Beam instabilities (I)

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in CERN Accelerator School, Advanced Level, Trondheim
Monday 21.08.2013
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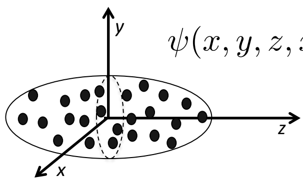
CAS

What is a beam instability?

- What is a beam instability?
  - A beam becomes unstable when a moment of its distribution exhibits an exponential growth (e.g. mean positions <x>, <y>, <z>, standard deviations σ_x, σ_y, σ_z, etc.) – resulting into beam loss or emittance growth!



$$\psi(x, y, z, x', y', \delta)$$

$$N = \int \psi(x, y, z, x', y', \delta) dx x' dy dy' dz d\delta$$

$$\langle x \rangle = \frac{1}{N} \int x \psi(x, y, z, x', y', \delta) dx x' dy dy' dz d\delta$$

$$\sigma_x = \frac{1}{N} \int (x - \langle x \rangle)^2 \, \psi(x, y, z, x', y', \delta) dx x' dy dy' dz d\delta$$

And similar definitions for $\langle y \rangle, \ \sigma_y, \ \langle z \rangle, \ \sigma_z$

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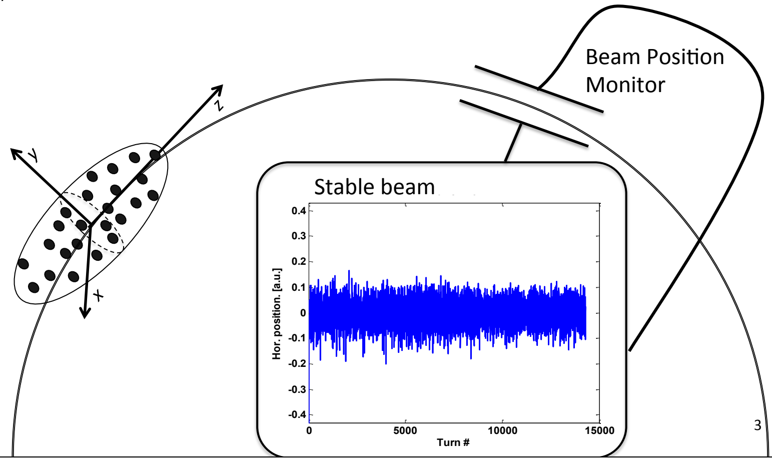

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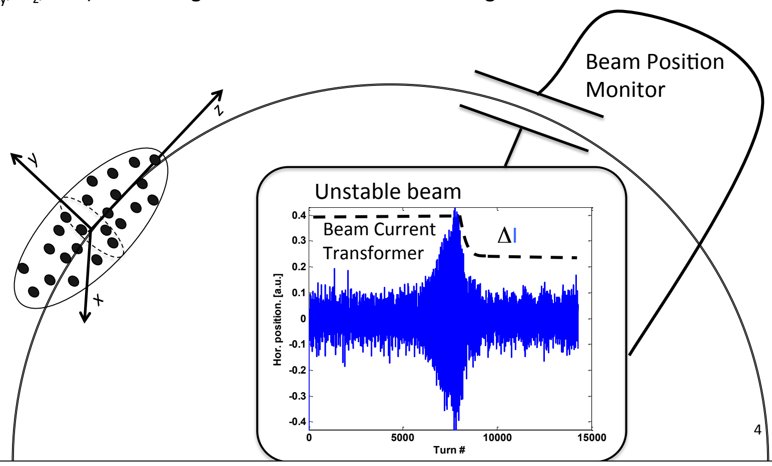

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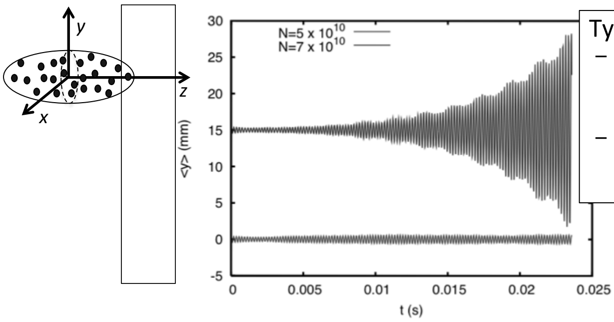
Why study beam instabilities?

- What is a beam instability?
  - A beam becomes unstable when a moment of its distribution exhibits an exponential growth (e.g. mean positions <x>, <y>, <z>, standard deviations σ_x, σ_y, σ_z, etc.) – resulting into beam loss or emittance growth!
- Why study beam instabilities?
  - The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (performance limitation)



Typical situation
- A beam centroid instability appears when the intensity is raised above a certain threshold
- It depends also on the ensemble of the machine settings

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- Why study beam instabilities?
  - The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (performance limitation)
  - Understanding the type of instability limiting the performance, and its underlying mechanism, is essential because it:
    - Allows identifying the source and possible measures to mitigate/suppress the effect
    - Allows dimensioning an active feedback system to prevent the instability

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Types of beam instabilities

⇒ Beam instabilities occur in both linear and circular machines
- Longitudinal plane (z,δ)
- Transverse plane (x,y,x',y')

⇒ Beam instabilities can affect the beam on different scales
- Cross-talk between bunches
  - → The unstable motion of subsequent bunches is coupled
  - → The instability is consequence of another mechanism that builds up along the bunch train
- Single bunch effect
- Coasting beam instabilities

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Example of multi-bunch instability

⇒ They can affect the beam on different scales
- Cross-talk between bunches
  - → **The unstable motion of subsequent bunches is coupled**
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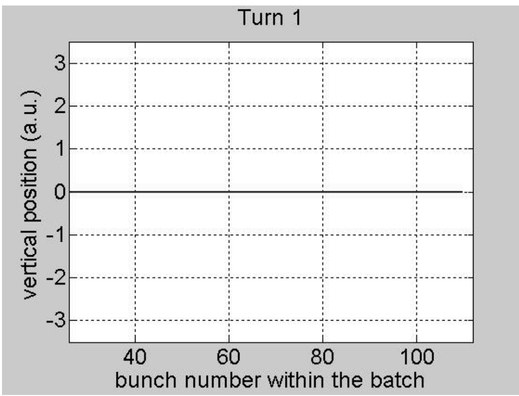

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Example of multi-bunch instability

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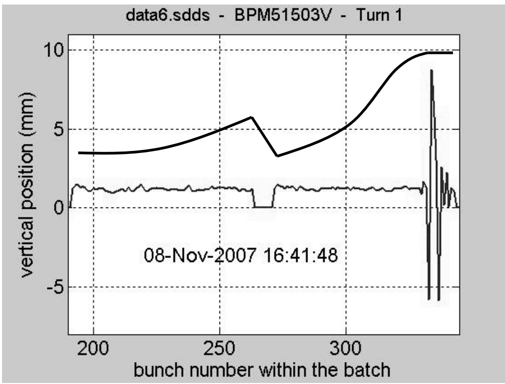
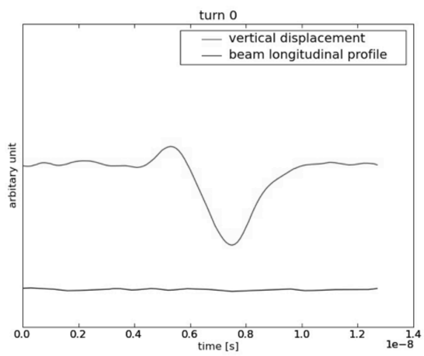
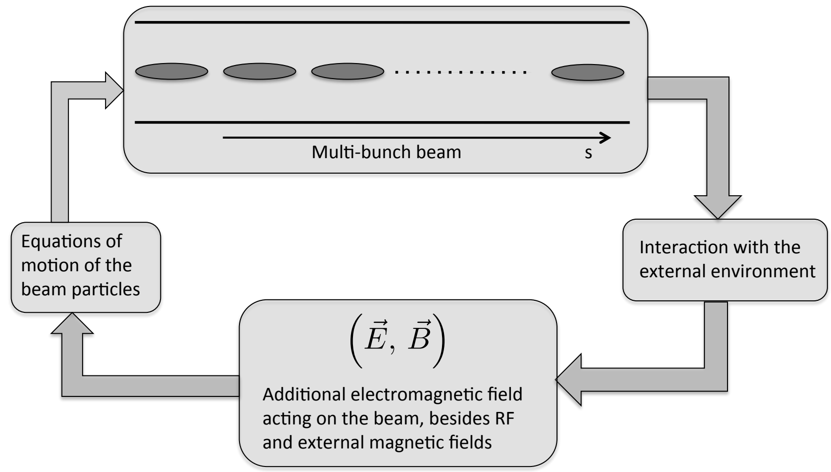


