\mathrm{NLO}} = \sigma_3^{\mathrm{NLO}} + \sigma_2^{\mathrm{NLO}}$$

- regularize both with subtraction:

$$\mathrm{d}\sigma_3^{\mathrm{NLO}} = \underbrace{\mathrm{d}\sigma^{\mathrm{R}} J_3 - \mathrm{d}\sigma^{\mathrm{A}} J_2}_{\text{integrable}} \qquad \mathrm{d}\sigma_2^{\mathrm{NLO}} = \underbrace{\left(\mathrm{d}\sigma^{\mathrm{V}} + \mathrm{d}\sigma^{\mathrm{A}}\right) J_2}_{\text{integrable}}$$

- requires special property of J_n:

$$\lim_{y_{13}, y_{23} \to 0} J_3 \to J_2$$

(IR safety expressed analytically)

Construction of $d\sigma^A$

$$\frac{y_{23}}{y_{13}}+\frac{y_{13}}{y_{23}}+\frac{2y_{12}}{y_{13}y_{23}}=\frac{y_{23}}{y_{13}}+\frac{1}{y_{13}}\frac{2y_{12}}{y_{13}+y_{23}}+(1\leftrightarrow 2)\quad\Leftarrow\frac{1}{ab}=\frac{1}{a}\frac{1}{a+b}+\frac{1}{b}\frac{1}{a+b}$$

$$=\frac{1}{y_{13}}\left[y_{23}+\left(2\overbrace{\frac{y_{12}+y_{13}+y_{23}}{y_{13}+y_{23}}}^{=1}-2\right)\right]+(1\leftrightarrow 2)$$

$$z_1=\frac{y_{12}}{y_{12}+y_{23}}\quad\Rightarrow\quad y_{13}+y_{23}=1-y_{12}=1-z_1(1-y_{13})$$

$$\frac{y_{23}}{y_{13}}-\frac{1}{y_{13}}=\frac{y_{23}(\cancel{1}-y_{13})-y_{12}-\cancel{y_{23}}}{y_{13}(y_{12}+y_{23})}=\frac{-y_{23}y_{13}-y_{12}}{y_{13}(y_{12}+y_{23})}=-\frac{y_{23}}{y_{12}+y_{23}}-\frac{z_1}{y_{13}}$$

$$=\left[\frac{\mathbf{1}}{\boldsymbol{y_{13}}}\left(\frac{\mathbf{2}}{\mathbf{1}-\boldsymbol{z_1}(\mathbf{1}-\boldsymbol{y_{13}})}-\mathbf{1}-z_1\right)-\frac{y_{23}}{y_{12}+y_{23}}\right]$$

$$+(\mathbf{1}\leftrightarrow\mathbf{2})$$

Approximate cross section

☛ rewrite the phase space using y_{13} and z_1

$$\mathrm{d}\sigma^{\mathrm{A}} = \left|\mathcal{M}_2\right|^2 \underbrace{S_\varepsilon \left(\frac{\mu^2}{s}\right)^{\varepsilon} \int_0^1 \mathrm{d}y_{13} \int_0^1 \mathrm{d}z_1 \left(1-y_{13}\right)^{1-2\varepsilon} y_{13}^{-\varepsilon} \left[z_1\left(1-z_1\right)\right]^{-\varepsilon} C_F V_{13,2}}_{= I(\varepsilon)\ |\mathcal{M}_2|^2}$$

$$+\left(1 \leftrightarrow 2\right)$$

☛ we shall see: this factorization is universal

$$V_{13,2}\left(p_1, p_2, p_3\right) = \frac{\alpha_s}{2\pi} \frac{1}{y_{13}} \left(\frac{2}{1-z_1\left(1-y_{13}\right)} - 1 - z_1 - \varepsilon\left(1-z_1\right)\right)$$

$$I\left(\varepsilon\right) = \left|\mathcal{M}_2\right|^2 C_F \frac{\alpha_s}{2\pi} \frac{1}{\Gamma\left(1-\varepsilon\right)} \left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon} \left[\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + 10 - \frac{\pi^2}{3} + \mathrm{O}\left(\varepsilon\right)\right]$$

Integrated subtraction term

$$\sigma^{\mathrm{A}} = \mathrm{I}(\varepsilon)\left|\mathcal{M}_2\right|^2 \int \mathrm{d}y_{13,2}\,\delta\left(1-y_{13,2}\right) J_2\left(\tilde{p}_{13},\tilde{p}_2\right) + \left(1 \leftrightarrow 2\right)$$

$$I(\varepsilon)\left|\mathcal{M}_2\right|^2 = \left|\mathcal{M}_2\right|^2 C_F \frac{\alpha_{\mathrm{s}}}{2\pi} S_\varepsilon \left(\frac{\mu^2}{s}\right)^\varepsilon \left[\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + 10 - \frac{\pi^2}{3} + \mathrm{O}(\varepsilon)\right]$$

$$\sigma^{\mathrm{V}} = \left|\mathcal{M}_2\right|^2 C_F \frac{\alpha_{\mathrm{s}}}{2\pi} S_\varepsilon \left(\frac{\mu^2}{s}\right)^\varepsilon \left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \pi^2 + \mathrm{O}(\varepsilon)\right]$$

$$\times \int_0^1 \mathrm{d}y_{12}\,\delta\left(1-y_{12}\right) J_2\left(p_1,p_2\right)$$

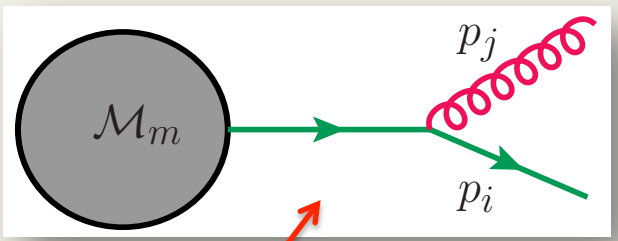
$$\sigma_2^{\mathrm{NLO}} = \left|\mathcal{M}_2\right|^2 C_F \frac{\alpha_{\mathrm{s}}}{\pi}\left[1 + \frac{\pi^2}{3} + \mathrm{O}(\varepsilon)\right] \int_0^1 \mathrm{d}y_{12}\,\delta\left(1-y_{12}\right) J_2\left(p_1,p_2\right)$$

Origin of singular behaviour

Can we construct $d\sigma^A$ universally?

