Stefan Liebler

SusHi: Cross sections in gluon fusion and bottom-quark annihilation

German-Egyptian school of
particle physics
Zewail City, Cairo, Egypt

25. February 2013

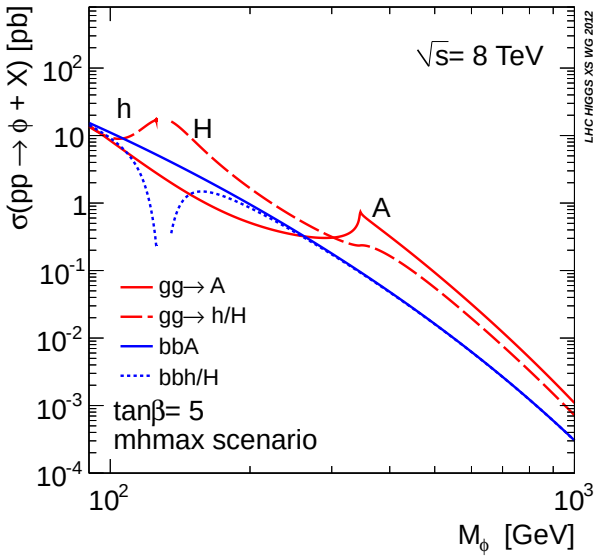
Theoretische Teilchenphysik
Fachbereich C
Bergische Universität Wuppertal

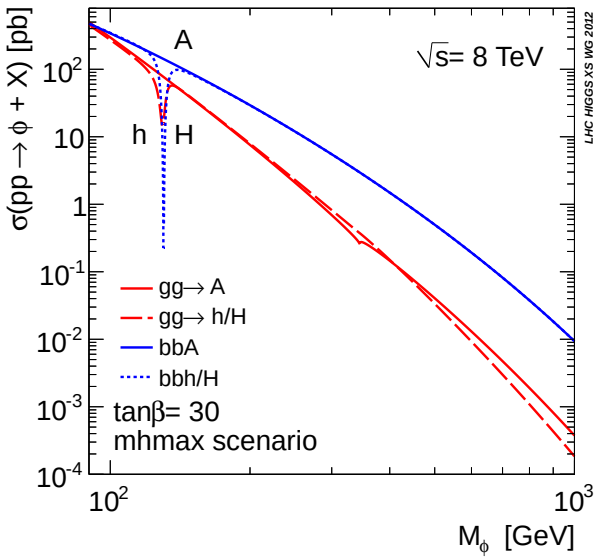
DFG
PHYSICS AT THE TERA SCALE
Helmholtz Alliance

Outline

1. Neutral MSSM Higgs XS - Theory
2. SusHi - A Fortran code
3. Conclusion

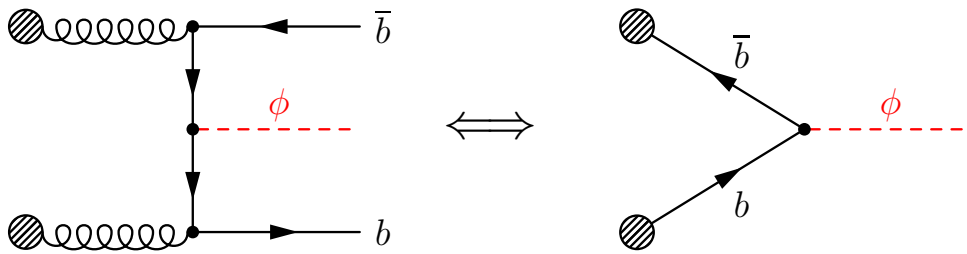
Neutral Higgs production in the MSSM:



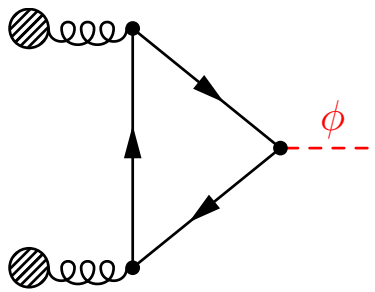


Production cross sections according to YR1 [arXiv:1101.0593]

Bottom-quark annihilation:



Gluon fusion:



Gluon fusion at LO using $\tau_\phi = m_\phi^2/s$:

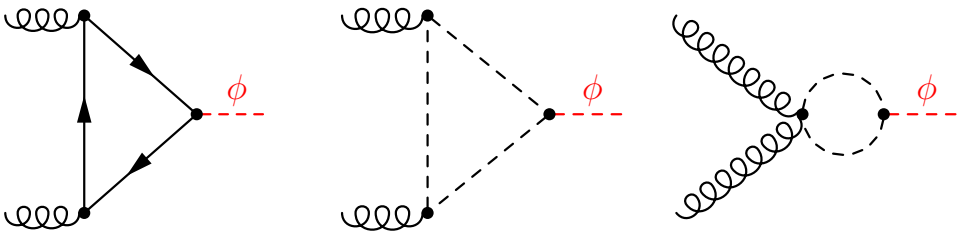
$$\sigma(pp \to \phi + X) = \sigma_0^\phi \tau_\phi \frac{d\mathcal{L}^{gg}}{d\tau_\phi}$$

Partonic $\to$ Hadronic XS:

$$\frac{d\mathcal{L}^{gg}}{d\tau} = \int_\tau^1 \frac{dx}{x} g(x) g(\tau/x)$$

LO partonic cross section (XS):

$$\sigma_0^\phi = \frac{G_F \alpha_s^2}{288\sqrt{2}\pi} |\mathcal{A}^\phi|^2$$



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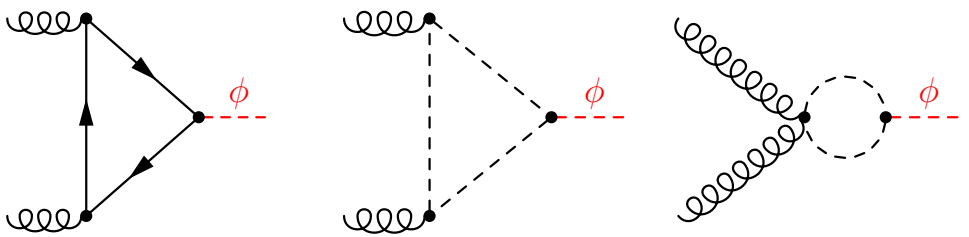


$$\mathcal{A}^\phi = \sum_{q \in \{t,b\}} \left(a_q^{\phi,(0)} + a_{\tilde{q}}^{\phi,(0)} \right) \quad \text{with} \quad a_q^{\phi,(0)} = g_q^\phi \frac{3\tau_q}{2} (1 + (1 - \tau_q^\phi) f(\tau_q^\phi))$$