Example of single bunch instability

⇒ They can affect the beam on different scales
- Cross-talk between subsequent bunches
  - → The instability builds up along a bunch train
  - → The instability is consequence of another mechanism that builds up along the bunch train
- **Single bunch effect**



Instability loop



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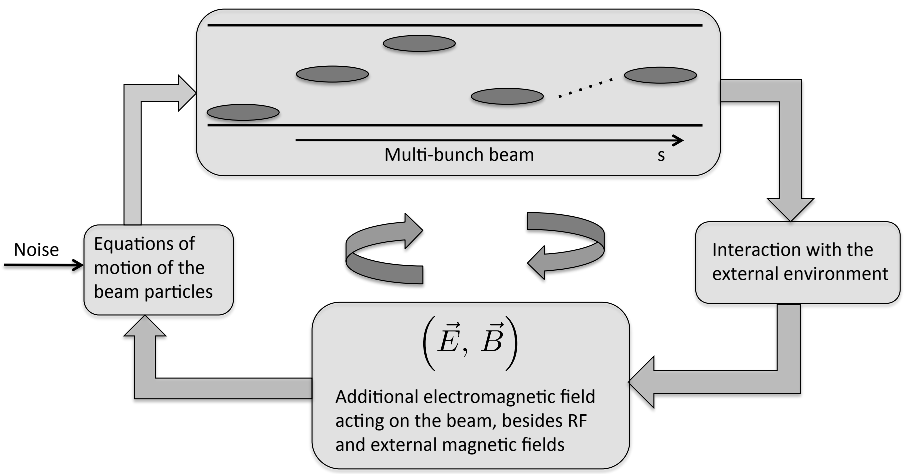
Instability loop



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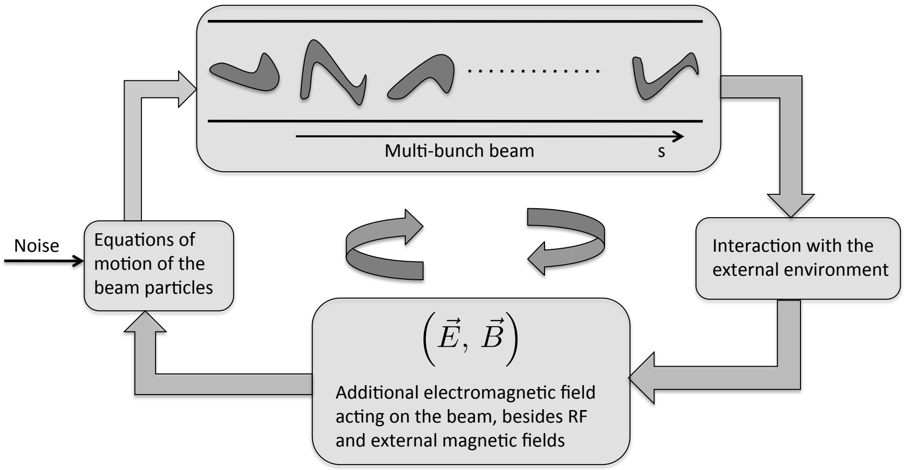

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Some examples of beam-environment interaction

Interaction of the beam with the external environment

Beam self-fields
- Set of Maxwell's equations with
  - The beam as the source term
  - Boundary conditions given by the accelerator component in which the beam propagates

Fields from another beam
- Set of Maxwell's equations with
  - The other beam as the source term
  - Boundary conditions given by the chamber in which the encounter between the beams takes place

The electron cloud
- Electron production and multipacting
- Poisson's equation with
  - The electron cloud as the source term
  - Boundary conditions given by the chamber in which the electron cloud builds up

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Electro-magnetic beam-environment interaction

Interaction of the beam with the external environment →

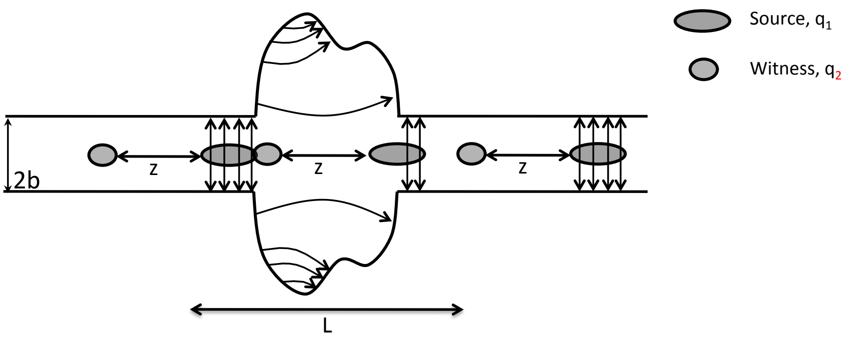
Beam self-fields
- Set of Maxwell's equations with
  - The beam as the source term
  - Boundary conditions given by the accelerator component in which the beam propagates

Need of a mathematical framework to describe the effects of the self-induced fields on the beam

Time domain: Wake functions/ potentials

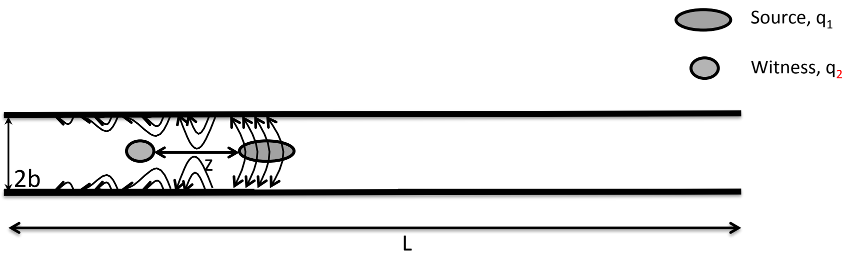
Frequency domain: Impedances

Wake fields (general)



- While source and witness ($q_i\ \delta(s-ct)$), distant by $z<0$, move in a perfectly conducting chamber, the witness does not feel any force ($\gamma >> 1$)
- When the source encounters a discontinuity (e.g., transition, device), it produces an electromagnetic field, which trails behind (wake field)
  - The source loses energy
  - The witness feels a net force all along an effective length of the structure, L

Wake fields (general)



- Not only geometric discontinuities cause electromagnetic fields trailing behind sources traveling at light speed.
- For example, a pipe with finite conductivity causes a delay in the image currents, which also produces delayed electromagnetic fields
  - No ringing, only slow decay
  - The witness feels a net force all along an effective length of the structure, L
- In general, also electromagnetic boundary conditions other than PEC can be the origin of wake fields.

