LECTURE 2.

THE MSSM & VARIATIONS

Lagrangian for $\mathcal{N}=1$ gauge theories

every SM fermion $\Psi_i \longrightarrow \Phi_i$ chiral SF

$$\Phi = \underbrace{\varphi(y)}_{\text{sfermion}} + \sqrt{2}\theta\,\Psi(y) + \theta^2 F(y)$$

$y^\mu \hat{=} x^\mu + i\theta\sigma^\mu\bar{\theta}$

Ψ: fermion

F: F-term auxiliary field

SM gauge fields $A_\mu \longrightarrow W_\alpha$ vector SF

$$W_\alpha = -i\lambda_\alpha(y) + \theta_\beta\left[\delta_\alpha^\beta D(y) - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu)^\beta_\alpha F_{\mu\nu}\right] + \theta^2\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\lambda}^{\dot{\alpha}}(y)$$

λ_α: gaugino

D: D-term auxiliary field

$F_{\mu\nu}$: gauge field

Lagrangian in superspace:

$$\int d^4\theta\,\Phi_i^+ e^{gV}\Phi_i + \frac{1}{4g^2}\int d^2\theta\,W^{\alpha a}W_\alpha^a + h.c. + \int d^2\theta\,W(\Phi) + h.c.$$

$\int d^4\theta\,\Phi_i^+ e^{gV}\Phi_i$: gauge invariant kinetic terms for matter fields

$W^{\alpha a}W_\alpha^a$: gauge + gaugino kinetic terms

$W(\Phi)$: superpotential

The Matter content of the MSSM

same gauge group as SM, but need 2 Higgs ~~ch~~ chiral superfields (otherwise $SU(2)^2 U(1)_Y$, $SU(2)^3$ (witten) anomaly) Write in terms of LH chiral superfields only to maintain holomorphy of SUSY...

	$SU(3)$	$\times SU(2)_L$	$\times U(1)_Y$	B	L
L	1	2	$-1/2$	0	1
E	1	1	$+1$	0	-1
Q	3	2	$1/6$	$1/3$	0
$\bar{U}$	$\bar{3}$	1	$-2/3$	$-1/3$	0
$\bar{D}$	$\bar{3}$	1	$1/3$	$-1/3$	0
H_u	1	2	$1/2$	0	0
H_d	1	2	$-1/2$	0	0

Possible superpotential terms:

$$W^{(good)} = \lambda_u^{ij} \underbrace{Q^i H_u \bar{U}^j + \lambda_d^{ij} Q^i H_d \bar{D}^j + \lambda_e^{ij} L^i H_d \bar{E}^j}_{\text{SUSY extensions of SM Yukawa couplings}}$$

$$+ \underbrace{\mu H_u H_d}_{\text{Supersymmetric Higgs mass (μ-term)}}$$

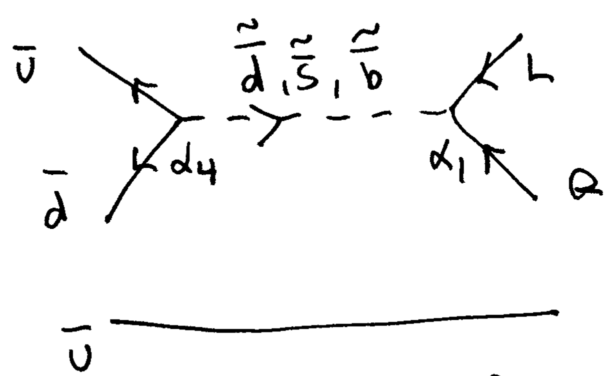
Need these:
- give masses to SM fermions
- give mass to Higgsinos (eliminate axion)

$$W^{(bad)} = \overbrace{\alpha_1^{ijk}\, Q^i L^j \bar{D}^k + \alpha_2^{ijk}\, L^i L^j \bar{E}^k}^{\Delta L = 1} + \underbrace{\alpha_3^{i}\, L^i H_u}_{\Delta L=1} + \underbrace{\alpha_4^{ijk}\, \bar{D}^i \bar{D}^j \bar{U}^k}_{\Delta B = 1}$$

would violate baryon & lepton #
renormalizable interaction

- VERY different from SM : in SM all terms allowed by gauge invariance also conserve B, L. B, L accidental global symmetry. In SM B, L violation $\propto \frac{1}{M}$ where M can be a very high scale.
- In MSSM : new fields (superpartners) that also carry B, L, more renormalizable terms.
  Need to forbid $W^{(bad)}$!

could give proton decay



couplings completely unsuppressed!

$$p^+ \to e^+ \pi^0 \quad \text{or} \quad \bar{\nu}\pi^+$$
$$\qquad e^+ K^0 \qquad\qquad \bar{\nu} K^+$$

Forbid $W^{(bad)}$ by matter parity, Z_2 symmetry

quark, lepton χSF $\quad P_M = -1$

Higgs $\quad P_M = +1$

~~gauge~~ Vector SF $\quad P_M = +1$

$W^{(good)}$: all have $P_M = +1$

$W^{(bad)}$: all have $P_M = -1$

can check:

$$P_M = (-1)^{3(B-L)}$$

variation:

R-parity: $\quad P_R = (-1)^{3(B-L)+2s}$

spin of field

If matter parity conserved, R-parity also conserved, $(-1)^{2s} \rightarrow$ always need even # of fermions by Lorentz.

R-parity:

$$\left.\begin{array}{l}(\text{SM fields}) \rightarrow +1 \\ (\text{superpartners}) \rightarrow -1\end{array}\right\} \text{like a } T\text{-parity.}$$

Forbids all tree-level EWP corrections, chance SUSY is right...

Important consequences of R-parity

(usually quoted as consequences of SUSY, but it really just follows from R-parity)

- Lightest R-parity odd particle stable
  $\equiv$ LSP lightest superpartner
  if LSP electronically neutral, color singlet:
  candidate for WIMP-like DM

- Each sparticle other than LSP will decay, at the end will contain odd # (usually one) LSP's

- Collider experiments: initial state $P_R = +1$ $\rightarrow$ only even # of superpartners can be produced, must be pair produced.
  At the end decay to LSP's $\rightarrow$ missing energy signal in colliders.

