

Linear Accelerators

Maurizio Vretenar, CERN
Advanced Accelerator Physics Course
Trondheim 2013



These 2 lectures:

- *recall the basic concepts introduced at the Introductory CAS course following an alternative more rigorous approach*
- *again, common treatment for electron and proton linacs (main concepts with little mathematics)*
- *go more deeply into 2 specific topics: applications of linacs, RFQs and small accelerators*

Lecture 1:

1. Main concepts, building blocks, synchronicity
2. Periodic accelerating structures
3. Applications of linacs

Lecture 2:

1. Beam dynamics fundamentals
2. Linac architecture
3. The Radio Frequency Quadrupole

1. Introduction: main concepts, building blocks, synchronicity



Variety of linacs

- ◆ The first and the smallest: Rolf Widerøe thesis (1923)
- ◆ The largest: Stanford Linear Collider (2 miles = 3.2 km) (but CLIC design goes to 48.3 km !)
- ◆ One of the less linear: ALPI at LNL (Italy)
- ◆ A limit case, multi-pass linacs: CEBAF at JLAB
- ◆ The most common: medical electron linac (more than 7'000 in operation around the world!)

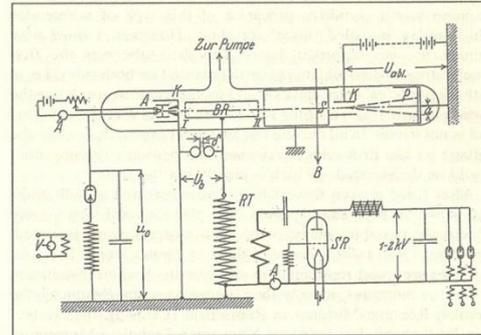
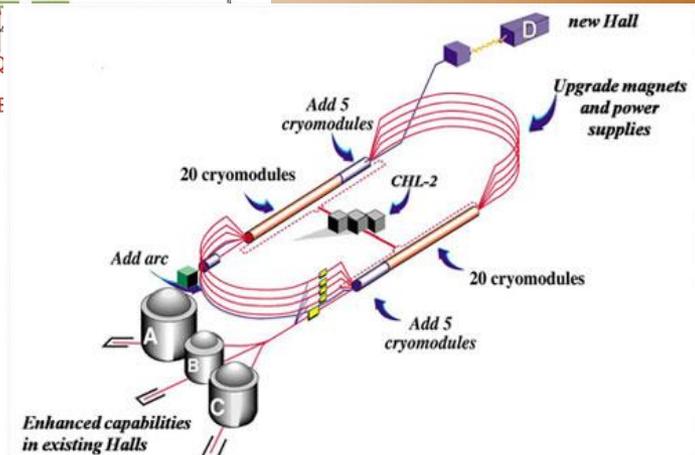
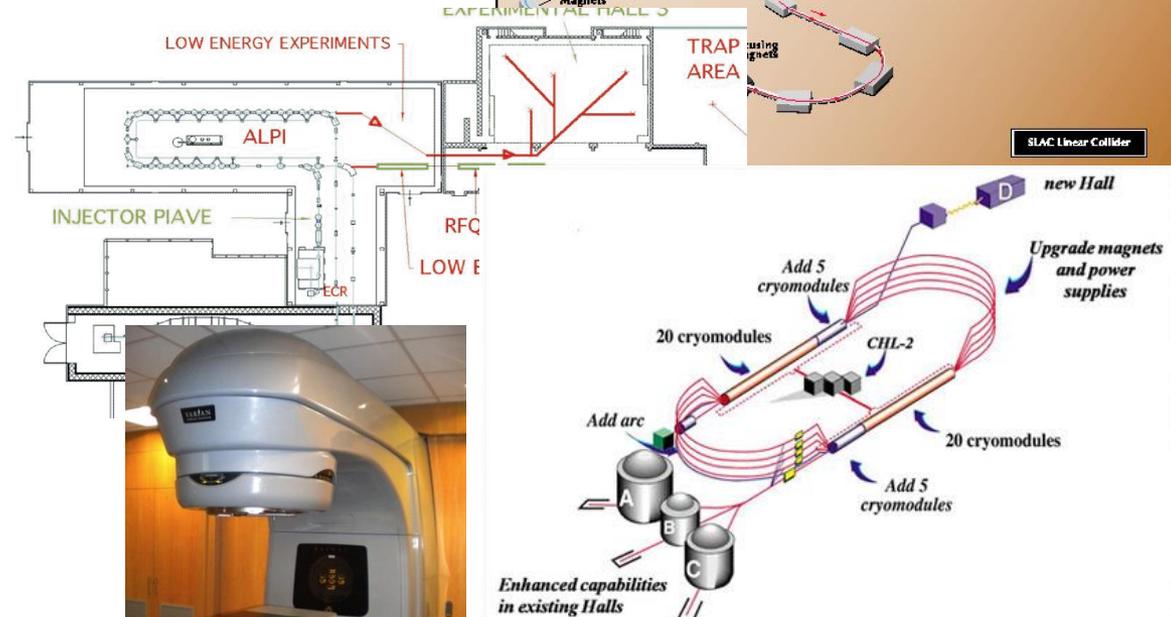
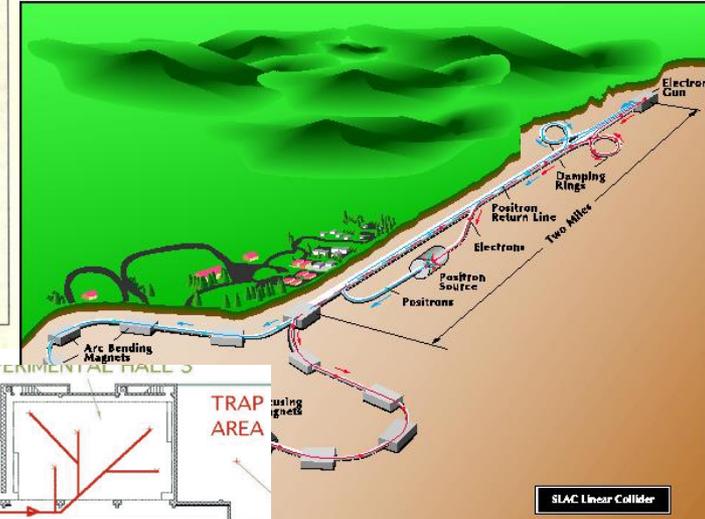


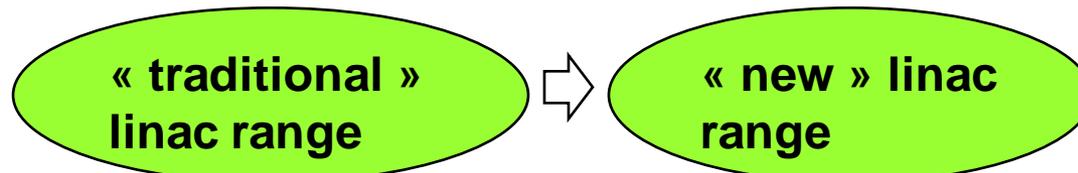
Fig. 3.6: Acceleration tube and switching circuits [Wi28].



Linacs:
Particles go only
once through the
accelerating
structure

LINACS

	Low Energy	High Energy
Protons, Ions	<p>Injectors to synchrotrons, stand alone applications</p> <p><i>synchronous with the RF fields in the range where velocity increases with energy.</i></p> <p>Protons : $\beta = v/c = 0.51$ at 150 MeV, 0.95 at 2 GeV.</p>	<p>Higher beam intensity (increased luminosity or production of secondary beams)</p> <p><i>higher cost/MeV than synchrotron but:</i></p> <ul style="list-style-type: none"> - <i>high average beam current</i> (high repetition rate, less resonances, easier beam loss) - <i>higher accumulated intensity</i> in the synchrotron for a higher energy linac.
Electrons	<p>Conventional e- linac</p> <p><i>simple and compact</i></p>	<p>Linear colliders</p> <p><i>do not lose energy because of synchrotron radiation – only option for high energy!</i></p>



In linear accelerators the beam crosses only once the accelerating structures.

Protons, ions: used in the region where the velocity of the particle beam increases with energy

Electrons: used at all energies.

The periodicity of the RF structure must match the particle velocity → development of a complete panoply of structures (NC and SC) with different features.

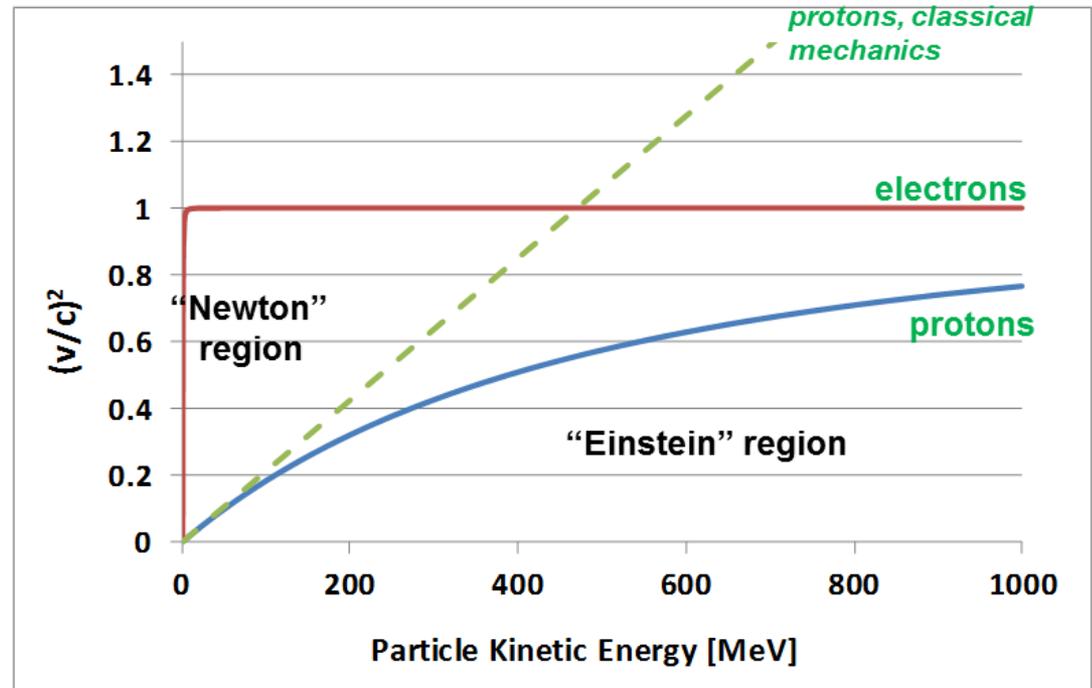
Protons and ions:

at the beginning of the acceleration, beta ($=v/c$) is rapidly increasing, but after few hundred MeV's (protons) relativity prevails over classical mechanics ($\beta \sim 1$) and particle velocity tends to saturate at the speed of light. The region of **increasing β** is the region of **linear accelerators**.

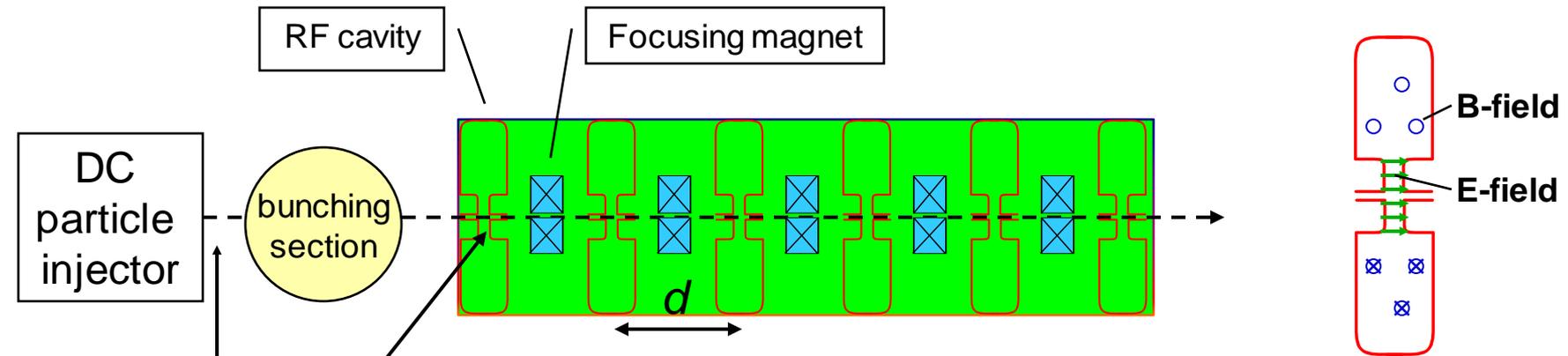
The region of **nearly constant β** is the region of **synchrotrons**.

Electrons:

Velocity=c above few keV

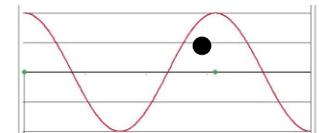


Basic linear accelerator structure



Protons: energy
~100 keV
 $\beta = v/c \sim 0.015$

Accelerating gap: $E = E_0 \cos(\omega t + \phi)$
en. gain $\Delta W = eV_0 T \cos \phi$



Acceleration \rightarrow the beam has to pass in each cavity on a phase ϕ near the crest of the wave

- \Rightarrow {
1. The beam must to be **bunched** at frequency ω
 2. **distance** between cavities and **phase** of each cavity must be correlated

Phase change from cavity i to $i+1$ is $\Delta\phi = \omega\tau = \omega \frac{d}{\beta c} = 2\pi \frac{d}{\beta\lambda}$

\Rightarrow For the beam to be synchronous with the RF wave ("ride on the crest") phase must be related to distance by the relation:

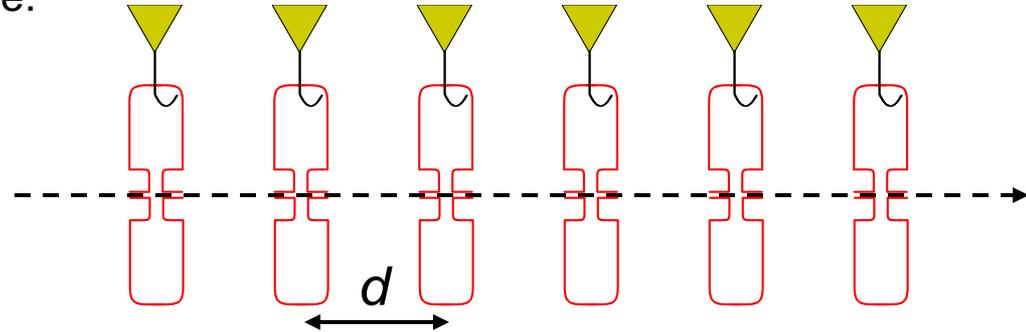
$$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

... and on top of acceleration, we need to introduce in our "linac" some **focusing elements**,
... and on top of that, we will couple a number of gaps in an "**accelerating structure**"

When β increases during acceleration, either the phase difference between cavities $\Delta\phi$ must decrease or their distance d must increase.