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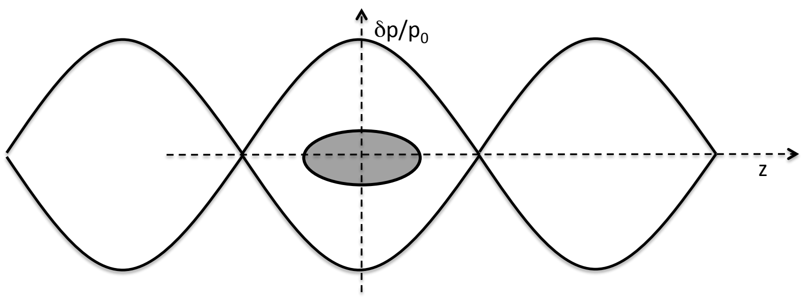
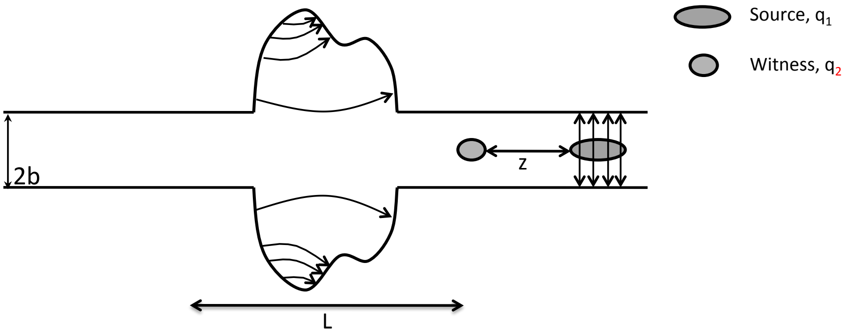


1. The longitudinal plane

Longitudinal wake function: definition



$$\int_0^L F_{\|}(s,z)ds = -q_1 q_2 W_{\|}(z)$$

$$F_{\|}(s,z) = q_2 E_z(s,z)$$

$$\Delta E_2 \quad \Rightarrow \quad \frac{\Delta E_2}{E_0} \approx \frac{\Delta p_2}{p_0}$$

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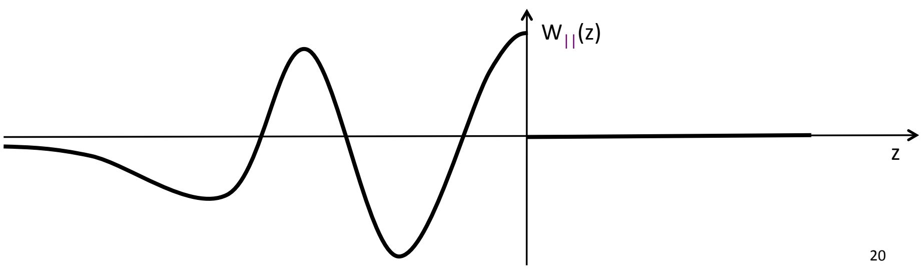
Longitudinal wake function: properties

$$W_{\|}(z) = -\frac{\Delta E_2}{q_1 q_2} \quad \xrightarrow[q_2 \to q_1]{z \to 0} \quad W_{\|}(0) = -\frac{\Delta E_1}{q_1^2}$$

- The value of the wake function in 0, ***W***$_{//}$***(0)***, is related to the energy lost by the source particle in the creation of the wake
- ***W***$_{//}$***(0)>0*** since ***ΔE***$_1$***<0***
- ***W***$_{//}$***(z)*** is discontinuous in z=0 and it vanishes for all z>0 because of the ultra-relativistic approximation



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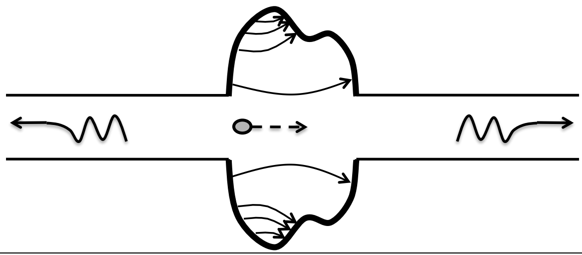
The energy balance

$$W_{\parallel}(0) = -\frac{\Delta E_1}{q_1^2}$$

What happens to the energy lost by the source?

- In the global energy balance, the energy lost by the source splits into
  - Electromagnetic energy of the modes that remain trapped in the object
    - ⇒ Partly dissipated on **lossy walls** or into purposely designed inserts or **HOM absorbers**
    - ⇒ Partly transferred to following particles (or the same particle over successive turns), possibly feeding into an instability!
  - Electromagnetic energy of modes that propagate down the beam chamber (above cut-off), which will be eventually lost on surrounding lossy materials



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Longitudinal impedance

- The wake function of an accelerator component is basically its Green function in time domain (i.e., its response to a pulse excitation)
  - ⇒ Very useful for macroparticle models and simulations, because it can be used to describe the driving terms in the single particle equations of motion!
- We can also describe it as a transfer function in frequency domain
- This is the definition of **longitudinal beam coupling impedance** of the element under study

$$\boxed{Z_{\parallel}(\omega)} = \int_{-\infty}^{\infty} \boxed{W_{\parallel}(z)} \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$

[Ω]   [Ω/s]

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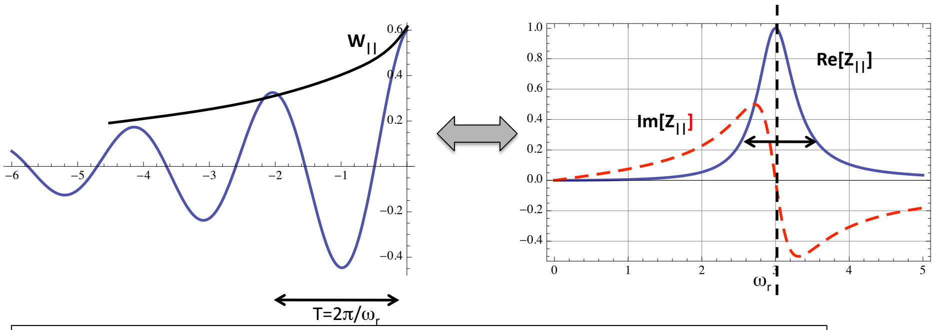
Longitudinal impedance: resonator

$$Z_{\parallel}(\omega) = \int_{-\infty}^{\infty} W_{\parallel}(z) \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$



– The frequency ω_r is related to the oscillation of E_z, and therefore to the frequency of the mode excited in the object
– The decay time depends on how quickly the stored energy is dissipated (quantified by a quality factor Q)

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Longitudinal impedance: resonator

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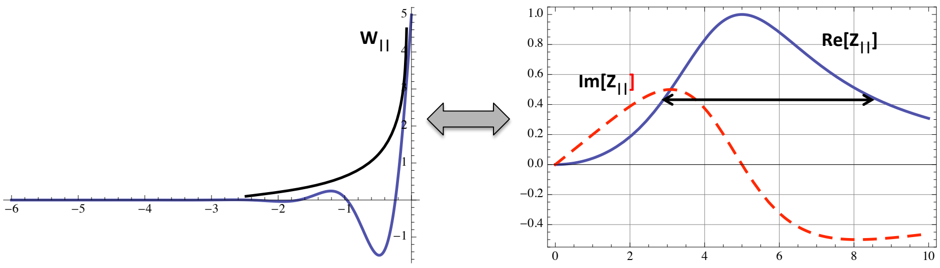

– In general, the impedance will be composed of several of these resonator peaks
– Other contributors to the global impedance can also have different shapes, e.g. the resistive wall