Quark contributions

Squark contributions

$$\tilde{a}_q^{\phi,(0)} = -\frac{3\tau_q^\phi}{8} \sum_{i=1}^{2} g_{\tilde{q}ii}^\phi (1 - \tau_{\tilde{q}i}^\phi f(\tau_{\tilde{q}i}^\phi))$$

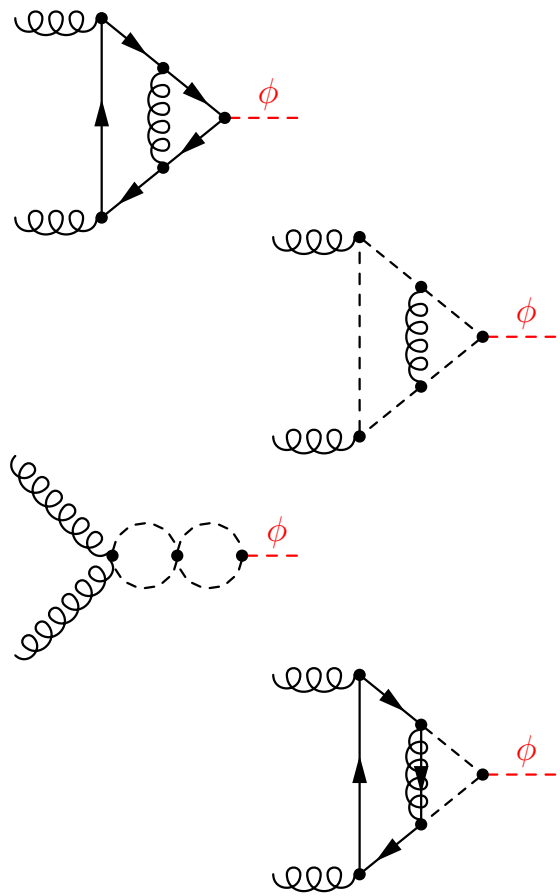
$$\tau_q^\phi = 4m_q^2/m_\phi^2, \quad \tau_{\tilde{q}i}^\phi = 4m_{\tilde{q}i}^2/m_\phi^2, \quad f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4} \left(\log \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi \right)^2 & \tau < 1 \end{cases}$$

Gluon fusion at NLO using $\tau_\phi = m_\phi^2/s$:

$$\sigma(pp \to \phi + X) = \sigma_0^\phi \tau_\phi \frac{d\mathcal{L}^{gg}}{d\tau_\phi}\left[1 + C^\phi \frac{\alpha_s}{\pi}\right] + \Delta\sigma_{gg}^\phi + \Delta\sigma_{gq}^\phi + \Delta\sigma_{q\bar{q}}^\phi$$

NLO amplitudes $a_q^{\phi,(1)}$ and $a_{\tilde{q}}^{\phi,(1)}, a_{\tilde{q}\tilde{g}}^{\phi,(1)}$ entering C^ϕ:

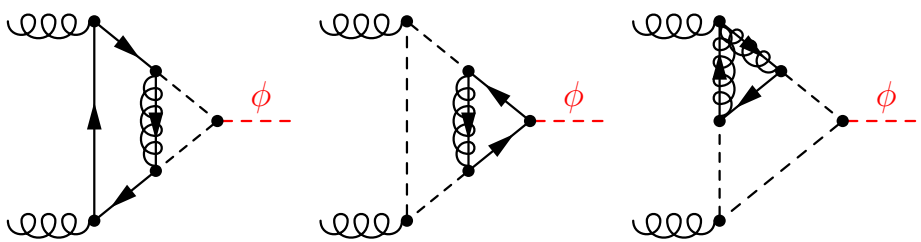
- ▷ gluon-quark: known analytically (higher orders)
  [Spira Djouadi Graudenz Zerwas '95; Harlander Kant '05; . . .]
- ▷ gluon-squark: known analytically/numerically
  [Anastasiou Beerli Bucherer Daleo Kunszt '06;
  Aglietti Bonciani Degrassi Vicini '06; Mühlleitner Spira '06;
  Bonciani Degrassi Vicini '07]
- ▷ gluino-squark-quark contributions:
  semi-analytically known $\longrightarrow$ no public code
  [Anastasiou Beerli Daleo '08; Mühlleitner Spira Rzehak '10]



Problem with gluino-quark-squark contributions:
up to 5 different masses in the 2-loop integrals
$\longrightarrow$ think of reasonable approximations

Knowledge of NLO amplitudes $a_q^{\phi,(1)}$ (12 diag.) and $a_{\tilde{q}}^{\phi,(1)}, a_{\tilde{q}\tilde{g}}^{\phi,(1)}$ (59 diag.):
Five different masses in the diagram: $m_q, m_{\tilde{q}1}, m_{\tilde{q}2}, m_{\tilde{g}}, p^2 = m_\phi^2$

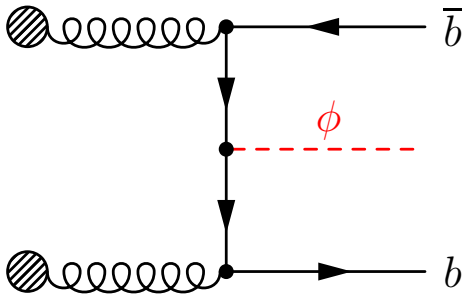
- Taylor expansion in external momentum/Higgs mass:
  $m_\phi \ll m_q, m_{\tilde{q}1}, m_{\tilde{q}2}, m_{\tilde{g}}$
  $\longrightarrow$ top-stop-gluino contribution using $m_\phi \ll m_t, m_{\tilde{t}1}, m_{\tilde{t}2}, m_{\tilde{g}}$
  [Harlander Steinhauser '03 '04 + Hofmann '05; Degrassi Slavich '08]
- Expansion in SUSY masses: $m_\phi, m_q \ll m_{\tilde{q}_1}, m_{\tilde{q}_2}, m_{\tilde{g}}$
  $\longrightarrow$ bottom-sbottom-gluino for $m_{\tilde{q}_1} = m_{\tilde{q}_2} = m_{\tilde{g}}$ [Harlander Hofmann Mantler '10]
  $\longrightarrow$ quark-squark-gluino [Degrassi Slavich '10 + Di Vita '11 '12]



3-loop top-stop-gluino contributions known for $m_\phi \ll m_t, m_{\tilde{t}1}, m_{\tilde{t}2}, m_{\tilde{g}}$
[Pak Steinhauser Zerf '10 '12]

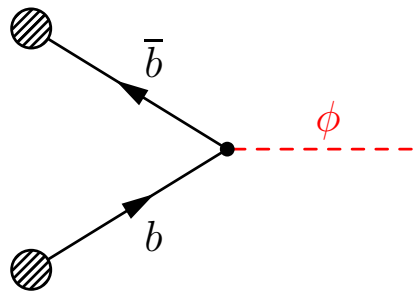
Bottom-quark annihilation:
$pp \to (b\bar{b})\phi + X$ for enhanced couplings g_b^ϕ to b-quarks relevant $\to$ MSSM!