Will postulate that MSSM has exact R-parity conservation (somewhat ad-hoc assumption)

SUPERSYMMETRY BREAKING

SUSY unbroken if $Q_\alpha|0\rangle = 0$

$\bar{Q}_{\dot\alpha}|0\rangle = 0$

Then using SUSY algebra $\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$

$$\rightarrow P^\nu = \tfrac{1}{4} (\bar\sigma^\nu)^{\dot\beta\alpha} \{Q_\alpha, \bar{Q}_{\dot\beta}\}$$

$$H = P^0 = \tfrac{1}{4} \left(Q_1 \bar{Q}_{\dot 1} + \bar{Q}_{\dot 1} Q_1 + Q_2 \bar{Q}_{\dot 2} + \bar{Q}_{\dot 2} Q_2 \right)$$

if SUSY unbroken $\langle 0|H|0\rangle = 0$

if SUSY broken $\langle 0|H|0\rangle > 0$

Scalar potential

$$V(\phi) = \underbrace{\sum_i{}' \left| \frac{\partial W}{\partial \phi_i} \right|^2}_{\sum_i |F_i|^2} + \sum_a \frac{1}{2} g^2 \left| \sum_i \phi_i^+ T^a \phi_i \right|^2$$

$$\underbrace{\sum_i |F_i|^2}_{V_F} + \underbrace{\sum_a \frac{1}{2} g D^a D^a}_{V_D}$$

SUSY breaking $\langle F_i \rangle \neq 0$ or $\langle D \rangle \neq 0$

F-type breaking D-type breaking

If SUSY breaking $\rightarrow \exists$ massless fermion

Goldstino.

For example if $\langle F \rangle \neq 0$, the SUSY transformation of ψ

$\delta_\xi \psi = 2\xi \langle F \rangle$ → shift symmetry for fermion → fermion in multiplet where $\langle F \rangle = 0$ <u>massless..</u>

If more than one field:

$$\delta \psi_i = 2\xi \langle F_i \rangle$$

$$\psi_{\text{Goldstino}} = \sum_i \frac{F_i}{\sqrt{\sum_i F_i^2}} \psi_i$$ → always just one Goldstino.

<u>How to apply to MSSM?</u>

<u>SUM rule for broken SUSY</u>

Fermion masses:

superpartner of D-terms

$$i\sqrt{2}g \, (T^a)^i{}_j \left(\varphi_i \bar{\lambda}^a \bar{\psi}^j - \varphi^* \lambda \psi \right)$$

$$- \frac{\partial^2 W}{\partial \varphi_i \partial \varphi_j} \psi_i \psi_j + \text{h.c.}$$

from superpotential

$$\boxed{\begin{gathered} F_i = \frac{\partial W}{\partial \varphi_i}, \quad \bar{F}^i = \frac{\partial \bar{W}}{\partial \bar{\varphi}^i} \\ D^a = g \sum_i \varphi_i^* T^a \varphi_i \end{gathered}}$$

Fermion mass matrix:

$$(\psi_i \; \lambda_a) \begin{pmatrix} F_{ij} & \sqrt{2} D_{bi} \\ \sqrt{2} D_{aj} & 0 \end{pmatrix} \begin{pmatrix} \psi_j \\ \lambda_b \end{pmatrix}$$

$$F_{ij} \equiv \frac{\partial F_i}{\partial \varphi_j} \quad , D_{ai} = \frac{\partial D_a}{\partial \varphi_i} = g\, \varphi_i^* T^a$$

$$m^{j=1/2} = \begin{pmatrix} F_{ij} & \sqrt{2}\, D_{bj} \\ D_{ai} & 0 \end{pmatrix}$$

<u>Scalar mass</u>

$$m^{2\; j=0}_{ij} = \begin{bmatrix} \dfrac{\partial^2 V}{\partial \varphi_i \partial \varphi^*_j} & \dfrac{\partial^2 V}{\partial \varphi_i \partial \varphi_j} \\ \dfrac{\partial^2 V}{\partial \varphi_i^* \partial \varphi_j^*} & \dfrac{\partial^2 V}{\partial \varphi_i^* \partial \varphi_j} \end{bmatrix}$$

$$= \begin{bmatrix} \bar{F}^{ik} F_{kj} + D_a^{\,i} D_{aj} + D_a{}^i{}_j D_a & \bar{F}^{ijk} F_k + D_a^{\,i} D_a^{\,j} \\ F_{ijk} \bar{F}^k + D_{ai} D_{aj} & F_{ik} \bar{F}^{jk} + D_{ai} D_a^{\,j} + D_a{}^j{}_i D_a \end{bmatrix}$$

<u>GB mass matrix</u>

$$\sum_i g^2 \left| A_\mu^a T^{a\alpha}{}_\beta \phi_{i\alpha} \right|^2 = \left| A_\mu^a D_a^{\,i} \right|^2$$

$$m^{2\,(j=1)}_{ab} = D_a^{\,i} D_{bi} + D_{ai} D_b^{\,i}$$

<u>Traces:</u> $\mathrm{Tr}\; m^{(j=1/2)} (m^+)^{(j=1/2)} = F_{ij} \bar{F}^{ij} + 4 |D_{ai}|^2$

$$\mathrm{Tr}\; m^{2\,(j=0)} = 2 F^{ij} \bar{F}_{ij} + 2 D_a^{\,i} D_{ai} + 2 D_a D_a{}^i{}_i$$

$$\mathrm{Tr}\left(m^{2(j=1)}\right) = 2 D_{ai} D_a^{\ i}$$

$$\mathrm{STr} M^2 = \mathrm{Tr}\,(2j+1)(-1)^{2j} M^2$$

$$= -2F\bar{F} - 8(D_{ai})^2 + 2F\bar{F} + 2D_a^{\ i} D_{ai}$$

$$+ 2 D_a D_a{}^i{}_i + 3\cdot 2\, D_{ai} D_a^{\ i}$$

$$= 2 D_a (D_a)^i{}_i$$

$\langle D_a \rangle \neq 0$ only for $U(1)$'s

$$= 2 D\, D^i{}_i \qquad D^i{}_i = \sum q_i$$

sum of all $U(1)$ charges

$$\mathrm{STr} M^2 = 2 D_\alpha \sum_i q_i^{\ \alpha}$$

α: $U(1)$ factors

usually $\sum q_i^\alpha = 0$ due to anomaly cancellation!