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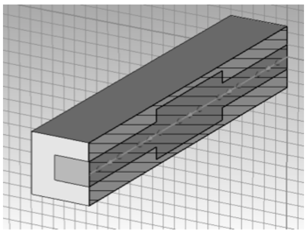
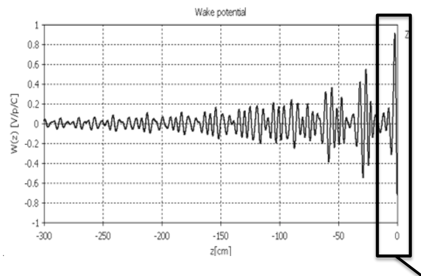
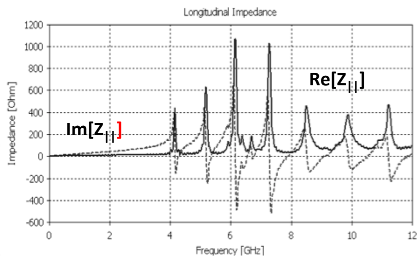
Longitudinal impedance: cavity



- A more complex example: a simple pill-box cavity with walls having finite conductivity
- Several modes can be excited
  - Below the pipe cut-off frequency the width of the peaks is only determined by the finite conductivity of the walls
  - Above, losses also come from propagation in the chamber





Note $W_{||}(0)$<0, **CST** uses opposite convention!!

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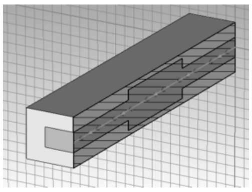
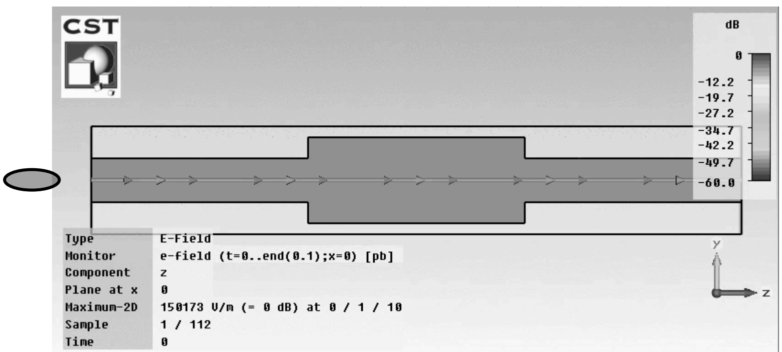
Longitudinal impedance: cavity



- Evolution of the electromagnetic fields (E_z) in the cavity while and after the beam has passed



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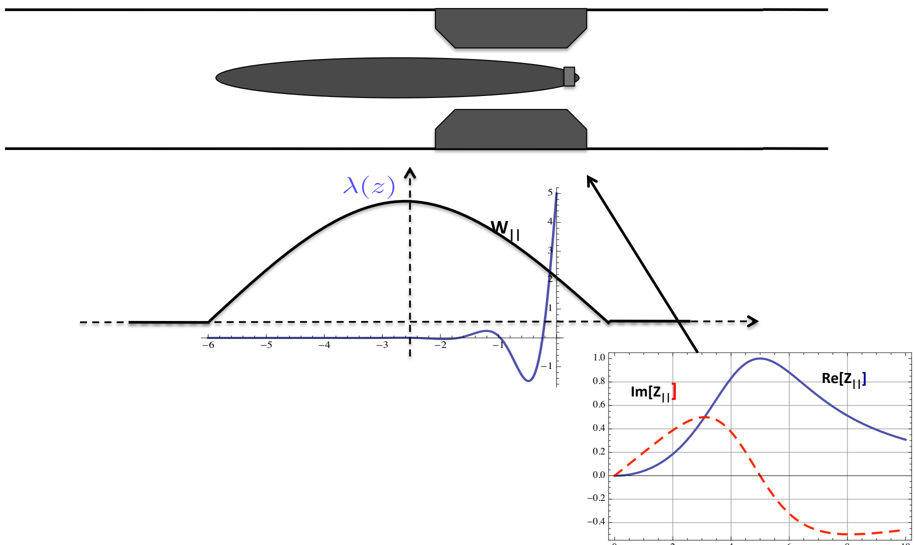
Single bunch effects



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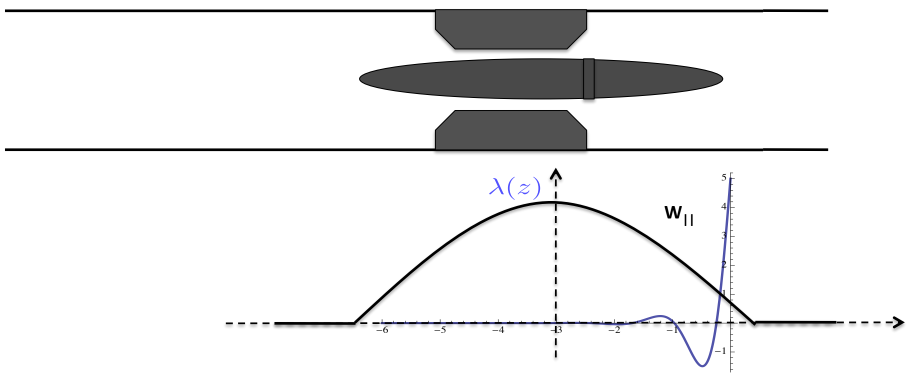
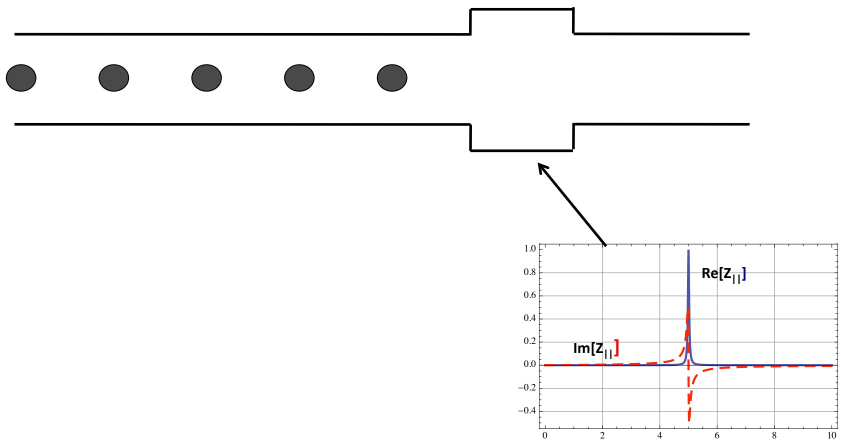


$$\Delta E(z) = -e^2 \int_z^{\hat{z}} \lambda(z') W_{\parallel}(z - z') dz'$$

Multi bunch effects



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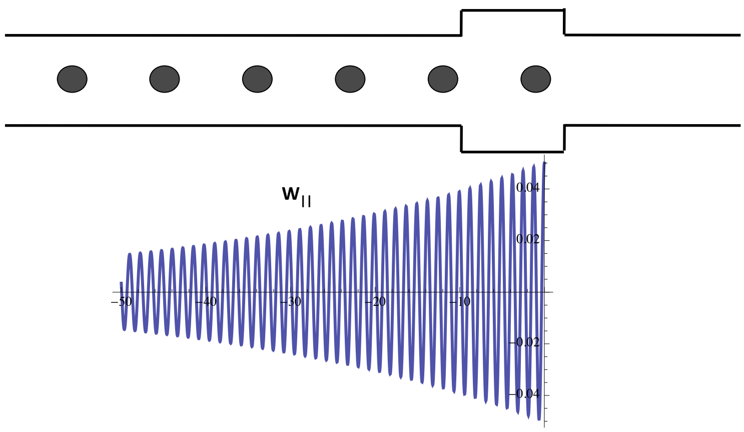
Multi bunch effects