4 flavour scheme (4FS)
Collinear logarithms $\propto \log(m_b/m_\phi)$

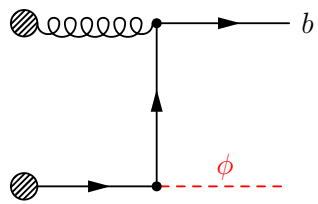


$\Longleftrightarrow$

5 flavour scheme (5FS)
Resummmation of logarithms
b quarks as partons



Calculation of inclusive cross section at NNLO in the 5FS:
`bbh@nnlo` [Harlander Kilgore '03]



Distributions for Higgs+jet(s) production in the 5FS [Harlander Ozeren Wiesemann '10]

$\to$ Bottom-quark annihilation included in SusHi

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What programs are on the market for gluon fusion/bottom-quark annihilation in the MSSM?

- Gluon fusion: SM differential/inclusive cross sections by
  Higlu [Spira Djouadi Graudenz Zerwas '95]
  $\rightarrow$ MSSM results with quarks implemented
- Gluon fusion/Bottom-quark annihilation: MSSM weighted results by
  FeynHiggs [Frank Degrassi Hahn Heinemeyer Hollik Rzehak Slavich Weiglein Williams]

However there is no code for the squark contributions in gluon fusion and differential cross sections for bottom-quark annihilation publicly available.

The Fortran code SusHi (Supersymmetric Higgs production) includes squark contributions and combines various other codes.

What does SusHi take into account in case of gluon fusion?

1. Calculate XS with quark contributions using resummation (later):

$$\sigma_{\text{NLO}}^{\text{MSSM}} = (g_t^\phi)^2 \sigma_{\text{SM},t}^{\text{NLO}} + (g_b^\phi)^2 \sigma_{\text{SM},b}^{\text{NLO}} + g_t^\phi g_b^\phi \sigma_{\text{SM},tb}^{\text{NLO}}$$

2. Add squark contributions of third generation:
   - top-stop-gluino contributions:
     Expansion in $m_\phi \ll m_t, m_{\tilde{t}1}, m_{\tilde{t}2}, m_{\tilde{g}}$ for $\phi = h$
     Expansion in SUSY masses $m_\phi, m_t \ll m_{\tilde{t}1}, m_{\tilde{t}2}, m_{\tilde{g}}$ for $\phi \in \{H, A\}$
   - sbottom-bottom-gluino contributions
     in SUSY masses expansion $m_b < m_\phi \ll m_{\tilde{b}1}, m_{\tilde{b}2}, m_{\tilde{g}}$

3. Add NNLO top effects and electroweak contributions:

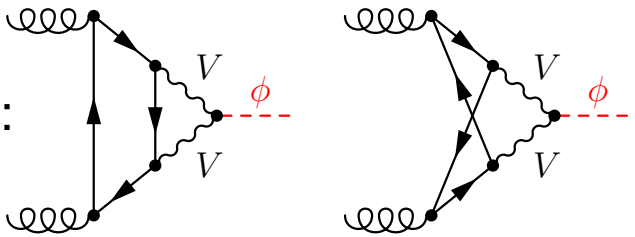
$$\sigma(pp \to \phi + X) = \sigma_{\text{NLO}}^{\text{MSSM}}(1 + \delta_{\text{EW}}^{lq}) + (g_t^\phi)^2 \left(\sigma_{\text{NNLO}}^{t,\text{SM},0} - \sigma_{\text{NLO}}^{t,\text{SM},0}\right)$$

Electroweak contributions by light quarks

ggh@nnlo

Electroweak contributions by light quarks: [Aglietti Bonciani Degrassi Vicini '04 '10]

Relevant diagrams with $V \in \{W, Z\}$:



Definition of SUSY electroweak correction factor:

$$\delta_{\rm EW}^{lq} = \frac{\alpha_{\rm EW}}{\pi} 2{\rm Re}(\mathcal{A}^{\phi}\mathcal{A}^{\phi,{\rm EW}})/|\mathcal{A}^{\phi}|^2$$

$$\mathcal{A}^{\phi,{\rm EW}} = -\frac{3}{8}\frac{x_W}{s_W^2}\left[\frac{2}{c_W^2}\left(\tfrac{5}{4} - \tfrac{7}{3}s_W^2 + \tfrac{22}{9}s_W^4\right)A[x_Z] + 4A[x_W]\right] g_V^{\phi}$$

Complex mass scheme: $x_V = (m_V - i\frac{\Gamma_V}{2})^2/m_\phi^2$

Supersymmetry enters g_V^{ϕ}:
$g_V^h = \sin(\beta-\alpha), \quad g_V^A = 0, \quad g_V^H = \cos(\beta-\alpha)$

For moderate masses of SM-like Higgs results in similiar correction as SM electroweak correction factor [Actis Passarino Sturm Uccirati '08].

