LECTURE 2.

THE MSSM & VARIATIONS
Lagrangian for \( N=1 \) gauge theories

\[
\phi = \psi(y) + \sqrt{2}\theta \psi(y) + \theta^2 F(y)
\]

F-term auxiliary field

SM gauge fields \( A_\mu \) \( \rightarrow \) \( W_\alpha \) vector SF

\[
W_\alpha = -c \lambda (y) + \theta \sigma^\mu \left[ \delta_{\alpha}^\beta D(y) - \frac{i}{2} \left( \sigma^\mu \bar{\psi} \right)^\beta \bar{F}_{\mu\nu} \right] + \theta^2 \sigma^\mu \partial_\mu \bar{\lambda} \lambda(y)
\]

D-term auxiliary field

Gaugino

Lagrangian in superspace:

\[
\int d^4\theta \, \phi_i^+ \epsilon^{i\nu} \phi_i + \frac{1}{4g^2} \int d^2\theta \, W_\alpha W^{\alpha} + \text{h.c.}
\]

\[
+ \int d^2\theta \, W(\phi) \, \text{h.c.}
\]

Gauge invariant kinetic terms for matter fields

Superpotential

Gauge + gaugino kinetic terms
The Matter Content of the MSSM

same gauge group as SM, but need
2 Higgs + chiral superfields (otherwise
SU(2)^2 x U(1)_Y), SU(2)_L anomaly). Write in
terms of LH chiral superfields only
to maintain holomorphy of SUSY...

<table>
<thead>
<tr>
<th></th>
<th>SU(3)</th>
<th>SU(2) × U(1)_Y</th>
<th>B</th>
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<td>EE</td>
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<td>$H_d$</td>
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Possible superpotential terms:

\[ W_{\text{good}} = \lambda^{ij} Q^i H_u \bar{U}^j + \lambda^{ij} Q^i H_d \bar{D}^j + \lambda e^{ij} L^i \bar{H}_d^j \]

SUSY extensions of SM

Yukawa couplings

Supersymmetric

Higgs mass ($\mu$-term)

Need these: - give masses to SM fermions
give mass to Higgsinos (eliminate axion)
\[
W^{\text{bad}} = a_1 \delta^{\text{ijk}} Q_i L_j \bar{D}_k + a_2 \delta^{\text{ijk}} L_i L_j \bar{E}_k \\
+ a_3 L_i H_u + a_4 \delta^{\text{ijk}} \bar{D}_i \bar{D}_j \bar{U}_k \]

\[\Delta L = 1\]  
\[\Delta B = 1\]

would violate baryon & lepton #

renormalizable interaction

- VERY different from SM: in SM all terms allowed by gauge invariance also conserve \(B, L\). \(B, L\) accidental global symmetry. In SM \(B, L\) violation

\[
\Delta \frac{1}{M} \quad \text{where } M \text{ can be a very high scale.}
\]

- In HSSM: new fields (superpartners) that also carry \(B, L\), more renormalizable terms. Need to forbid \(W^{\text{bad}}\)!

could give proton decay

\[\overline{u} \rightarrow \frac{2}{3} s \rightarrow \frac{2}{3} b \rightarrow \nu \]
\[d \rightarrow \nu \]
\[\overline{u} \rightarrow e^+ \nu^0 \quad \text{or} \quad \overline{\nu} \pi^+ \quad \text{or} \quad \overline{\nu} K^+ \]

\[
\begin{cases}
\text{couplings completely unsuppressed!} \\
\end{cases}
\]
Forbid $W^{\text{bad}}$ by $\text{waller parity}$, $\mathbb{Z}_2$ symmetry.

- quark, lepton $\times$ $F$
- Higgs
- gauge, vector $\times$ $F$

$W^{\text{good}}$:
- all have $P_M = +1$

$W^{\text{bad}}$:
- all have $P_M = -1$

Can check:

$$P_M = (-1)^3 (B-L)$$

Variation:

$$R\text{-parity: } P_R = (-1)^{3(B-L)+2S}$$

If waller parity conserved, $R\text{-parity}$ also conserved. $(-1)^{2S} \rightarrow \text{always need even } #\text{ of fermions by Lorente.}$

$R\text{-parity:}$

\[
\begin{align*}
\text{(SM fields)} & \rightarrow +1 & \text{like a } t\text{-parity,} \\
\text{(superpartners)} & \rightarrow -1
\end{align*}
\]

Forbids all tree-level EW corrections, chance SUSY is right...
Important consequences of R-parity

(usually quoted as consequences of SUSY, but it really just follows from R-parity)

- Lightest R-parity odd particle stable
  \[ \equiv \text{LSP} \quad \text{lightest superpartner} \]
  if LSP electrically neutral, color singlet: candidate for WIMP-like DM

- Each sparticle other than LSP will decay, at the end will contain odd # (usually one) LSP’s

- Collider experiments: initial state \( P_{e^+} \rightarrow \) only even # of superpartners can be produced, must be pair produced.
  At the end decay to LSP’s \( \rightarrow \) missing energy signal in colliders.