$$\rightarrow \boxed{\mathrm{STr} M^2 = 0}$$

This is a very bad relation for the MSSM
Tells that SOME superpartners lighter than SM masses

Application to the MSSM (Dimopoulos & Georgi)

Assume sum rule applies. Consequence:
One squark lighter than m_u or m_d (experimentally impossible)

Scalar mass matrix:

$$M^{2\,i}{}_j = \begin{bmatrix} \bar{F}^{ik} F_{kj} + \frac{1}{2} D_a^i D_{aj} + \frac{1}{2} D^i_{ja} D_a & \bar{F}^{ijk} F_k + \frac{1}{2} D_a^i D_a^j \\ \bar{F}^k F_{ijk} + \frac{1}{2} D_{ia} D_{ja} & F_{jk} \bar{F}^{ki} + \frac{1}{2} D_{ai} D^{aj} + \frac{1}{2} D_a{}^j{}_i D_a \end{bmatrix}$$

Specify to squark mass matrix. Squarks should <u>NOT</u> get VEV (color not broken) $D_a^i = 0$

quarks <u>only</u> get mass from superpotential, since squark VEV = 0

$D_{color} = 0, \quad D_{1,2} = 0 \qquad$ only $D_3, D_Y \neq 0$

$$M^2_{2/3} = \begin{bmatrix} m_{2/3} m_{2/3}^{+} + \left(\frac{1}{2} g D_3 + \frac{1}{6} g' D_Y\right) \mathbb{1} & \Delta \\ \Delta^{+} & m_{2/3}^{+} m_{2/3} - \frac{2}{3} g' D_Y \end{bmatrix}$$

$$M^2_{1/3} = \begin{bmatrix} m_{1/3} m_{1/3}^{+} + \left(-\frac{1}{2} g D_3 + \frac{1}{6} g' D_Y\right) \mathbb{1} & \Delta' \\ \Delta'^{+} & m_{1/3}^{+} m_{1/3} + \frac{1}{3} g' D_Y \mathbb{1} \end{bmatrix}$$

sum of all D-terms $= 0 \qquad$ at least one ≤ 0

Assume for example $\frac{1}{2} g D_3 + \frac{1}{6} g' D_Y \leq 0$

If β eigenvector of $m_{2/3}$ $(\beta^\dagger, 0)\, M^2_{2/3} \begin{pmatrix} \beta \\ 0 \end{pmatrix} \leq m_0^2$

There must be a squark mass less than m_u or m_d
$\rightarrow$ not possible.

<u>SUM RULES must be broken!</u>

Need to <u>violate</u> assumption leading to sum rule

- renormalizable
- tree-level

Need to assume that no renormalizable interaction between $\cancel{SUSY}$ sector & SM

For example:
- only transmitted through gravity structure of SUGRA Lagrangian (non-renormalizable) allows more terms
- a "messenger sector" mediates between SM fields & $\cancel{SUSY}$ sector

If we don't want to specify, try to parametrize what kind of terms will we get from non-renormalizable interactions that violate SUM rule?

Assume ~~SUSY~~ field S, has only

non-renormalizable couplings to visible sector (either through gravity, quantum loops, ...) What operators could be generated?

$$\langle S \rangle = \ldots \quad + \theta^2 \langle F \rangle$$

Possible terms:

- $\int \Phi^+ \Phi \, \frac{S^+ S}{M^2} \, d^4\theta \quad \longrightarrow \quad \varphi^* \varphi \left(\frac{F}{M}\right)^2$

Scale at which new physics is integrated out
M_{PL} for gravity
M_{mess} for GM

Scalar mass

$m^2 \sim \left(\frac{F}{M}\right)^2$

(of course could also add terms like
$\int \Phi^+ \Phi^+ \Phi \, \frac{S^+ S}{M^3} \, d^4\theta$ get much more suppressed terms...)

- $\int \Phi^2 S \, d^2\theta \quad \longrightarrow \quad F \, (\varphi^2 + \varphi^{*2})$

b-term, natural size $\sim F$ w/o symmetry.

- $\int \frac{S}{M} \, \Phi^3 \, d^2\theta \quad \longrightarrow \quad \frac{F}{M} \, (\varphi^3 + \varphi^{*3}) \rightarrow A \, (\varphi^3 + \varphi^{*3})$

$A \sim m$, same order as scalar mass

$$-\int W_\alpha W^\alpha \frac{S}{M}\, d^2\theta \quad \rightarrow \quad \frac{F}{M} \lambda\lambda + h.c.$$

gaugino mass

$$m_\lambda \sim \frac{F}{M} \sim m \sim A$$

Find:

- scalar mass
- gaugino mass
- scalar holomorphic cubic (A) & quadratic (b) terms

Note: $\quad STrM^2 = \underbrace{2\sum_i m_i^2 - 2\sum_a m^2_{\lambda_a}}_{\text{no reason to vanish!}}$

This is the rationale for

SOFT breaking terms for the MSSM!