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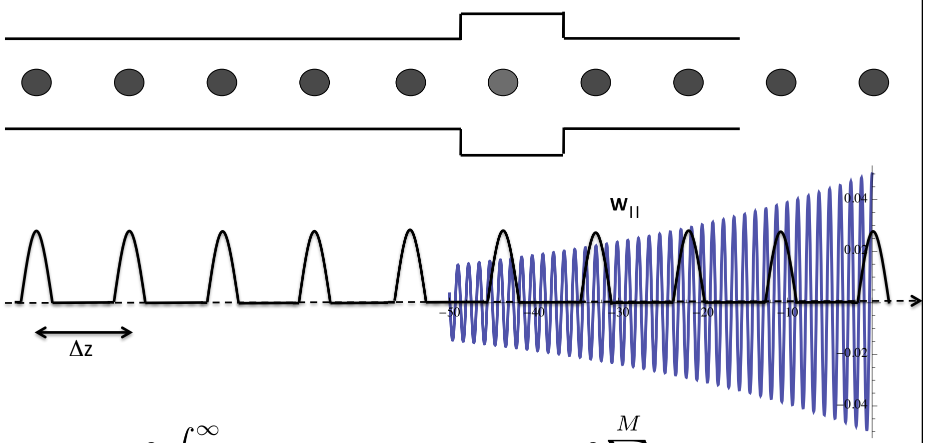
Multi bunch effects



$$\Delta E_j = -e^2 \int_{z_j}^{\infty} \lambda(z') W_{||}(z_j - z') dz' \approx N_j e^2 \sum_{i=0}^{M} N_i W[(j-i)\Delta z]$$

$$\Delta E_j = -e^2 \int_{z_j}^{\infty} \lambda(z') W_{||}(z_j - z') dz' \approx N_j e^2 \sum_{k=0}^{\infty} \sum_{i=0}^{M} N_i W[(j-i)\Delta z + kC]$$

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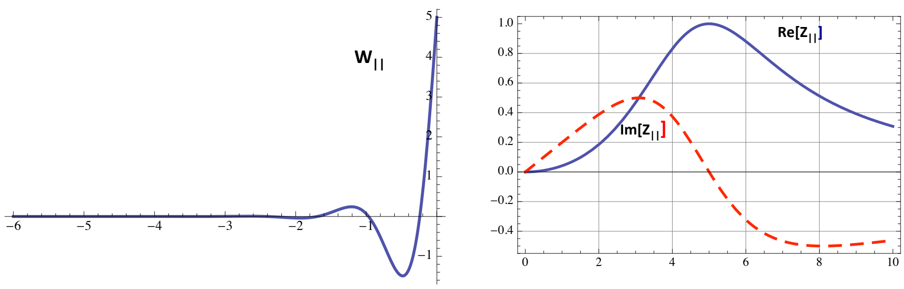
Single bunch vs. Multi bunch

- A short-lived wake, decaying over the length of one bunch, can only cause intra-bunch (head-tail) coupling
- It can be therefore responsible for single bunch instabilities



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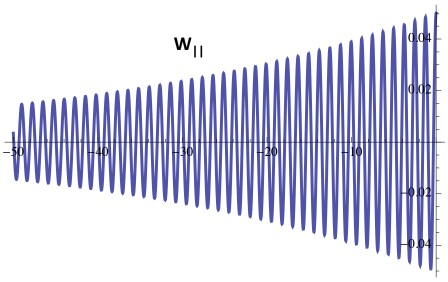
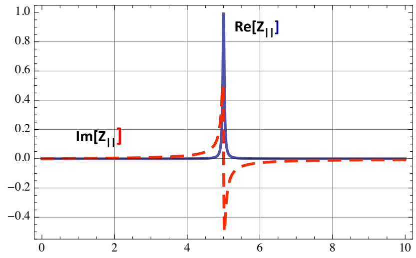
Single bunch vs. Multi bunch

- A long-lived wake field, decaying over the length of a bunch train or even more than a turn, causes bunch-to-bunch or multi-turn coupling
- It can be therefore responsible for multi-bunch or multi-turn instabilities





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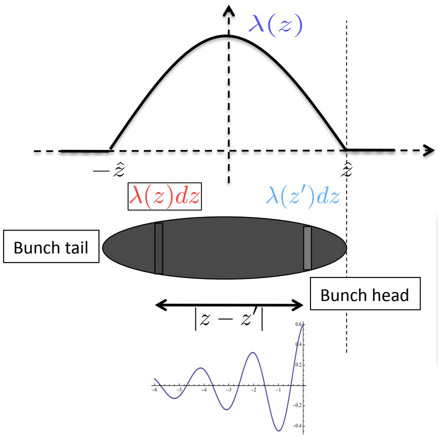
- Detailed calculations:
  - Energy loss
  - Robinson instability
- Qualitative descriptions:
  - Coupled bunch instabilities
  - Single bunch modes

[1] "Physics of Collective Beam Instabilities in High Energy Accelerators", A. W. Chao

Energy loss of a bunch (single pass)

- The energy kick ΔE(z) on each particle e in the witness slice λ(z)dz is the integral of the contributions from the wakes left behind by all the preceding eλ(z')dz slices (sources)
- The total energy loss ΔE of the bunch can then be obtained by integrating ΔE(z) λ(z) over the full bunch extension



$$\Delta E(z) = -e^2 \int_z^{\hat{z}} \lambda(z') W_{\|}(z-z')dz'$$

$$\Delta E = \int_{-\hat{z}}^{\hat{z}} \lambda(z) \Delta E(z) dz$$

$$\Delta E = -\frac{e^2}{2\pi} \int_{-\infty}^{\infty} \tilde{\lambda}(\omega)\tilde{\lambda}^*(\omega) Z_{\|}(\omega) d\omega = -\frac{e^2}{2\pi} \int_{-\infty}^{\infty} |\tilde{\lambda}(\omega)|^2 \mathrm{Re}\left[Z_{\|}(\omega)\right] d\omega$$

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Energy loss of a bunch (multi-pass)

- The total energy loss ΔE of the bunch can still be obtained by integrating ΔE(z) over the full bunch extension
- ΔE(z) this time also includes contributions from all previous turns, spaced by multiples of the ring circumference C

$$\Delta E = -\frac{e^2}{2\pi} \int_{-\infty}^{\infty} \lambda(z) dz \int_{-\infty}^{\infty} dz' \lambda(z') \sum_{k=-\infty}^{\infty} W_{\|}(kC + z - z')dz'$$

$$\sum_{k=-\infty}^{\infty} W_{\|}(kC + z - z') = \frac{c}{C} \sum_{p=-\infty}^{\infty} Z_{\|}(p\omega_0) \exp\left[-\frac{ip\omega_0(z-z')}{c}\right]$$

$$\Delta E = -\frac{e^2\omega_0}{2\pi} \boxed{\sum_{p=-\infty}^{\infty}} Z_{\|}(p\omega_0) \underbrace{\int_{-\infty}^{\infty} \lambda(z) \exp\left(\frac{-ip\omega_0 z}{c}\right) dz}_{\tilde{\lambda}(p\omega_0)} \underbrace{\int_{-\infty}^{\infty} \lambda(z') \exp\left(\frac{ip\omega_0 z'}{c}\right) dz'}_{\tilde{\lambda}^*(p\omega_0)}$$

$$\Delta E = -\frac{e^2\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} |\tilde{\lambda}(p\omega_0)|^2 \mathrm{Re}\left[Z_{\|}(p\omega_0)\right]$$