Will postulate that MSSM has exact R-parity conservation (somewhat ad-hoc assumption)
**SUPERSYMMETRY BREAKING**

Susy unbroken if $Q_a|0\rangle = 0$

Then using Susy algebra $\{Q_a, \bar{Q}_a\} = 2\delta^a_\mu P_\mu$

$P^\mu = \frac{1}{4} (\bar{\psi} \gamma^\mu \psi)$

$H = P^0 = \frac{1}{4} (Q_1 \bar{Q}_1 + \bar{Q}_2 Q_2 + \bar{Q}_2 Q_2 + Q_2 \bar{Q}_2)$

Susy unbroken if $\langle 0 | H | 0 \rangle = 0$

Susy broken if $\langle 0 | H | 0 \rangle > 0$

Scalar potential

$V(\phi) = \sum_i |\frac{\partial W}{\partial \phi_i}|^2 + \sum_a \frac{1}{2} g^2 |\sum_i \phi_i^+ T^a \phi_i|^2$

$\sum_i |F_i|^2 + \sum_a \frac{1}{2} g D^a D^a$

Susy breaking if $\langle F_i \rangle \neq 0$ or $\langle D \rangle \neq 0$

F-type breaking

D-type breaking

If Susy breaking $\Rightarrow$ massless fermion Goldstino
For example if $\langle F \rangle \neq 0$, the SUSY
transformation of $\psi$
$\delta \psi_i = 2 \delta \langle F \rangle \rightarrow$ shift symmetry for
fermion $\rightarrow$ fermion in
multiplet where $\langle F \rangle = 0$
massless.

If more than one field:
$\delta \psi_i = 2 \delta \langle F_i \rangle$

$\psi_{\text{Goldstino}} = \sum_i \frac{F_i}{\sqrt{2} \langle F \rangle} \psi_i \rightarrow$ always just
one Goldstino.

How to apply to MSSM?

$\text{SUSY rule for broken SUSY}$

Fermion masses:
$\frac{i}{\sqrt{2}} g \left( T^a \right)_{ij} \left( \psi_i \gamma^\alpha \overline{\psi}_j - \overline{\psi}_i \gamma^\alpha \psi_j \right)$

$-\frac{\partial W}{\partial \psi_i \partial \psi_j} \psi_i \psi_j \text{ th.c.} \leftarrow \text{from superpotential}$

Fermion mass matrix:
$\left( \psi_i \gamma^\alpha \right) \left( \begin{array}{cc} F_{ij} & \sqrt{2} D_{bi} \\ \sqrt{2} D_{ai} & 0 \end{array} \right) \left( \psi_j \right)$

$F_i = \frac{\partial W}{\partial \psi_i}$

$F_i = \frac{\partial W}{\partial \overline{\psi}_i}$

$D^a = g \sum_i \psi_i^* T^a \psi_i$
\[ F_{ij} = \frac{\partial F_i}{\partial \psi_j}, \quad D_a : = \frac{\partial D_a}{\partial \psi_i} = g \psi_i \times T^a \]

\[ m_{ij}^{(1/2)} = \begin{pmatrix} F_{ij} & \sqrt{2} D_b_j \\ D_a : & 0 \end{pmatrix} \]

Scalar mass

\[ m^2_{ij} = \begin{pmatrix} \frac{\partial^2 V}{\partial \psi_i \partial \psi_j} & \frac{\partial^2 V}{\partial \psi_i \partial \psi_j^*} \\ \frac{\partial^2 V}{\partial \psi_i \partial \psi_j^*} & \frac{\partial^2 V}{\partial \psi_j \partial \psi_j^*} \end{pmatrix} \]

\[ F_{ij} F_{kj} + D_a^i D_{aj} + D_a^j D_{ai} \]

\[ F_{ijk} F_{kj} + D_a^i D_{aj} D_a^j \]