(independently of process and observable)

☛ need study the origin of singularities in $|\mathcal{M}_m|^2$



$\theta \to 0$: $|\mathcal{M}_{m+1}|^2 \simeq |\mathcal{M}_m|^2/\theta^2$

$E_j \to 0$: $|\mathcal{M}_{m+1}|^2 \simeq |\mathcal{M}_m|^2/E_j^2$

$$\frac{1}{(p_i+p_j)^2} = \frac{1}{2\,p_i\cdot p_j} = \frac{1}{2E_i\,E_j\,(1-\cos\theta)} \simeq \frac{1}{E_i\,E_j\,\theta^2}$$

With phase space factor

$$\frac{\mathrm{d}^3 p_j}{2E_j} = \tfrac{1}{2}E_j\,\mathrm{d}E_j\,\mathrm{d}\cos\theta\,\mathrm{d}\phi \simeq \tfrac{1}{4}E_j\,\mathrm{d}E_j\,\mathrm{d}\theta^2\,\mathrm{d}\phi$$

we find log singularities $\mathrm{d}E_j/E_j$ & $\mathrm{d}\theta/\theta$

Origin of singular behaviour

arising at kinematically degenerate phase space configurations, i.e. we cannot distinguish (at NLO accuracy)

- ☞ a single hard parton
- ☞ single hard parton splitting into two collinear partons
- ☞ single hard parton emitting a soft gluon

The singularities are artificial, appear if we integrate from 0 (in pQCD), can be tamed into log-enhanced contributions if integrated from carefully chosen lower cut-off ➔ 4th lecture

Are there less inclusive IR safe observables?

Kinoshita-Lee-Nauenberg (KLN) theorem:

In massless, renormalized QFT in 4 dimensions transition rates are IR safe if summation over kinematically degenerate (initial) and final states is carried out.

☛ IR safe observables must be insensitive to soft and/or collinear parton branching:

- ☞ event shape variables (rarely used for hadron collisions)
- ☞ jet cross sections (several definitions exist)

Event shapes

thrust, thrust major/minor, C- & D-parameters, oblateness, sphericity, aplanarity, jet masses, jet-broadening, energy-energy correlation, differential jet rates

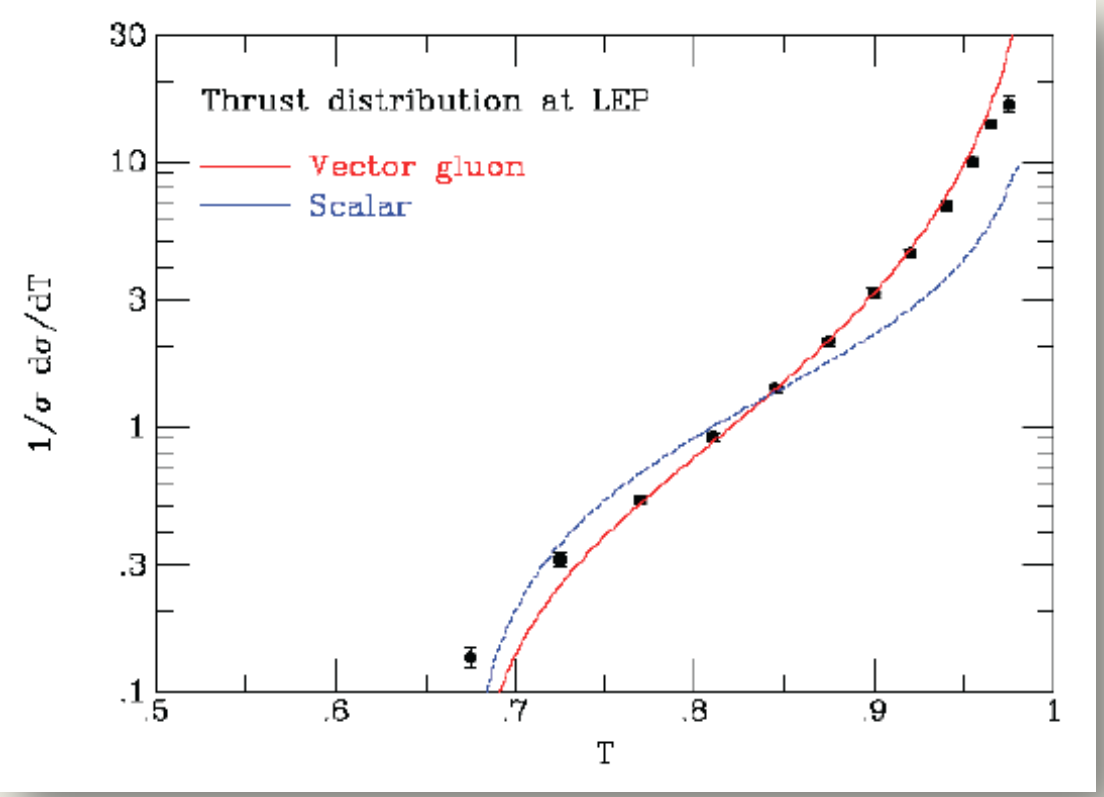
☛ thrust: $$T = \max_{\vec{n}} \frac{\sum_{i=1}^{m} |\vec{p}_i \cdot \vec{n}|}{\sum_{i=1}^{m} |\vec{p}_i|}$$

IR safe:

- ☞ $p_j \to 0$ drops from the sums
- ☞ replacing p_i with $z\, p_i + (1-z) p_i$ does not change the sums

Thrust at LO

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}T} = C_F \frac{\alpha_s}{2\pi}\left[\frac{2\left(3T^2-3T+2\right)}{T(1-T)}\ln\left(\frac{2T-1}{1-T}\right) - 3(3T-2)\frac{2-T}{1-T}\right]$$

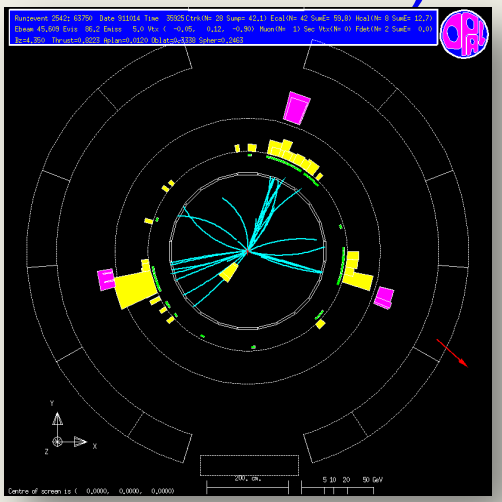


- singular at T=1 (2-jet region)
- logs at *n*th order $\alpha_s^n \log^m(1/(1-T))\,,$ $m \leq n+1$ may become large at T → 1

needs resummation (beyond our scope)

Jet algorithms

- ☛ to quantify the jet-like structure of the final states (how many jets can you see here?)
- ☛ has a long history with slow convergence
- ☛ experimental and theoretical requirements differ
  - ☞ exp: define cones that include almost all hadronic tracks at cheap CPU price
  - ☞ theo: needs to be IR safe and resummable



Experimenters prefered cone jet algorithms ("Snowmass accord")