So full MSSM Lagrangian

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}$$

$$\begin{aligned}
\mathcal{L}_{soft} = & -\frac{1}{2}\left(M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B}\right) + h.c. \\
& -\left(a_u \tilde{Q} H_u \tilde{\bar{u}} + a_d \tilde{Q} H_d \tilde{\bar{d}} + a_e \tilde{L} H_d \tilde{\bar{e}}\right) + h.c. \\
& - \tilde{Q}^\dagger m_Q^2 \tilde{Q} - \tilde{L}^\dagger m_L^2 \tilde{L} - \tilde{\bar{u}}^\dagger m_u^2 \tilde{\bar{u}} \\
& - \tilde{\bar{d}}^\dagger m_d^2 \tilde{\bar{d}} - \tilde{\bar{e}}^\dagger m_e^2 \tilde{\bar{e}} - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d \\
& - (b H_u H_d + h.c.)
\end{aligned}$$

$a_{u,d,e}$: 3×3 matrices in flavor space,
1–1 correspondence to Yukawa matrices

$m^2_{Q,L,u,d,e}$: 3×3 matrices in flavor space

<u>We assume:</u> $M_{1,2,3}$. $a_{u,d,e}$ $\sim m_{soft}$

$m^2_{QudLe,H_uH_d,b}$ $\sim m_{soft}^2$

m_{soft} $\sim$ few $\times$ 100 GeV – TeV

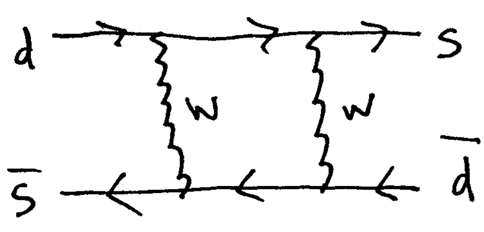
A LOT of new parameters : 105 new masses, phases, mixing angles on top of SM.
<u>BUT</u>: most of it ALREADY excluded from flavor & CP violating processes!

In SM: no tree-level FCNC's, loop level suppressed by GIM mechanism.
Similarly lepton flavor # violation strongly suppressed

<u>Example:</u> FCNC in K-$\bar{K}$ mixing (one of the best tested processes).

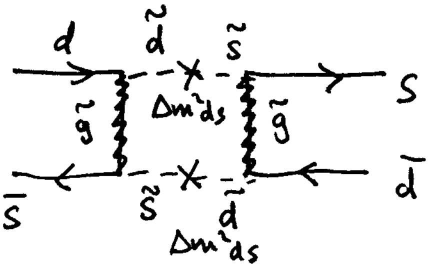
$K = d\bar{s}$
$\bar{K} = \bar{d}s$

in SM



CKM unitarity implies additional suppression

In MSSM :



Additional strongly coupled contribution. No GIM suppression

$$\sim \int \frac{d^4 p}{p^{10}} \sim \frac{1}{m_{SUSY}^6}$$

$$\mathcal{M}_{K\bar{K}}^{MSSM} \propto \alpha_3^2 \left(\frac{\Delta m^2_{ds}}{m_{SUSY}^2} \right)^2 \frac{1}{m_{SUSY}^2}$$

compare to exp'l bound

$$\frac{\Delta m_{ds}^2}{M_{SUSY}^2} \lesssim 4 \cdot 10^{-3} \left(\frac{m_{SUSY}}{500\ GeV} \right)$$

Off-diagonal terms need to be strongly suppressed...

Similar constraints from $\mu \to e\gamma$, CP violating phases...

Organizing "principle":

<u>Soft-breaking universality</u>

1.) Soft breaking masses are universal ($\propto \mathbb{1}$) for all types of particles

$$m_Q^2 = \mathbb{1}\, m_Q^2$$
$$m_{\bar{u}}^2 = \mathbb{1}\, m_{\bar{u}}^2$$
$$\vdots$$

2.) If A-terms not flavor universal, after Higgs VEV will induce similar mixings

$$A\,(Q\bar{U}H_u + Q\bar{D}H_d + L\bar{E}H_d)$$

assume A itself proportional to Yukawa matrix! Whatever rotation you do on quarks, can also do on squarks → will be diagonal in same basis!

$$A_{ij}\, Q_i \bar{U}_j H_u \rightarrow A_u\, \lambda^u_{ij} Q_i \bar{U}_j H_u$$

3.) to avoid CP violation, assume all non-trivial phases beyond SM CKM vanishes

Ultimately want to <u>explain</u> this, for example gauge mediation!

ELECTROWEAK SYMMETRY BREAKING IN MSSM, LITTLE HIERARCHY

Need Higgs potential (assume squarks, sleptons don't get VEVs)

quartic: only from D-terms

$$V_D = \frac{1}{2} g^2 \left| H_u^\dagger H_d \right|^2 + \frac{1}{8}(g^2+g'^2)\left(|H_u|^2 - |H_d|^2\right)$$

Important: higgs quartic $\sim g^2, g'^2$
Higgs mass $\sim \sqrt{\lambda}\, v$ $\rightarrow$ Higgs mass related to M_Z!

Full Higgs potential:

$$V_H = (\mu^2 + m_{Hu}^2)\,|H_u|^2 + (\mu^2 + m_{Hd}^2)\,|H_d|^2$$
$$- B_\mu H_u H_d + h.c. + \frac{1}{2} g^2 \left|H_u^\dagger H_d\right|^2$$
$$+ \frac{1}{8}(g^2+g'^2)\left(|H_u|^2 - |H_d|^2\right)$$

~~NB quartic along~~

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$$

only neutral comp's can get VEV!

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$$

In terms of H_u^0, H_d^0:

$$V_H = (|\mu|^2 + m_{Hu}^2)\,|H_u^0|^2 + (|\mu|^2 + m_{Hd}^2)\,|H_d^0|^2$$

$$-B_\mu \left(H_u^0 H_d^0 + h.c. \right) \quad + \frac{1}{8}\left(g^2+g'^2\right)\left(|H_u^0|^2 - |H_d^0|^2 \right)^2$$

no quartic along
$H_u^0 = H_d^0$

condition for EWSB: one direction along origin destabalized, but direction with no quartic has positive $(\text{mass})^2$!

$$\begin{vmatrix} |\mu|^2 + m_{H_u}^2 & -B_\mu \\ -B_\mu & |\mu|^2 + m_{H_d}^2 \end{vmatrix} < 0$$

$$B_\mu^2 > \left(|\mu|^2 + m_{H_u}^2\right)\left(|\mu|^2 + m_{H_d}^2\right) \qquad \text{negative } m^2$$

$$2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 - 2B_\mu > 0 \qquad \text{stability}$$

NO solution for $m_{H_u}^2 = m_{H_d}^2$.