GB mass matrix

\[ \sum_i g^2 | A_{\mu}^a T^{a \beta} \phi_{id} |^2 = | A_{\mu}^a D_{ai} |^2 \]

\[ m_{ab}^{(G=1)} = D_a^i D_{bi} + D_a^i D_{bi} \]

Traces:

\[ \text{Tr} \ m_{ij}^{(G=1/2)} (m^+)(j=1/2) = F_{ij} F_{ij}^* + 4 | D_a : |^2 \]

\[ \text{Tr} \ m_{ij}^{(G=0)} = 2 F_{ij} F_{ij}^* + 2 D_a^i D_{ai} + 2 D_a D_a \]
\[ \text{Tr} \left( m^2 (g=1) \right) = 2 D_a : D_a \]

\[ S \text{Tr} H^2 = \text{Tr} \left( (2j+1) \left( -1 \right)^{I_e} M^2 \right) \]

\[ = -2 FF - 8 (D_a :)^2 + 2 FF + 2 D_a : D_a : + 2 D_a : D_a : + 3 \cdot 2 \]

\[ = 2 D_a (D_a :) \]

\[ \langle D_a \rangle \neq 0 \text{ only for } U(1)'s \]

\[ = 2 D D_c \]

\[ D_c = \Sigma q_i \text{ sum of all } U(1) \text{ charges} \]

\[ S \text{Tr} H^2 = 2 D_a \Sigma q_i \cdot q_i \cdot \alpha \]

\[ \alpha: U(1) \text{ factors} \]

\[ \Sigma q_i \cdot q_i = 0 \text{ due to anomaly cancellation!} \]

\[ \rightarrow \boxed{S \text{Tr} H^2 = 0} \]

This is a very bad relation for the MSSM. Tells that some superpartners lighter than SM masses.
Application to the MSSM (Dino, Paulos & Georgi)

Assume sum rule applies. Consequence:
One squark lighter than \( w_\mu \) or \( w_d \) (experimentally impossible)

Scalar mass matrix:

\[
M^2_{ij} = \begin{bmatrix}
F_{ik} F_{kj} \left( + \frac{1}{2} \right) L_i L_j + \frac{1}{2} D_i^a D_j^a & F_{ijk} F_k \left( + \frac{1}{2} \right) L_i L_j D_j^a \\
F_k F_{ijk} \left( + \frac{1}{2} \right) L_i L_j D_j^a & F_{jk} F_k \left( + \frac{1}{2} \right) L_i L_j D_j^a + \frac{1}{2} D_i^a D_j^a L_j^a
\end{bmatrix}
\]

Specify 5 squark mass matrix. Squarks should not get VEV (color not broken) \( D_\ell = 0 \)
Quarks only get mass from superpotential, since squark VEV =
\( D_\text{color} = 0 \)
\( D_{1,2} = 0 \) only \( D_3, D_4 \neq 0 \)

\[
M^2_{243} = \begin{bmatrix}
m_{243} m_{24}^+ \left( + \frac{1}{2} g_{D_3} + \frac{1}{2} g_{D_4} \right) & \Delta \\
\Delta^+ & m_{243} m_{24}^+ - \frac{2}{3} g_{D_4}^2
\end{bmatrix}
\]

\[
M^2_{143} = \begin{bmatrix}
m_{143} m_{14}^+ \left( - \frac{1}{2} g_{D_3} + \frac{1}{2} g_{D_4} \right) & \Delta^+ \\
\Delta^+ & m_{143} m_{14}^+ + \frac{1}{3} g_{D_4}^2
\end{bmatrix}
\]

Sum of all D-terms = 0 \( \Delta \) at least one \( \leq 0 \)
Assume for example \( \frac{1}{2} g_{D_3} + \frac{1}{6} g_{D_4} \leq 0 \)
If $\beta$ eigenvector of $m_{1/3}$ ($\beta^T \eta_0$) $N_{3/2}^2 \langle \beta \rangle \leq m^2$

There must be a squark mass less than $m_c$ or $m_d$ → not possible.

**SUM RULES** must be broken!

Need to **update** assumption leading to sum rule

- renormalizable
- free - level

Need to **assume** that no renormalizable interaction between SUSY sector & SM

For example:
- only transmitted through gravity
  - structure of SUGRA Lagrangian (non-renormalizable) allows more terms
  - a "messenger sector" mediates between SM fields & SUSY sector

If we don't want to specify, try to parametrize what kind of terms will we get from non-renormalizable interactions that violate SUM rule?
Assume SUSY field $S_i$ has only non-renormalizable couplings to visible sector (either through gravity, quant um loops,...)