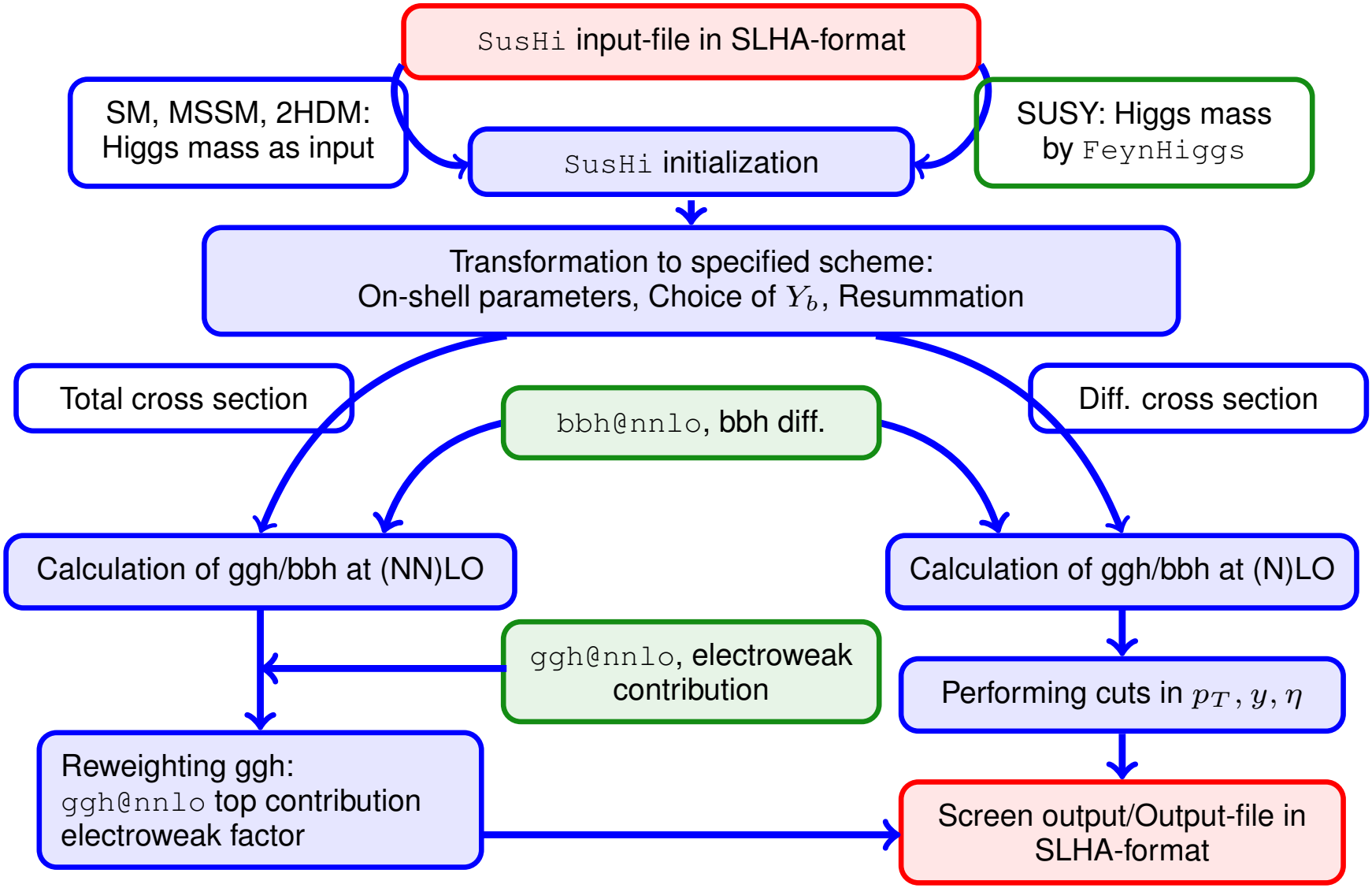
SusHi contains/links to:

- Higgs mass calculation with FeynHiggs
- PDF sets from LHAPDF [Whalley Bourilkov Group]
- inclusive/differential cross sections for $(gg \to \phi)$@NLO including squark contributions from [Harlander Steinhauser '03 '04; Degrassi Slavich '08 '10 + Di Vita '11 '12; Harlander Hofmann Mantler '10]
- NNLO effects from ggh@nnlo
- electroweak effects [Actis Passarino Sturm Uccirati '08 '09, Vicini et al. '04 '10]
- inclusive/differential cross sections for $(bb \to \phi)$@(N)NLO using reweighted results from bbh@nnlo and code from [Harlander Ozeren Wiesemann '10]

How to install?
After downloading and extracting SusHi adjust some links in the Makefile:

- Link to LHAPDF libraries, implying that LHAPDF should be installed
- Link to FeynHiggs, if used for MSSM Higgs masses

Compile using Fortran compiler (tested: gfortran, ifort).
Provide a SUSHI dish (an input file)!


SusHi input-file in SLHA-format
SM, MSSM, 2HDM: Higgs mass as input
SusHi initialization
SUSY: Higgs mass by FeynHiggs
Transformation to specified scheme: On-shell parameters, Choice of Y_b, Resummation
Total cross section
bbh@nnlo, bbh diff.
Diff. cross section
Calculation of ggh/bbh at (NN)LO
Calculation of ggh/bbh at (N)LO
ggh@nnlo, electroweak contribution
Performing cuts in p_T, y, η
Reweighting ggh: ggh@nnlo top contribution electroweak factor
Screen output/Output-file in SLHA-format

Specify model, collider, $\sqrt{s}$, Higgs, processes

Give SM input values

Give MSSM values:

- Specify Higgs sector
- Use `Block FEYNHIGGS`

Specify PDF sets

Specify VEGAS input

```
Block SUSHI
  1    1          # model: 0 = SM, 1 = MSSM, 2 = 2HDM
  2    0          # 0 = scalar (h), 1 = pseudoscalar (A), 2 = scalar (H)
  3    0          # collider: 0 = p-p, 1 = p-pbar
  4    8000.d0    # center-of-mass energy in GeV
  5    2          # order ggh: -1 = off, 0 = LO, 1 = NLO, 2 = NNLO
                  #              3 = ~NNLO stop
  6    2          # order bbh: -1 = off, 0 = LO, 1 = NLO, 2 = NNLO
  7    1          # electroweak cont. for ggh:
                  # 0 = no, 1 = light quarks at NLO, 2 = SM EW factor
Block SMINPUTS             # Standard Model inputs
  1    1.27934000e+02     # alpha_em^(-1)(MZ) SM MSbar
.....
Block MINPAR               # SUSY breaking input parameters
  3    1.00000000e+01     # tanb
Block EXTPAR
  3    800.d0             # M_3
 11    2000.d0            # A_t
.....
Block ALPHA
 -8.3691204e-01           # mixing in Higgs sector
Block MASS
 25    120.76695d0        # Higgs mass h
.....
.....
Block PDFSPEC
  1    MSTW2008lo68cl.LHgrid              # name of pdf (lo)
  2    MSTW2008nlo68cl.LHgrid             # name of pdf (nlo)
  3    MSTW2008nnlo_asmzrange.LHgrid      # name of pdf (nnlo)
  4    0          # set number
Block VEGAS
  1    10000      # number of points
  2    5          # number of iterations
  3    10         # print: 0 = no output, 1 = prettyprint, 10 = table
```

Example output file

All relevant internal masses and parameters given.