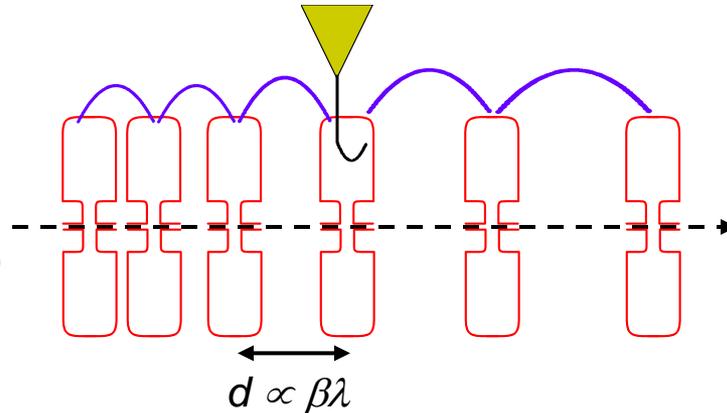
$$\frac{\Delta\phi}{d} = \frac{2\pi}{\beta\lambda}$$

**$d = \text{const.}$
 ϕ variable**



Individual cavities – distance between cavities constant, each cavity fed by an individual RF source, phase of each cavity adjusted to keep synchronism, used for linacs required to operate with different ions or at different energies. Flexible but expensive!

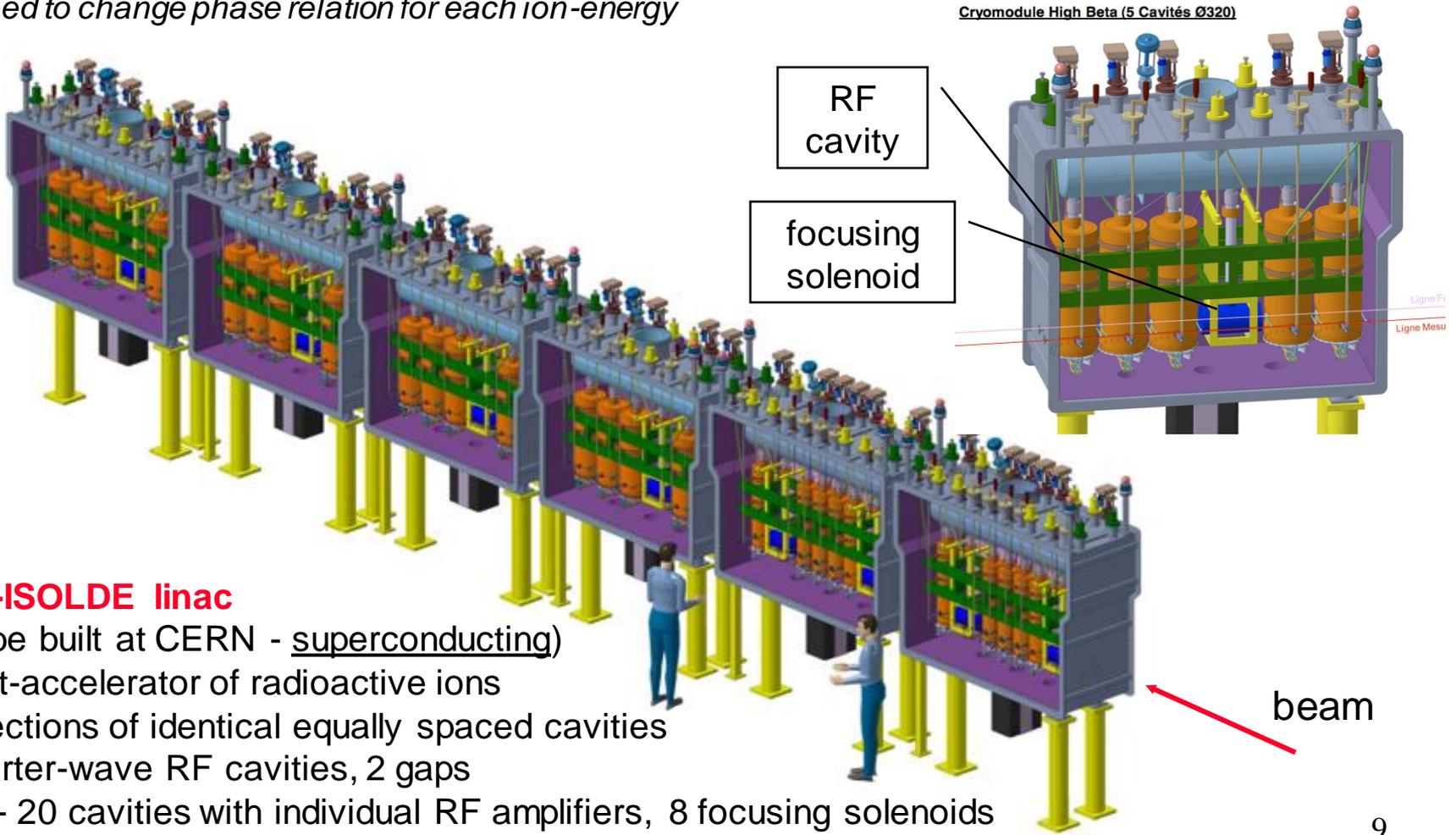
**$\phi = \text{const.}$
 d variable**



Better, but 2 problems:
1. create a "**coupling**";
2. create a mechanical and RF structure with increasing cell length.

Coupled cell cavities - a single RF source feeds a large number of cells (up to ~100!) - the phase between adjacent cells is defined by the coupling and the distance between cells increases to keep synchronism. Once the geometry is defined, it can accelerate only one type of ion for a given energy range. Effective but not flexible.

The goal is flexibility: acceleration of different ions (e/m) at different energies
 → need to change phase relation for each ion-energy



HIE-ISOLDE linac

(to be built at CERN - superconducting)

Post-accelerator of radioactive ions

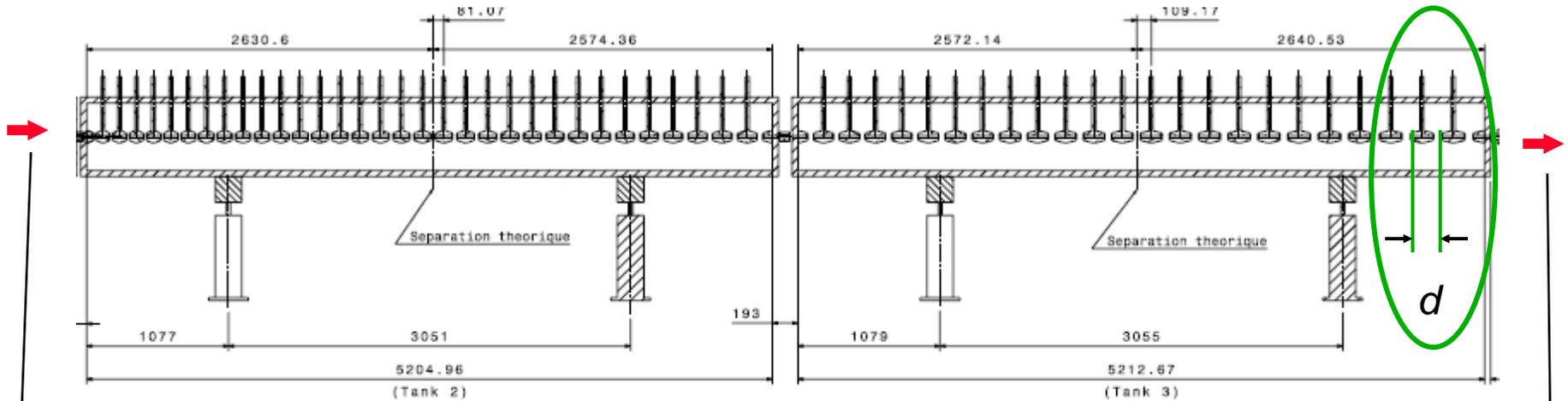
2 sections of identical equally spaced cavities

Quarter-wave RF cavities, 2 gaps

12 + 20 cavities with individual RF amplifiers, 8 focusing solenoids

Energy 1.2 → 10 MeV/u, accelerates different A/m

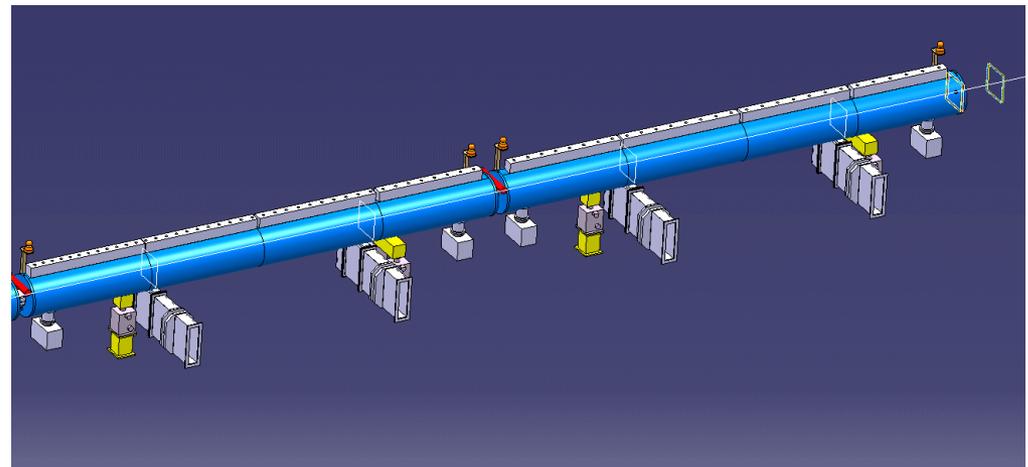
Case 2 : a Drift Tube Linac



10 MeV,
 $\beta = 0.145$

50 MeV,
 $\beta = 0.31$

Tank 2 and 3 of the new Linac4 at CERN:
57 coupled accelerating gaps
Frequency 352.2 MHz, $\lambda = 85$ cm
Cell length ($d = \beta\lambda$) from 12.3 cm to 26.4 cm (factor 2 !).



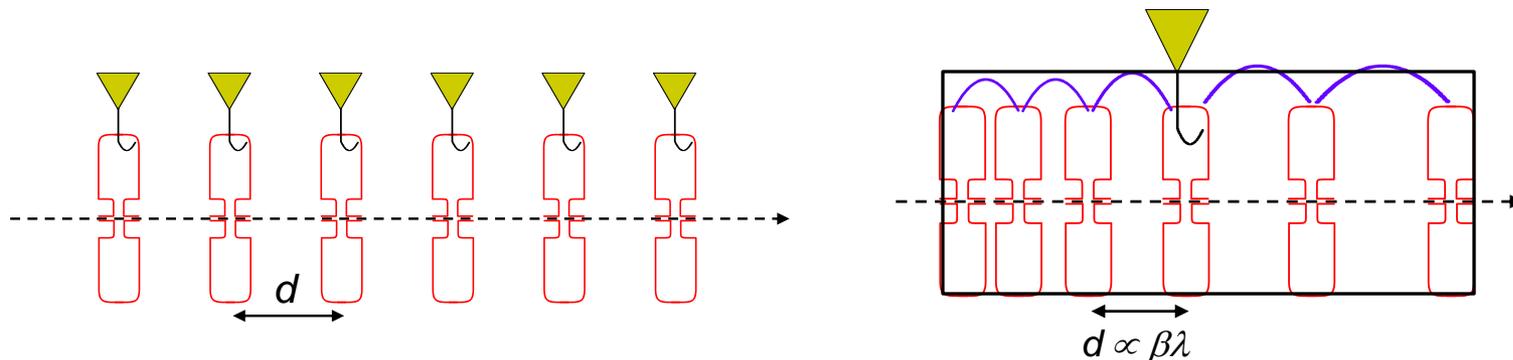
But:

Between these 2 “extremes” there are many “intermediate” cases, because:

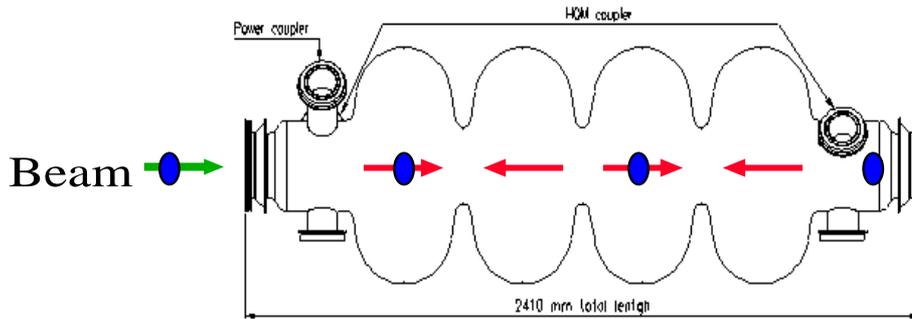
- a. Single-gap cavities are expensive (both cavity and RF source!).
- b. Structures with each cell matched to the beta profile are mechanically complicated and expensive.

→ as soon as the increase of beta with energy becomes small ($\Delta\beta/\Delta W$) we can accept a small error and:

1. Use multi-gap cavities with constant distance between gaps.
2. Use series of identical cavities (standardised design and construction).