- start from a cone seed (center) in $\eta-\varphi$ plane: (η_c, φ_c)
- define distance of track from the seed:

$$d_{ij} = \sqrt{(\eta_i - \eta_c)^2 + (\varphi_i - \varphi_c)^2}$$

track i belongs to the cone if $d_{ij} < R$ (a predefined value, usually 0.7)

- IR unsafe
- problem with overlapping cones

➔abandoned

Theory prefers iterative jet algorithms

- define
  - a distance between two momenta (of partons or tracks): d_{ij}
  - a rule to combine two momenta, $i + j \rightarrow (ij)$
- select a value for resolution d_{cut} and consider all pairs of momenta:

  IF min $d_{ij} < d_{\mathrm{cut}}$ THEN

  $i + j \rightarrow (ij)$; replace i and j with (ij); start again

  ELSE

  remaining momenta are the jet momenta

  STOP

Becomes very expensive computationally for many patricles

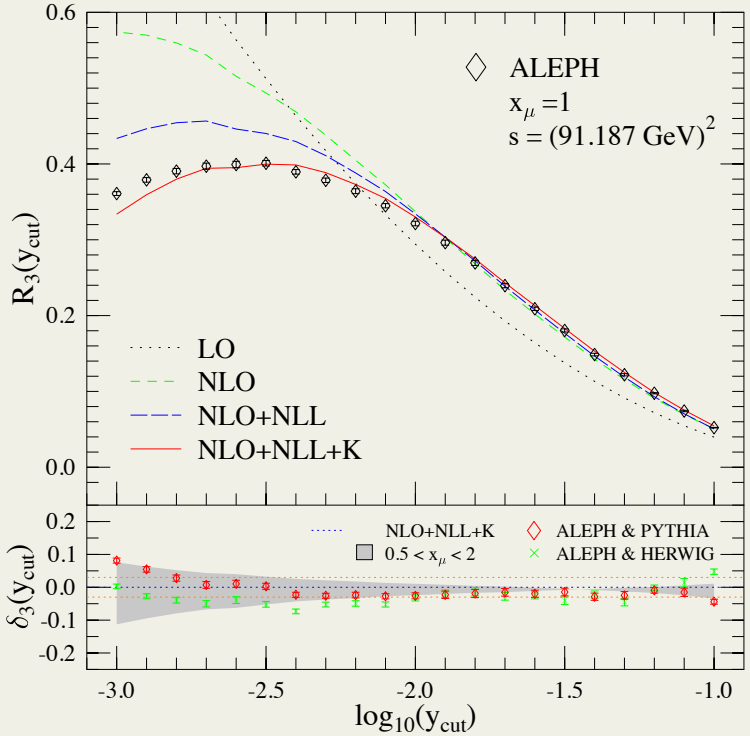
Preferred jet algorithm at e^+e^- colliders

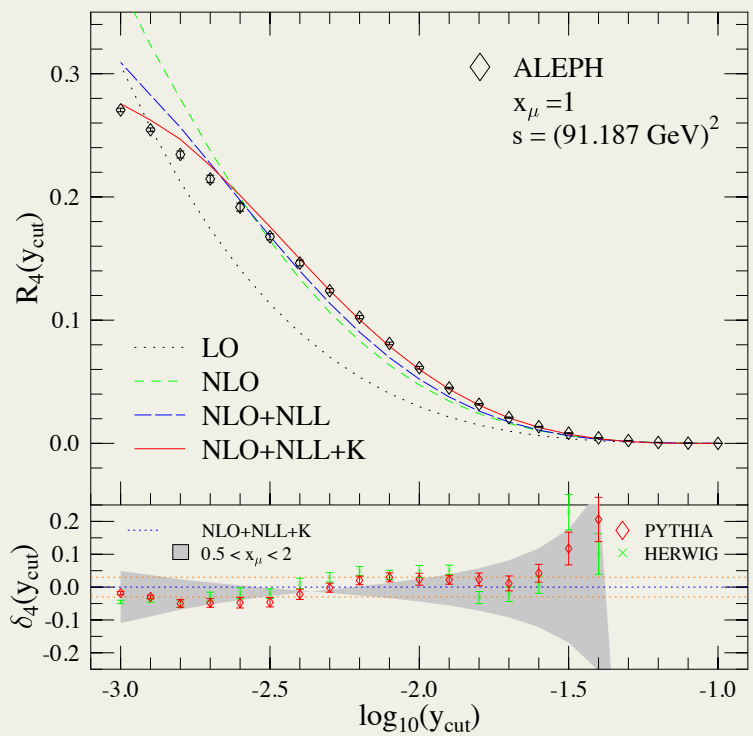
- theory won: Durham (or k_T) algorithm
  - d_{ij}: $d_{ij} = 2\frac{\min\left(E_i^2, E_j^2\right)}{s}\underbrace{(1-\cos\theta_{ij})}_{= R_{ij}}$
  - $i + j \to (ij)$: $p_{ij}^\mu = p_i^\mu + p_j^\mu$
  - resolution paramter $0 < y_{\rm cut} < 1$
- log-enhanced terms ($0.118 \ln^2(10^2) = 2.5$)