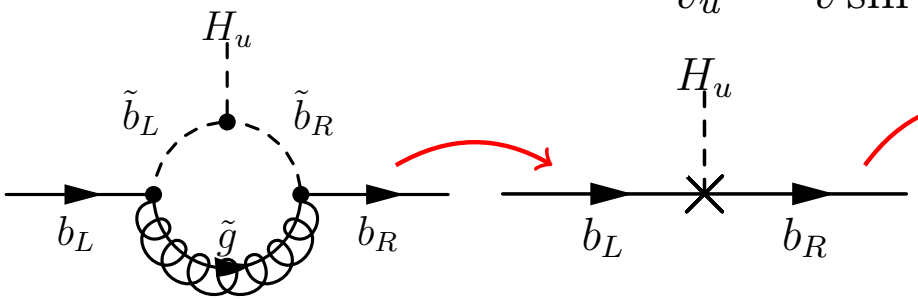
```
Block SUSHIggh # Bon appetit
         1      1.59566551E+01   # ggh XS in pb
Block SUSHIbbh # Bon appetit
         1      2.67975811E-01   # bbh XS in pb
Block XSGGH # ggh MSSM-Cross sec. in pb (w/o EW)
         2      1.13849524E+01   # NLO
        21      1.14675980E+01   # NLO gg
        22     -1.02733041E-01   # NLO qg
        23      2.00874475E-02   # NLO qq
Block XSGGHEFF # ggh MSSM-Cross sec.
         1      1.49056433E+01   # ggh@NLO MSSM
         2      1.88154287E+01   # ggh@NNLO MSSM
         3      5.81396602E-02   # electroweak factor
Block XSBBH # bbh MSSM-Cross sec. in pb
         1      3.72508379E-01   # LO
         2      3.35738356E-01   # NLO
         3      2.67975811E-01   # NNLO
......
Block MASSOUT
         5      4.21300000E+00   # m_b(m_b), MSbar
         6      1.73200000E+02   # m_t(pole)
        23      9.11876000E+01   # m_Z
        24      8.03980311E+01   # m_W
        25      1.23484421E+02   # MSSM-Mh in GeV
   1000005      4.93350465E+02   # sbottom1 mass in GeV
   2000005      5.18897088E+02   # sbottom2 mass in GeV
   1000006      3.23616710E+02   # stop1 mass in GeV
   2000006      6.71663496E+02   # stop2 mass in GeV
```

Interlude: Resummation of large $\tan\beta$-enhanced terms in the MSSM

$$\mathcal{L} \supset -Y_t H_u Q t_R + Y_b H_d Q b_R$$

Using $\langle H_u \rangle = v_u, \langle H_d \rangle = v_d$ and $v_d^2 + v_u^2 = v^2, \tan\beta = v_u/v_d$ we define

$$Y_t = \frac{m_t}{v_u} = \frac{m_t}{v \sin\beta}, \qquad Y_b = \frac{m_b}{v_d} = \frac{m_b}{v \cos\beta}$$



$$\mathcal{L}^{\text{eff}} \supset Y_b H_d Q b_R - \tilde{Y}_b H_u^* Q b_R$$

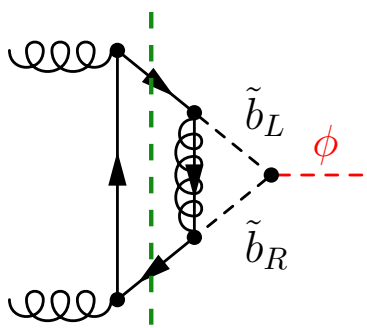
$$\Delta_b = \frac{\tilde{Y}_b v_u}{Y_b v_d} =: \epsilon \tan\beta$$

The effective Lagrangian motivates:

$$m_b = Y_b v_d + \tilde{Y}_b v_u = Y_b v_d (1 + \epsilon \tan\beta)$$

$$\rightarrow \quad Y_b = \frac{m_b}{v_d(1+\Delta_b)}$$

This replacement implies a resummation of large $\tan\beta$-enhanced terms:



Specify m_b/Y_b

$\tan\beta$-resummation

Renormalization scheme

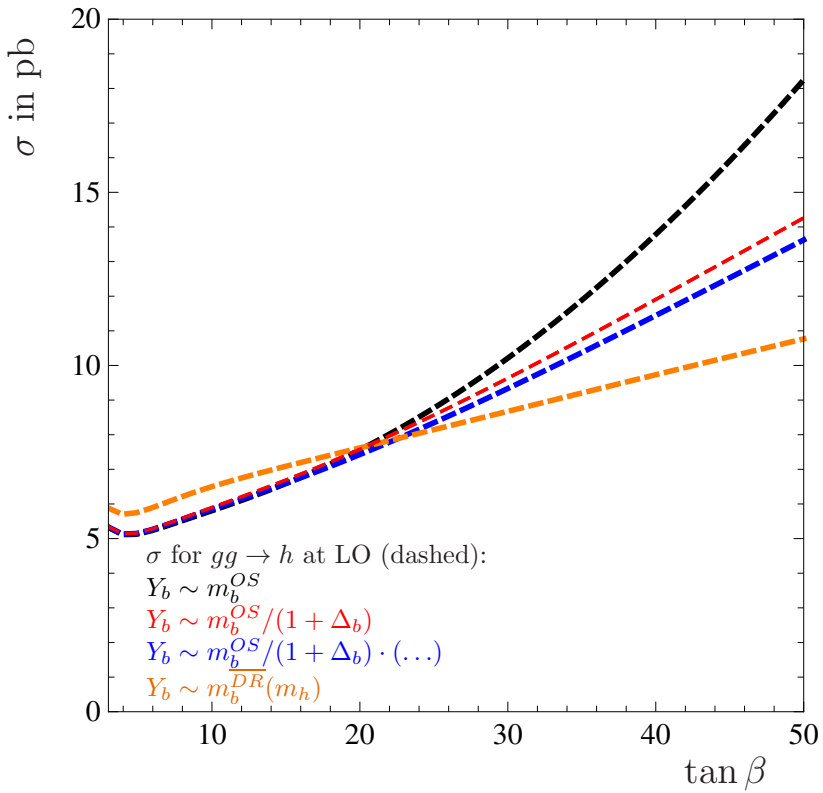
```
Block RENORMBOT # Renormalization of the bottom sector
  1    0          # m_b used for Y_b:  0 = OS, 1 = MSbar(m_b)...
  2    1          # tan(beta)-res. of Y_b: 0 = no, 1 = naive, 2 = full...
  3    1          # Delta_b: Take Delta_b from FeynHiggs:0 = no, 1 = yes
  4    4.75d0     # Fixed value of m_b^OS
Block RENORMSBOT # Renormalization of the sbottom sector
  1    2          # m_b:      0 = OS, 1 = DRbar, 2 = dep; recommended: 2
  2    0          # A_b:      0 = OS, 1 = DRbar, 2 = dep; recommended: 0
  3    0          # theta_b: 0 = OS, 1 = DRbar          ; recommended: 0
```

Discussion of different effects:

- Choice of the bottom Yukawa coupling Y_b
  - $Y_b^{h,H} \propto m_b^{\text{OS}}$
  - $Y_b^{h,H} \propto \frac{m_b^{\text{OS}}}{1+\Delta_b}$
  - $Y_b^{h} \propto \frac{m_b^{\text{OS}}}{1+\Delta_b}\left(1-\Delta_b\frac{\cot\alpha}{\tan\beta}\right), \quad Y_b^{H} \propto \frac{m_b^{\text{OS}}}{1+\Delta_b}\left(1+\Delta_b\frac{\tan\alpha}{\tan\beta}\right)$
  - $Y_b^{h,H} \propto \frac{m_b^{\overline{\text{MS}}}(\mu_b)}{1+\Delta_b}$ with $\mu_b \in \{m_b, m_{h,H}\}$

