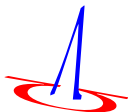


# WHIZARD

## Introduction & Tutorial



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Theoretical Challenges

Matrix Elements

Adaptive Monte Carlo

Usage

Exercise

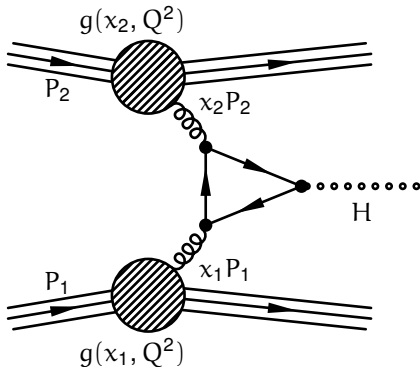
- ▶ **efficiently** and **reliably** compute **scattering probabilities**

$$|\langle q_1, q_2, \dots; \text{out} | T | p_1, p_2; \text{in} \rangle|^2$$

with lots of **gauge cancellations** among contributions

- ▶ for a **multitude** of physics models with **qualitatively** different **particle content** and **interactions**
  - ▶ standard model
  - ▶ supersymmetric extensions of the SM
  - ▶ SM with anomalous couplings
  - ▶ SM with extended gauge sector
  - ▶ SM with strongly interacting gauge bosons
  - ▶ additional space dimensions
  - ▶ ...
- ▶ such that their **parameter space** can be scanned and compared with experimental observations
- ▶ **efficiently sample** the multi particle phase space
  - ▶ scattering probabilities typically have **many** overlapping **narrow peaks** and **integrable boundary singularities**

- ▶ **Caveat:** not everything can be calculated in **perturbation theory** when **hadrons** (i. e. **strongly interacting particles**) are in play
- ▶ e. g. Higgs production at the LHC depends not only on the  $gg \rightarrow H$  cross section, but also on the composition of the protons:

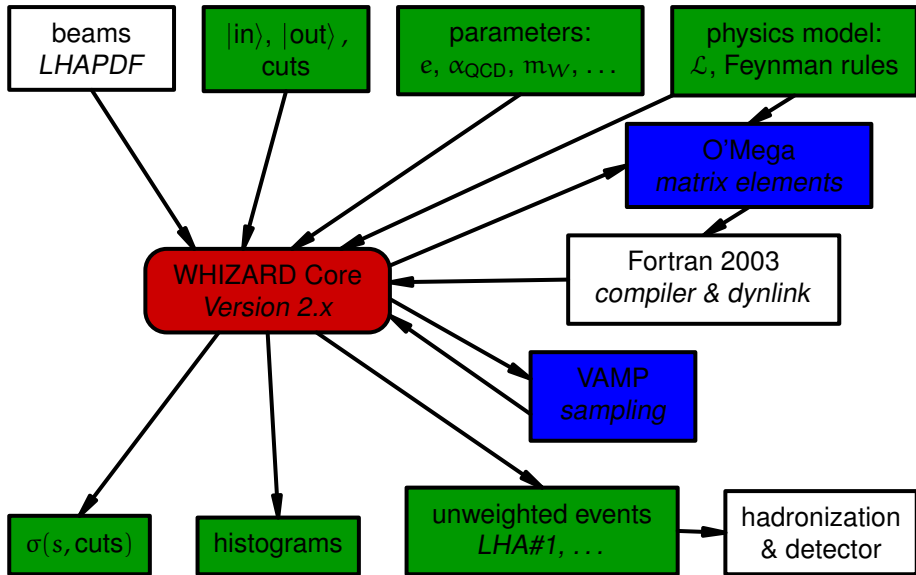


- ▶ **asymptotic freedom** and **factorization** allow to separate

$$\sigma(s) = \sum_{i_1 i_2} \int dx_1 dx_2 D_{i_1}(x_1, \mu) D_{i_2}(x_2, \mu) \hat{\sigma}(x_1 x_2 s; \mu)$$

**independent** of the scale  $\mu$ !

- ▶ **weakly coupled short distance/high energy** phenomena, calculable in **perturbation theory**
    - ▶ **hard scattering cross sections**  $\hat{\sigma}(\hat{s}; \mu)$
  - ▶ **universal strongly coupled long distance/low energy** phenomena, described by **parametrizations**
    - ▶ **parton distributions**  $D_{i_j}(x_j; \mu)$   
and/or **phenomenological models**
    - ▶ **fragmentation** and **hadronization**
  - ▶ a series of **Les Houches Accords** defines interfaces implementing this separation
- ∴ studies of **new physics** can concentrate on the hard interactions!



- ▶ desired: a **computer program** implementing the function

$$(\mathcal{L}, \{\text{incoming}\}, \{\text{outgoing}\}) \mapsto \mathcal{M}(\alpha_1, \dots; p_1, \dots; s_1, \dots)$$

where

- ▶  $\mathcal{L}$ : **Lagrangian** (or **Feynman rules**) of a **model** (SM, MSSM, ...)
- ▶  $\mathcal{M}(\alpha_1, \dots; p_1, \dots; s_1, \dots)$ : a function

$$\underbrace{\mathbf{R} \times \dots \times \mathbf{R}}_{\text{masses, couplings, ...}} \times \underbrace{\mathbf{V}^+ \times \dots \times \mathbf{V}^+}_{\text{4-momenta (forward light cones)}} \times \underbrace{\mathbf{Z} \times \dots \times \mathbf{Z}}_{\text{helicities, colors}} \rightarrow \underbrace{\mathbf{C}}_{\text{amplitude}}$$

in a form that can be evaluated **numerically**, typically as C, C++ or Fortran code in that can be compiled and linked to Monte Carlo phase space integrators and generators

- ▶ NB: in some cases only  $\mathcal{L} \rightarrow \sum |\mathcal{M}(\alpha_1, \dots; p_1, \dots; s_1, \dots)|^2$  is required. It is often better defined (**infrared/collinear cancellations**) and sometimes more compact (**spin/polarization sums**).
- ▶ first robust and usable examples in the early 1990s: **CompHEP**, **FeynArts**, **Grace**, **MadGraph**, ...