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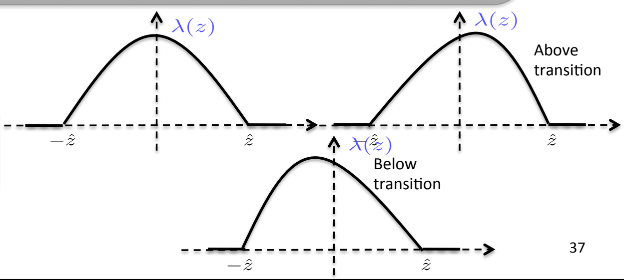
Energy loss per turn: stable phase shift

- If we divide the total impedance of a given ring into a narrow band component $Z^{NB}_{||}(\omega)$ and a broad band component $Z^{BB}_{||}(\omega)$, then we can write a general expression for the energy loss per bunch and per turn
- To compensate for this energy loss, the equilibrium distribution of the bunch must move in the bucket from being centered to a synchronous angle $\Delta\Phi_s$

$$\Delta E_{\rm turn} = -\frac{e^2}{2\pi}\left\{\omega_0 \sum_{p=-\infty}^{\infty} |\tilde{\lambda}(p\omega_0)|^2 {\rm Re}\left[Z_{||}^{\rm NB}(p\omega_0)\right] + + \int_{-\infty}^{\infty} |\tilde{\lambda}(\omega)|^2 {\rm Re}\left[Z_{||}^{\rm BB}(\omega)\right] d\omega\right\}$$

$$\sin\Delta\Phi_s = \frac{\Delta E_{\rm turn}}{NeV_m}$$



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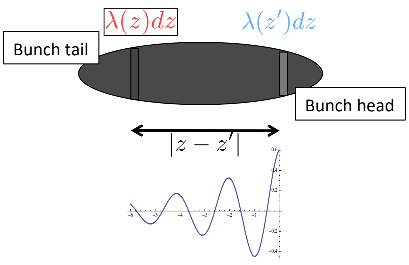
Single particle equations of the longitudinal motion in presence of wake fields

- The single particle in the witness slice $\lambda(z)dz$ will feel the force from the RF, from the bunch own space charge, and that associated to the wake
- The wake contribution can extend to several turns



$$\underbrace{\frac{d^2z}{dt^2} + \frac{\eta e V_{\rm rf}(z)}{m_0\gamma C}}_{\text{External RF}} = \underbrace{\frac{\eta e^2}{m_0\gamma C}\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\lambda(z'+kC)W_{||}(z-z'-kC)dz'}_{\text{Wake fields}}$$

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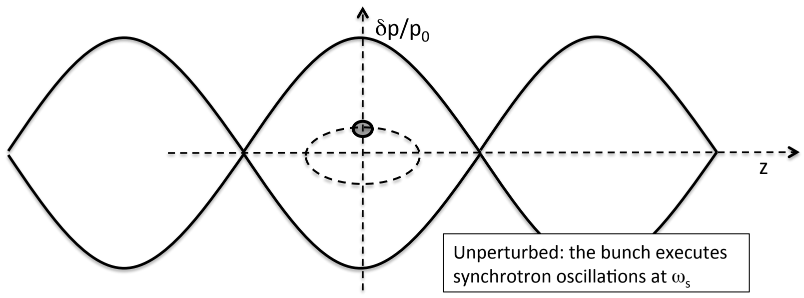
The Robinson instability

– To illustrate the Robinson instability we will use some simplifications:
  - ⇒ The bunch is point-like and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  - ⇒ The bunch additionally feels the effect of a multi-turn wake



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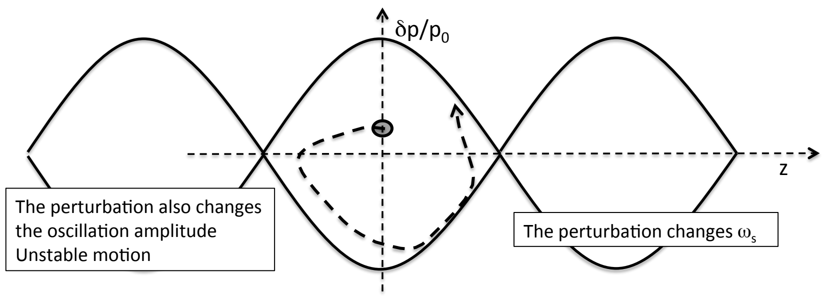

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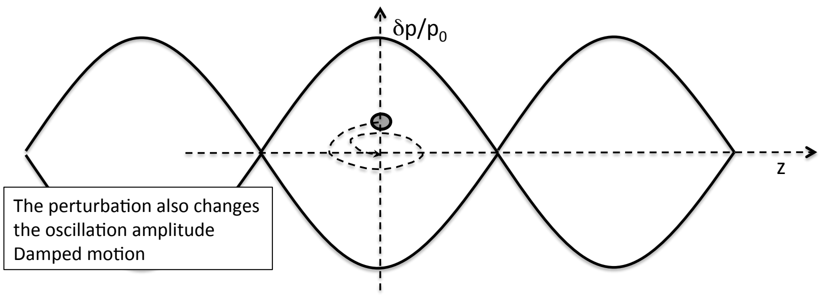
  ⇒ The bunch is point-like and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  ⇒ The bunch additionally feels the effect of a multi-turn wake



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The Robinson instability

– To illustrate the Robinson instability we will use some simplifications:
  ⇒ The bunch is point-like and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  ⇒ The bunch additionally feels the effect of a multi-turn wake

$$\frac{d^2 z}{dt^2} + \omega_s^2 z = \frac{Ne^2\eta}{Cm_0\gamma} \sum_{k=-\infty}^{\infty} W_{\parallel}\left[z(t) - z(t - kT_0) - kC\right]$$

We assume that the wake can be linearized on the scale of a synchrotron oscillation

$$W_{\parallel}\left[z(t) - z(t - kT_0) - kC\right] \approx W_{\parallel}(kC) + W'_{\parallel}(kC) \cdot \left[z(t) - z(t - kT_0)\right]$$

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The Robinson instability

$$W_{\parallel}\left[z(t) - z(t-kT_0) - kC\right] \approx W_{\parallel}(kC) + W_{\parallel}'(kC)\cdot[z(t) - z(t-kT_0)]$$

⇒ The term $\Sigma\, W_{||}(kC)$ only contributes to a constant term in the solution of the equation of motion, i.e. the synchrotron oscillation will be executed around a certain z_0 and not around 0. This term represents the stable phase shift that compensates for the energy loss

⇒ The dynamic term is proportional to $z(t)-z(t-kT_0) \approx kT_0 dz/dt$, so it will introduce a "friction" term in the equation of the oscillator, which can lead to instability!

$$z(t) \propto \exp(-i\Omega t)$$

$$\Omega^2 - \omega_s^2 = -\frac{Ne^2\eta}{Cm_0\gamma}\sum_{k=-\infty}^{\infty}\left[1-\exp(-ik\Omega T_0)\right]\cdot W_{\parallel}'(kC)$$

$$i\cdot\frac{1}{C}\sum_{p=-\infty}^{\infty}\left[p\omega_0 Z_{\parallel}(p\omega_0) - (p\omega_0+\Omega)Z_{\parallel}(p\omega_0+\Omega)\right]$$

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The Robinson instability

⇒ We assume a small deviation from the synchrotron tune
⇒ $\mathrm{Re}(\Omega - \omega_s)$ → Synchrotron tune shift
⇒ $\mathrm{Im}(\Omega - \omega_s)$ → Growth/damping rate, only depends on the dynamic term, if it is positive there is an instability!