Cross section for $gg \to h$

$$M_{\tilde{q}_{L,R}} = 750\text{GeV}, A_q = 2\text{TeV},$$
$$m_{\tilde{g}} = 800\text{GeV}, \mu = 500\text{GeV},$$
$$m_A = 150\text{GeV}$$



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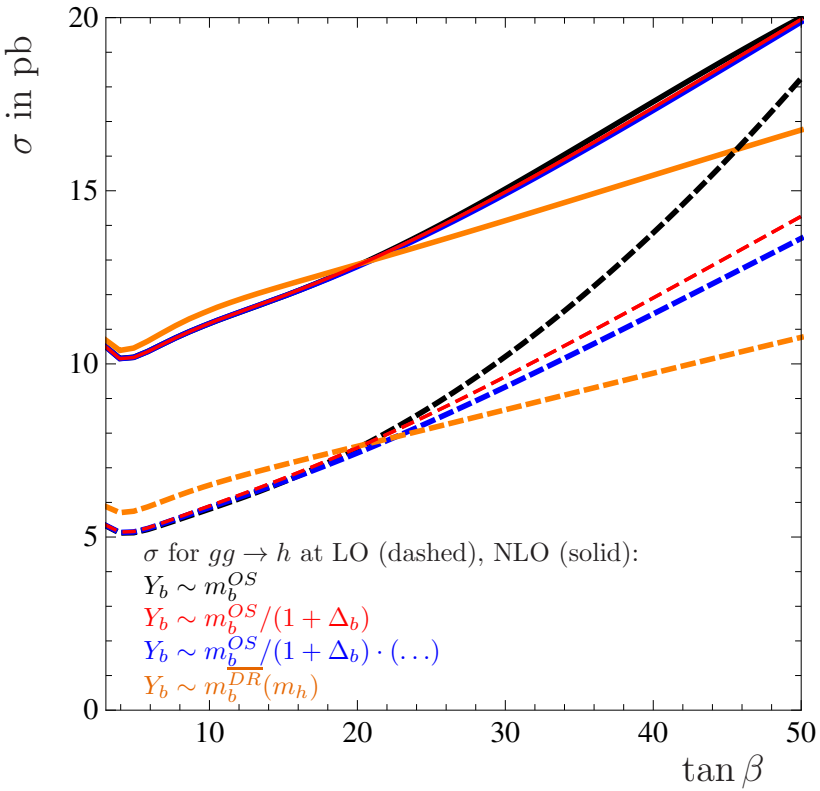
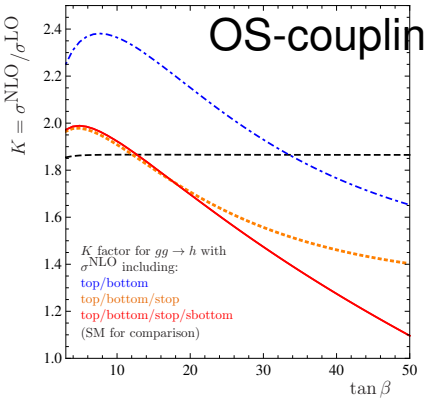
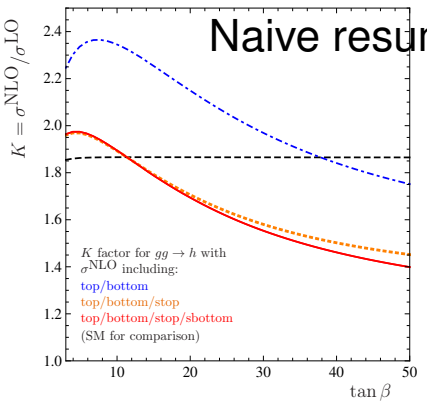
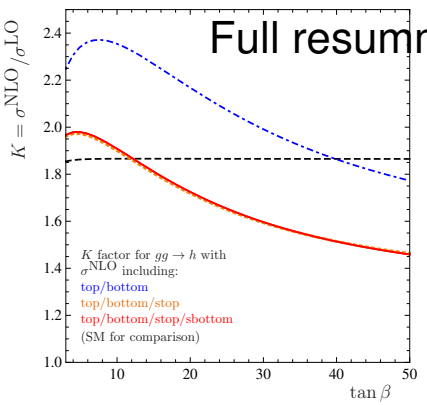








Inclusive/Differential XS

```
Block  DISTRIB
  1    0        # distribution : 0 = sigma_total, 1 = dsigma/dpt,
                #                2 = dsigma/dy,    3 = d^2sigma/dy/dpt
                #               (values for pt and y: 22 and 32)
  2    0        # pt-cut: 0 = no, 1 = pt > ptmin, 2 = pt < ptmax,
                #         3 = ptmin < pt < ptmax
 21    30.d0    # minimal pt-value ptmin in GeV
 22    100.d0   # maximal pt-value ptmax in GeV
  3    0        # rapidity-cut: 0 = no, 1 = Abs[y] < ymax,
                #    2 = Abs[y] > ymin, 3 = ymin < Abs[y] < ymax
 31    0.5d0    # minimal rapidity ymin
 32    1.5d0    # maximal rapidity ymax
  4    0        # 0 = rapidity, 1 = pseudorapidity
```

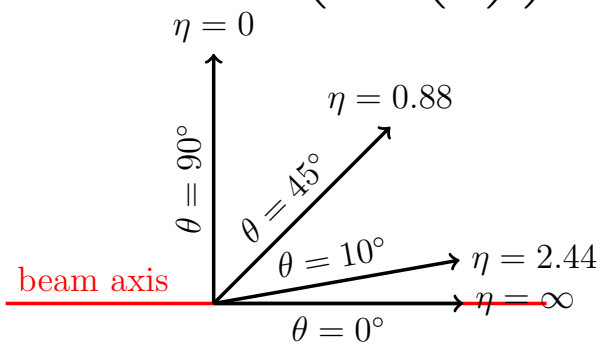
Available differential distributions for $gg \to \phi$ and $bb \to \phi$:
Cuts in transverse momentum p_T, pseudorapidity η and rapidity y

Pseudorapidity

$$\eta = -\ln\left(\tan\left(\frac{\theta}{2}\right)\right) = \frac{1}{2}\left(\frac{|\vec{p}| + p_L}{|\vec{p}| - p_L}\right)$$