- ▶ for simplicity:  $e^+e^- \rightarrow \mu^+\mu^-$  at PETRA (i. e. QED, mostly)
- ▶ just one Feynman diagram

$$i\mathcal{M} = \bar{v}(p_2) (-ie\gamma_\rho) u(p_1) \frac{-ig_{\rho\sigma}}{(p_1 + p_2)^2 + i\epsilon} \bar{u}(q_1) (-ie\gamma_\sigma) v(q_2)$$

- ▶ analytical expression

$$\begin{aligned}
 i\mathcal{M} &= \bar{v}(p_2) (-ie\gamma^\rho) u(p_1) \frac{-ig_{\rho\sigma}}{(p_1 + p_2)^2 + i\epsilon} \bar{u}(q_1) (-ie\gamma^\sigma) v(q_2) \\
 &= ie^2 \frac{1}{s} [\bar{v}(p_2) \gamma_\rho u(p_1)] [\bar{u}(q_1) \gamma^\rho v(q_2)]
 \end{aligned}$$



- ▶ corresponding Fortran95 code  
(using a library for **vector** and **spinor products** and **states**)

```

pure function eleposmuoamu (k, s) result (amp)
  real(kind=omega_prec), dimension(0,:), intent(in) :: k
  integer, dimension(:), intent(in) :: s
  complex(kind=omega_prec) :: amp
  type(momentum) :: p1, p2, p3, p4
  type(spinor) :: muo_4, ele_1
  type(conjspinor) :: amu_3, pos_2
  type(vector) :: gam_12
  type(momentum) :: p12
  p1 = - k(:,1) ! incoming e-
  p2 = - k(:,2) ! incoming e+
  p3 =  k(:,3) ! outgoing m-
  p4 =  k(:,4) ! outgoing m+
  ele_1 = u (mass(11), - p1, s(1))           ! u s_1 (k_1)
  pos_2 = vbar (mass(11), - p2, s(2))       ! v-bar s_2 (k_2)
  amu_3 = ubar (mass(13), p3, s(3))         ! u-bar s_3 (k_3)
  muo_4 = v (mass(13), p4, s(4))           ! v s_4 (k_4)
  p12 = p1 + p2
  gam_12 = pr_feynman(p12, + v_ff(qlep,pos_2,ele_1)) ! (1/s) e v(k_2) gamma_mu u(k_1)
  amp = 0
  amp = amp + gam_12*( + v_ff(qlep,amu_3,muo_4)) ! (1/s) e v(k_2) gamma_mu u(k_1) e u(k_3) gamma^mu v(k_4)
  amp = - amp ! 2 vertices, 1 propagators
end function eleposmuoamu

```

- ▶ the usual rules for **manual** calculations are **algorithmic**
- ∴ can be implemented in a computer program

The number of tree Feynman diagrams w/  $n$  legs grows like a **factorial**, e. g. in  $\phi^3$ -theory:  $F(n) = (2n - 5)!! = (2n - 5) \cdot (2n - 7) \cdot \dots \cdot 3 \cdot 1$

$n$	$F(n)$	$P(n)$
4	3	3
5	15	10
6	105	25
7	945	56
8	10 395	119
9	135 135	246
10	2 027 025	501
11	34 459 425	1 012
12	654 729 075	2 035
13	13 749 310 575	4 082
14	316 234 143 225	8 177
15	7 905 853 580 625	16 368

- ▶ computational **costs** grow beyond all reasonable limits
- ▶ **gauge cancellations** cause **loss of precision**

Number of possible momenta in tree diagrams grows only **exponentially**

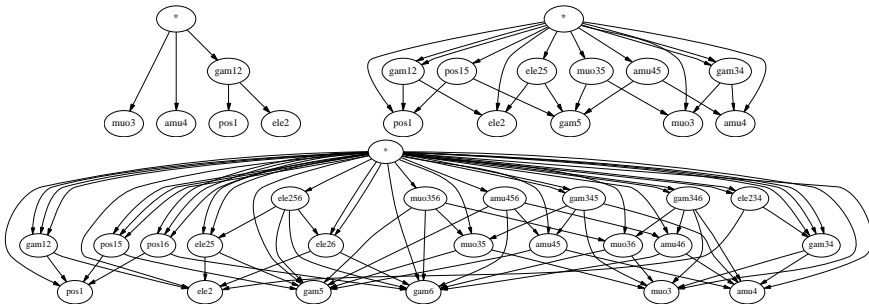
$$P(n) = \frac{2^n - 2}{2} - n = 2^{n-1} - n - 1$$

∴ Feynman diagrams **redundant** for many external particles!