What operators could be generated?

$$\langle S \rangle = \ldots + \theta^2 F$$

Possible terms:

$$- \int \frac{\phi^2}{M^2} \frac{S^2}{d^4 \theta} \rightarrow \frac{\phi^2}{M^2} \left( \frac{F}{M} \right)^2$$

Scale at which new physics is integrated out

$M$ for gravity

$M_{\text{mess}}$ for GM

$m^2 \sim \left( \frac{F}{M} \right)^2$

(of course could also add terms like

$$\int \frac{\phi^2}{M^2} \frac{S^2}{d^4 \theta} \rightarrow \int \frac{\phi^2}{M^2} \left( \frac{F}{M} \right)^2$$

get much more suppressed terms...)

$$- \int \phi^2 \phi^2 \frac{d^4 \theta}{M^2} \rightarrow \frac{\phi^2}{M^2} \left( \frac{F}{M} \right)^2$$

b-term, natural size is $F$

w/o symmetry

$$- \int \frac{S}{M} \phi^3 \frac{d^2 \theta}{M} \rightarrow \frac{\phi^2}{M} \left( \frac{F}{M} \right)^2 \rightarrow \frac{\phi^2}{M} \left( \frac{F}{M} \right)^2$$

$A \sim M$, same order as scalar mass
\[ - \int W \cdot W^d \frac{S}{M} \, d^2 \theta \rightarrow \frac{F}{M} \text{ gaugino mass} \]
\[ m_\alpha \sim \frac{F}{\kappa} \sim m_\alpha \sim A \]

Find:
- scalar mass
- gaugino mass
- scalar holomorphic cubic (A) & quadratic (b) terms

Note:
\[ \text{STR} \, M^2 = 2 \frac{\Sigma}{2} m_\alpha^2 - 2 \frac{\Sigma}{2} m_\alpha^2 \]
\[ \text{no reason } \Sigma \text{ vanish!} \]

This is the rationale for

SOFT breaking terms for the MSSM.

So full HSSM Lagrangian

\[ L = L_{\text{susy}} + L_{\text{soft}} \]

\[ L_{\text{soft}} = - \frac{1}{2} (N_3 \tilde{g} \tilde{g}^i + N_2 \tilde{W} \tilde{W} + H_1 \tilde{B} \tilde{B}) + \text{h.c.} \]

\[ -(a_u \tilde{Q}_L \tilde{H}_u \tilde{u} + a_d \tilde{Q}_d \tilde{H}_d \tilde{d} + a_e \tilde{L} \tilde{H}_d \tilde{e}) + \text{h.c.} \]

\[ - \frac{Q^+}{2} m_a^2 \tilde{Q} - \tilde{\nu}^+ m_e^2 \tilde{\nu} - \tilde{\nu}^+ m_u^2 \tilde{u} \]

\[ - d + m_d^2 \tilde{d} - \tilde{e} + m_e^2 \tilde{e} - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d \]

\[ -(b \tilde{H}_u \tilde{H}_d + \text{h.c.}) \]
andle : $3 \times 3$ matrices in flavor space, 
1-1 correspondence to Yukawa matrices

$m^2_{\text{andle}} : 3 \times 3$ matrices in flavor space

We assume:

$H_{12,13} \sim m_{\text{soft}}$

$m_{\text{andle}, H_u H_d, b} \sim m_{\text{soft}}^2$

$m_{\text{soft}} \sim \text{few x 100 GeV - TeV}$

A lot of new parameters: 105 new masses, phases, mixing angles on top of SM.

BUT: most of it ALREADY excluded from flavor & CP violating processes!

In SM: no tree-level FCNC's.

Loop level suppressed by GIM mechanism. Similarly, lepton flavor violation strongly suppressed.

Example: FCNC in K-$\bar{K}$ mixing (one of the best tested processes).