Rapidity

$$\leftrightarrow \quad y = \frac{1}{2}\left(\frac{E + p_L}{E - p_L}\right)$$



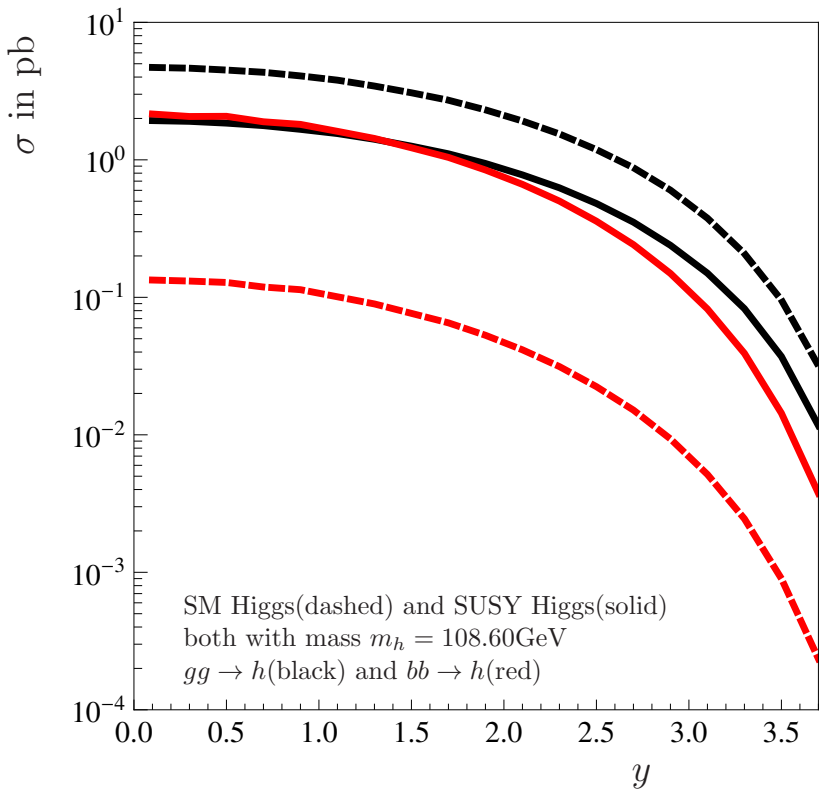
Relation:

$$y = \ln \frac{\sqrt{m^2 + p_T^2 \cosh^2 \eta} + p_T \sinh \eta}{\sqrt{m^2 + p_T^2}}$$

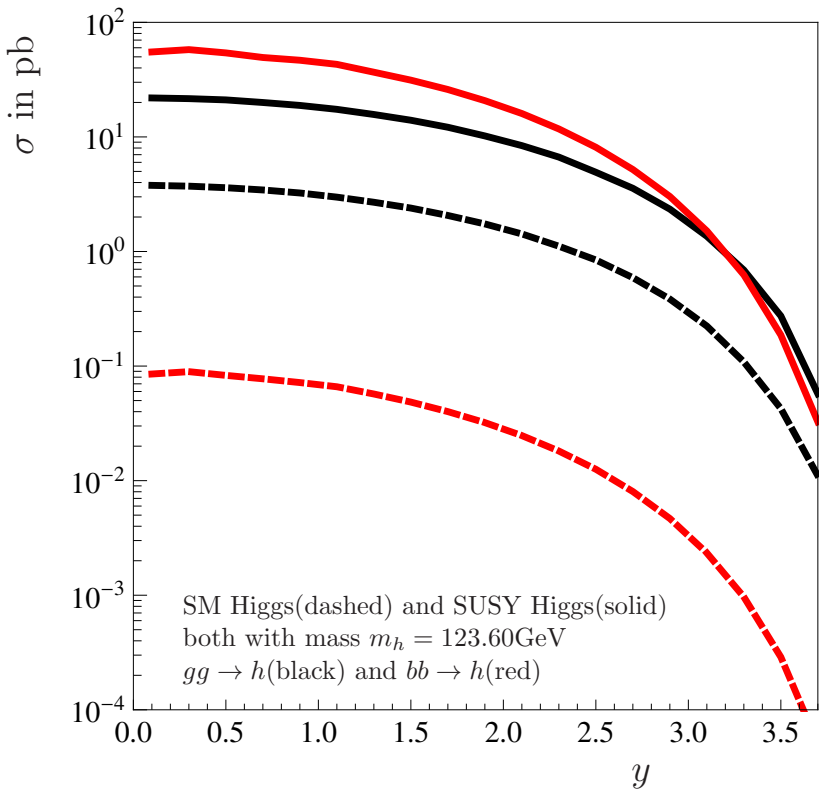
Differential cross section $gg/bb \to \phi$ in rapidity y

mhmax: $M_{\tilde{q}_{L,R}} = 750\text{GeV}, A_q = 2\text{TeV}, m_{\tilde{g}} = 800\text{GeV}, \mu = 500\text{GeV}, m_A = 150\text{GeV}$

$\tan\beta = 5$



$\tan\beta = 30$



$bb \to \phi$ can dominate $gg \to \phi$ in SUSY showing a different slope!

Outline

1 Neutral MSSM Higgs XS - Theory

2 SusHi - A Fortran code

3 Conclusion

We presented `SusHi`, which allows for:

- calculation of gluon fusion XS @NLO
  including all supersymmetric contributions.
- calculation of weighted bottom-quark annihilation XS.
- differential cross sections with cuts in p_T, η, y.

`SusHi` itself is a Fortran code controlled by input files in `SLHA`-style. It is linked to `FeynHiggs` for the calculation of Higgs masses in the real MSSM. `SusHi` also includes the mentioned 2 Higgs Doublet models.

In case you want to use it, ask me or have a look in the manual arXiv:1212.3249.
`SusHi` is available from:

http://sushi.hepforge.org

Many thanks for your attention!

The neutral components of the Higgs doublets $H_u = (H_u^+, H_u^0)^T$ and $H_d = (H_d^0, H_d^-)^T$ mix as follows

CP-even Higgs CP-odd Higgs

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h \\ H \end{pmatrix} + \frac{i}{\sqrt{2}} R_\beta \begin{pmatrix} G \\ A \end{pmatrix} \quad .$$

The mixing matrix is expressed in terms of the "Higgs mixing angle α"

$$R_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \quad .$$

The Higgs sector at LO is determined by fixing $\tan\beta = \frac{v_u}{v_d}$ and m_A^2:

$$m_{h,H} = \frac{1}{2}\left(m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 - m_Z^2)^2 + 4 m_Z^2 m_A^2 \sin^2(2\beta)} \right)$$

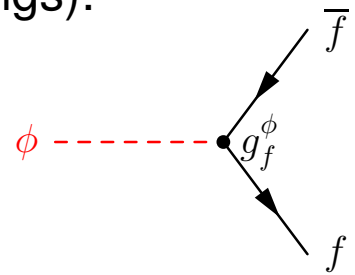
$$\tan(2\alpha) = \tan(2\beta) \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2}$$

The lightest Higgs h mass obtains large corrections at higher orders:
`FeynHiggs` [Frank Degrassi Hahn Heinemeyer Hollik Rzehak Slavich Weiglein Williams]
3-loop [Martin '07; Kant Harlander Mihaila Steinhauser '08 '10].