Typically $\quad m_{H_u}^2 < 0$

$\quad m_{H_d}^2 > 0$

$$\langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}} = \frac{v \sin\beta}{\sqrt{2}}$$

$$\langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}} = \frac{v \cos\beta}{\sqrt{2}} \qquad \tan\beta = \frac{v_u}{v_d}$$

Minimizing potential we find:

$$\sin 2\beta = \frac{2 B_\mu}{2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2}$$

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

really weird equation! Connects

M_Z , m_{H_u, H_d}^2 μ

z-mass soft-breaking SUSY mass.

Origin of Little hierarchy!

Evaluate higgs masses, lightest CP-even higgs ~ SM higgs,

$$m_{h^0}^2 = \frac{1}{2}\left[M_Z^2 + m_A^2 \pm \sqrt{(M_Z^2 + m_A^2)^2 - 4 m_A^2 M_Z^2 \cos^2 2\beta}\right]$$

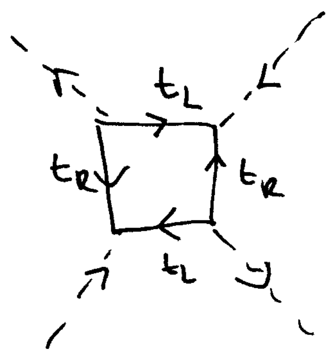
$$m_A^2 = \frac{B_\mu}{s_\beta c_\beta}$$

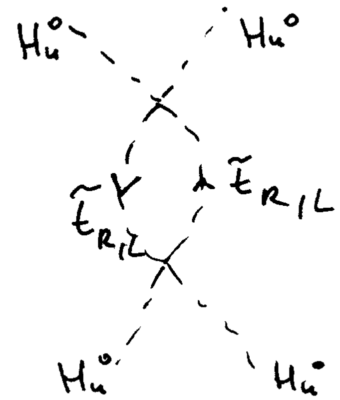
$$m_{h^0} \leq M_Z |\cos 2\beta| \leq M_Z$$

Tree-level upper bound on m_h.
But already know from LEP $m_h \gtrsim 114$ GeV!

Tree-level MSSM excluded. Need a large correction to quartic self-coupling. Main effect from top-stop loops!

1.) Want tree-level quartic maximized → large $\tan\beta$, VEV mostly in H_u. Light higgs $\sim H_u$. So need mostly H_u^4 coupling. 1 loop:





result: $$\lambda(m_t) = \lambda_{SUSY} + \frac{2N_c (y_t)^4}{16\pi^2} \log\left(\frac{m_{\tilde{t}_1} \tilde{m}_{t_2}}{m_t^2}\right)$$

fixed.

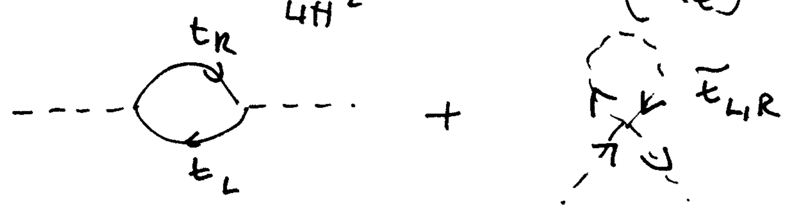
To push up higgs mass → need to increase $m_{\tilde{t}}$!

$$\Delta m_{h^0}^2 = \frac{3}{4\pi^2} v^2 y_t^4 \sin^2\beta \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) \lesssim 130\ \text{GeV}$$

Little hierarchy of MSSM

- At tree-level $\quad m_{h^0} \leq M_Z$
- Need a large $\quad m_{\tilde{t}} \sim 1-1.4$ TeV to increase quartic to push $\quad m_{h^0} > 114$ GeV
- But then also get corrections to quadratic in $m_{H_u}^2$

$$\Delta m_{H_u}^2 \sim -\frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \log\left(\frac{\Lambda}{m_{\tilde{t}}}\right)$$

from 

The bigger $m_{\tilde{t}}$, the larger the shift in $m_{H_u}^2$. But remember weird equation

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

$$\rightarrow \quad m_{H_u}^2 \sim \frac{M_Z^2}{2}$$

but loop correction $\quad \Delta m_{H_u}^2 = \frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \log\left(\frac{\Lambda}{m_{\tilde{t}}}\right)$

$$FT \sim \left(\frac{\Delta m_{H_u}^2}{M_Z^2/2}\right) \sim 800 \quad \text{for} \quad m_{\tilde{t}} = 1.2\,\text{TeV}, \ \Lambda = 10^{16}$$

$\rightarrow 0.1\,\%$ tuning.

Little hierarchy of MSSM!

Gauge mediated ~~SUSY~~

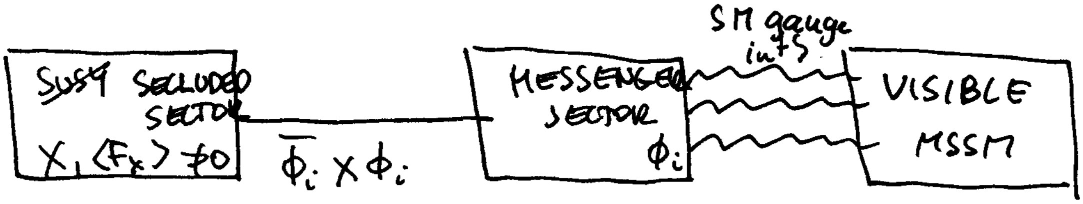
Flavor problem: in SM in limit when Yukawa $\to 0$ $U(3)^5$ flavor symmetry (3 gen's completely equivalent, 5 types of particles $\underbrace{Q\,u\,d\,L\,e}_{U(3)^5}$)

This flavor symmetry broken at SOME scale Λ_F, below which only imprint is Yukawas.
Λ_F could be very large, so effects of flavor breaking $\propto \frac{1}{\Lambda_F}$ $\to$ could all be very small!