The distance between accelerating gaps is proportional to particle velocity

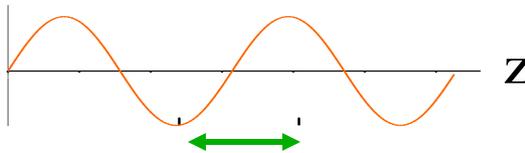


Example: a linac superconducting 4-cell accelerating structure

Synchronism condition bw. particle and wave
 t (travel between centers of cells) = $T/2$

$$\frac{d}{\beta c} = \frac{1}{2f} \quad \Rightarrow \quad d = \frac{\beta c}{2f} = \frac{\beta \lambda}{2}$$

Electric field
(at time t_0)



d = distance between centres of consecutive cells

The synchronism condition is valid only for one β :
 if β changes (acceleration!) in a cavity made of
 identical cells only the central cell will be
 synchronous, and in the other cells the beam will
 have a phase error.

“phase slippage”

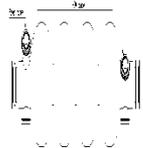
$$\Delta\phi = \omega\Delta t = \pi \frac{\Delta\beta}{\beta}$$

= phase error on a gap for a particle with
 $\beta + \Delta\beta$ crossing a cell designed for β

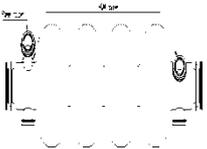
*High and unacceptable for low energy,
 becomes lower and acceptable for high β*

Sections of identical cavities: a superconducting linac (medium β)

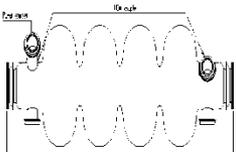
The same superconducting cavity design can be used for different proton velocities. The linac has different sections, each made of cavities with cell length matched to the average beta in that section. At "medium energy" (>150 MeV) we are not obliged to dimension every cell or every cavity for the particular particle beta at that position, and we can accept a slight "asynchronicity" → phase slippage + reduction in acceleration efficiency from the optimum one.



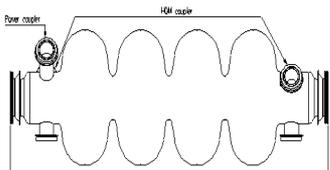
$\beta=0.52$



$\beta=0.7$

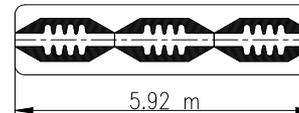


$\beta=0.8$

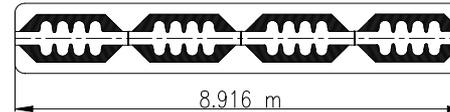


$\beta=1$

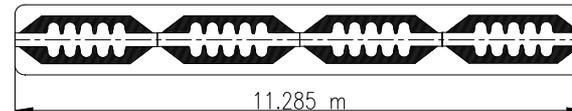
A). $\beta=0.52$



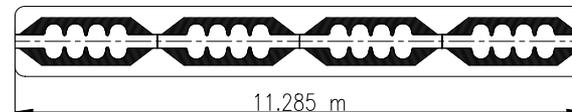
B). $\beta=0.7$

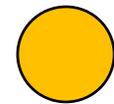


C). $\beta=0.8$, LEP cryostat



D). $\beta=1$, LEP cryostat

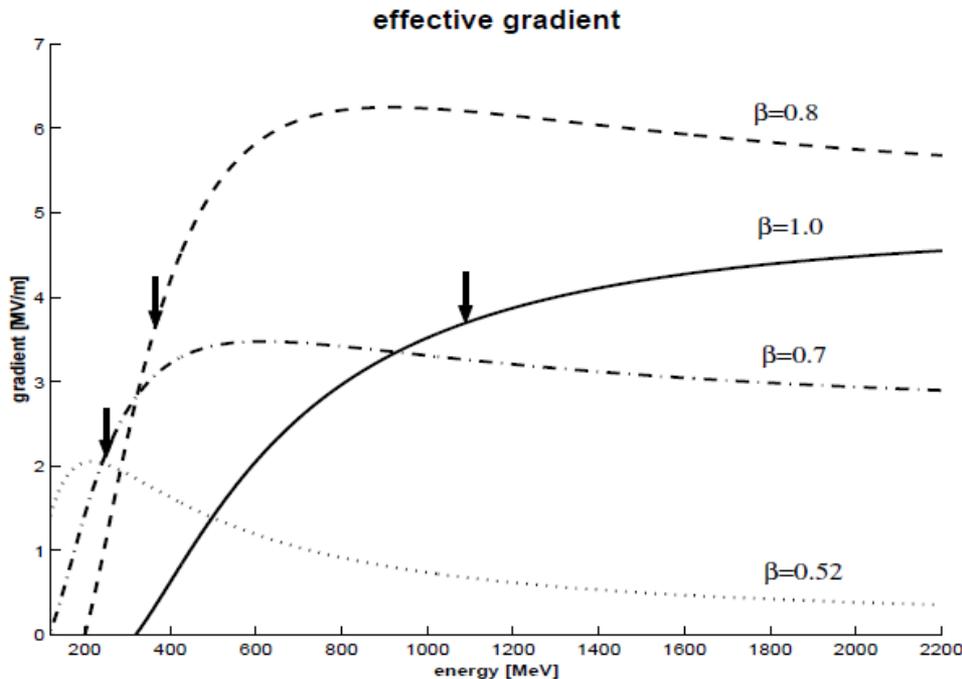




When sequences of cells are not matched to the particle beta → phase slippage

$$\Delta\phi = \omega\Delta t = \pi \frac{\Delta\beta}{\beta} \quad \rightarrow$$

1. The effective gradient seen by the particle is lower.
2. The phase of the bunch centre moves away from the synchronous phase → can go (more) into the non-linear region, with possible longitudinal emittance growth and beam loss.



$$\Delta\phi = \pi \frac{\Delta\beta}{\beta} = \pi \frac{1}{\gamma(\gamma-1)} \frac{\Delta W}{W}$$

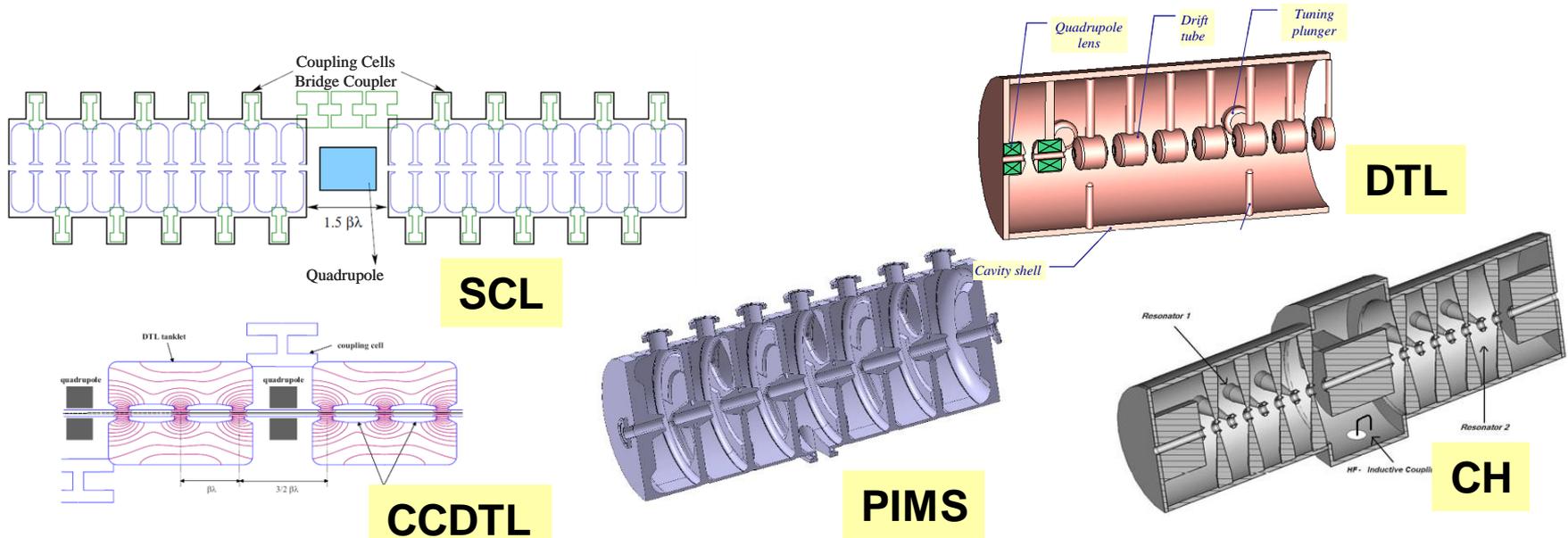
Very large at small energy ($\gamma \sim 1$)
 becomes negligible at high energy
 ($\sim 2.5^\circ/m$ for $\gamma \sim 1.5$, $W = 500$ MeV).

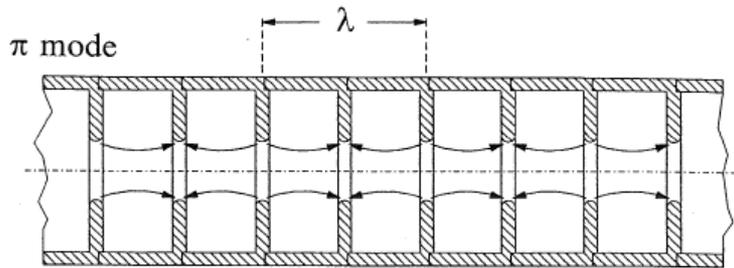
Curves of effective gradient
 (gradient seen by the beam for a
 constant gradient in the cavity)
 for the previous case (4 sections
 of beta 0.52, 0.7, 0.8 and 1.0).

Multi-gap coupled-cell cavities (for protons and ions)

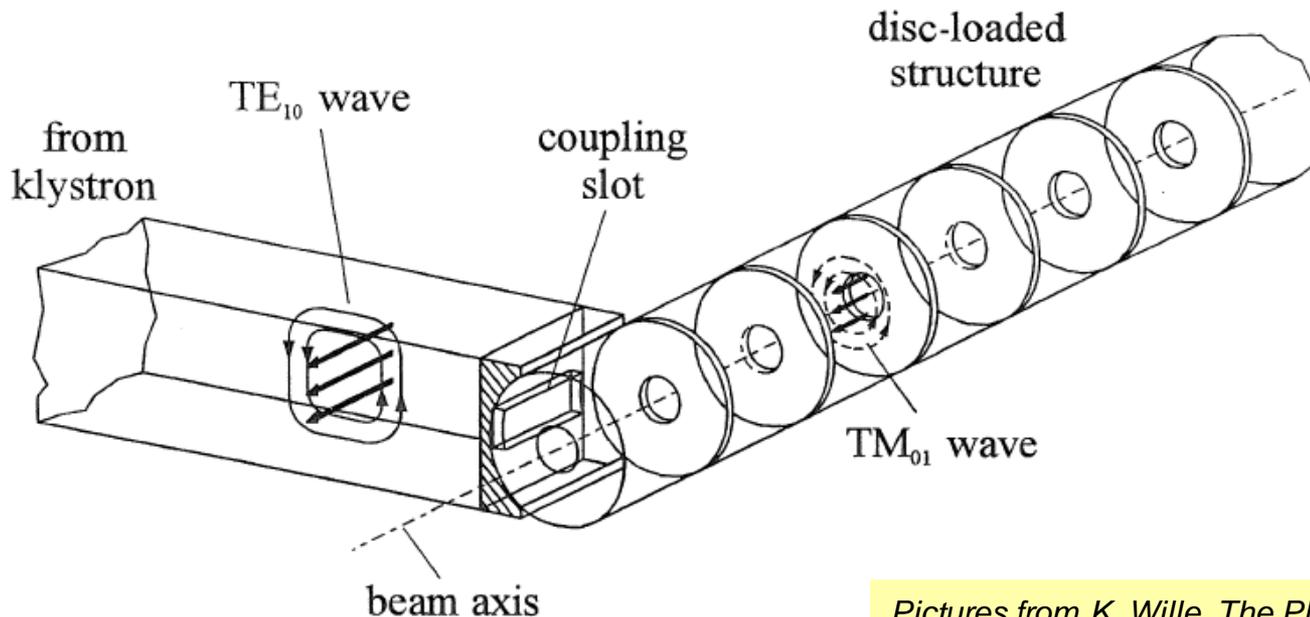
Between the 2 extreme cases (array of independently phased single-gap cavities / single long chain of coupled cells with lengths matching the particle beta) there can be a large number of variations (number of gaps per cavity, length of the cavity, type of coupling) each optimized for a certain range of energy and type of particle.

The goal of this lecture is to provide the background to understand the main features of these different structures...