$$\alpha_{\rm s}^n \log^m(1/y_{\rm cut})\,, \quad m = 2n\,,\ 2n-1$$

can be resummed to all orders

3- and 4-jet rates at LEP





Predictions include hadronization corrections

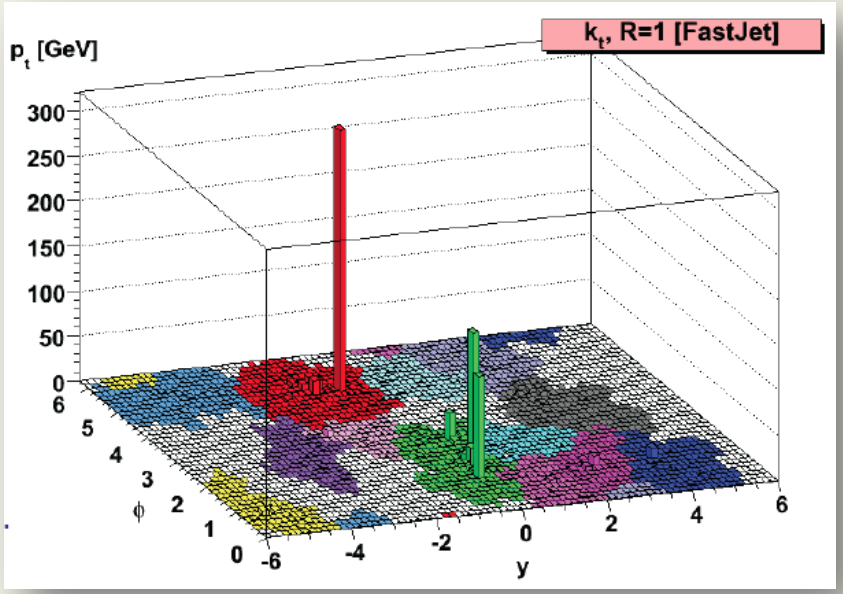
k_T algorithms at hadron colliders

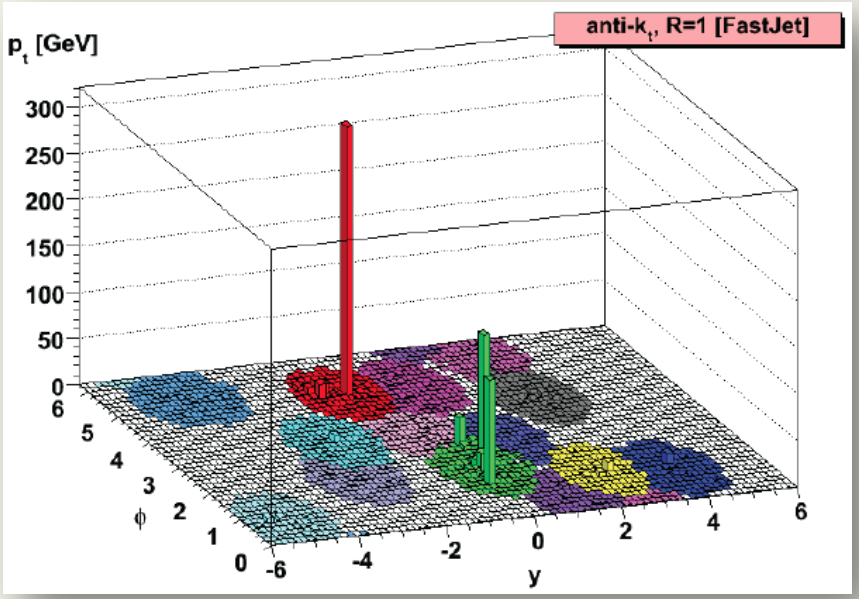
- $$d_{ij} = \min(p_{\perp,i}^{2n}, p_{\perp,j}^{2n}) \frac{R_{ij}^2}{R^2}\,, \quad n = -1, 0, 1$$
- need distance from beam too: $d_{iB} = p_{\perp,i}^{2n}$

  IF smallest is d_{ij} THEN $i + j \to (ij)$

  ELSE d_{iB} is moved to jets
- unlike the cone algorithms: IR safe
- was very expensive computationally – improved algorithm made it accessible (and standard): FastJet http://fastjet.fr/

k_T algorithms at hadron colliders





n = 1: k_T n = -1: anti-k_T

☛ jets (n = 0: Cambridge/Aachen)

with weird shapes more cone-like

NLL resummation does not depend on n

k_T algorithm: good features

- ☛ R plays a similar role as R in cone algorithms:
  - ☞ particles within angular separation R tend to combine
  - ☞ particles separated by more than R from all other particles become jets
- ☛ assigns a clustering sequence to particles within jets ➔ can look at jet substructure (can also be done with C/A, but not with anti-k_T)

k_T algorithm: not so good features

- arbitrary soft particles can form jets
- soft particles tend to cluster first (➔irregular shapes)
- non-linear dependence on soft particles
- energy calibration difficult
- underlying event correction depends on area
- acceptance corrections are more difficult to compute

anti-k_T algorithm

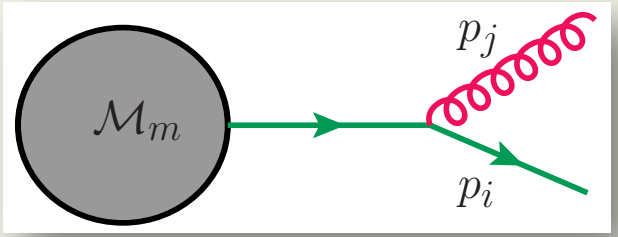
- ☛ particles close in angle cluster first ➔ regular, cone-like shapes
- ☛ has become the standard at LHC, but does not provide information on jet substructure

…and now a deep breath

Factorization of $|\mathcal{M}_m|^2$ in the soft limit

☛ gluon with momentum $p_s^\mu = \lambda\, q^\mu$, with

$\lambda \to 0$, q^μ fixed



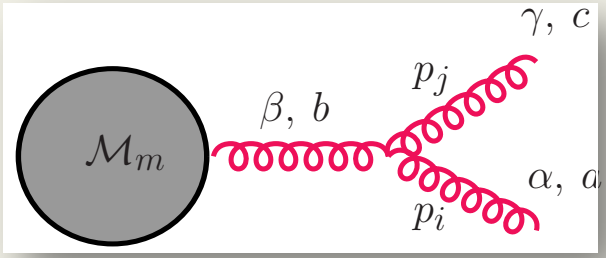
$$\propto T_i^j\, g_s\, \bar{u}(p_i, s_i)\gamma^\mu \frac{\not{p}_i + \not{p}_j}{s_{ij}}$$

$$\gamma^\mu \not{p}_i = -\not{p}_i \gamma^\mu + 2p_i^\mu$$

$$\bar{u}(p_i)\not{p}_i = 0$$

$$\stackrel{p_j \to 0}{\simeq} T_i^j\, g_s\, \frac{p_i^\mu}{p_i \cdot p_j}\, \bar{u}(p_i, s_i)$$

try to prove:



$$\stackrel{p_j \to 0}{\simeq} T_i^c\, g_s\, \frac{p_i^\mu}{p_i \cdot p_j}\, \epsilon_\beta(p_i, n)$$

Factorization of $|\mathcal{M}_m|^2$ in the soft limit

- $\boldsymbol{S}_j$ is an operator taking the soft limit, keeping the leading $1/\lambda$ singular term:

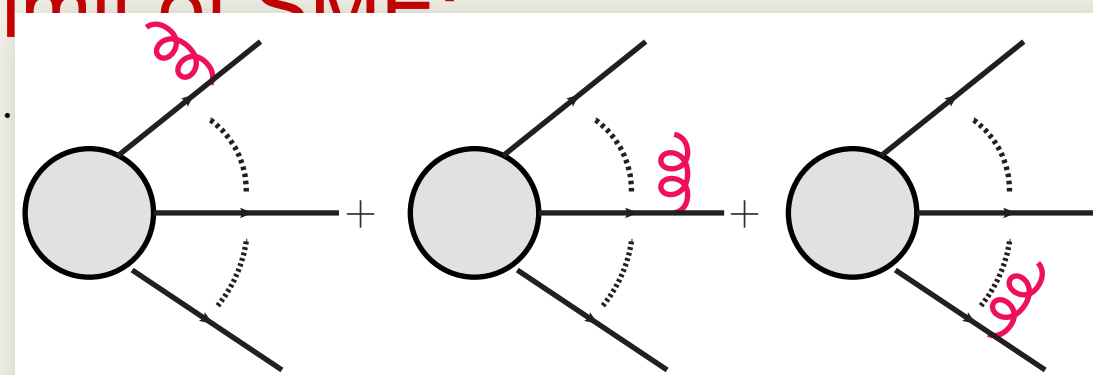
$$\boldsymbol{S}_j \langle c|\mathcal{M}_{m+1}(p_j,\ldots)\rangle = g_s \epsilon^\mu(p_j)\, \boldsymbol{J}_\mu(p_j)|\mathcal{M}_m(\ldots)\rangle$$

with $\boldsymbol{J}_\mu$ soft gluon current: $\boldsymbol{J}_\mu(p_j) = \sum_{k=1}^{m} \boldsymbol{T}_k^c \frac{p_{k\,\mu}}{p_k \cdot p_j}$

- $\boldsymbol{J}_\mu$ conserved: $p_j^\mu\, \boldsymbol{J}_\mu(s)|\mathcal{M}_m\rangle = \sum_{k=1}^{m} \boldsymbol{T}_k^c |\mathcal{M}_m\rangle = 0$

- soft limit of SME:

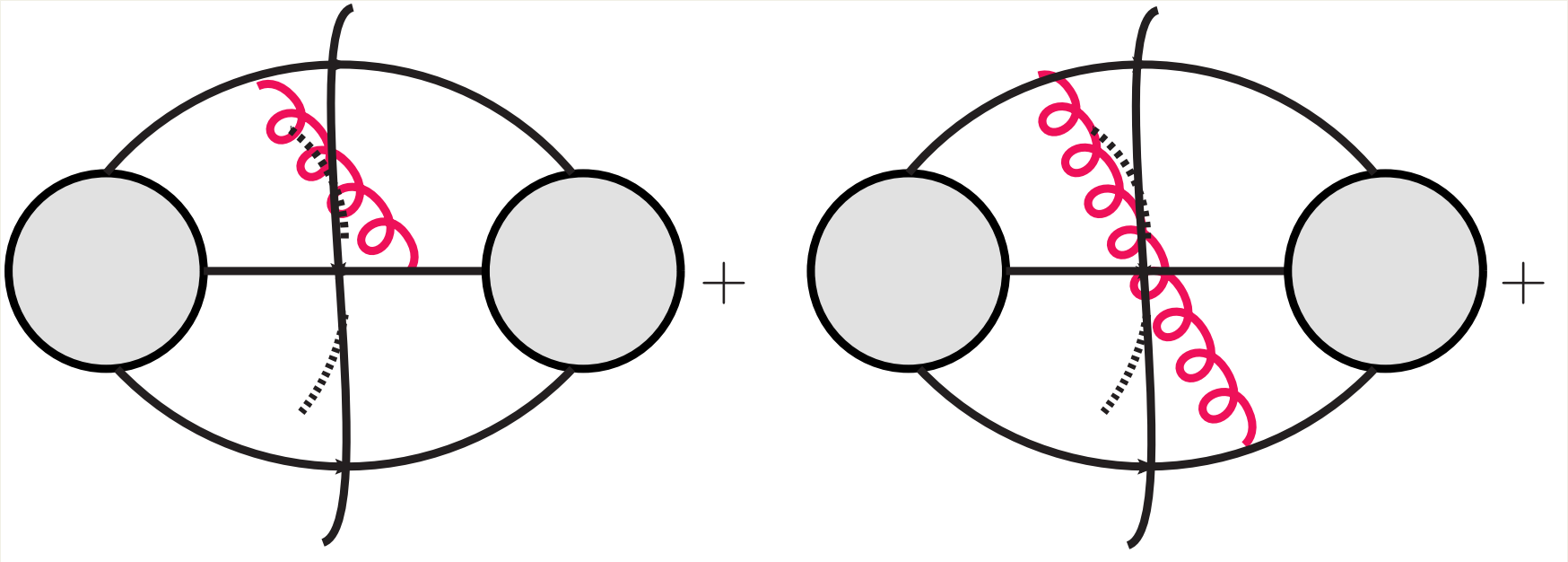
$\boldsymbol{S}_j|\mathcal{M}_{m+1}(p_j,\ldots$



$_m|\boldsymbol{T}_i \cdot \boldsymbol{T}_k|\mathcal{M}_m\rangle$

gauge terms.

Factorization of $|\mathcal{M}_m|^2$ in the soft limit



Insert colour connection with eikonal current squeared

Factorization of $|\mathcal{M}_m|^2$ in the collinear limit

☛ defined by Sudakov parametrization:

$$p_i^\mu = z_i p^\mu + k_{i\perp}^\mu - \frac{k_{i\perp}^2}{z_i}\frac{n^\mu}{2\,p\cdot n}$$

$$p_j^\mu = z_j p^\mu + k_{j\perp}^\mu - \frac{k_{j\perp}^2}{z_j}\frac{n^\mu}{2\,p\cdot n}$$

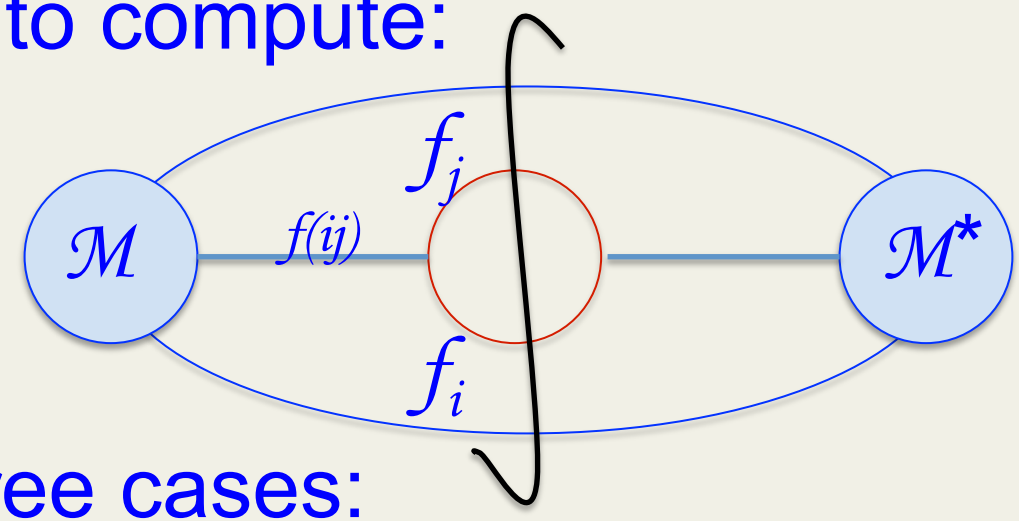
with $k_{i\perp}^\mu + k_{j\perp}^\mu = 0, \qquad z_i + z_j = 1$ and

$$p_i^2 = p_j^2 = p^2 = 0 \qquad n^2 = 0 \qquad k_{i\perp}\cdot p = k_{j\perp}\cdot n = 0$$