However, if ~~SUSY~~ mediated by gravity, ~~SUSY~~ happens at $M_{Pl} > \Lambda_F$, really NO reason for soft breaking terms to not have $\mathcal{O}(1)$ flavor violation. Even if at tree-level for some reason they are flavor invariant, loop effects of flavor breaking sector will be large.
Would like theory where scale of ~~SUSY~~ mediation $\ll \Lambda_F$. Need to lower relevant mass scale for mediation (& physics of mediation itself should be flavor universal!)

Most important example:

GAUGE MEDIATION



<u>Idea:</u> – generate mass splittings OBEYING sum rules for messengers
– only through messengers in loop will MSSM feel $\cancel{\text{SUSY}}$

- Effectively: generate non-renormalizable ops. connecting MSSM & $\cancel{\text{SUSY}}$ sector.
- relevant scales M messenger mass
  $\langle F_X \rangle$ $\cancel{\text{SUSY}}$ VEV
- below M integrate out messenger sector, generate soft $\cancel{\text{SUSY}}$ masses
- since interactions ~~of~~ mediating $\cancel{\text{SUSY}}$ SM gauge interactions → will be flavor universal (if $\Lambda_F > M$)

<u>Minimal gauge mediation</u>

SM gauge singlet X $\qquad \langle X \rangle = M$
$\qquad \langle F_X \rangle \neq 0$

$\cancel{\text{SUSY}}$ sector

<u>Messengers:</u> N_f flavors $\phi_i, \bar{\Phi}_i$

$$W = \lambda \bar{\Phi} X \phi$$

messenger scalar mass matrix:

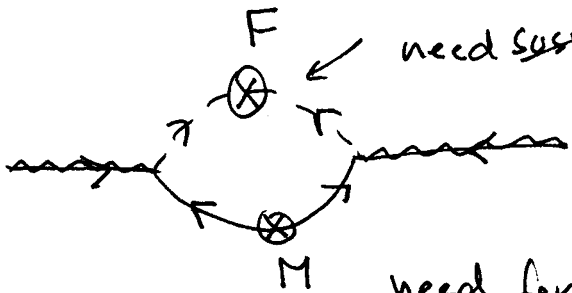
$$\left|\frac{\partial W}{\partial \phi}\right|^2 = |\lambda \langle X\rangle|^2 |\phi|^2 + |\lambda \langle X\rangle|^2 |\bar{\Phi}|^2$$

$$+ \quad \lambda \bar{\Phi}\phi F_X$$

masses: $m^2 = \lambda^2 M^2 \pm \lambda F$ scalar
λM fermion

$\Big\}$ obey sum rule

SM gaugino mass:



need ~~SUSY~~ mass for scalar

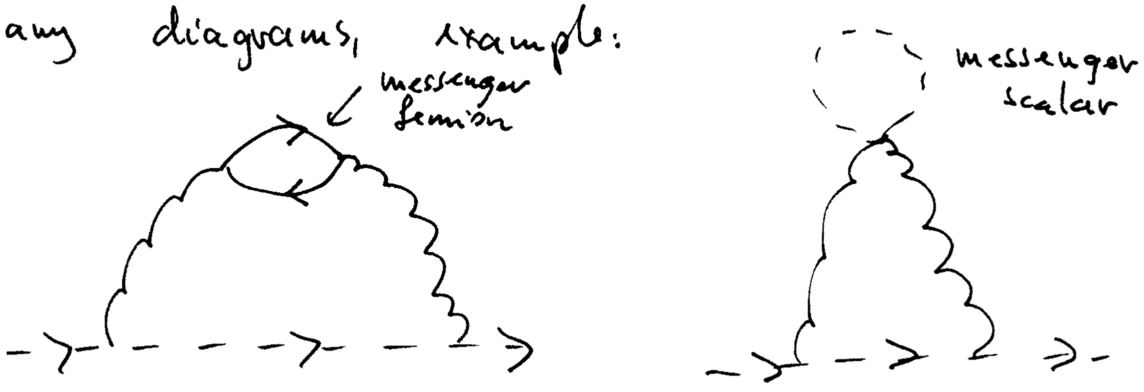
need fermion mass for helicity flip

$$m_\lambda = \frac{F \cdot M}{M^2} \frac{g^2}{16\pi^2} \cdot N_m \qquad \text{exact}$$

$$m_{\lambda_i} = \frac{\alpha_i}{4\pi} N_m \frac{F}{M}$$

Scalar mass: generated @ 2loop only
need both gauge boson & messenger to run in loop

many diagrams, example:



+ ...

result

$$m_{soft}^2 \propto \frac{g^4}{(16\pi^2)^2} N_m \frac{F^2}{M^2}$$

note $m_{soft}^2 \sim (m_{gaugino})^2$

<u>Important phenomenological consequence:</u>

LSP = gravitino. Why?

~~mass~~

$$m_{3/2} = \frac{F}{M_P}$$ always set by M_P (like $M_W \sim gv$)

But now F very small

$$\frac{\alpha}{4\pi} N_m \frac{F}{M} \sim m_{EW}$$

$$m_{3/2} = \left(\frac{\sqrt{F}}{100\,\text{TeV}}\right) 2.4\,\text{eV}$$

For relevant F's $m_{3/2} \ll m_{EW}$
Very light, but very weakly coupled!