- ∴ Replace the forest of tree diagrams by the **Directed Acyclical Graph (DAG)** of the algebraic expression.

$$ab(ab + c) = \begin{array}{c} \times \\ \diagup \quad \diagdown \\ a \quad b \quad + \\ \quad \quad \quad \diagup \quad \diagdown \\ \quad \quad \quad a \quad b \quad c \end{array} = \begin{array}{c} \times \\ \diagup \quad \diagdown \\ \quad \quad \quad \diagup \quad \diagdown \\ \quad \quad \quad a \quad b \quad c \end{array}$$

- ▶ simplest examples:  $e^+e^- \rightarrow \mu^+\mu^-$ ,  $e^+e^- \rightarrow \mu^+\mu^-\gamma$  and  $e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$  (only QED)



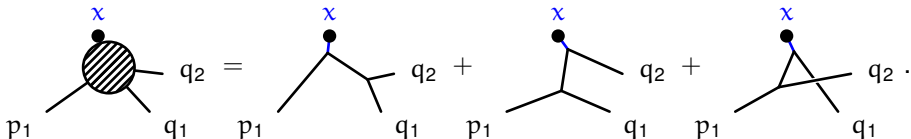
## Efficient tree amplitudes

- ▶ **Berends-Giele Recursion Relations** [Berends, Giele]
  - ▶ manual calculations
- ▶ **HELAS** [Hagiwara et al.],
  - ▶ manual partial common subexpression elimination
- ▶ **Madgraph** [Stelzer et al.], **AMEGIC++**, **COMIX** [Krauss et al.]:
  - ▶ partial common subexpression elimination
  - ∴ **partial** elimination of redundancy
- ▶ **ALPHA** [Caravaglios & Moretti]:
  - ▶ tree level scattering amplitude is Legendre transform of Lagrangian
  - ▶ can be performed **numerically**, using only  $P^*(n)$  independent variables
- ▶ **HELAC** [Papadopoulos et al.]:
  - ▶ ALPHA algorithm can be reformulated as recursive **numerical** solution of Schwinger-Dyson equations
- ▶ **O'Mega** [TO et al.]:
  - ▶ systematic elimination of **all** redundancies
  - ▶ symbolic, generation of compilable code

One particle off-shell wave functions (**1POWs**) are obtained from by applying the LSZ reduction formula to all but one line:

$$W(x; p_1, \dots, p_n; q_1, \dots, q_m) = \langle \phi(q_1), \dots, \phi(q_m); \text{out} | \Phi(x) | \phi(p_1), \dots, \phi(p_n); \text{in} \rangle .$$

E. g.  $\langle \phi(q_1), \phi(q_2); \text{out} | \Phi(x) | \phi(p_1); \text{in} \rangle$  in  $\phi^3$ -theory at tree level



- ▶ the set of **all** 1POWs at tree level grows **exponentially** and can be constructed **recursively** from other 1POWs at tree level.

There exists a well defined set of **keystones**  $K$  that allow to express the sum of **Feynman diagrams** through **1POWs**:

$$T = \sum_{i=1}^{F(n)} D_i = \sum_{k,l,m=1}^{P(n)} K_{f_k f_l f_m}^3(p_k, p_l, p_m) W_{f_k}(p_k) W_{f_l}(p_l) W_{f_m}(p_m)$$

Even for vector particles, the 1POWs are 'almost' physical objects and satisfy simple **Ward Identities** in unbroken gauge theories

$$\frac{\partial}{\partial x_\mu} \langle \text{out} | A_\mu(x) | \text{in} \rangle_{\text{amp.}} = 0$$

and in spontaneously gauge theories in  $R_\xi$ -gauge

$$\frac{\partial}{\partial x_\mu} \langle \text{out} | W_\mu(x) | \text{in} \rangle_{\text{amp.}} = \xi_W m_W \langle \text{out} | \phi_W(x) | \text{in} \rangle_{\text{amp.}} .$$

- ▶ code for matrix elements can optionally be instrumented to check these Ward identities, testing the **consistency** a particular model and the **numerical stability** of expressions.

Amplitudes can be continued off-shell:

- ▶ **Slavnov-Taylor Identities** can be checked numerically by adding **operator insertions** implementing BRS transformations.

Slightly simplified Model.T signature that **all** models must implement:

```
module type Model.T =
  sig
    type flavor (* all quantum numbers *)
    val flavor_symbol : flavor -> string
    val conjugate : flavor -> flavor (* antiparticles *)
    val lorentz : flavor -> Coupling.lorentz (* spin *)
    val fermion : flavor -> int (* fermion, boson, antifermion *)
    val width : flavor -> Coupling.width (* scheme, not value! *)
    type gauge (* parametrized gauges *)
    val gauge_symbol : gauge -> string
    val propagator : flavor -> gauge Coupling.propagator
    type constant (* coupling constants *)
    val constant_symbol : constant -> string
    val fuse2 : flavor -> flavor ->
      (flavor * constant Coupling.t) list (*  $A_\mu(p_{12}) \leftarrow g\bar{\psi}(p_1)\gamma_\mu\psi(p_2)$  *)
    val fuse3 : flavor -> flavor -> flavor ->
      (flavor * constant Coupling.t) list (*  $\phi(p_{123}) \leftarrow g\phi(p_1)\phi(p_2)\phi(p_3)$  *)
    val fuse : flavor list -> (flavor * constant Coupling.t) list
  end
```

- ▶ For interfacing to parton shower and hadronization programs, we need the amplitudes for all possible **color flows** or color connections.

E. g. in  $q\bar{q} \rightarrow q\bar{q}$

$$\Rightarrow \sum_a (T^a)_i^j (T^a)_k^l = \frac{1}{2} \left( \delta_i^l \delta_k^j - \frac{1}{N} \delta_i^j \delta_k^l \right)$$

- ▶ This can be expressed by two diagrams, one for gluon and one for **phantom** exchange

$$\frac{1}{\sqrt{2}} \delta_i^{l'} \delta_{k'}^j + \frac{-1}{\sqrt{2}} \delta_i^j \dots \left( \frac{-1}{N} \right) \dots \frac{-1}{\sqrt{2}} \delta_k^l$$

- ▶ the sum over all colors can be written as a sum over all color flows

$$\sum_{IJ} N_C^{\lambda(J,J)} A_I A_J^*$$

with  $\lambda(J, J)$  the number of closed color loops in  $A_I A_J^*$ .