$$\Omega^2 - \omega_s^2 \approx 2\omega_s\,(\Omega-\omega_s)$$

Complex frequency shift

$$\Delta\omega_s = \mathrm{Re}(\Omega-\omega_s) = \left(\frac{e^2}{m_0c^2}\right)\frac{N\eta}{2\gamma T_0^2\omega_s} \times$$

$$\times \sum_{p=-\infty}^{\infty}\left[p\omega_0 \mathrm{Im}Z_{\parallel}(p\omega_0) - (p\omega_0+\omega_s)\mathrm{Im}Z_{\parallel}(p\omega_0+\omega_s)\right]$$

$$\tau^{-1} = \mathrm{Im}(\Omega-\omega_s) = \left(\frac{e^2}{m_0c^2}\right)\frac{N\eta}{2\gamma T_0^2\omega_s}\sum_{p=-\infty}^{\infty}(p\omega_0+\omega_s)\mathrm{Re}Z_{\parallel}(p\omega_0+\omega_s)$$

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The Robinson instability

$$\tau^{-1} = \text{Im}(\Omega - \omega_s) = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta}{2\gamma T_0^2 \omega_s} \sum_{p=-\infty}^{\infty} (p\omega_0 + \omega_s) \text{Re} Z_{\parallel}(p\omega_0 + \omega_s)$$

- ⇒ We assume the impedance to be peaked at a frequency ω_r close to $h\omega_0 >> \omega_s$ (e.g. RF cavity fundamental mode or HOM)
- ⇒ Only two dominant terms are left in the summation at the RHS of the equation for the growth rate
- ⇒ Stability requires that η and $\Delta[\text{Re } Z_{||}(h\omega_0)]$ have different signs

$$\tau^{-1} = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta h\omega_0}{2\gamma T_0^2 \omega_s} \underbrace{\left[\text{Re}Z_{\parallel}(h\omega_0 + \omega_s) - \text{Re}Z_{\parallel}(h\omega_0 - \omega_s)\right]}_{\Delta\left[\text{Re}Z_{\parallel}(h\omega_0)\right]}$$

Stability criterion → $\eta \cdot \Delta\left[\text{Re}Z_{\parallel}(h\omega_0)\right] < 0$

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The Robinson instability

Stability criterion → $\eta \cdot \Delta\left[\text{Re}Z_{\parallel}(h\omega_0)\right] < 0$

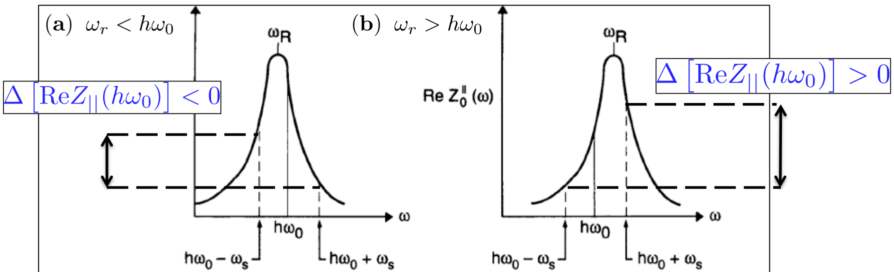

Figure 4.4. Illustration of the Robinson stability criterion. The rf fundamental mode is detuned so that ω_R is (a) slightly below $h\omega_0$ and (b) slightly above $h\omega_0$. (a) is Robinson damped above transition and antidamped below transition. (b) is antidamped above transition and damped below transition.

	$\omega_r < h\omega_0$	$\omega_r > h\omega_0$
Above transition (η>0)	stable	unstable
Below transition (η<0)	unstable	stable

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The Robinson instability

$$\tau^{-1} = \mathrm{Im}(\Omega - \omega_s) = \left(\frac{e^2}{m_0 c^2}\right) \frac{N\eta}{2\gamma T_0^2 \omega_s} \sum_{p=-\infty}^{\infty} (p\omega_0 + \omega_s) \mathrm{Re} Z_{\parallel}(p\omega_0 + \omega_s)$$

⇒ Other types of impedances can also cause instabilities through the Robinson mechanism

⇒ However, a smooth broad-band impedance with no narrow structures on the ω_0 scale cannot give rise to an instability

- ✓ Physically, this is clear, because the absence of structure on ω_0 scale in the spectrum implies that the wake has fully decayed in one turn time and the driving term in the equation of motion also vanishes

$$\sum_{p=-\infty}^{\infty} (p\omega_0 + \omega_s) \mathrm{Re} Z_{\parallel}(p\omega_0 + \omega_s) \rightarrow \frac{1}{\omega_0} \int_{-\infty}^{\infty} \omega \mathrm{Re} Z_{\parallel}(\omega) d\omega \rightarrow 0$$

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Other longitudinal instabilities

- The Robinson instability occurs for a single bunch under the action of a multi-turn wake field
  - It contains a term of coherent synchrotron tune shift
  - It results into an unstable rigid bunch dipole oscillation
  - It does not involve higher order moments of the bunch longitudinal phase space distribution
- Other important collective effects can affect a bunch in a beam
  - Potential well distortion (resulting in synchronous phase shift, bunch lengthening or shortening, synchrotron tune shift/spread)
  - Coupled bunch instabilities
  - High intensity single bunch instabilities (e.g. microwave instability)
  - Coasting beam instabilities (e.g.negative mass instability)
- To be able to study these effects we would need to resort to a more detailed description of the bunch(es)
  - Vlasov equation (kinetic model)
  - Macroparticle simulations

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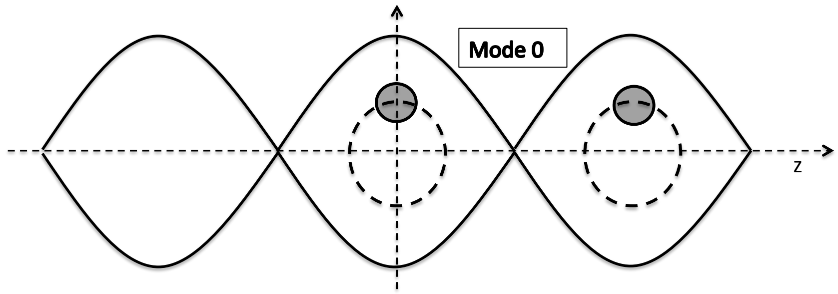
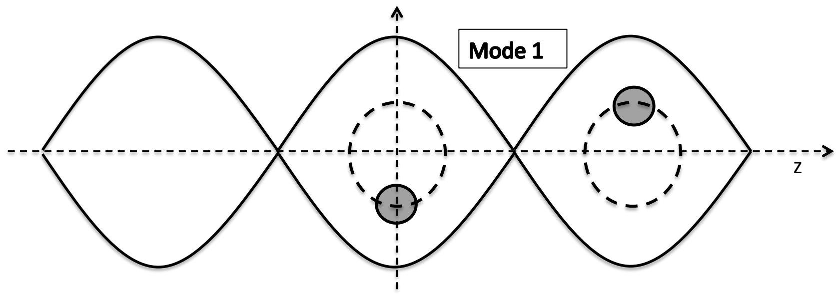
Coupled bunch modes

- M bunches can exhibit M different modes of coupled rigid bunch oscillations in the longitudinal plane