If $\sqrt{F} \gtrsim 10^6$ GeV: ~~More~~ NLSP lives so long, for collider physics like ordinary LSP

if $\sqrt{F} \lesssim 10^6$ GeV NLSP decays within detector
→ quite unique signal displaced photons $+\not{E}_T$

The μ-B_μ problem of gauge mediation

μ param: only SUSY preserving mass term. Need to forbid it, then relate to ~~SUSY~~

$$\left.\begin{aligned} H_u &\to e^{i\alpha} H_u \\ H_d &\to e^{i\alpha} H_d \end{aligned}\right\} \text{PQ symmetry forbids it}$$

Assume ~~SUSY~~ breaks PQ symmetry.

In gravity mediation works perfectly

$$\int d^4\theta \; \frac{X^\dagger H_u H_d}{M_{Pl}}$$

$\langle X \rangle = \theta^2 F \to$
get effective μ

$$\mu \sim \frac{F}{M_{Pl}}$$

$$\int d^4\theta \; \frac{X^\dagger X}{M_{Pl}^2} H_u H_d \qquad B_\mu \sim \frac{F^2}{M_{Pl}^2} \sim \mu^2$$

$B_\mu \sim \mu^2$ good. Only in gravity mediation

In gauge mediation: $F \ll 10^{11}\,\text{GeV}$
μ, B_μ way too small. Need to couple Higgs directly to messengers, but then

$$\left.\begin{aligned} \mu &= \frac{1}{16\pi^2} F/M \\ B_\mu &= \frac{1}{16\pi^2} \left(F/M\right)^2 \end{aligned}\right\} \text{both at 1-loop,}$$

$B_\mu \gg \mu^2$!

no good EWSB...

VARIATIONS BEYOND THE MSSM

- SUSY UNDER PRESSURE FROM THE LHC

1.) HIGGS MASS $m_h = 125$ GeV
HARD TO ACHIEVE IN MSSM

2.) NO SIGN OF SUPERPARTNERS.
WITH SIMPLEST ASSUMPTIONS $m_{\tilde{q}} \sim m_{\tilde{g}}$
FIND $m_{\tilde{q}} \gtrsim 1.2$ TeV ...
NO LONGER NATURAL...

WAYS OUT : LIFT THE HIGGS MASS

- NMSSM
- RAISE D-TERMS BY CHARGING HIGGS UNDER ADDITIONAL $U(1)_x$

ABSENCE OF SUPER PARTNERS

- ONLY STOP, HIGGSINO, GAUGINO LIGHT, OTHER SUPERPARTNERS 2-3TeV
- R-PARITY VIOLATION

⋮

THE NMSSM

ADD ADDITIONAL SINGLET S TO HIGGS SECTOR.

MOTIVATIONS: - SOLVE μ- PROBLEM
- RAISE HIGGS QUARTIC - HIGGS MASS

$$W_{NMSSM} = Y_u H_u Q \bar{u} + Y_d H_d Q \bar{d} + Y_e H_d L \bar{e}$$

$$+ \lambda S H_u H_d + \frac{1}{3} \kappa S^3$$

(if we make simplifying assumption of Z_3 symmetry...)

$$V_{soft} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2$$
$$+ m_S^2 |S|^2 + \lambda A_\lambda (S H_u H_d + h.c.)$$
$$+ \frac{1}{3} \kappa A_\kappa (S^3 + h.c.)$$

+SUSY part: $|\lambda H_u H_d + \kappa S^2|^2$

$$+ \lambda^2 |S|^2 (|H_u|^2 + |H_d|^2) + \text{usual D-terms}$$

VEVs: $\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}$ $\langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}$

$$\langle S \rangle = s$$

$\boxed{\mu_{eff} = \lambda s}$ solves μ-problem...

Approximate expression of Higgs mass:

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta - \frac{\lambda^2}{\kappa^2} v^2 (\lambda - \kappa \sin 2\beta)^2 + \frac{3 m_t^4}{4\pi^2 v^2}\left(\log\left(\frac{m_T^2}{m_t^2}\right) + \frac{A_t^2}{m_T^2}\left(1 - \frac{A_t^2}{12 m_T^2}\right)\right)$$

Can be higher than in MSSM depending on λ. λ itself can't be too high to not loose perturbativity, but 125 GeV is fine...

"Natural SUSY"