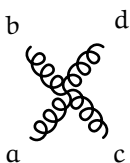
- ▶ one can use the completeness relation  $2T^a \otimes T^a = \delta \otimes \delta - \delta \otimes \delta$  **repeatedly** to compute all the color flow amplitudes
- ▶ instead, one can formulate **equivalent** Feynman rules that give the color flow amplitudes **directly**
- ▶ **Propagators**

$$\begin{array}{l}
 a \text{ --- } \underbrace{\text{wavy line}}_{-i\delta^{ab}} \text{ --- } b \iff j \text{ --- } \underbrace{\text{double line}}_{-i\delta_k^i \delta_j^l} \text{ --- } k + \dots \dots \dots -i \left( \frac{-1}{N} \right) \\
 a \text{ --- } \underbrace{\text{dashed line}}_{-i\delta^{ab}} \text{ --- } b \iff j \text{ --- } \underbrace{\text{dashed double line}}_{-i\delta_k^i \delta_j^l} \text{ --- } k \\
 i \text{ --- } \underbrace{\text{arrow}}_{i\delta_j^i} \text{ --- } j \iff j \text{ --- } \underbrace{\text{arrow}}_{i\delta_j^i} \text{ --- } i
 \end{array}$$

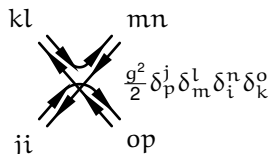
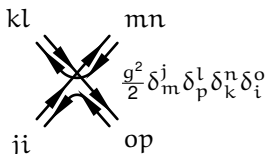
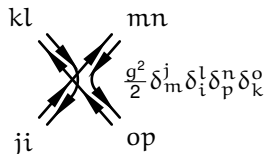
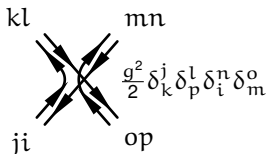
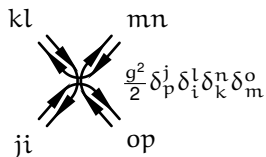
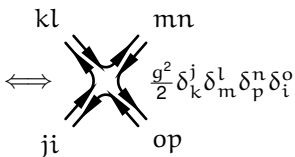
- ▶ The price to pay in the introduction of **phantom** particles that subtract the trace part of the gluons
- ▶ **NB:** they're **not** required for the **Faddeev-Popov ghosts**, because the trace of the gluons behaves as if it was abelian.



▶ Quartic vertices:



$$g^2(fabe fcde + face fdbe + fade fbce)$$



Example from **supersymmetry**: electroproduction of **chargino pairs** with **bremstrahlung**, i. e.  $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \gamma$ :

```

pure function l1b1l1cplcmla (k, s) result (amp)
  real(kind=omega_prec), dimension(0:,:), intent(in) :: k
  integer, dimension(:), intent(in) :: s
  complex(kind=omega_prec) :: amp
  type(momentum) :: p1, p2, p3, p4, p5
  type(bispinor) :: cp1_4, l1_2
  type(bispinor) :: cm1_3, l1b_1
  type(vector) :: a_5
  complex(kind=omega_prec) :: sn1_24, snc1_13
  type(bispinor) :: cp1_45, l1_25
  type(bispinor) :: cm1_35, l1b_15
  type(vector) :: a_34, a_12, z_34, z_12
  type(momentum) :: p12, p13, p15, p24, p25, p34, p35, p45
  p1 = - k(:,1) ! incoming e+
  p2 = - k(:,2) ! incoming e-
  p3 = k(:,3) ! outgoing ch1+
  p4 = k(:,4) ! outgoing ch1-
  p5 = k(:,5) ! outgoing A
  l1b_1 = u (mass(11), - p1, s(1))
  l1_2 = u (mass(11), - p2, s(2))
  cm1_3 = v (mass(69), p3, s(3))
  cp1_4 = v (mass(69), p4, s(4))
  a_5 = conjg (eps (mass(22), p5, s(5)))
  p12 = p1 + p2
  a_12 = pr_feynman(p12, + v_ff(qlep,l1b_1,l1_2))
  z_12 = pr_unitarity(p12,mass(23),wd_t1(p12,width(23)), &
    + va_ff(gnclep(1),gnclep(2),l1b_1,l1_2))
  p13 = p1 + p3
  snc1_13 = pr_phi(p13,mass(54),wd_t1(p13,width(54)), &
    + sr_ff(g_yuk_ch1_sn1_1_c,l1b_1,cm1_3))
  p24 = p2 + p4

```

```

sn1_24 = pr_phi(p24,mass(54),wd_tl(p24,width(54)), &
  + sl_ff(g_yuk_ch1_sn1_1,1l1_2,cp1_4))
p34 = p3 + p4
a_34 = pr_feynman(p34, + v_ff(qchar,cml_3,cp1_4))
z_34 = pr_unitarity(p34,mass(23),wd_tl(p34,width(23)), &
  + va_ff(-gczc_1l1(1),-gczc_1l1(2),cml_3,cp1_4))
p15 = p1 + p5
1l1b_15 = pr_psi(p15,mass(11),wd_tl(p15,width(11)), + f_vf(-qllep,a_5,1l1b_1))
p25 = p2 + p5
1l1_25 = pr_psi(p25,mass(11),wd_tl(p25,width(11)), + f_vf(qllep,a_5,1l1_2))
p35 = p3 + p5
cml_35 = pr_psi(p35,mass(69),wd_tl(p35,width(69)), &
  + f_vf(-qchar,a_5,cml_3))
p45 = p4 + p5
cp1_45 = pr_psi(p45,mass(69),wd_tl(p45,width(69)), + f_vf(qchar,a_5,cp1_4))
amp = 0
amp = amp + sn1_24*( + sr_ff(g_yuk_ch1_sn1_1_c,cml_35,1l1b_1))
amp = amp + snc1_13*( - sl_ff(g_yuk_ch1_sn1_1,1l1_25,cp1_4) &
  + sl_ff(g_yuk_ch1_sn1_1,cp1_45,1l1_2))
amp = amp + 1l1_25*( - f_vf(-qllep,a_34,1l1b_1) &
  - f_vaf(-gnclep(1),gnclep(2),z_34,1l1b_1))
amp = amp + 1l1b_15*( - f_srf(g_yuk_ch1_sn1_1_c,sn1_24,cml_3) &
  + f_vf(qllep,a_34,1l1_2) + f_vaf(gnclep(1),gnclep(2),z_34,1l1_2))
amp = amp + z_12*( - va_ff(-gczc_1l1(1),-gczc_1l1(2),cp1_45,cml_3) &
  + va_ff(-gczc_1l1(1),-gczc_1l1(2),cml_35,cp1_4))
amp = amp + a_12*( - v_ff(-qchar,cp1_45,cml_3) + v_ff(qchar,cml_35,cp1_4))
end function 1l1b1cp1cmla