$K = d \bar{s}$

$\bar{K} = \bar{d} s$

in SM

CKM unitarity implies additional suppression
In HSSM:

\[ \text{Diagram with fermion and Yukawa interactions} \]

Additional strongly coupled contribution, No GIM suppression

\[ \sim \int \frac{d^4 p}{p_\mu} \sim \frac{1}{m_{\text{SOY}}} \]

\[ M_{\text{SOY}} \propto d_s \left( \frac{\Delta m_{d}\Delta m_{s}}{m_{\text{SOY}}} \right)^2 \frac{1}{m_{\text{SOY}}^2} \]

Compare to exp'1 bound

\[ \frac{\Delta m_d^2}{m_{\text{SOY}}^2} \lesssim 4 \times 10^{-3} \left( \frac{m_{\text{SOY}}}{500 \text{ GeV}} \right) \]

Off-diagonal terms need to be strongly suppressed...

Similar constraints from mu \to e\gamma (CP violating phases...)

Organizing "principle": Soft-breaking universality
1.) Soft breaking masses are universal (2-11)
for all types of particles

\[ m_u^2 = \Delta m_u^2 \]
\[ m_d^2 = \Delta m_d^2 \]

2.) If A-terms not flavor universal, after Higgs VEV will induce similar mixings

\[ A ( Q H_u + Q H_d + L E H_d) \]

\[ \text{assume } A \text{ itself proportional to Yukawa matrix! Whatever rotation you do on quarks, can also do on squarks } \rightarrow \text{ will be diagonal in same basis!} \]

\[ A_{ij} Q_i \bar{U}_j H_u \rightarrow A_{uv} X_{ij} Q_i \bar{U}_j H_u \]

3.) To avoid CP violation, assume all non-trivial phases beyond SM CKM vanishes

Ultimately want to explain Higgs, for example gauge mediation.
Need Higgs potential (assume squarks, sleptons don’t get VEVs)

Quartic: only from D-terms

\[ V_0 = \frac{1}{2} g^2 \left( |H_u^+ H_d^-|^2 + \frac{1}{8} (g^2+g'^2) (|H_u|^2 - |H_d|^2) \right) \]

Important: higgs quartic \( \propto g^2 \sqrt{2} v \rightarrow \) Higgs mass related to \( M_Z \).

Full Higgs potential:

\[ V_H = (\mu^2 + m_{H_u}^2) |H_u|^2 + (\mu^2 + m_{H_d}^2) |H_d|^2 \]

\[ - B_{\mu} H_u H_d + h.c. + \frac{1}{2} g^2 |H_u^+ H_d^-|^2 \]

\[ + \frac{1}{8} (g^2+g'^2) (|H_u|^2 - |H_d|^2) \]

\[ H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \quad \text{only neutral comp's can get VEV!} \]

\[ H_d = \begin{pmatrix} H_d^+ \\ H_d^- \end{pmatrix} \]

In terms of \( H_u^0, H_d^0 \):

\[ V_H = (\mu^2 + m_{H_u}^2) |H_u^0|^2 + (\mu^2 + m_{H_d}^2) |H_d^0|^2 \]
\(-B_n (H^0 H^0 \text{ th.c.}) + \frac{1}{8} (g^2 + g'^2) (|H^0|^2 - |H^0|^2)\)

no quartic along

\(H^0 = H^0\)

condition for EWSB: one direction along origin destabilized, but direction with no quartic has positive mass^2.

\[
\begin{vmatrix}
\mu^2 + m_{H^0}^2 & -B_n \\
-B_n & (\mu^2 + m_{H^0}^2)
\end{vmatrix} \leq 0
\]

\(B_n^2 \geq (\mu^2 + m_{H^0}^2) (\mu^2 + m_{H^0}^2)\)

negative \(m^2\)

\[-2\mu^2 + m_{H^0}^2 + m_{H^0}^2 - 2B_n > 0\]

stability

NO solution for \(m_{H^0}^2 = m_{H^0}^2\).

Typically \(m_{H^0}^2 \leq 0\)

\(m_{H^0}^2 > 0\)

\(\langle H^0 \rangle = \frac{\phi_u}{\sqrt{2}} = \frac{\nu \sin \beta}{\sqrt{2}}\)

\(\langle H^0 \rangle = \frac{\phi_d}{\sqrt{2}} = \frac{\nu \cos \beta}{\sqrt{2}}\)

\(\tan \beta = \frac{\nu}{\sqrt{2}}\)
Minimizing potential we find:

\[ \sin 2\beta = \frac{2B_m}{2\mu^2 + m_{H_u}^2 + m_{H_d}^2} \]

\[ \frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} \]

really weird equation! Connects

\[ M_Z \quad m_{H_u}, m_{H_d} \quad \mu \]

\[ z\text{-mess} \quad \text{soft-breaking} \quad \text{SUSY mass} \]

Origin of Little hierarchy!