Theorem: in a physical gauge, leading collinear singularities ($k_{(i\,\&\,j)\perp} \to 0$) are due to the collinear splitting of an external parton

Factorization of $|\mathcal{M}_m|^2$ in the collinear limit

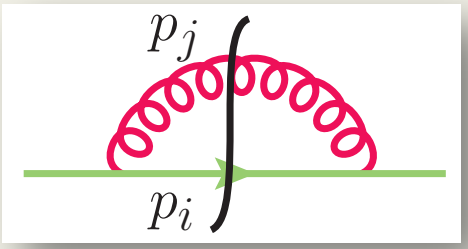
☛ need to compute:



for three cases:

$f_{(ij)}$	$\rightarrow$	$f_i + f_j$
q	$\rightarrow$	$q + q$
g	$\rightarrow$	$q + \bar{q}$
g	$\rightarrow$	$g + g$

Quark-gluon splitting



$$= C_F g_s^2 \mu^{2\epsilon} \frac{\not{p}_i + \not{p}_j}{s_{ij}} \gamma^\mu \not{p}_i \gamma^\nu \, d_{\mu\nu}(p_j, n) \frac{\not{p}_i + \not{p}_j}{s_{ij}}$$

The computation is lengthy but doable on paper (try it). In the collinear limit

$$= \frac{1}{s_{ij}} C_F \, 8\pi\alpha_s \mu^{2\epsilon} \underbrace{\left[2\frac{z_i}{z_j} + (1-\epsilon) z_j \right]}_{\frac{1+z_i^2}{1-z_i} - \epsilon(1-z_i)} \not{p}$$

and the collinear limit of SME:

$$\boldsymbol{C}_{ij} \left| \mathcal{M}_{m+1}^{(0)} \right|^2 = 8\pi\alpha_s \mu^{2\epsilon} \frac{1}{s_{ij}} \left\langle \mathcal{M}_m^{(0)}(p, \ldots) \right| \hat{P}_{qg}^{(0)}(z_i, z_j, k_\perp; \epsilon) \left| \mathcal{M}_m^{(0)}(p, \ldots) \right\rangle$$

Splitting kernels

☛ $q \to qg$: $$\langle s|\hat{P}^{(0)}_{qg}|s'\rangle = C_F\left[2\frac{z_i}{z_j} + (1-\epsilon)z_j\right]\delta_{ss'}$$
$$= C_F\left[\frac{2}{1-z_i} - 1 - z_i - \epsilon(1-z_i)\right]\delta_{ss'}$$

☛ $g \to q\bar{q}$:

$$\left\langle \mu \left| \hat{P}^{(0)}_{q\bar{q}}(z_i, z_j, k_\perp; \epsilon) \right| \nu \right\rangle = T_R\left[-g^{\mu\nu} + 4z_i z_j \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}\right]$$

☛ $g \to gg$:

$$\left\langle \mu \left| \hat{P}^{(0)}_{gg}(z_i, z_j, k_\perp; \epsilon) \right| \nu \right\rangle =$$
$$2C_A\left[-g^{\mu\nu}\left(\frac{z_i}{z_j} + \frac{z_j}{z_i}\right) - 2(1-\epsilon)z_i z_j \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}\right]$$

Splitting kernels

- Note my different notation from the literature:
  - usually $P_{ij}(z)$ denotes the splitting

$$f_i(p) \to f_j(z\,p)+f_k((1-z)\,p)$$

  - for me $P_{ij}(z)$ denotes the splitting

$$f_k(\mathrm{p}) \to f_i(z_i\,p)+f_j(z_j\,p)$$

  where the parton type f_k is determined uniquely by the flavour summation rules:
  - $q + g = q$
  - $q + \bar{q} = g$
  - $g + g = g$
- The reason is visible in $1 \to 3$ splittings only

Towards general subtractions
follow 4 sketchy slides, understand structure

☛ compare

$$\langle s|\hat{P}^{(0)}_{qg}|s'\rangle = C_F\left[\frac{2}{1-z_i} - 1 - z_i - \epsilon(1-z_i)\right]\delta_{ss'}$$

to

$$V_{ij,k}\left(p_i,p_j,p_k\right) = \frac{\alpha_s}{2\pi}\frac{1}{y_{ij}}\left(\frac{2}{1-z_i\left(1-y_{ij}\right)} - 1 - z_i - \varepsilon\left(1-z_i\right)\right)$$

➔ in the collinear limit

$$C_F V_{ij,k} \to \frac{\alpha_s}{2\pi}\frac{1}{y_{ij}}\hat{P}^{(0)}_{ij}(z_i, z_j, k_\perp; \epsilon)$$

in the soft limit (recall $z_i = \frac{s_{ik}}{s_{ik}+s_{kj}}$)

$$V_{ij,k} \to \frac{\alpha_s}{2\pi}\frac{2s_{ik}}{s_{ij}+s_{kj}}$$

Subtraction terms (not unique)

☛ define

$$\mathcal{D}_{ij,k}(p_1,\ldots,p_{m+1}) = -\frac{1}{s_{ij}}\langle\widetilde{\mathcal{M}}_m^{(0)}\left|\frac{\boldsymbol{T}_k\cdot\boldsymbol{T}_{ij}}{\boldsymbol{T}_{ij}^2}V_{ij,k}\right|\widetilde{\mathcal{M}}_m^{(0)}\rangle$$

then prove:

$$\left|\mathcal{M}_{m+1}^{(0)}\right|^2 J_m^{(m+1)} \overset{s_{ij}\to 0}{\simeq} \sum_{k\neq i,j}\mathcal{D}_{ij,k}(p_1,\ldots,p_{m+1})\,J_m^{(m)}(\tilde{p}_1,\ldots,\tilde{p}_{m+1})$$

tilde means phase space mapping that preserves momentum conservation with on-shell new momenta (needed for gauge inv.)

$$\tilde{p}_{ir}^{\mu} = p_i^{\mu} + p_j^{\mu} - \frac{y}{1-y}p_k^{\mu} \quad \tilde{p}_k^{\mu} = \frac{1}{1-y}\,p_k^{\mu}$$