```

28 fusions, 10 propagators, 12 diagrams

- ▶ readable code, can be edited for **exotic models** or **NLO** vertex functions

Remaining problem:

$$I(f) = \int_M d\mu(p) f(p)$$

1. **non-trivial geometry** of multi particle phase space

$$d\mu(p) = \delta^4(\sum_n k_n - P) \prod_n d^4k_n \delta(k_n^2 - m_n^2)$$

2. **ill-behaved function**, i. e. squared matrix element w/ kinematical cuts

$$f(p) = |T(k_1, k_2, \dots)|^2 \cdot C(k_1, k_2, \dots)$$

Choose a (pseudo-)random sequence  $p_g = \{p_1, p_2, \dots, p_N\}$  distributed according to  $d\mu_g(p) = g(p)d\mu(p)$ , then an estimator of  $I(f)$  is

$$E(f) = \left\langle \frac{f}{g} \right\rangle_g = \frac{1}{N} \sum_{i=1}^N \frac{f(p_i)}{g(p_i)}$$

The sampling error is estimated by the square root of the **variance**

$$V(f, g) = \frac{1}{N-1} \left( \left\langle \left\langle \left( \frac{f}{g} \right)^2 \right\rangle \right\rangle_g - \left\langle \frac{f}{g} \right\rangle_g^2 \right)$$

which **depends on  $g$** , even after averaging over  $p$ .

Conflicting goals for  $g$

1. make  $d\mu_g(p)$  simple enough, so that  $p_g$  can be generated w/ reasonable effort
2. choose  $g$  to minimize  $V(f, g)$ : **importance sampling** or **stratified sampling**
  - ▶ for multi particle phase space,  $d\mu(p)$  is very intricate and the generation of  $p_g$  is not trivial even for  $g(p) = 1/\text{Vol}(M)$ .
  - ⋮ RAMBO: elegant trick only for  $m_n = 0$  and  $g$  constant
  - ⋮ parametrizations  $]0, 1[^{\otimes \dim(M)} \rightarrow M$ : require compensation of bad Jacobians



## Practical considerations for particle physics:

- ∴ only a small number of different manifolds  $M$ :
  - ▶ number of particles 2, 3, 4, 5, 6, 7, ...
  - ▶ massless vs. massive particles
- ∴ it makes sense to invest manpower into an optimal treatment of the geometry, i. e.  $d\mu$
- ∴  $f$  changes w/
  - ▶ physics model du jour
  - ▶ physical parameters in the model
  - ▶ changing external cuts that can affect singular regions
- ∴ automatic and computer aided adaptive approaches, i. e. numerical optimizations, are appropriate

For over a quarter century, [Peter Lepage's VEGAS](#) has been the workhorse for adaptive Monte Carlo in particle physics.

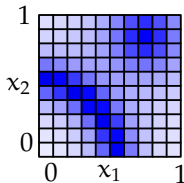


For simplicity

$$x \in M = ]0, 1[^{\otimes n}, \quad d\mu(x) = dx_1 \wedge dx_2 \wedge \dots \wedge dx_n$$

How can we implement efficiently a variable weight  $g$  in  $d\mu_g(x) = g(x)d\mu(x)$ ?

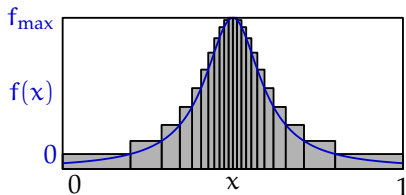
- ▶ optimization of expansion coefficients  $\alpha$  in  $g(x) = \sum_l \alpha_l g_l(x)$  popular, but not exciting for **generator generation**
  - ∴ selection of  $g_l$  requires expert human input
  - ∴ can't deal very efficiently w/ cuts
- ▶ fixed grid w/ variable weights



$x$  (i. e. characteristic functions as  $g_l$ )  
not useful at all

- ∴ locally fixed resolution can **not** adapt to the typical power law singularities over orders of magnitude

- ▶ alternative in **one** dimension: instead of adjusting weights of fixed bins, adjust density of equal weight bins
  - ∴ globally fixed resolution can nevertheless adjust locally over many orders of magnitude:



iteratively adjust grid, use estimates to either

- ▶ approximate  $f$  locally (importance sampling  $\implies$  event generation)
- ▶ or equidistribute the variance (stratified sampling  $\implies$  high precision integration).