Evaluate higgs masses, lightest CP-even higgs ~ SM higgs,

\[ m_{h_0} = \frac{1}{2} \left[ M_Z^2 + m_A^2 \pm \sqrt{(M_Z^2 + m_A^2)^2 - 4 m_{A}^2 M_Z^2 \cos^2 2\beta} \right] \]

\[ m_A^2 = \frac{B_m}{\sin \beta \cos \beta} \]

\[ m_{h_0} \leq M_Z |\cos 2\beta| \leq M_Z \]

Tree-level upper bound on \( m_h \).

But already know from LEP \( m_h > 114 \text{ GeV} \).
Tree-level HSSM excluded. Need a large correction to quartic self-coupling. Have effect from top-stop loops!

1) Want tree-level quartic maximized → large tan β, VEV mostly in H₂. Light higgses ~ H_u. So need mostly H_u coupling. 

\[
\text{result: } \lambda(m_h) = \lambda_{\text{susy}} + \frac{2Nc(Y_t)^4}{16\pi^2} \log\left(\frac{m_{\tilde{e}_1} m_{\tilde{e}_2}}{m_e^2}\right)
\]

fixed.

To push up higgs mass → need to increase m_e.^2

\[
\Delta m_{h^0}^2 = \frac{3}{4\pi^2} v^2 Y_t^4 \sin^2 \beta \log\left(\frac{m_{\tilde{e}_1} m_{\tilde{e}_2}}{m_e^2}\right) \approx 130 \text{ GeV}
\]
Little hierarchy of MSSM

- At tree-level \( m_{h_0} \leq H_2 \)
- Need a large \( m_{\tilde{e}} \approx 1-1.4 \) TeV to increase quartic to push \( m_{h_0} > 114 \) GeV
- But then also get corrections to quadratic in \( m_{H^u} \)

\[
\Delta m_{H^u} \sim -\frac{3y_{t\ell}^2}{4H^2} \frac{m_{\tilde{e}}^2}{m_{\tilde{e}}} \log \left( \frac{\Lambda}{m_{\tilde{e}}} \right)
\]

from

\[
\frac{m_{\tilde{t}L}^2}{\Lambda} + \frac{m_{\tilde{t}R}^2}{\Lambda}
\]

The bigger \( m_{\tilde{e}} \), the larger the shift in \( m_{H^u} \). But remember weird equation:

\[
\frac{m_{\tilde{e}}^2}{2} = -\mu^2 + \frac{m_{H^d}^2 - m_{H^u}^2 \tan^2 \beta}{\tan^2 \beta - 1}
\]

\[\Rightarrow m_{H^u} \sim \frac{H_2}{2}\]

but loop correction

\[\Delta m_{H^u} = \frac{3y_{t\ell}^2}{4H^2} \frac{m_{\tilde{e}}^2}{m_{\tilde{e}}} \log \left( \frac{\Lambda}{m_{\tilde{e}}} \right)\]

\[FT \sim \left( \frac{\Delta m_{H^u}^2}{H_2/2} \right) \sim 800 \text{ for } m_{\tilde{e}} = 1.2 \text{ TeV} \]

\[\Lambda = 10^{16}\]

\[\Rightarrow 0.1\% \text{ tuning.}\]
Gauge mediated SUSY

Flavor problem: in SM in limit when
Yukawa \to U(3)^5 flavor symmetry
(3 gen's completely equivalent, 5 types of particles
Qu d L e)
\underline{U(3)^5}

This flavor symmetry broken at some scale \( \Lambda_F \),
below which only imprint is Yukawas.
\( \Lambda_F \) could be very large, so effects of
flavor breaking \( 2 \frac{1}{\Lambda_F} \) could all be
very small!

However, if SUSY mediated by gravity,
SUSY happens at \( M_{pl} > \Lambda_F \), really no
reason for soft breaking terms to not have
\( \Theta(1) \) flavor violation. Even if at tree-level
for some reason they are flavor invariant,
loop effects of flavor breaking sector will be
large.

Would like theory where scale of SUSY
mediation \( \ll \Lambda_F \). Need to lower relevant
mass scale for mediation (\& physics of
mediation itself should be flavor universal!)