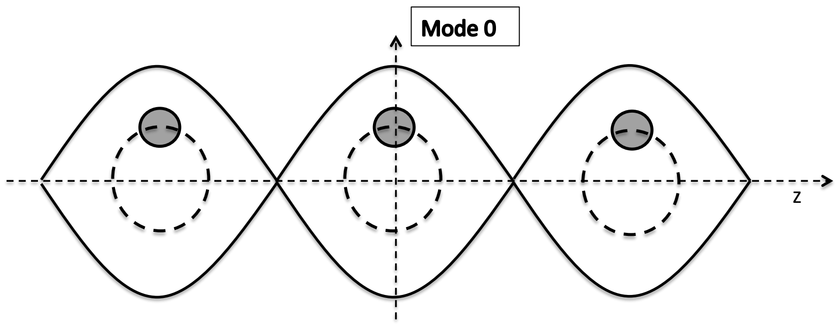
Coupled bunch modes

- M bunches can exhibit M different modes of coupled rigid bunch oscillations in the longitudinal plane



Coupled bunch modes

- M bunches can exhibit M different modes of coupled rigid bunch oscillations in the longitudinal plane
- Any rigid coupled bunch oscillation can be decomposed into a combination of these basic modes



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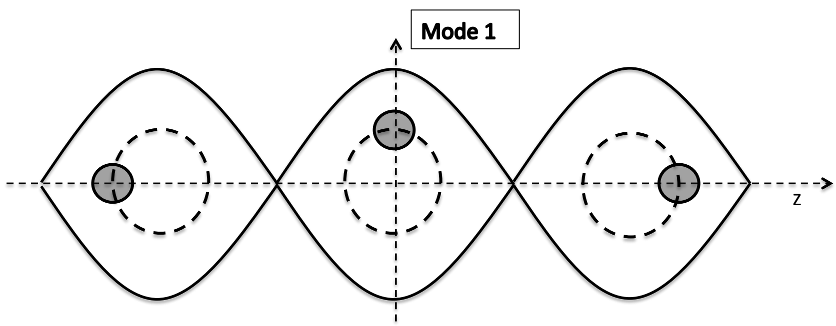
Coupled bunch modes

- M bunches can exhibit M different modes of coupled rigid bunch oscillations in the longitudinal plane
- Any rigid coupled bunch oscillation can be decomposed into a combination of these basic modes



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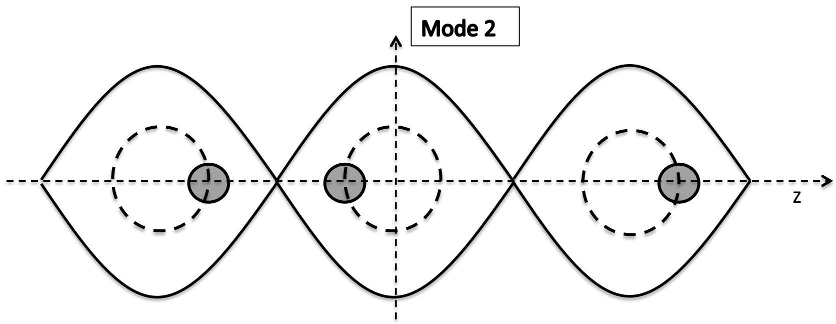
Coupled bunch modes

- M bunches can exhibit M different modes of coupled rigid bunch oscillations in the longitudinal plane
- Any rigid coupled bunch oscillation can be decomposed into a combination of these basic modes
- This modes can become unstable under the effect of long range wake fields



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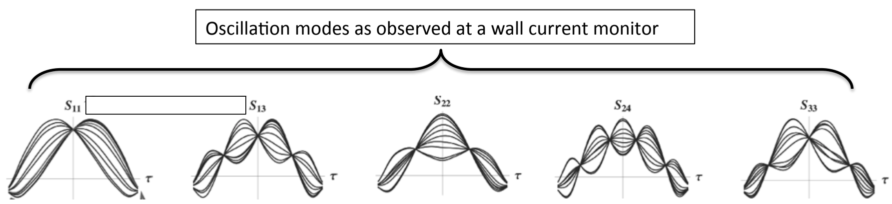
Single bunch modes

- In a similar fashion, a single bunch exhibits a double infinity of natural modes of oscillation, with rather complicated phase space portraits.
- Whatever perturbation on the bunch phase space distribution can be expanded as a series of these modes



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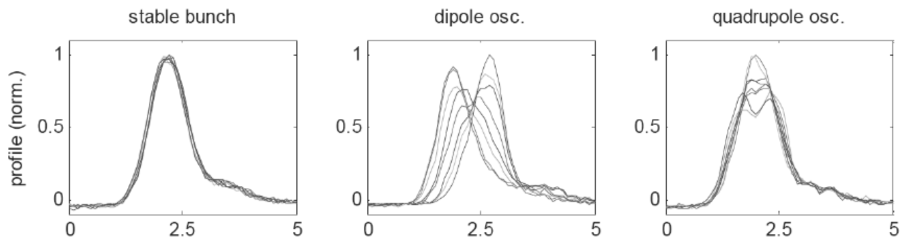
Single bunch modes

- In a similar fashion, a single bunch exhibits a double infinity of natural modes of oscillation, with rather complicated phase space portraits.
- Whatever perturbation on the bunch phase space distribution can be expanded as a series of these modes
- One of these modes or a combination of them can become unstable under the effect of a short range wake field
- In particular, the frequencies of these modes shift with intensity, and two of the modes can merge above a certain threshold, causing a microwave instability!

Observations in the CERN SPS in 2007



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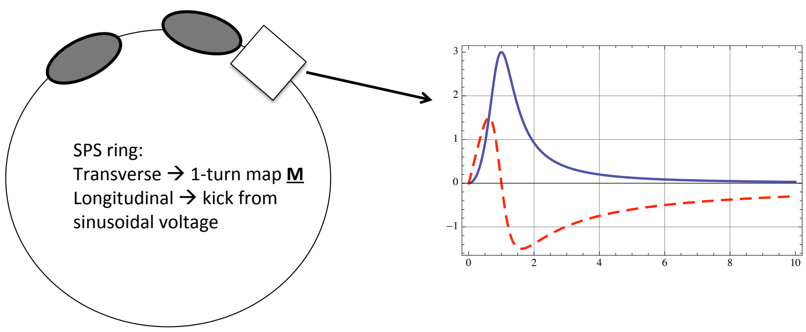
Macroparticle simulation

- We have simulated the evolution of an SPS bunch under the effect of a longitudinal broad band impedance lumped in one point of the ring



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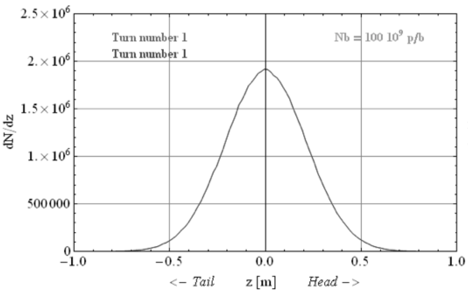
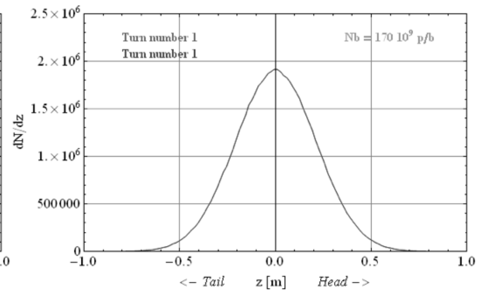
Macroparticle simulation

- We have simulated the evolution of an SPS bunch under the effect of a longitudinal broad band impedance lumped in one point of the ring
- Two different intensity values have been simulated
  - Low intensity → below instability threshold, but potential well distortion is visible in terms of stable phase shift and bunch lengthening
  - High intensity → above microwave instability threshold





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