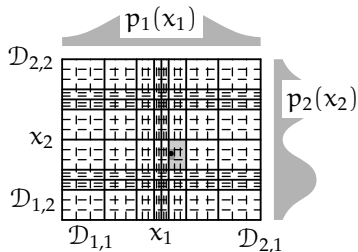
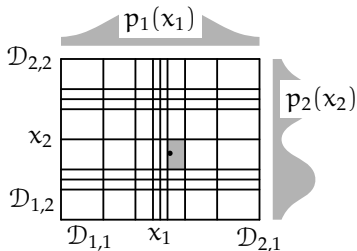
Factorized **ansatz**

$$g(\mathbf{x}) = g_1(x_1)g_2(x_2) \cdots g_n(x_n)$$

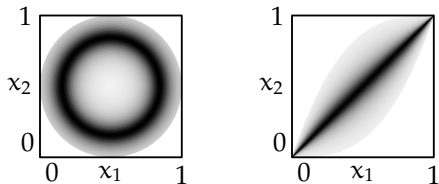
- ▶ guarantees hypercubic properties and simple one-dimensional formulae (w/ averaging over other dimensions)
- ▶ computational costs grow only **linearly** w/ the number of dimensions

VEGAS grid structure for **importance sampling**:

for genuinely **stratified** sampling, used in low dimensions:

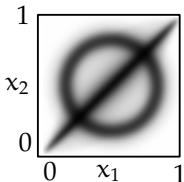


## VEGAS' factorized ansatz handles



separately after mappings.

- ▶ fails for overlapping singularities



which is the common case  
(for more than one diagram)

∴ adaptive multichannel approach

$$I(f) = \int_{\mathcal{M}} d\mu(\mathbf{p}) f(\mathbf{p})$$

$$I(f) = \sum_{i=1}^{N_c} \alpha_i \int_0^1 g_i(x) d^n x \frac{f(\phi_i(x))}{g(\phi_i(x))}$$

with

$$g = \sum_{i=1}^{N_c} \alpha_i \cdot (g_i \circ \phi_i^{-1}) \left| \frac{\partial \phi_i^{-1}}{\partial \mathbf{p}} \right|$$

- ▶ works with **factorized**  $g_i$  adapted by **VEGAS** and  $\alpha_i$  adapted by variance reduction.

- ▶ in general,  $g \circ \phi_i$  **does not factorize**, even if all  $g_i$  factorize.
- ▶  $\pi_{ij} = \phi_j^{-1} \circ \phi_i$ : coordinate transformations among coordinate systems in which different singularities factorize.
- ▶ pure geometry: **economical studies** of dependence on **cuts** and **parameters**
  - ∴  $\pi_{ij}$  universal and are calculated automatically by **WHIZARD**
  - ∴ **VEGAS** can optimize the  $g_i$  for each set of parameters and cuts

### However:

- ∴ singularity structure determined by **Feynman diagrams**
- ▶ **naive application** brings the **combinatorial explosion** in through the back door!
- ∴ **WHIZARD** uses heuristics to select the important channels
  - ▶ s-channel resonances
  - ▶ 1/t-poles for massless particlesand permutation symmetries to eliminate equivalent channels

- ▶ In Monte Carlo **integration** of integrals of observables  $f$  on phase space weighted with the cross section

$$\Sigma(f) = \int d\sigma(\Phi) f(\Phi) = \int d\Phi \frac{d\sigma}{d\Phi}(\Phi) f(\Phi)$$

it suffices to generate **weighted** phase space configurations

$$\mathcal{W} = \{(\phi_i, w_i)\}_{i \in \mathbb{N}}$$

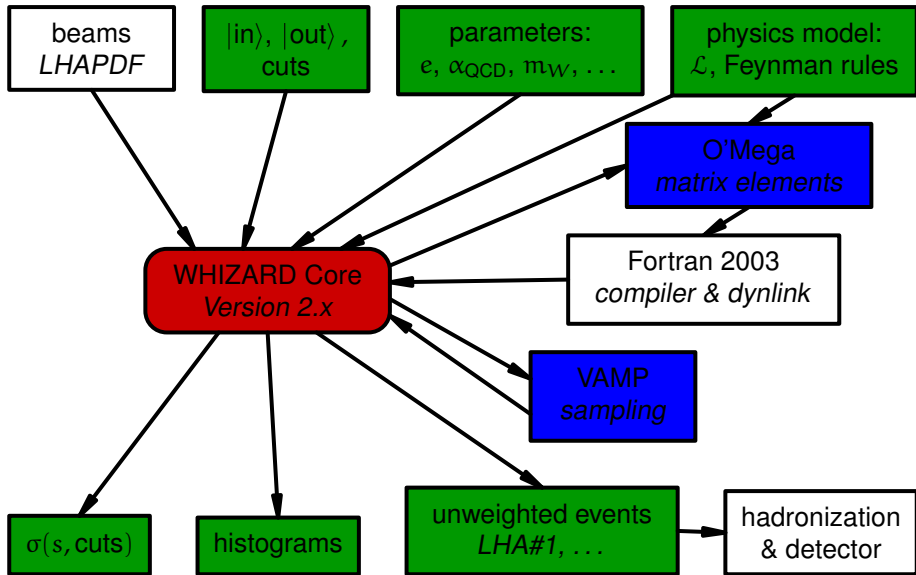
such that the integral is approximated by the **weighted sum**

$$\Sigma(f) \approx \sum_{i \in \mathbb{N}} w_i f(\phi_i).$$

- ▶ In the case of wildly varying cross sections, this is often **much simpler** than generating generate **unweighted** phase space configurations  $\mathcal{U} = \{\phi_i\}_{i \in \mathbb{N}}$  with

$$\Sigma(f) \approx \sum_{i \in \mathbb{N}} f(\phi_i),$$

- ▶ ... by they **are** required for realistic **detector simulations**.