Integrated subtraction terms

☛ phase space factorizes exactly:

$$\mathrm{d}\phi_3(p_i, p_j, p_k; Q) = \mathrm{d}\phi_2(\tilde{p}_{ij}, \tilde{p}_k; Q)[\mathrm{d}p_j(\tilde{p}_{ij}, \tilde{p}_k)]$$

where (this is tricky to derive):

$$\int [\mathrm{d}p_j] = \frac{(2\tilde{p}_{ij}\cdot\tilde{p}_k)^{1-\epsilon}}{16\pi^2}\int\frac{\mathrm{d}^{d-3}\Omega}{(2\pi)^{1-2\epsilon}}$$

$$\times\int_0^1 \mathrm{d}z_j[z_j(1-z_j)]^{-\epsilon}\int_0^1 \mathrm{d}y(1-y)^{1-2\epsilon}y^{-\epsilon}$$

so we can integrate the splitting matrices (without reference to the jet function!)

$$\mathcal{V}_{ij,k} = \int[\mathrm{d}r]\frac{1}{s_{ij}}\langle V_{ij,k}\rangle$$

Insertion operator

☛ the rest is (cumbersome) algebra, giving

$$\int_{m+1} \mathrm{d}\sigma^A = \int_m \mathrm{d}\sigma^B \otimes \boldsymbol{I}_m(\epsilon)$$

$$|\mathcal{M}_m|^2 \otimes \boldsymbol{I}_m(\epsilon) = \langle \mathcal{M}_m | \boldsymbol{I}_m(\epsilon) | \mathcal{M}_m \rangle$$

where $\boldsymbol{I}_m$ is a matrix in colour space

$$\boldsymbol{I}_m = -\frac{\alpha_\mathrm{s}}{2\pi} S_\epsilon \sum_{i=1}^{m} \frac{1}{T_i^2} \mathcal{V}_i(\epsilon) \sum_{k \neq i} \boldsymbol{T}_i \cdot \boldsymbol{T}_k \left(\frac{\mu^2}{s_{ik}} \right)^\epsilon$$

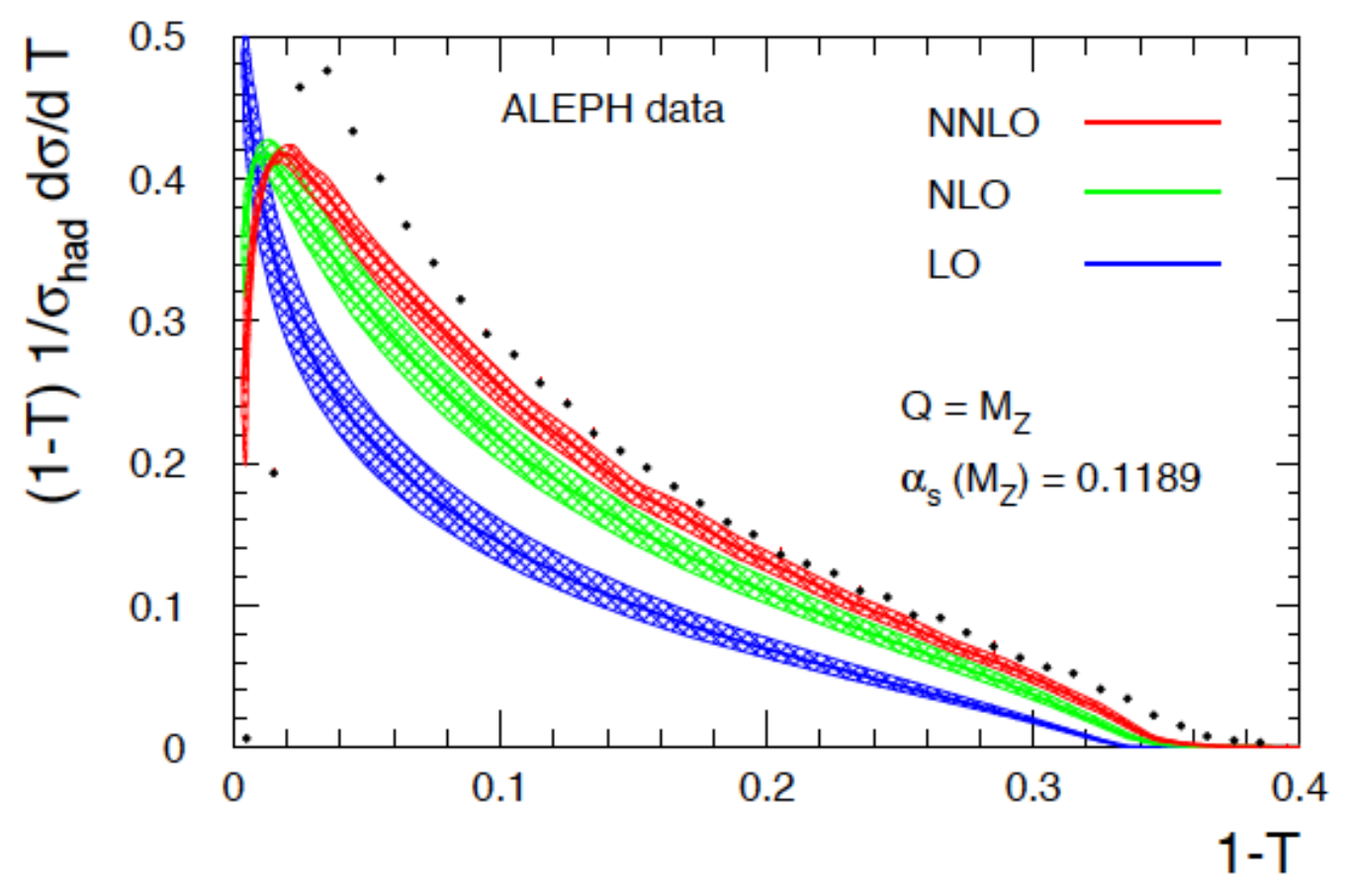
$$\mathcal{V}_i(\epsilon) = \boldsymbol{T}_i^2 \left(\frac{1}{\epsilon^2} - \frac{\pi^2}{3} \right) + \frac{1}{\epsilon} \gamma_i + \gamma_i + K_i$$