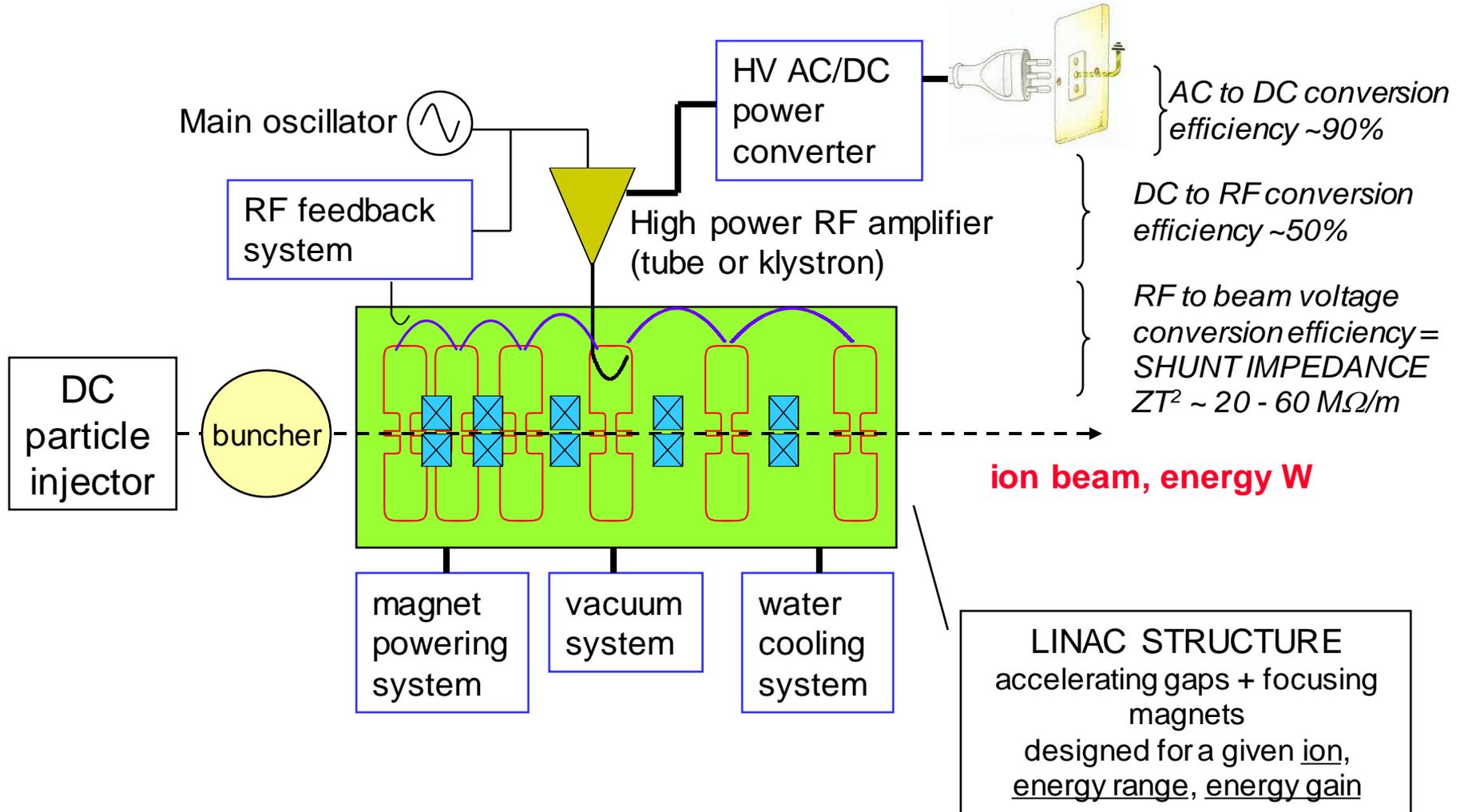




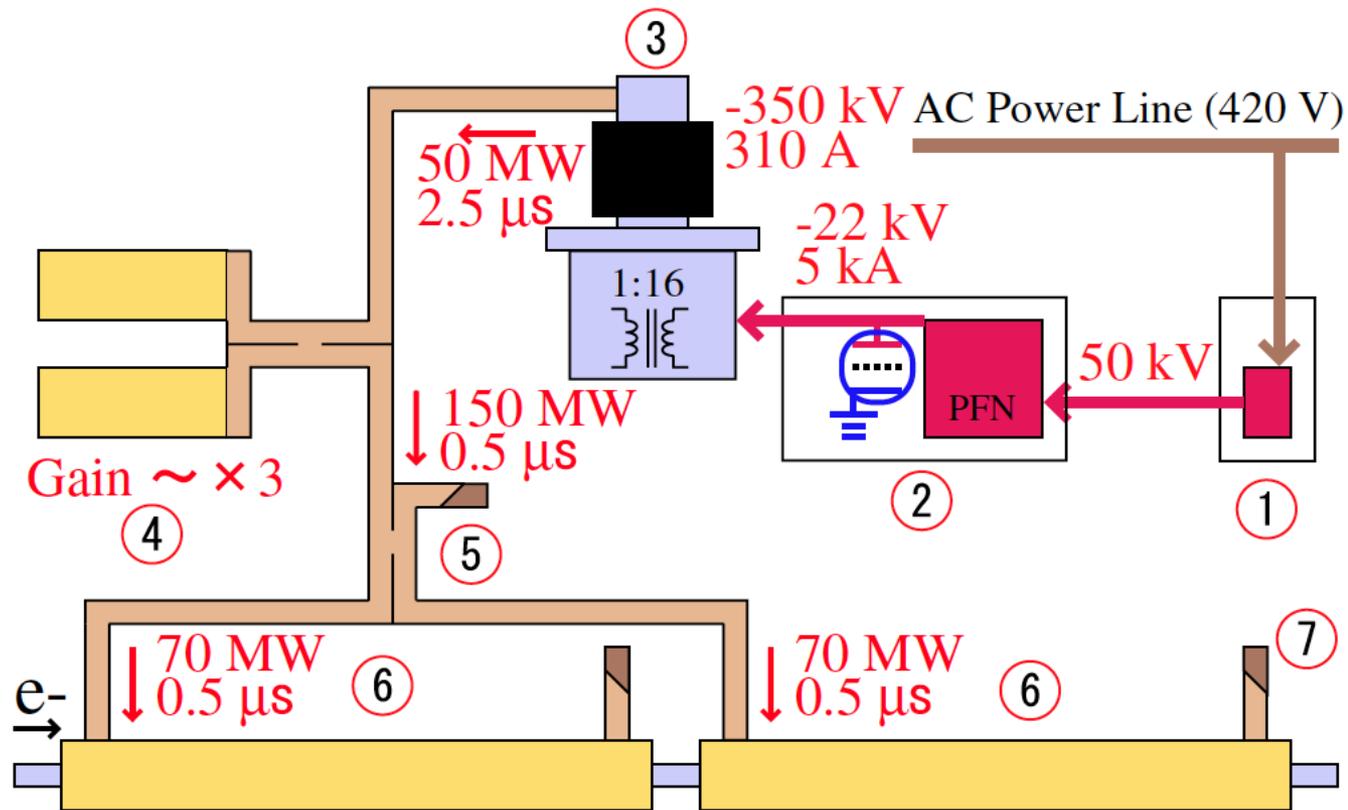
1. In an electron linac velocity is \sim constant. To use the fundamental accelerating mode cell length must be $d = \beta\lambda / 2$.
2. the linac structure will be made of a **sequence of identical cells**. Because of the limits of the RF source, the cells will be grouped in cavities operating in **travelling wave mode**.



(Proton) linac building blocks



TSUMORU SHINTAKE *et al.*

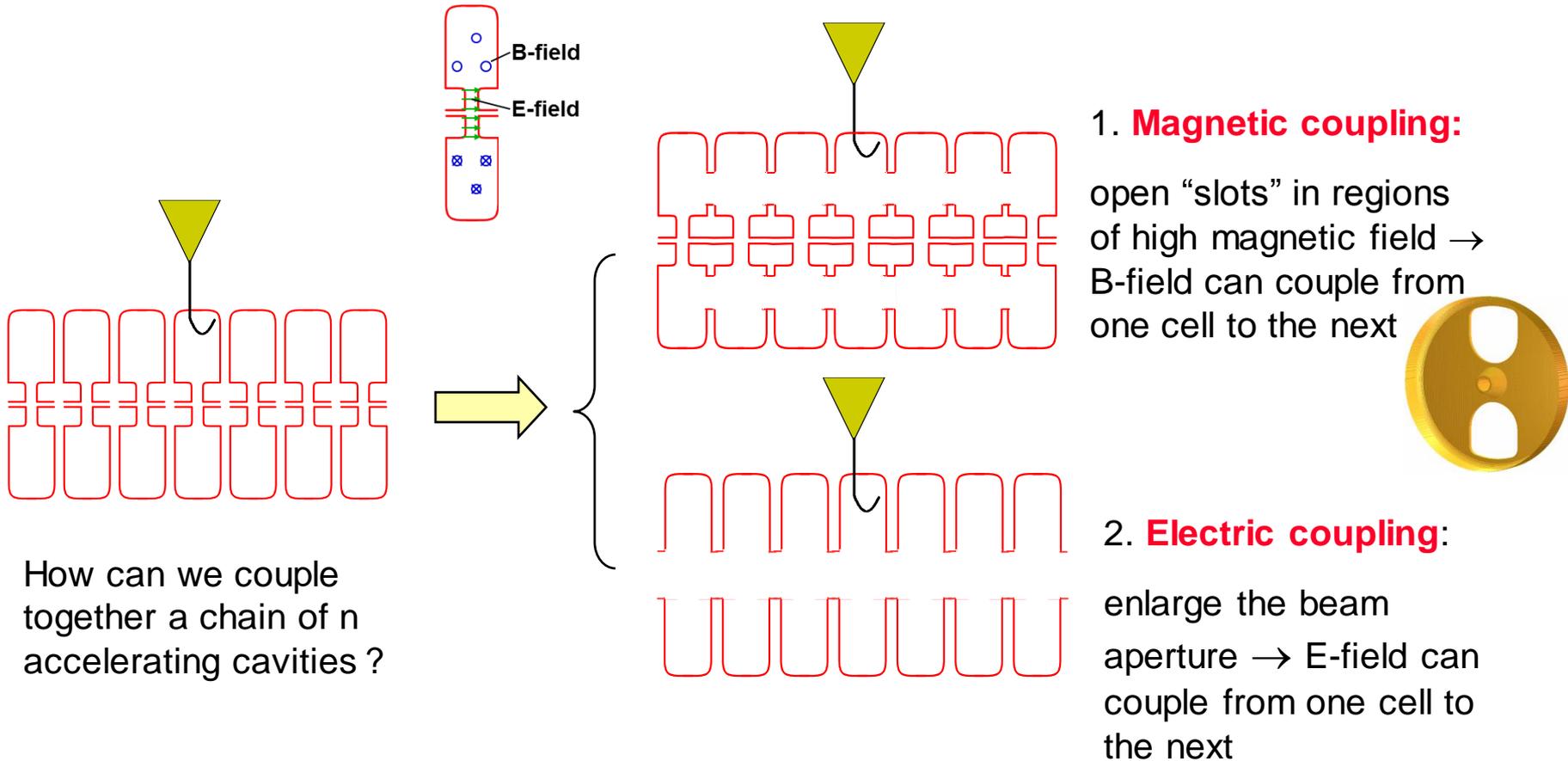


What did we learn?

1. A linac is composed of an array of accelerating gaps, interlaced with focusing magnets (quadrupoles or solenoids), following an ion source with a DC extraction and a bunching section.
2. When beam velocity is increasing with energy (“Newton” regime), we have to match to the velocity (or to the relativistic β) either the phase difference or the distance between two subsequent gaps.
3. We have to compromise between synchronicity (distance between gaps matched to the increasing particle velocity) and simplicity (number of gaps on the same RF source, sequences of identical cavities). At low energies we have to follow closely the synchronicity law, whereas at high energies we have a certain freedom in the number of identical cells/identical cavities.
4. Proton and electron linacs have similar architectures; proton linacs usually operate in standing wave mode, and electron linacs in traveling wave mode.

2 - Accelerating Structures



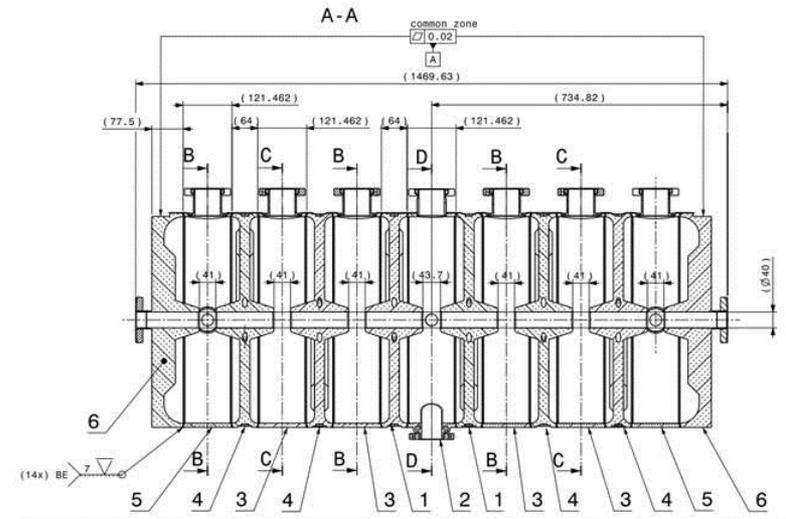
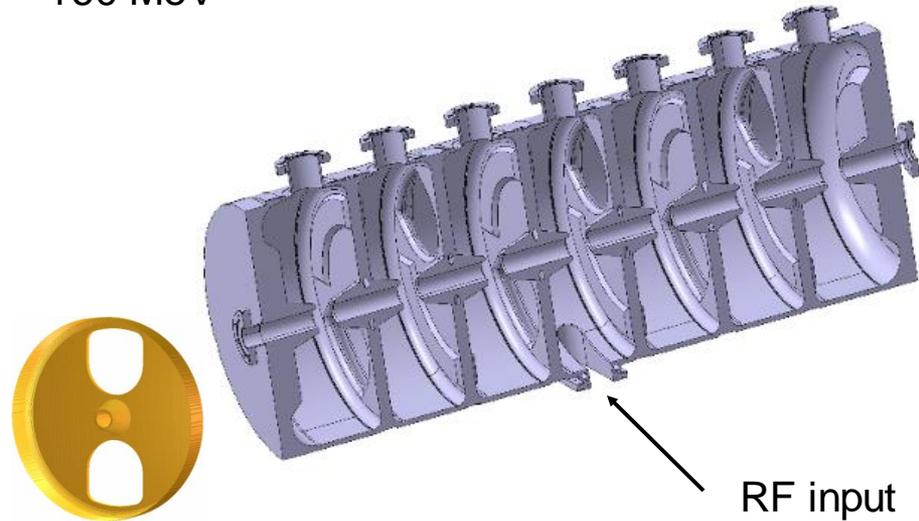


How can we couple together a chain of n accelerating cavities ?