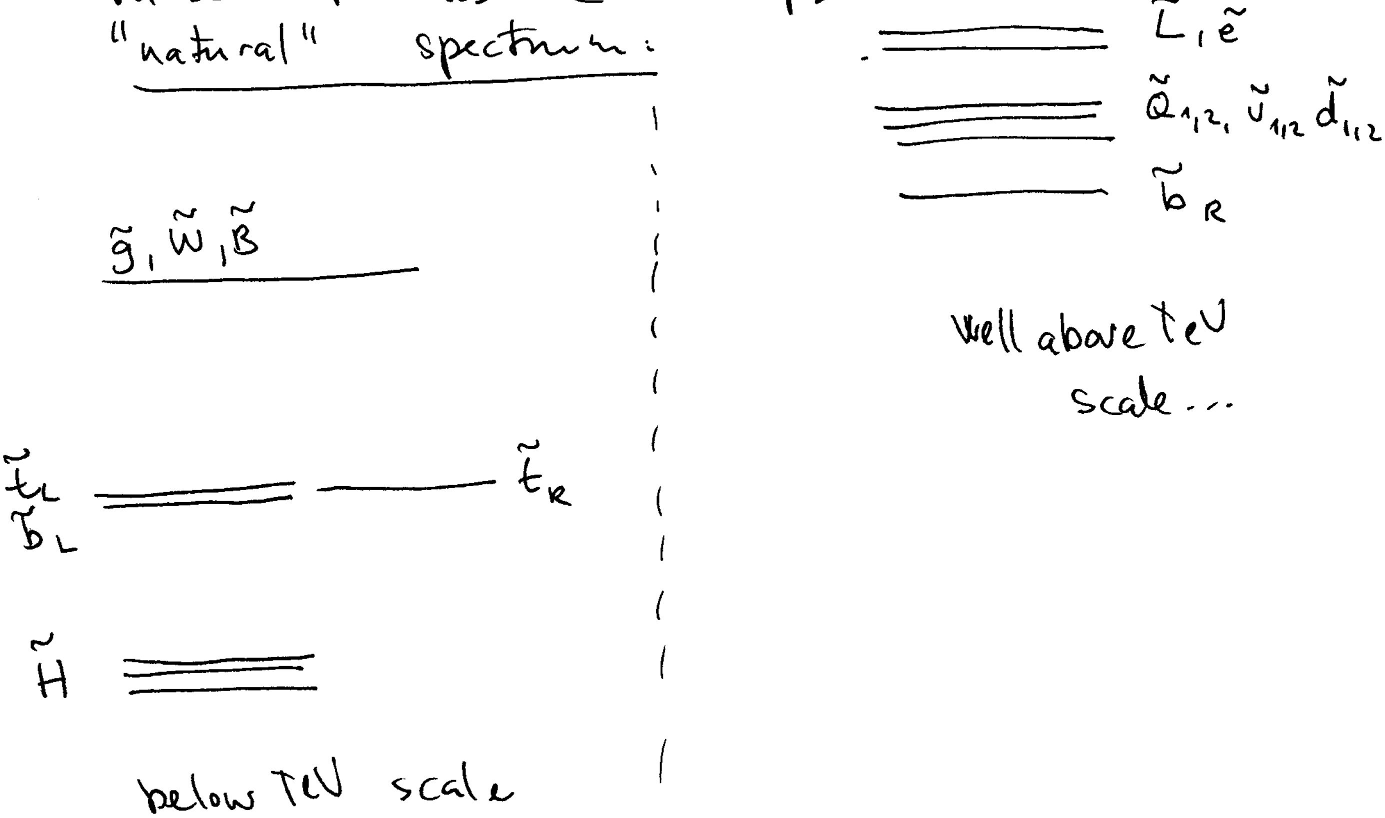
For cancellation of quadratic divergences only need

- stop
- gauginos (EWK)

In addition, to have natural EWSB need μ to be small $\rightarrow$ higgsinos need to be light

If gluino too heavy $\rightarrow$ will feed into higgs soft breaking mass at 2-loops (& also raise stop mass @ 1 loop). So

"natural" spectrum:



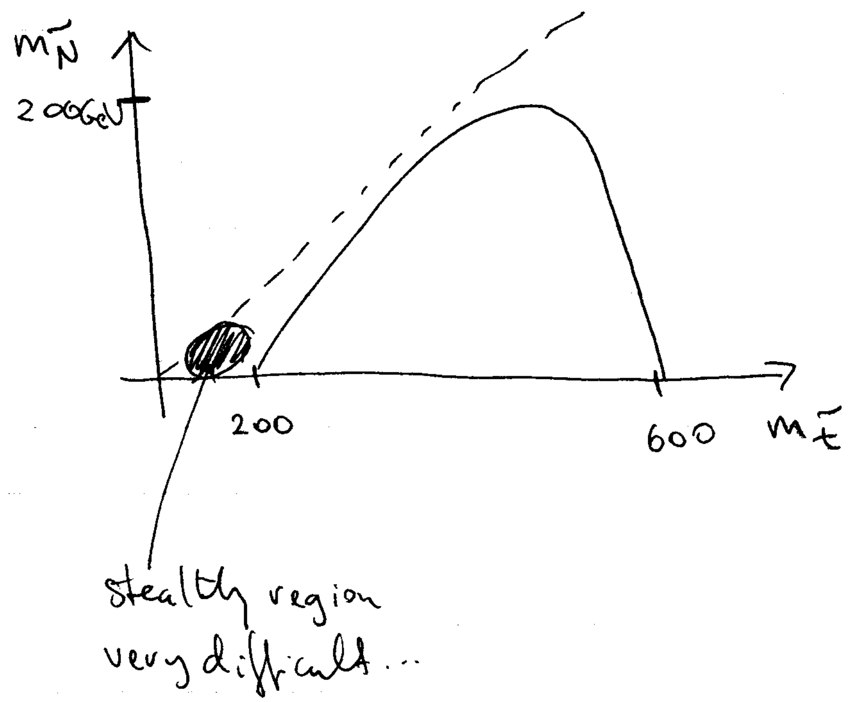
Simplest model:

$$\left.\begin{array}{l}\tilde{t}_L \longrightarrow t + \tilde{N} \\ \quad\;\; \longrightarrow b + \tilde{C}\end{array}\right\}$$ if kinematically available

or possibly $\tilde{t} \to t + \tilde{G}$

gravitino

bounds now : depends strongly on $\tilde{N}$ mass

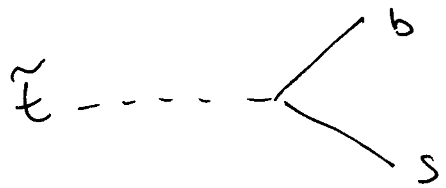


R- PARITY VIOLATION

ARGUED:

$$W_{RPV} = \lambda L L \bar{e} + \lambda' Q L \bar{d} + \lambda'' \bar{u}\bar{d}\bar{d}$$

should be suppressed. **But:** small RPV could be there make LSP decay. Simplest possibility: ONLY $\bar{u}\bar{d}\bar{d}$ BNV operator allowed. If no LNV $\rightarrow$ no proton decay. Still need to check n-$\bar{n}$ oscillation period, di-nucleon decay... Main point: $\tilde{t}$ LSP



$\tilde{t} \rightarrow \bar{b} + \bar{s}$

$\tilde{t} \rightarrow 2j$

no missing energy! Instead just get jets, large QCD background, hard to find!

Interesting model building possibility:

λ'' proportional to ordinary Yukawa couplings! Could explain why λ'' small...