- ▶ the **names** of particles and couplings depend on the model and can be looked up in the share/whizard/models directory, e. g. in  
`/home/HEP/toolbox-1.1.7/whizard/share/whizard/models/SM.mdl`  
for the standard model

- ▶ **input parameters**

```
parameter GF      = 1.16639E-5   # Fermi constant
parameter mZ      = 91.1882      # Z-boson mass
parameter mW      = 80.419       # W-boson mass
parameter mH      = 125          # Higgs mass
parameter alphas  = 0.1178       # Strong coupling constant (Z point)
parameter me      = 0.000510997 # electron mass
...
```

- ▶ **derived parameters**

```
derived v        = 1 / sqrt (sqrt (2.) * GF)   # v (Higgs vev)
derived cw       = mW / mZ                     # cos(theta-W)
...
```

- ▶ **particles**

```
particle D_QUARK 1 parton
  spin 1/2 charge -1/3 isospin -1/2 color 3
  name d down
  anti dbar D "d~"
  tex_anti "\\bar{d}"
```

## ▶ WHIZARD input xsect\_eemm.sin

```

model = SM
process mumu = e1, E1 => e2, E2
compile
sqrt_s = 90 GeV
beams = e1, E1
integrate (mumu) { iterations = 2:1000, 3:5000 }

```

## ▶ run WHIZARD

```
$ /home/HEP/toolbox-1.1.7/whizard/bin/whizard xsect_eemm.sin
```

## ▶ console output

```
| Integrating process 'mumu':
|=====|
| It      Calls  Integral[fb]  Error[fb]   Err[%]   Acc  Eff[%]  Chi2  N[It] |
|=====|
| 1       1000   1.0605313E+06  8.45E+02   0.08    0.03*  60.96  |
| 2       1000   1.0597762E+06  5.35E+02   0.05    0.02*  61.25  |
|-----|
| 2       2000   1.0599922E+06  4.52E+02   0.04    0.02   61.25  0.57  2  |
|-----|
| 3       5000   1.0601102E+06  1.34E+02   0.01    0.01*  61.12  |
| 4       5000   1.0599916E+06  9.36E+01   0.01    0.01*  78.78  |
| 5       5000   1.0598832E+06  7.58E+01   0.01    0.01*  67.98  |
|-----|
| 5      15000   1.0599559E+06  5.39E+01   0.01    0.01   67.98  1.19  3  |
|-----|
| 5      15000   1.0599559E+06  5.39E+01   0.01    0.01   67.98  1.19  3  |
|=====|
```

## ▶ WHIZARD input

```

model = SM
alias parton = u:U:d:D:g
process tt = parton, parton => b, Wp, B, Wm
compile
sqrts = 8 TeV
beams = p, p => pdf_builtin
cuts = all Pt > 10 GeV [b:Wp:B:Wm]
integrate (tt) { iterations = 2:10000, 5:20000 }

```

## ▶ console output

```

[1/5] gl gl -> b W+ bbar W- ... allowed.
[2/5] d dbar -> b W+ bbar W- ... allowed.
[3/5] dbar d -> b W+ bbar W- ... allowed.
[4/5] u ubar -> b W+ bbar W- ... allowed.
[5/5] ubar u -> b W+ bbar W- ... allowed.

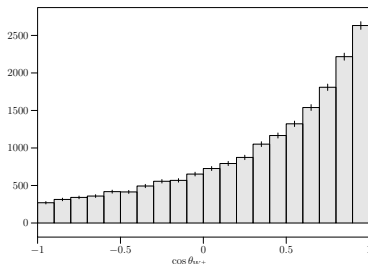
```

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	10000	9.9611255E+04	2.89E+04	29.06	29.06*	2.02		
2	10000	1.1805759E+05	1.65E+04	13.95	13.95*	2.63		
2	20000	1.1354494E+05	1.43E+04	12.61	17.83	2.63	0.31	2
3	20000	1.1757383E+05	1.64E+04	13.91	19.67	1.45		
4	20000	1.2032373E+05	8.96E+03	7.45	10.53*	2.32		
5	20000	1.1948028E+05	5.69E+03	4.76	6.73*	3.09		
6	20000	1.2279117E+05	4.20E+03	3.42	4.84*	3.86		
7	20000	1.2270293E+05	3.64E+03	2.97	4.20*	3.42		
7	100000	1.2190186E+05	2.36E+03	1.94	6.13	3.42	0.09	5
7	100000	1.2190186E+05	2.36E+03	1.94	6.13	3.42	0.09	5

time estimate for generating 10000 unweighted events: 14m:18s

```
▶ model = SM
process ww = e1, E1 => Wp, Wm
compile
$x_label = "$\cos\theta_{W^+}$"
?draw_errors = true
histogram costh (-1, 1, 0.1)
sqrts = 180 GeV
beams = e1, E1
luminosity = 1000 / 1 pbarn
integrate (ww) { iterations = 2:1000, 5:50000 }
simulate (ww) { analysis = record costh (eval cos (Theta) [Wm]) }
compile_analysis { $out_file = "ww.dat" }
```

▶



$pp \rightarrow gg \rightarrow \mu^- \bar{\nu}_e u \bar{d}$  as a **Les Houches Event File (LHEF)**:

```

model = SM
process hWW = g, g => e2, N2, u, D
compile
sqrts = 14 TeV
beams = p, p => pdf_builtin
sample_format = lhef
simulate (hWW) { n_events = 1 }

```

```
<LesHouchesEvents version="1.0">
```

```
<header>
```

```
<generator_name>WHIZARD</generator_name>
```

```
<generator_version>2.1.1</generator_version>
```

```
</header>
```

```
<init>
```

```

      2212      2212  7000.0000000000000000  7000.0000000000000000  -1      -1      -1      -1
420.46515665414375  249.25905165127301  1.00000000000000000000  1