In the MSSM, Higgs couplings to the b-quark can be enhanced by $\tan\beta$:
Relative strength of the Higgs boson couplings g_f^ϕ with $\phi \in \{h, H, A\}$ to the SM fermions (with respect to the SM Higgs boson couplings):

$$g_u^h = \frac{\cos\alpha}{\sin\beta} \qquad g_u^H = \frac{\sin\alpha}{\sin\beta} \qquad g_u^A = \frac{1}{\tan\beta}$$

$$g_d^h = -\frac{\sin\alpha}{\cos\beta} \qquad g_d^H = \frac{\cos\alpha}{\cos\beta} \qquad g_d^A = \tan\beta$$

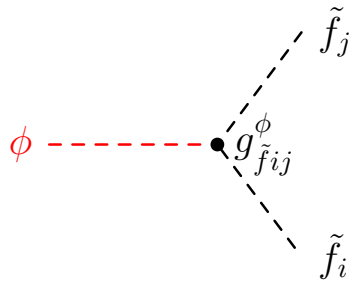


In addition, the superpartners of the quarks, the squarks, are relevant

$$\mathcal{L} \supset -(\tilde{q}_L^\dagger, \tilde{q}_R^\dagger)\mathcal{M}_{\tilde{q}}^2 \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

with the mass matrix:

$$\mathcal{M}_{\tilde{q}}^2 = \begin{pmatrix} M_{qL}^2 + m_q^2 + m_{E1}^2 & m_q(A_q - \mu\kappa) \\ m_q(A_q - \mu\kappa) & M_{qR}^2 + m_q^2 + m_{E2}^2 \end{pmatrix}$$



They form two mass eigenstates:

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{q}} & \sin\theta_{\tilde{q}} \\ -\sin\theta_{\tilde{q}} & \cos\theta_{\tilde{q}} \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

$m_{E1}^2 = m_z^2\cos(2\beta)(T_q^3 - Q_q\sin^2\theta_W)$; $\quad m_{E2}^2 = m_Z^2\cos(2\beta)Q_q\sin^2\theta_W$; $\quad \kappa = \tan\beta(d), \cot\beta(u)$

We have to tackle the question how to renormalize our parameters.
This involves in particular:

- How do we renormalize the squark sectors?
  Remember the mass matrix of the squarks in gauge eigenstates $\tilde{q}_L, \tilde{q}_R$:

$$\mathcal{M}^2_{\tilde{q}} = \begin{pmatrix} {M_{qL}}^2 + {m_q}^2 + m^2_{E1} & m_q(A_q - \mu\kappa) \\ m_q(A_q - \mu\kappa) & {M_{qR}}^2 + {m_q}^2 + m^2_{E2} \end{pmatrix}$$

  They form two mass eigenstates $\tilde{q}_1, \tilde{q}_2$:

$$\begin{matrix} m_{\tilde{q}_1} \\ m_{\tilde{q}_2} \end{matrix} \rightarrow \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{q}} & \sin\theta_{\tilde{q}} \\ -\sin\theta_{\tilde{q}} & \cos\theta_{\tilde{q}} \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

  Relations between gauge/mass eigenstates:
  $M_{qL}, M_{qR}, A_q \leftrightarrow m_q, m_{\tilde{q}_1}, m_{\tilde{q}_2}, \theta_{\tilde{q}}$
  Should those parameters be taken on-shell?

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  Relations between gauge/mass eigenstates:
  $M_{qL}, M_{qR}, A_q \leftrightarrow m_q, m_{\tilde{q}_1}, m_{\tilde{q}_2}, \theta_{\tilde{q}}$
  Should those parameters be taken on-shell?

- Are we resumming large $\tan\beta$-enhanced terms in Y_b?

Renormalization of the stop and the sbottom sector

Typically in the stop sector $m_t, m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}$ are taken to be on-shell parameters and A_t is derived from

$$\sin(2\theta_t) = \frac{2m_t(A_t - \mu/\tan\beta)}{{m_{\tilde{t}_1}}^2 - {m_{\tilde{t}_2}}^2} \quad .$$

However in the sbottom sector a similar procedure induces large corrections:

$$\delta A_b \propto \alpha_s \mu^2 \tan^2\beta / m_{\tilde{g}}$$

Alternative scheme: [Brignole et al. '02; Rzehak et al. '04 '10; Degrassi Slavich '10]
Define A_b from $\tilde{b}_1\tilde{b}_2 A$ vertex and use $A_b, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$ as on-shell parameters.
$\longrightarrow m_b^{\text{sbot}}$ is a derived quantity.

$\rightarrow$ We need a proper renormalization of all the parameters at LO!
Relations between gauge/mass eigenstates $M_{qL}, M_{qR}, A_q \leftrightarrow m_q, m_{\tilde{q}_1}, m_{\tilde{q}_2}, \theta_{\tilde{q}}$

$$\sin(2\theta_{\tilde{q}}) = \frac{2m_q(A_q - \mu\kappa) + \delta^{(1)}}{m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}}$$

$$\cos(2\theta_{\tilde{q}}) = \frac{M^2_{qL} - M^2_{qR} + m^2_Z \cos(2\beta)(T^3_q - 2Q_q \sin^2\theta_W) + \delta^{(2)}}{m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}}$$

$$m^2_{\tilde{q}_1} + m^2_{\tilde{q}_2} = M^2_{qL} + M^2_{qR} + 2m^2_q + m^2_Z \cos(2\beta)T^3_q + \delta^{(3)}$$

"Canonical scheme":
Choose $m_q, m_{\tilde{q}_1}, m_{\tilde{q}_2}, \theta_{\tilde{q}}$ to be on-shell parameters and $\delta^{(i)} = 0$!

Various possibilities:

- Choose m_b not to be renormalized on-shell, but $\overline{DR}$ at scale μ_r.
- Choose A_b to be renormalized on-shell, resulting in a dependent $\theta_{\tilde{b}}/m_b$.
- ..... $\rightarrow$ to appear in the code later!

A resummation of large $\tan\beta$ terms might be on order: $Y_b \rightarrow \frac{m_b}{1+\epsilon_b \tan\beta}$

Note: Due to $M^B_{bL} = M^B_{tR}$ we add $\Delta M^2_L = \delta M^2_{bL} - \delta M^2_{tL}$ to $\mathcal{M}^2_{\tilde{b}}(1,1)$.