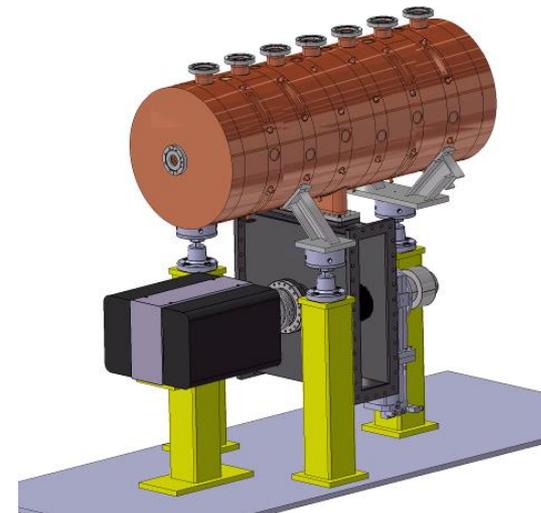
The effect of the coupling is that the cells no longer resonate independently, but will have common resonances with well defined field patterns.

A 7-cell magnetically-coupled structure: the PIMS

PIMS = Pi-Mode Structure, will be used in Linac4 at CERN to accelerate protons from 100 to 160 MeV

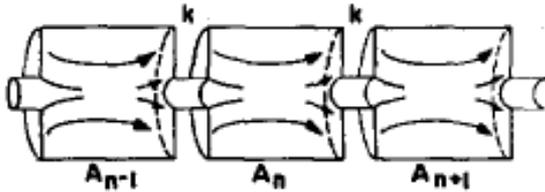


This structure is composed of 7 accelerating cells, magnetically coupled.
 The cells in a cavity have the same length, but they are longer from one cavity to the next, to follow the increase in beam velocity.



Linac cavities as chains of coupled resonators

What is the relative phase and amplitude between cells in a chain of coupled cavities?

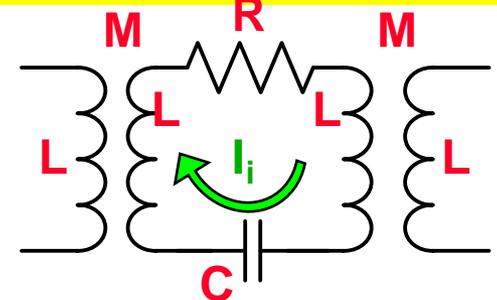


COUPLED CAVITIES

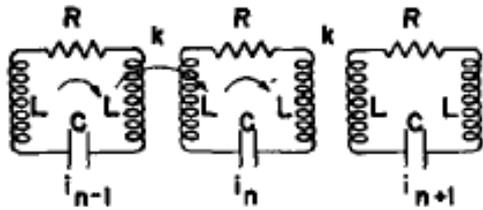
A linear chain of accelerating cells can be represented as a chain of resonant circuits magnetically coupled.

Individual cavity resonating at $\omega_0 \rightarrow$ frequenci(es) of the coupled system ?

Resonant circuit equation for circuit i ($R \cong 0, M = k\sqrt{L^2}$):



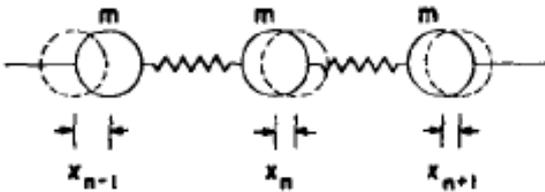
$$\omega_0 = 1/\sqrt{2LC}$$



COUPLED CIRCUITS

$$I_i(2j\omega L + \frac{1}{j\omega C}) + j\omega k L(I_{i-1} + I_{i+1}) = 0$$

Dividing both terms by $2j\omega L$:



LINEAR LATTICE

$$X_i(1 - \frac{\omega_0^2}{\omega^2}) + \frac{k}{2}(X_{i-1} + X_{i+1}) = 0$$

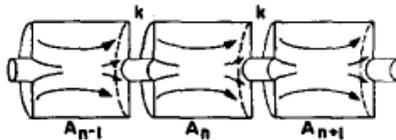
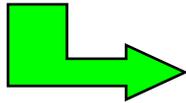
General response term,
 \propto (stored energy)^{1/2},
 can be voltage, E-field,
 B-field, etc.

General
 resonance term

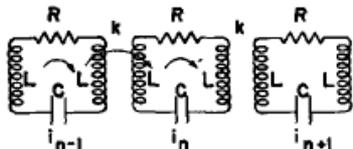
Contribution from
 adjacent oscillators

$$X_i \left(1 - \frac{\omega_0^2}{\omega^2}\right) + \frac{k}{2} (X_{i-1} + X_{i+1}) = 0$$

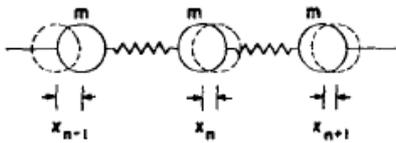
$$i = 0, \dots, N$$



COUPLED CAVITIES



COUPLED CIRCUITS



LINEAR LATTICE

A chain of $N+1$ resonators is described by a $(N+1) \times (N+1)$ matrix:

$$\begin{vmatrix} 1 - \frac{\omega_0^2}{\omega^2} & \frac{k}{2} & 0 & \dots \\ \frac{k}{2} & 1 - \frac{\omega_0^2}{\omega^2} & \frac{k}{2} & \dots \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \frac{k}{2} & 1 - \frac{\omega_0^2}{\omega^2} \end{vmatrix} \begin{vmatrix} X_0 \\ X_2 \\ \dots \\ X_N \end{vmatrix} = 0 \quad \text{or} \quad M X = 0$$

This matrix equation has solutions only if $\det M = 0$

Eigenvalue problem!

1. System of order $(N+1)$ in $\omega \rightarrow$ only $N+1$ frequencies will be solution of the problem ("eigenvalues", corresponding to the resonances) \rightarrow a system of N coupled oscillators has N resonance frequencies \rightarrow an *individual resonance opens up into a band of frequencies*.
2. At each frequency ω_i will correspond a set of relative amplitudes in the different cells (X_0, X_2, \dots, X_N) : the "eigenmodes" or "modes".

We can find an analytical expression for eigenvalues (frequencies) and eigenvectors (modes):

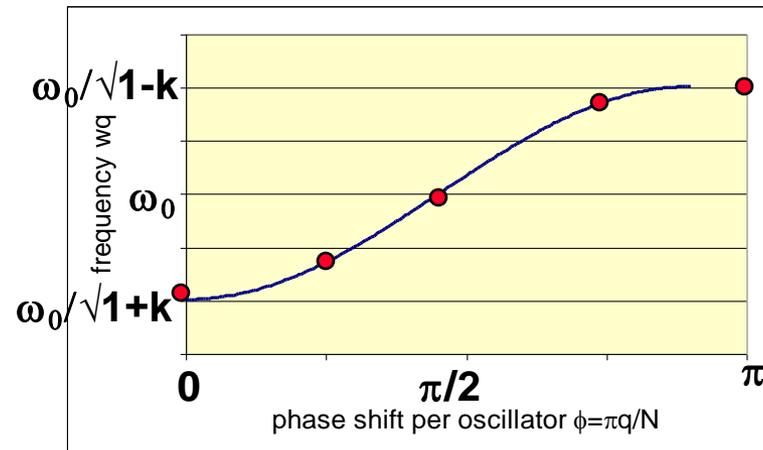
Frequencies of the coupled system :

$$\omega_q^2 = \frac{\omega_0^2}{1 + k \cos \frac{\pi q}{N}}, \quad q = 0, \dots, N$$

the index q defines the number of the solution \rightarrow is the “mode index”

\rightarrow Each mode is characterized by a phase $\pi q/N$. Frequency vs. phase of each mode can be plotted as a “dispersion curve” $\omega=f(\phi)$:

1. each mode is a point on a sinusoidal curve.
2. modes are equally spaced in phase.



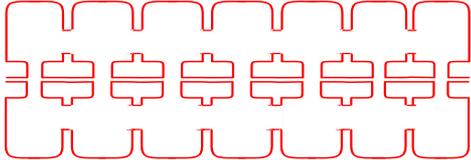
The “eigenvectors = relative amplitude of the field in the cells are:

$$X_i^{(q)} = (const) \cos \frac{\pi qi}{N} e^{j\omega_q t} \quad q = 0, \dots, N$$

STANDING WAVE MODES, defined by a phase $\pi q/N$ corresponding to the phase shift between an oscillator and the next one $\rightarrow \pi q/N = \Phi$ is the phase difference between adjacent cells that we have introduced in the 1st part of the lecture.

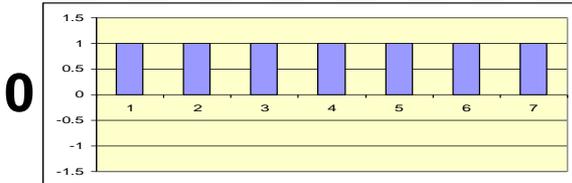


Example: Acceleration on the normal modes of a 7-cell structure



$$X_i^{(q)} = (\text{const}) \cos \frac{\pi q i}{N} e^{j\omega_q t} \quad q = 0, \dots, N$$

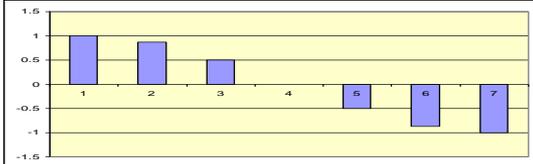
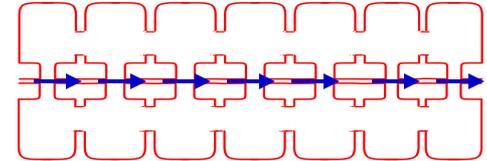
$$\Delta\phi = 2\pi \frac{d}{\beta\lambda}$$



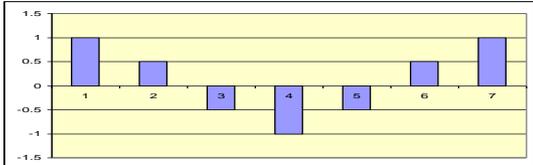
$$\Phi = 2\pi, \quad 2\pi \frac{d}{\beta\lambda} = 2\pi, \quad d = \beta\lambda$$

$$\omega = \omega_0 / \sqrt{1+k}$$

0 (or 2π) mode, acceleration if $d = \beta\lambda$



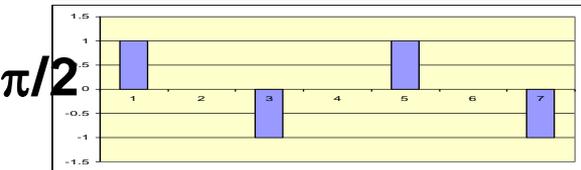
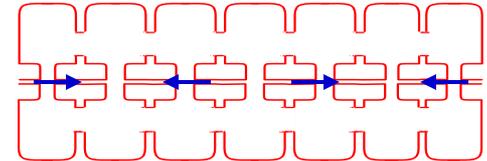
Intermediate modes



$$\Phi = \frac{\pi}{2}, \quad 2\pi \frac{d}{\beta\lambda} = \frac{\pi}{2}, \quad d = \frac{\beta\lambda}{4}$$

$$\omega = \omega_0$$

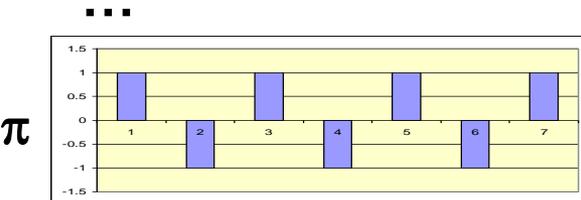
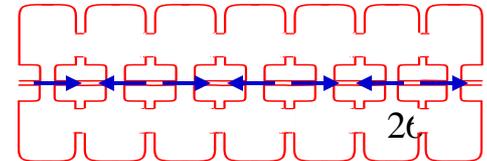
$\pi/2$ mode, acceleration if $d = \beta\lambda/4$



$$\Phi = \pi, \quad \pi \frac{d}{\beta\lambda} = 2\pi, \quad d = \frac{\beta\lambda}{2}$$

$$\omega = \omega_0 / \sqrt{1-k}$$

π mode, acceleration if $d = \beta\lambda/2$,



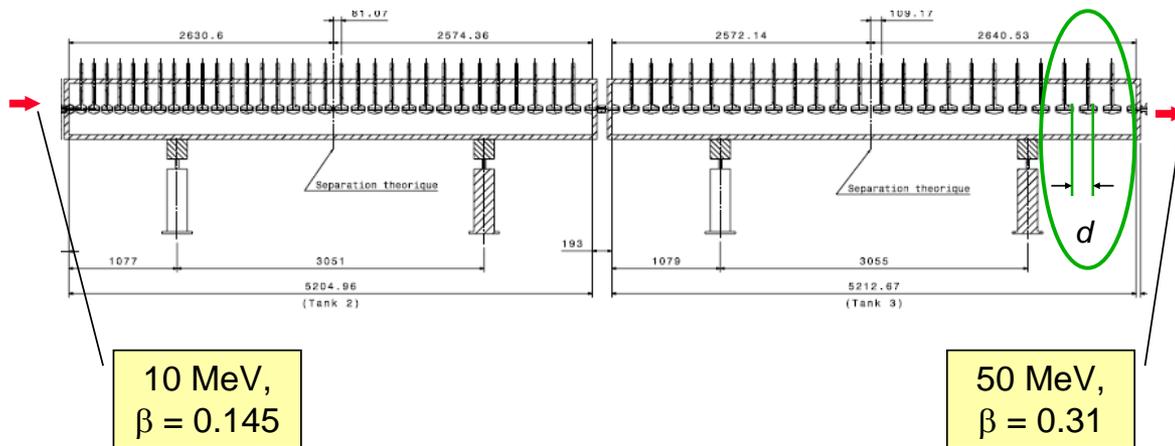
Note: Field always maximum in first and last cell!

Note: our equations depend only on the cell frequency ω , not on the cell length d !!!

$$\omega_q^2 = \frac{\omega_0^2}{1 + k \cos \frac{\pi q}{N}}, \quad q = 0, \dots, N$$

$$X_n^{(q)} = (\text{const}) \cos \frac{\pi q n}{N} e^{j\omega_q t} \quad q = 0, \dots, N$$

→ As soon as we keep the frequency of each cell constant, we can change the cell length following any acceleration (β) profile!