Most important example:
**Gauge Mediation**

Idea: - generate mass splittings **obey**ing sum rules for messengers
- only through messengers in loop will MSSM feel SUSY

- Effectively: generate non-renormalizable ops. connecting MSSM & SUSY sector.
- relevant scales: $M$ messenger mass
  - below $M$ integrate out messenger sector, generate soft SUSY masses
- since interactions of mediating SUSY SM gauge interactions \( \rightarrow \) will be flavor universal (if $\Lambda_F > M$)

Minimal gauge mediation

SM gauge singlet $\times$ SUSY sector

\[ \langle F_x \rangle = \lambda \]

\[ \langle F_x \rangle \neq 0 \]
Mesengers: $N_f$ flavors $\Phi_i, \bar{\Phi}_i$

\[ W = \lambda \bar{\Phi} \times \Phi \]

**Mesenger Scalar Mass Matrix:**

\[ \left( \frac{\partial W}{\partial \Phi} \right)^2 = \lambda^2 \langle \chi \rangle^2 |\Phi|^2 + (2 \langle \chi \rangle^2 |\bar{\Phi}|^2 + \lambda \bar{\Phi} \Phi F_x \]

**Masses:**

\[ m^2 = 2^2 M^2 \pm 2 F \]

\[ \frac{2M}{\lambda} \]

**Scalar:** $\frac{2}{3}$ obey sum rule

**SM Gaugino Mass:**

\[ M = \frac{F \cdot M}{M^2} = \frac{g^2}{16 \pi^2} \cdot N \cdot m \] exact

\[ m_a = \frac{F \cdot M}{M^2} \frac{g^2}{16 \pi^2} \cdot N \cdot m \]

\[ m_{\tilde{\chi}} = \frac{M}{N \cdot m} \cdot \frac{F}{M} \]
Scalar mass generated at 2-loop only
need both gauge boson & messenger to run in loop

many diagrams, example:

\[
\begin{align*}
ms_{\text{soft}}^2 & \sim \frac{g^4}{(16\pi^2)}\alpha Nm \frac{F^2}{M^2} \\
\text{note } ms_{\text{soft}}^2 & \sim (m_{\text{gaugino}})^2
\end{align*}
\]

Important phenomenological consequence:
LSP = gravitino. Why?

\[
m_{3/2} = \frac{F}{M_p}
\]
always set by \( M_p \)
(like \( M_W \approx 94 \))

But now \( F \) very small
\[ \frac{1}{\sqrt{\pi}} \frac{N_m}{M} \sim m_{\text{ew}} \]

\[ m_{3/2} = \left( \frac{\sqrt{F}}{100 \text{eV}} \right) 2.4 \text{eV} \]

For relevant \( F \)'s, \( m_{3/2} \ll m_{\text{ew}} \)

Very light, but very weakly coupled!

If \( \sqrt{F} \gtrsim 10^6 \text{ GeV} \):

NLSP lives so long,

for collider physics like ordinary LSP

\[ \sqrt{F} \lesssim 10^6 \text{ GeV} \]

NLSP decays

within detector

\rightarrow quite unique

signal displaced

photons + \( \gamma \)

The \( \mu-B_\mu \) problem of gauge mediation

\( \mu \) param:

only SUSY preserving mass term.

Need to forbid it; then relate to SUSY

\[ H_u \rightarrow e^{i\phi} H_u \]

\[ H_d \rightarrow e^{i\phi} H_d \]

\( \gamma \) PR symmetry forbids it

Assume SUSY breaks PR symmetry.
In gravity mediation works perfectly

\[ \mathcal{L} \propto \frac{X^+ H u H d}{M_{Pl}} \]

\[ \mu \propto \frac{F}{M_{Pl}} \]

\[ \mathcal{L} \propto \frac{X^4 X}{M_{Pl}^2} H u H d \]

\[ B_m \propto \frac{F^2}{M_{Pl}^2} \sim \mu^2 \]

\[ B_m \sim \mu^2 \quad \text{good. Only in gravity mediation} \]

In gauge mediation: \[ F \ll 10^{10} \text{GeV} \]

\[ \mu, B_m \text{ way too small. Need to couple Higgs directly to messengers, but then} \]

\[ M = \frac{1}{16 \pi^2} \frac{F}{M} \]

\[ B_m = \frac{1}{16 \pi^2} (\frac{F}{M})^2 \]

Both at 1-loop,

\[ B_m \Rightarrow \mu^2 \quad \text{no good EWSB} \ldots \]
VARIATIONS BEYOND THE MSSM

- SUSY UNDER PRESSURE FROM THE LHC

1) Higgs Mass $m_h = 125$ GeV
   Hard to achieve in MSSM

2) No sign of superpartners.
   With simplest assumptions $m_{\tilde{g}} \sim m_{\tilde{g}}$
   Find $m_{\tilde{g}} \approx 1.2$ TeV ...
   No longer natural...