```

```
</init>
```

```
<event>
```

```

      6      1  1.00000000000000000000  136.31643354892731  -1.00000000000000000000  0.11780000000000000000
21      -1      0      0      501      502  0.00000000000000000000  0.00000000000000000000  64.59551
21      -1      0      0      503      501  0.00000000000000000000  0.00000000000000000000  -71.91741
13      1      1      2      0      0  16.068546922717953  -15.231242240310792  28.41895
-14     1      1      2      0      0  -35.486359371670360  14.938719772529632  -38.73337
2      1      1      2      503      0  14.687109514917077  13.162760335459950  -13.89070
-1     1      1      2      0      502  4.7307029340353255  -12.870237867678787  16.88323

```

```
</event>
```

```
</LesHouchesEvents>
```

pp  $\rightarrow$  gg  $\rightarrow$   $\mu^- \bar{\nu}_e u \bar{d}$  as a HepMC file:

```
model = SM
process hWW = g, g => e2, N2, u, D
compile
sqrts = 14 TeV
beams = p, p => pdf_builtin
sample_format = hepmc
simulate (hWW) { n_events = 1 }
```

HepMC::Version 2.06.09

HepMC::IO\_GenEvent-START\_EVENT\_LISTING

```
E 1 -1 1.3631643354892731e+02 1.1780000000000000e-01 -1.0000000000000000e+00 1 0 3 10001 10002 0 4 1.0000000000000000e+00 7.08284436471
N 4 "0" "1" "2" "3"
U GEV MM
C 4.2046515665414375e+02 2.4925905165127301e+02
V -1 0 0 0 0 0 1 2 0
P 10001 2212 0 0 7.0000000000000000e+03 7.0000000000000000e+03 0 4 0 0 -1 0
P 10003 21 0 0 6.4595518178633455e+01 6.4595518178633455e+01 0 3 0 0 -3 2 1 1 2 2
P 10005 93 0 0 6.9354044818213670e+03 6.9354044818213670e+03 0 3 0 0 2 1 2 2 1
V -2 0 0 0 0 0 1 2 0
P 10002 2212 0 0 -7.0000000000000000e+03 7.0000000000000000e+03 0 4 0 0 -2 0
P 10004 21 0 0 -7.1917412304487158e+01 7.1917412304487158e+01 0 3 0 0 -3 2 1 3 2 1
P 10006 93 0 0 -6.9280825876955132e+03 6.9280825876955132e+03 0 3 0 0 2 1 1 2 3
V -3 0 0 0 0 0 0 4 0
P 10007 13 1.6068546922717953e+01 -1.5231242240310792e+01 2.8418956865688791e+01 3.6025507816263499e+01 1.0565838899979969e-01 1 0 0 0
P 10008 -14 -3.5486359371670360e+01 1.4938719772529632e+01 -3.8733375441153328e+01 5.4614296873280757e+01 0 1 0 0 0 0
P 10009 2 1.4687109514917077e+01 1.3162760335459950e+01 -1.3890707675246947e+01 2.4123043035054472e+01 4.7683715820312500e-07 1 0 0 0 1
P 10010 -1 4.7307029340353255e+00 -1.2870237867678787e+01 1.6883232124857773e+01 2.1750082758521934e+01 4.1295309247228556e-07 1 0 0 0
HepMC::IO_GenEvent-END_EVENT_LISTING
```

```

model = SM
process hWW = g, g => e2, N2, u, D
compile
sqrts = 14 TeV
beams = p, p => lhpdf
sample_format = hepmc, lhef
integrate (hWW) { iterations = 1:1000 }
?ps_fsr_active = true
?ps_isr_active = true
?hadronization_active = true
?ps_use_PYTHIA_shower = true
simulate (hWW) { n_events = 1 }

```

```

<event>
310      1  1.0000000000000000      103.17887552082534      -1.0000000000000000      0.117800000000000000
21       -1      0      0      501      502      0.0000000000000000      0.0000000000000000      22.60643
21       -1      0      0      502      503      0.0000000000000000      0.0000000000000000      -117.7306
93        3      0      0      502      501      0.0000000000000000      0.0000000000000000      6977.393
93        3      0      0      503      502      0.0000000000000000      0.0000000000000000      -6882.269
-14       1      1      2      0      0      -27.571846966921186      11.001180372709559      -41.40772
13        1      1      2      0      0      8.2564499233431601      -44.666087683248804      -32.04015
22        1      1      2      0      0      -0.60053536107337491      -4.8109348517593622      -4.105068
211       1      1      2      0      0      0.18025914014147792      0.23261429271929662      0.4834233
-211      1      1      2      0      0      -0.83369544321947398      0.25818955197077087      1.536586
2112      1      1      2      0      0      -0.34678766523372612      2.0518746878398826      0.9660215
-2212     1      1      2      0      0      -7.7893814158812913E-002      1.0446949824101728      2.571061
2212     1      1      2      0      0      0.46265670377713941      2.0358472993730437      3.421707
-2212     1      1      2      0      0      0.71417103301686868      1.6232688240042823      2.751320
211       1      1      2      0      0      2.7762381422163100      3.4505228032902853      9.001631
-211      1      1      2      0      0      0.65565726609788855      0.75407218229487294      1.674591
211       1      1      2      0      0      1.1650307289886463      0.58079731055743034      4.041057
-211      1      1      2      0      0      0.10194216370164123      -4.8888582506378710E-002      7.609617
-321     1      1      2      0      0      1.2505517529312573      0.24270629084564452      -1.790215

```

...



- ▶ Compute the cross section for **single top production**

$$pp \rightarrow tb$$

in the standard model at the Tevatron and at the 14 TeV LHC.

- ▶ Plot the rapidity distribution of the produced top quark.
- ▶ Include the two-particle decay of the top quark.
- ▶ Include the irreducible backgrounds.
- ▶ Estimate the influence of bottom-quark parton distribution functions.