Example:

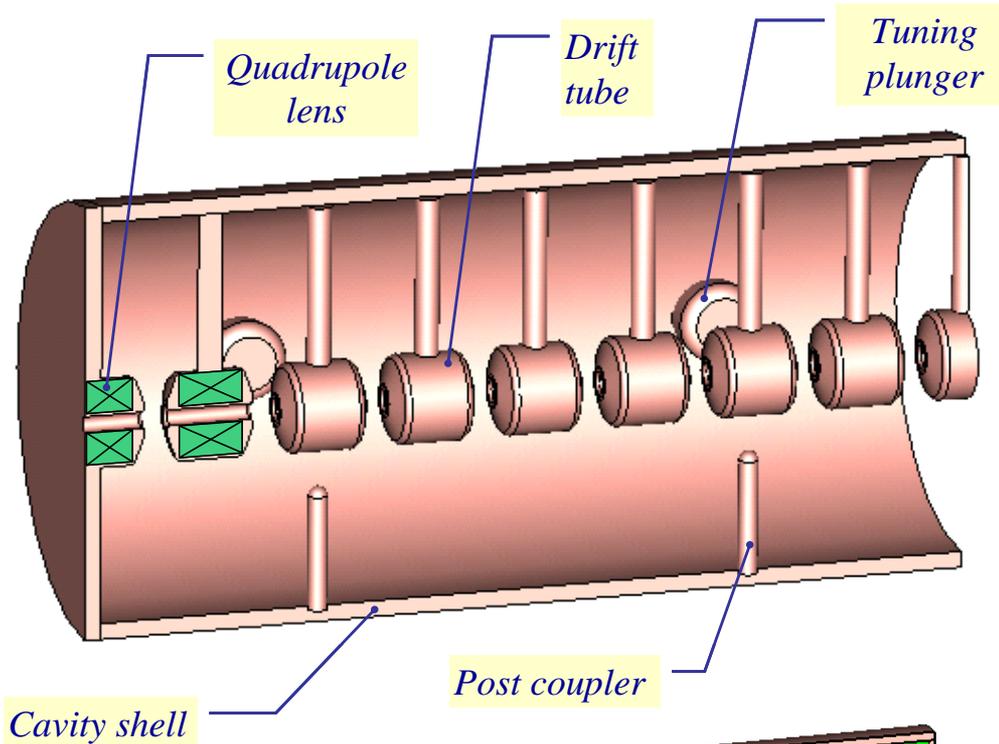
The Drift Tube Linac (DTL)

Chain of many (up to 100!) accelerating cells operating in the 0 mode. The ultimate coupling slot: no wall between the cells!

Each cell has a different length, but the cell frequency remains constant → “the EM fields don’t see that the cell length is changing!” 27

$d \uparrow \rightarrow (L \uparrow, C \downarrow) \rightarrow LC \sim \text{const} \rightarrow \omega \sim \text{const}$

The DTL

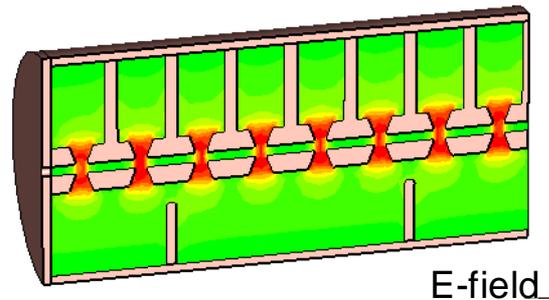


Standing wave linac structure for protons and ions, $\beta=0.1-0.5$, $f=20-400$ MHz

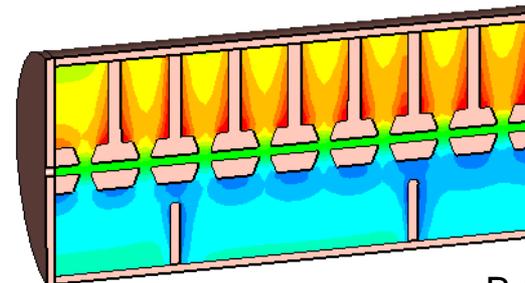
Drift tubes are suspended by stems (no net RF current on stem)

Coupling between cells is maximum (no slot, fully open !)

The 0-mode allows a long enough cell ($d=\beta\lambda$) to house focusing quadrupoles inside the drift tubes!

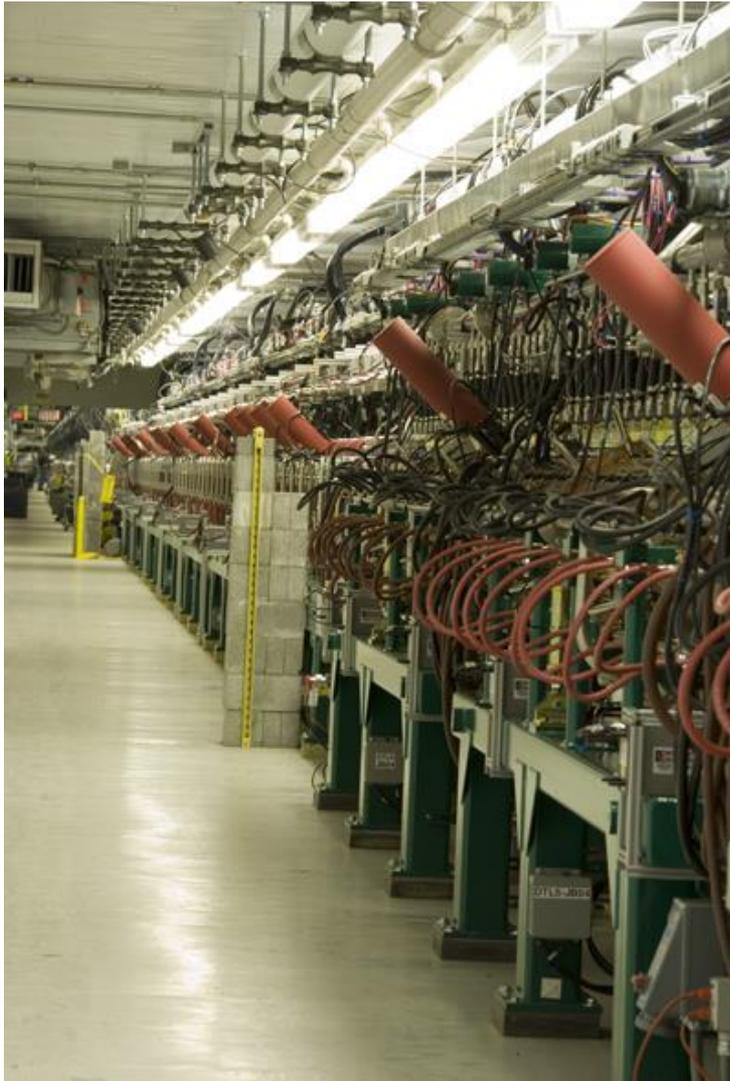


E-field



B-field

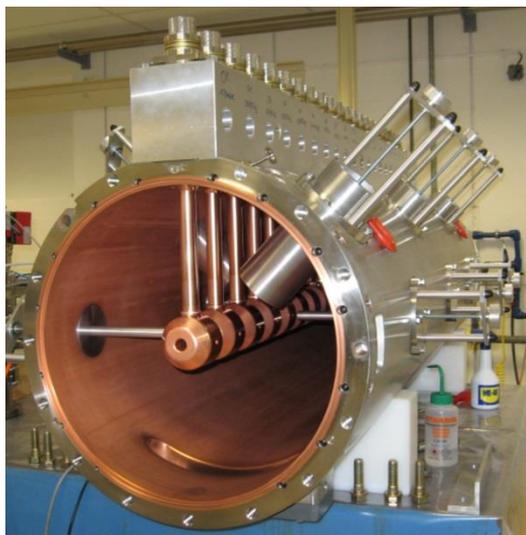
Examples of DTL



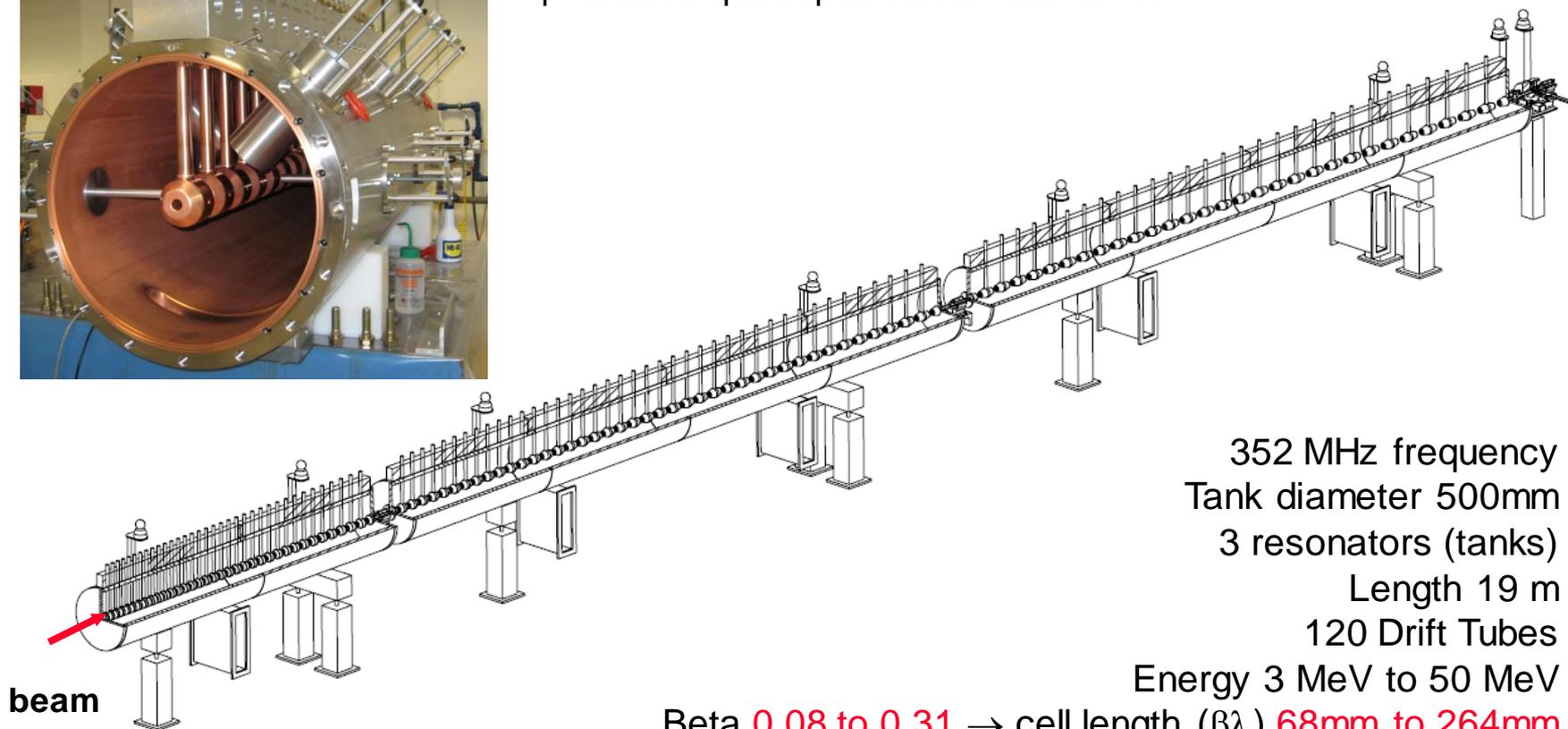
Top; CERN Linac2 Drift Tube Linac: 1978, 202.5 MHz, 3 tanks, final energy 50 MeV, tank diameter 1 meter.

Left: The Drift Tube Linac of the SNS at Oak Ridge (USA): 402.5 MHz, 6 tanks, final energy 87 MeV.

The Linac4 DTL



DTL tank 1 fully equipped: focusing by small permanent quadrupoles inside drift tubes.



352 MHz frequency
Tank diameter 500mm
3 resonators (tanks)

Length 19 m

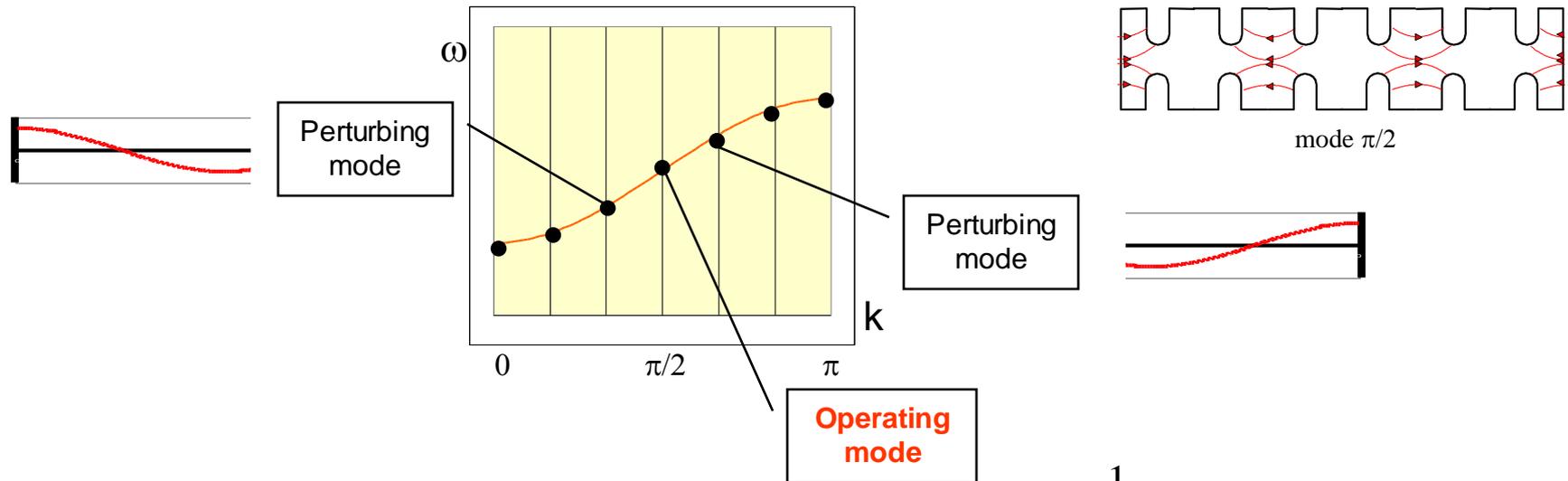
120 Drift Tubes

Energy 3 MeV to 50 MeV

Beta 0.08 to 0.31 → cell length ($\beta\lambda$) 68mm to 264mm

→ factor 3.9 increase in cell length

Long chains of linac cells can be operated in the $\pi/2$ mode, which is **intrinsically insensitive** to mechanical errors = differences in the cell frequencies.
In presence of errors, the E-field will have components from the adjacent modes, with amplitude proportional to the error and to the mode separation.