Ways out: LIFT THE HIGGS MASS

- NMSSM
- Raise D-terms by charging Higgs under additional $U(1)_X$

Absence of superpartners

- Only stop, higgsino, gaugino light, other superpartners 2-3 TeV
- R-parity violation
THE NMSSM

ADD ADDITIONAL SINGLET S TO HIGGS SECTOR.

MOTIVATIONS: - SOLVE $\mu$-PROBLEM
- RAISE HIGGS QUARTIC - HIGGS MASS

$W_{\text{NMSSM}} = Y_u H_u Q \bar{u} + Y_d H_d Q \bar{d} + Y_e H_d \bar{L} \bar{e} + \lambda S H_u H_d + \frac{1}{3} \kappa S^3$

(if we make simplifying assumption of $Z_3$ symmetry...)

$V_{\text{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2$
$+ m_s^2 |S|^2 + \lambda A_\kappa (S H_u H_d + \text{h.c.})$
$+ \frac{1}{3} \kappa A_\kappa (S^3 + \text{h.c.})$

+SUSY part: $|\lambda H_u H_d + \kappa S^2|^2$
$+ \lambda^2 (S)^2 (|H_u|^2 + |H_d|^2) + \text{usual D-terms}$

VEVs: $\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}$
$\langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}$

$\zeta_S \approx \zeta_S^0$

$\mu_{\text{eff}} = \lambda S$

solves $\mu$-problem...
Approximate expression of Higgs mass:

\[ m_h^2 = H_2^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta \]

\[ - \frac{\lambda^2}{m_t^2} v^2 (\lambda - \kappa \sin^2 \beta)^2 + \frac{3m_t^4}{4 \Pi^2 v^2} \left( \log \left( \frac{m_\tau^2}{m_t^2} \right) \right) \]

\[ + \frac{A_\tau^2}{m_t^2} \left( 1 - \frac{A_\tau^2}{12m_t^2} \right) \]

Can be higher than in MSSM depending on \( \lambda \). \( \lambda \) itself can't be too high to not loose perturbativity, but 125 GeV is fine...
"Natural SUSY"

For cancellation of quadratic divergences only need
- stop
- gauginos (EWK)

In addition, to have natural EWSB need μ to be small \( \Rightarrow \) higgsinos need to be light

If higgsino too heavy \( \Rightarrow \) will feed uβ higgs soft breaking mass at 2-loops (\& also raise stop mass \( @ \) 1 loop). So

"natural" spectrum:

\[ \tilde{g}, \tilde{W}, \tilde{B} \]

\[ \tilde{t}_L, \tilde{b}_L \]

\[ \tilde{e}_L, \tilde{e}_R \]

\[ \tilde{\nu}_{e1,2}, \tilde{\nu}_{\mu,2}, \tilde{\nu}_{\tau,2} \]

\[ \tilde{d}_{L1,2} \]

Well above TeV scale...

below TeV scale
Simplest model:

$$\bar{t}_L \rightarrow t + N \rightarrow b + \tilde{N}$$ if kinematically available

or possibly $$\bar{t} \rightarrow t + \tilde{G}$$

$$\uparrow$$
gravitino

bounds now: depends strongly on $\tilde{N}$ mass

![Graph with points and lines indicating mass vs. other parameter.](image-url)
R- PARITY VIOLATION

ARGUED:

\[ W_{RPV} = \lambda LL\bar{e} + \lambda' QL\bar{d} + \lambda'' \bar{u}\bar{d}\bar{d} \]

should be suppressed. \underline{But: small RPV could be there} make LSP decay. Simplest possibility: \underline{ONLY} \bar{u}\bar{d}\bar{d} BNU operator allowed. If no LNU \rightarrow no \underline{proton decay}. Still need to check \underline{n̅-n̅ oscillation period}, di-nucleon decay... Main point: \underline{E LSP}

\[ \bar{\nu} \rightarrow b+s \]

\[ \bar{\nu} \rightarrow 2\nu \]

no missing energy! \underline{Instead just get jets}, large QCD background, hard to find!

Interesting model building possibility:

\[ \lambda'' \] proportional to ordinary Yukawa couplings! Could explain why \[ \lambda'' \] small...