Other uses of

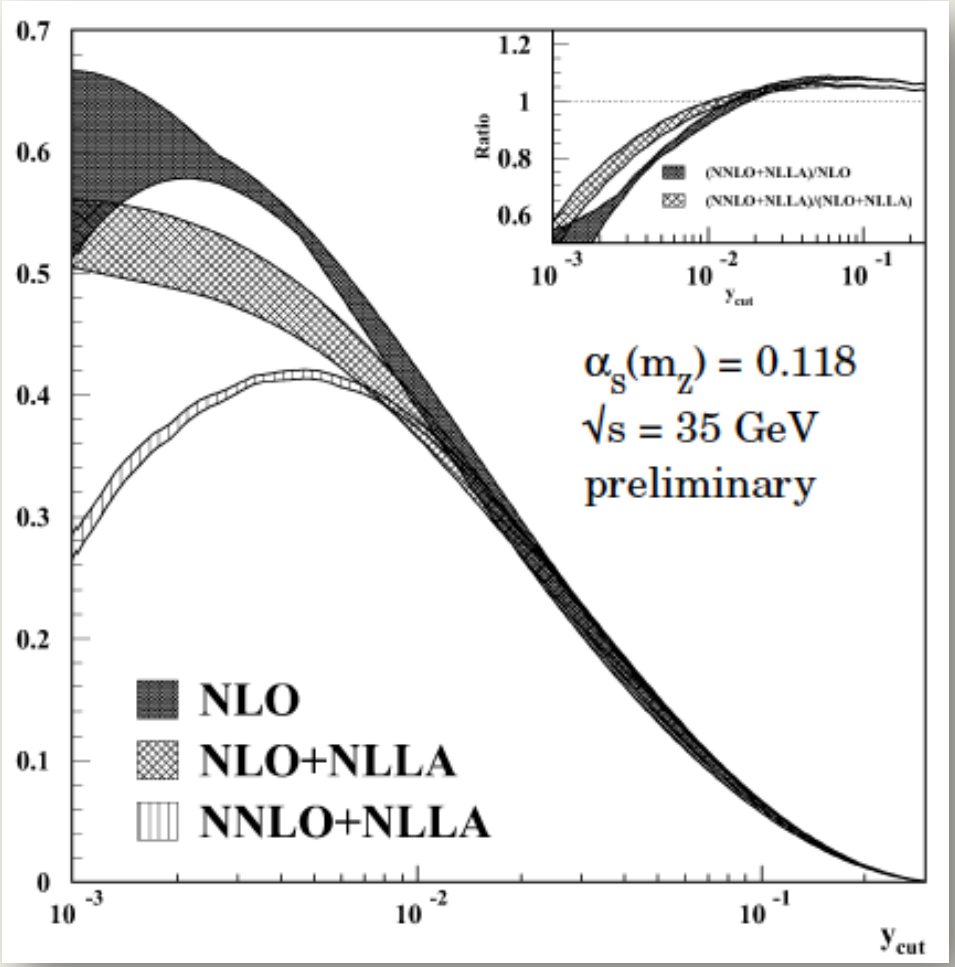
- factorization in the soft/collinear limits
  - in checking numerical implementations of SME
  - can also be derived for loop SME
  - in all order resummation of log-enhanced terms
  - in devising a parton shower (PS) algorithm splitting kernels:
  - evolution equations for PDF's (see next lecture)

Thrust at NLO and NNLO

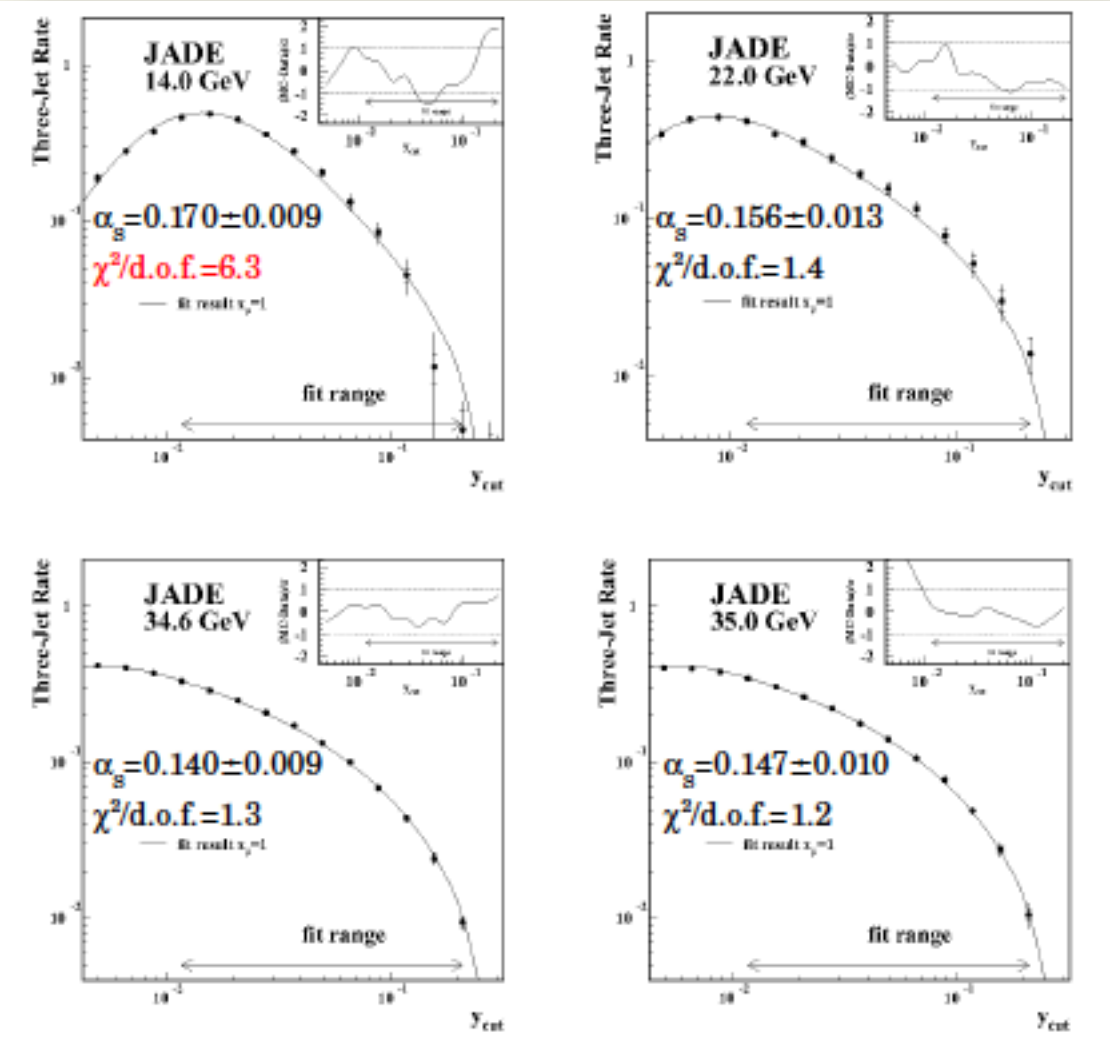
Effect of hadronization is significant



Three-jet rate in PT



Jade data for three-jet rate vs PT



Summary

- ☛ Identified sources of IR singularities
  - ☞ understood how to cancel those in the final state
  - ☞ Showed that pQCD can provide approx-imations to distributions of IR safe observables (event shapes, jet xsections)

Let us try to collide with protons