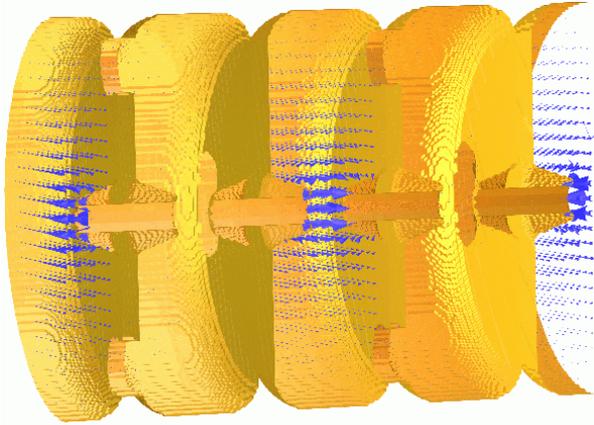
→ Contribution from adjacent modes proportional to $\frac{1}{f^2 - f_0^2}$ **with the sign !!!**

The perturbation will add a component $\Delta E/(f^2 - f_0^2)$ for each of the nearest modes.

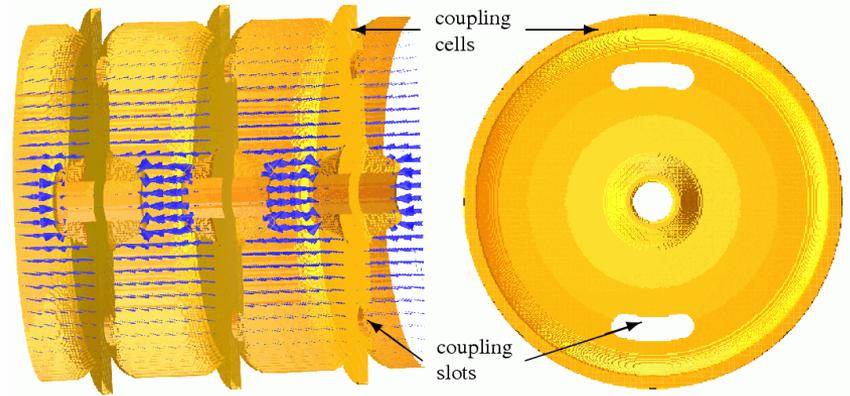
Contributions from equally spaced modes in the dispersion curve will cancel each other !!

Examples of $\pi/2$ structures

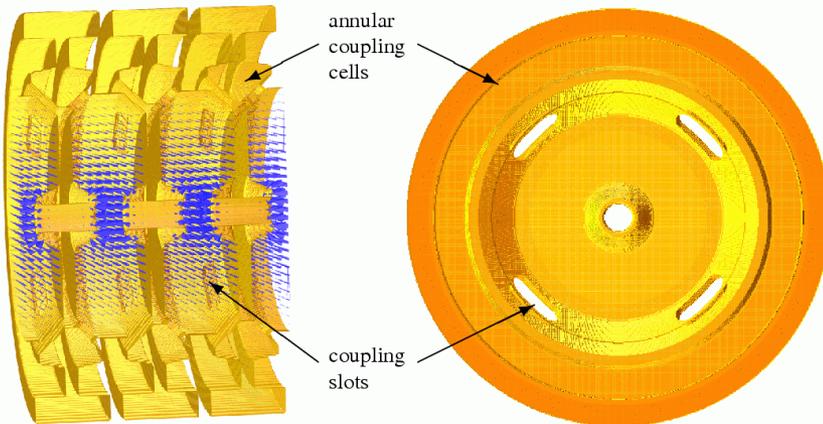
$\pi/2$ -mode in a coupled-cell structure



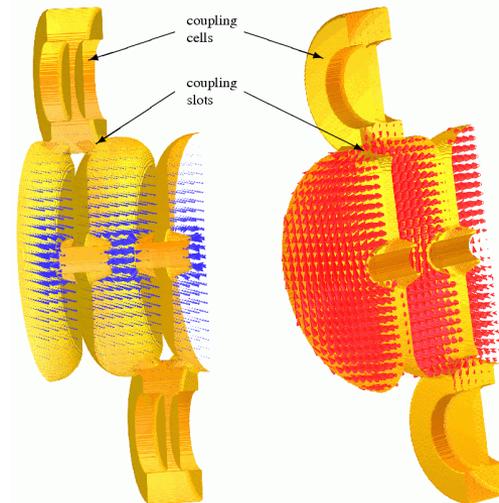
On axis Coupled Structure (OCS)



Annular ring Coupled Structure (ACS)



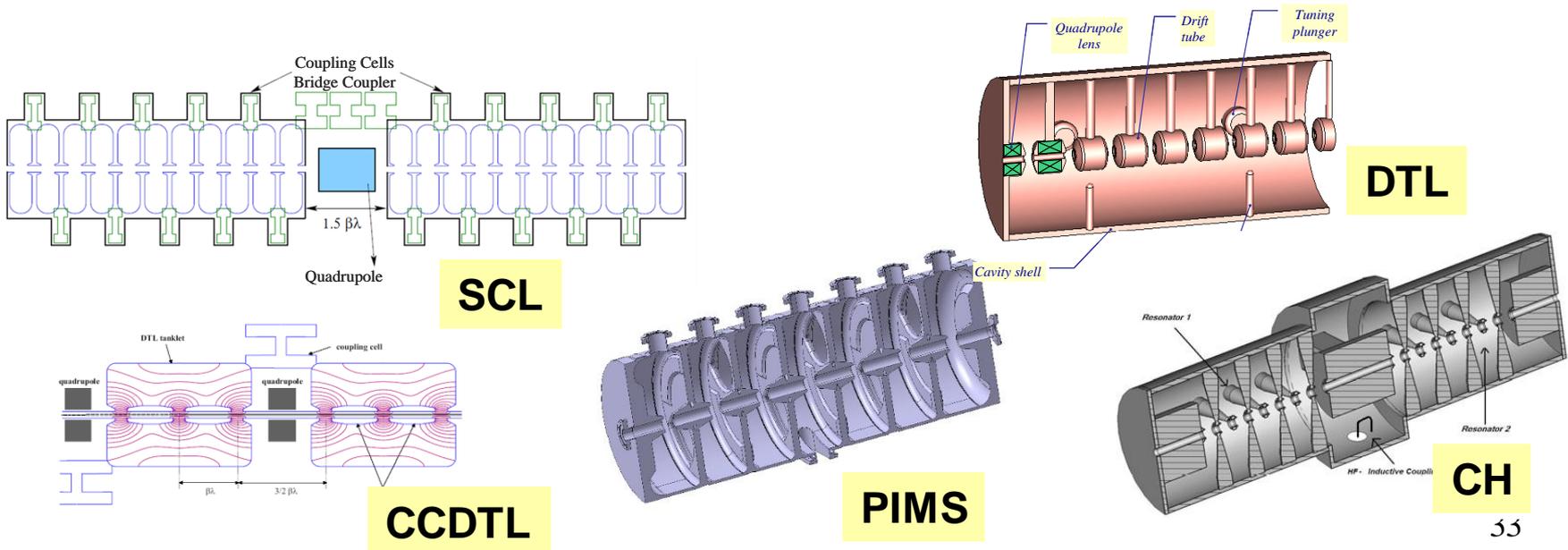
Side Coupled Structure (SCS)



For normal-conducting, the goal is designing high-efficiency structures with a large number of cells (higher power RF sources are less expensive).

Two important trends:

1. Use $\pi/2$ modes for stability of long chains of resonators \rightarrow CCDTL (Cell-Coupled Drift Tube Linac), SCL (Side Coupled Linac), ACS (Annular Coupled Structure),....
2. Use alternative modes: H-mode structures (TE band) \rightarrow Interdigital IH, CH



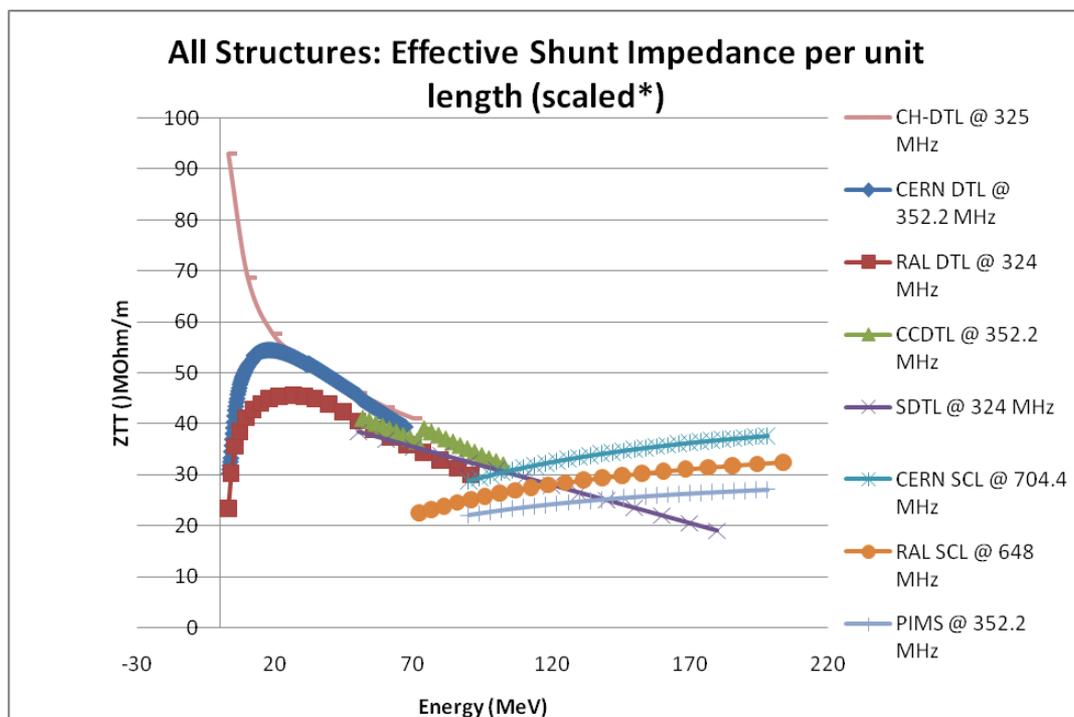
How to choose the best linac structure?

Main figure of merit is the power efficiency = **shunt impedance**

Ratio between energy gain (square) and power dissipation, is a measure of the energy efficiency of a structure.

Depends on the beta, on the energy and on the mode of operation.

But the choice of the best accelerating structure for a certain energy range depends as well on **beam dynamics** and on construction **cost**.

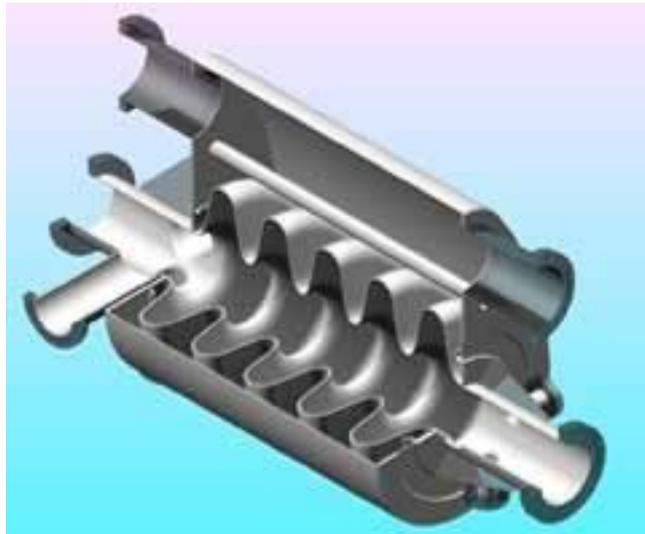


Comparison of shunt-impedances for different low-beta structures done in 2005-08 by the “HIPPI” EU-funded Activity.

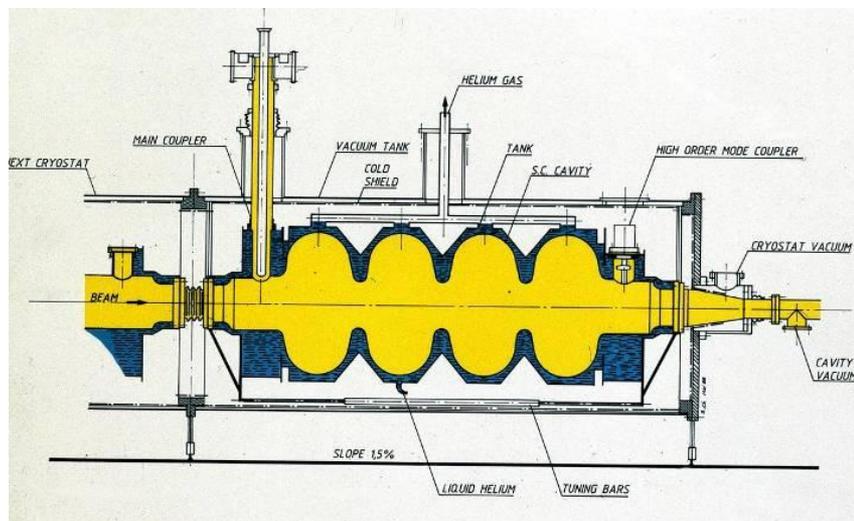
In general terms, a DTL-like structure is preferred at low-energy, and π -mode structures at high-energy.

CH is excellent at very low energies (ions).

Multi-gap Superconducting linac structures (elliptical)

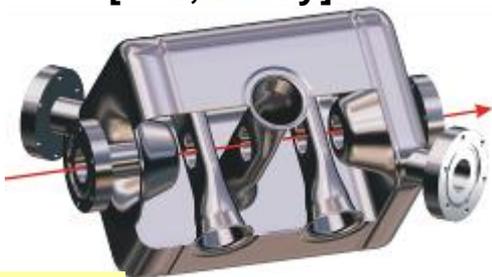


Standing wave structures for particles at $\beta > 0.5-0.7$, widely used for protons (SNS, etc.) and electrons (ILC, etc.)
 $f = 350-700$ MHz (protons),
 $f = 350$ MHz - 3 GHz (electrons)
 Chain of cells electrically coupled, large apertures (ZT^2 not a concern).
 Operating in π -mode, cell length $\beta\lambda/2$
 Input coupler placed at one end.



The superconducting zoo

Spoke (low beta)
[FZJ, Orsay]



4 gaps

CH (low/medium beta)
[IAP-FU]



10 gaps

QWR (low beta)
[LNL, etc.]



2 gaps

HWR (low beta)
[FZJ, LNL, Orsay]



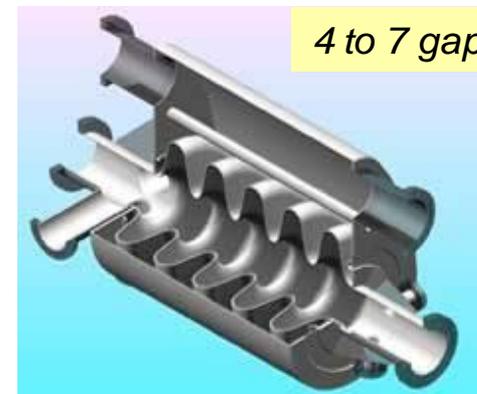
2 gaps

Re-entrant
[LNL]



1 gap

4 to 7 gaps



Elliptical cavities [CEA, INFN-MI, CERN, ...]

Superconducting structure for linacs can have a small number of gaps → used for low and medium beta.

Elliptical structures with more gaps (4 to 7) are used for medium and high beta.

What happens if we have an infinite chain of oscillators?

$$\omega_q^2 = \frac{\omega_0^2}{1 + k \cos \frac{\pi q}{N}}, \quad q = 0, \dots, N$$

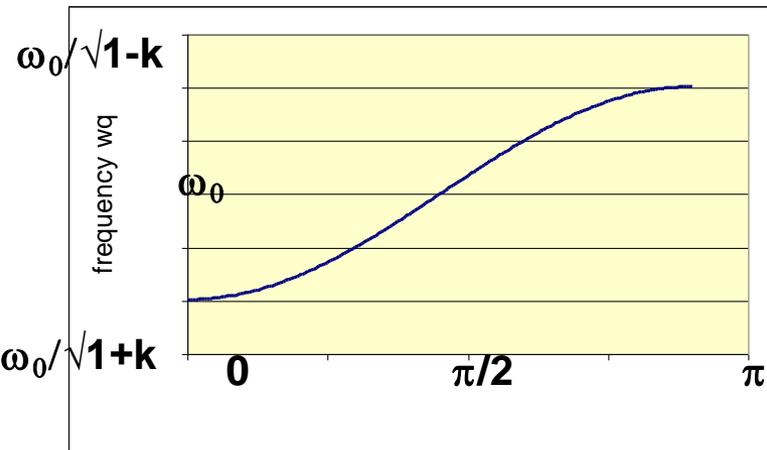
becomes ($N \rightarrow \infty$)

$$\omega^2 = \frac{\omega_0^2}{1 + k \cos \varphi}$$

$$X_n^{(q)} = (\text{const}) \cos \frac{\pi q n}{N} e^{j\omega_q t} \quad q = 0, \dots, N$$

becomes ($N \rightarrow \infty$)

$$X_i = (\text{const}) e^{j\omega_q t}$$



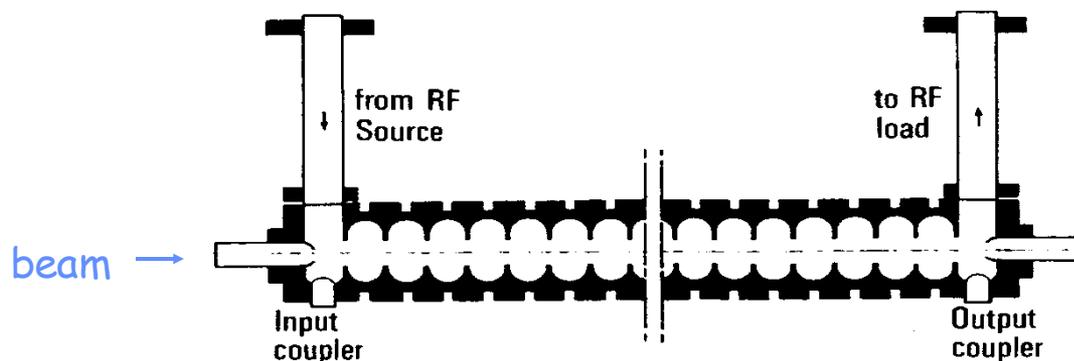
All modes in the dispersion curve are allowed, the original frequency degenerates into a continuous band. The field is the same in each cell, there are no more standing wave modes → only “traveling wave modes”, if we excite the EM field at one end of the structure it will propagate towards the other end.

But: our dispersion curve remains valid, and defines the velocity of propagation of the travelling wave, $v_\phi = \omega d / \Phi$

For acceleration, the wave must propagate at $v_\phi = c$

→ for each frequency ω and cell length d we can find a phase Φ where the apparent velocity of the wave v_ϕ is equal to c

How to “simulate” an infinite chain of resonators? Instead of a single input, exciting a standing wave mode, use an input + an output for the RF wave at both ends of the structure.



“Disc-loaded waveguide” or chain of electrically coupled cells characterized by a continuous band of frequencies. In the chain is excited a “traveling wave mode” that has a propagation velocity $v_{ph} = \omega/k$ given by the dispersion relation.

For a given frequency ω , $v_{ph} = c$ and the structure can be used for particles traveling at $\beta=1$

The “**traveling wave**” structure is the standard linac for **electrons from $\beta \sim 1$** .

→ Can **not** be used for protons at $v < c$:

1. constant cell length does not allow synchronism
2. structures are long, without space for transverse focusing

Example: the 3 GHz electron linac



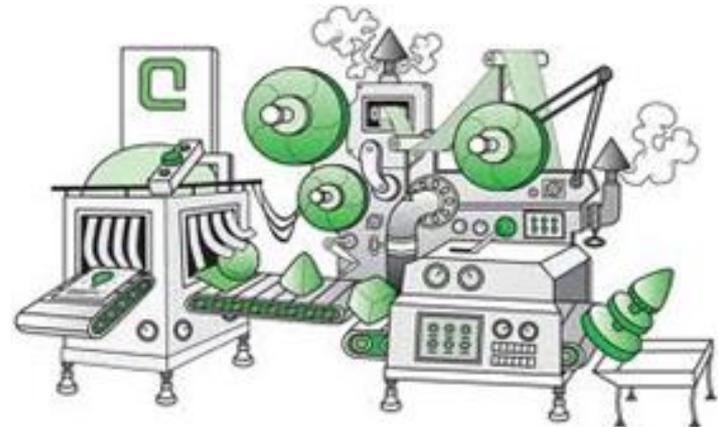
A 3 GHz LIL accelerating structure used for CTF3. It is 4.5 meters long and provides an energy gain of 45 MeV. One can see 3 quadrupoles around the RF structure.

What did we learn?

1. Coupling together accelerating cells (via the magnetic or electric field) is a way to fix their phase relation.
2. A chain of N coupled resonators will always have N modes of oscillation. Each mode will have a resonance frequency and a field pattern with a corresponding phase shift from cell to cell.
3. Choosing the excitation frequency, we can decide in which mode to operate the structure, and we can select a mode with a phase advance between cells suitable for acceleration. If we change the length of a cell without changing its frequency, we can follow the increase the particle velocity.
4. Practical linac structures operate either on mode 0 (DTL), less efficient but leaving space for internal focusing elements, or on mode π , standard for multi-cell cavities. More exotic modes ($\pi/2$, TE) are used in special cases.
5. Electron linacs operate with long chains of identical cells excited by a traveling wave, propagating at the (constant) velocity of the beam.

3. Applications of linacs

beyond nuclear and particle physics: materials
and life science, energy, medicine, industry



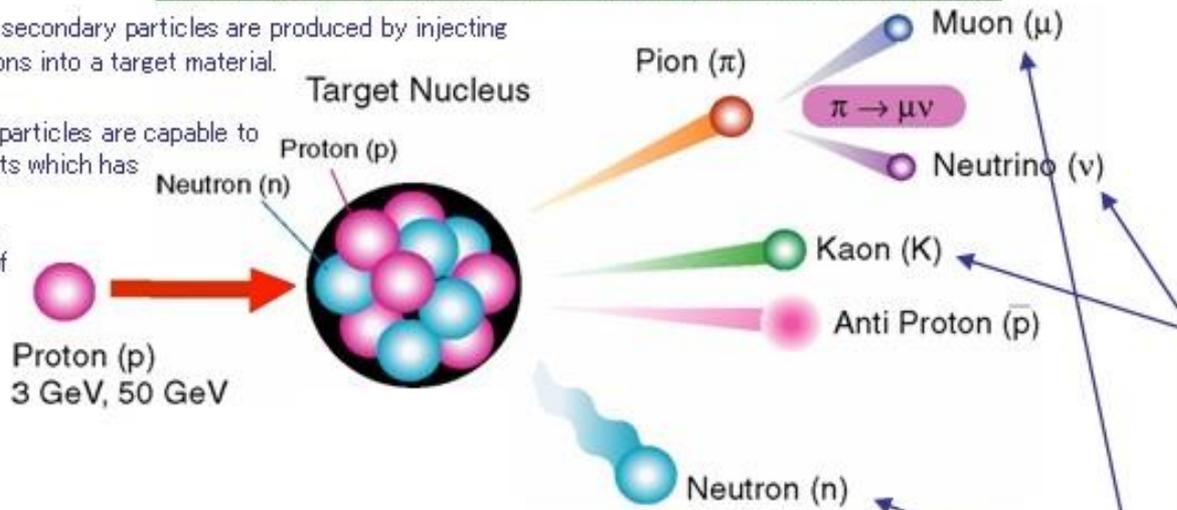
Linacs as powerful sources of secondary particle beams

Proton linacs can operate at high repetition frequencies with long pulses and currents up to 100 mA → can accelerate large average currents to energies above spallation threshold (about 1 GeV) and produce intense beams of secondary particles on a target (neutrons, pions, etc.)

Secondary Particles Beams via High Power Proton Beam

Various types of secondary particles are produced by injecting high energy protons into a target material.

Those new born particles are capable to investigate objects which has not been visible. To see it clearly, high brightness of the particles is essential.



Above a threshold, the amount of secondary particles is proportional to beam power ($P=W \times I$)

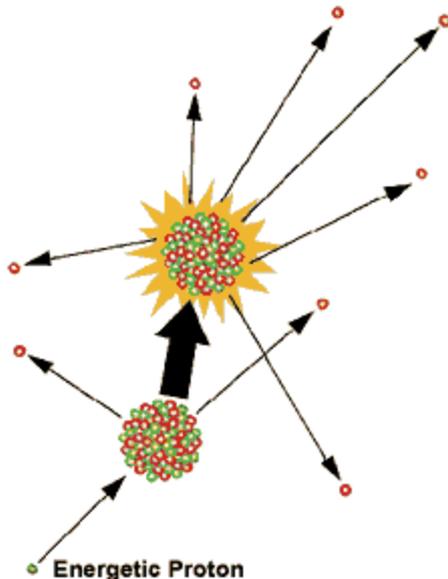
Need to have high-power proton beams

→ MW-class proton accelerator (current frontier is about 0.1 MW)

- Materials & Life Sciences at 3 GeV
- Nuclear & Particle Physics at 50 GeV
- R&D toward Transmutation at 0.6 GeV

- Energy production (Accelerator Driven Systems), > 2 GeV
- Radiative Ion Beams (ISOL), > 1 GeV

Neutrons are ideal probes to map the molecular and magnetic structure and behavior of materials (high-temperature superconductors, polymers, metals), and biological samples → Application to [fundamental physics](#), [structural biology](#) and [biotechnology](#), [magnetism and superconductivity](#), [chemical and engineering materials](#), [nanotechnology](#), [complex fluids](#), ...

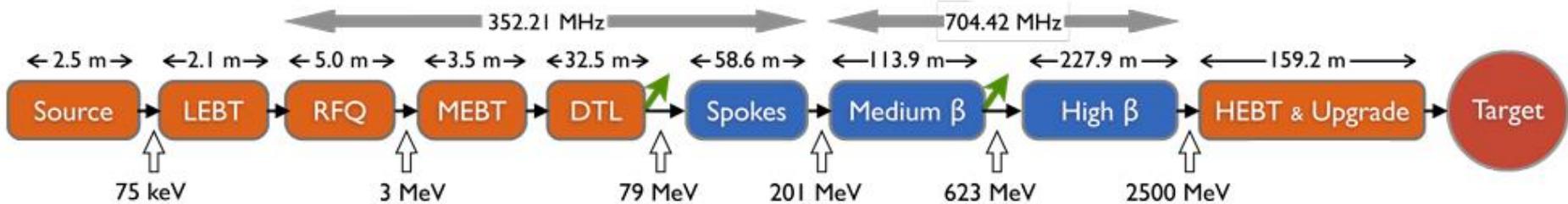


Traditionally, neutrons were produced by **nuclear reactors**; nowadays, are used **linear accelerators** at energy ≥ 1 GeV

2 linac-based facilities operational (SNS at Oak Ridge - USA and J-PARC at Tokai - Japan) and a third starting construction (**European Spallation Source at Lund - Sweden**).

The ESS linac

FDSL_2012_05_15



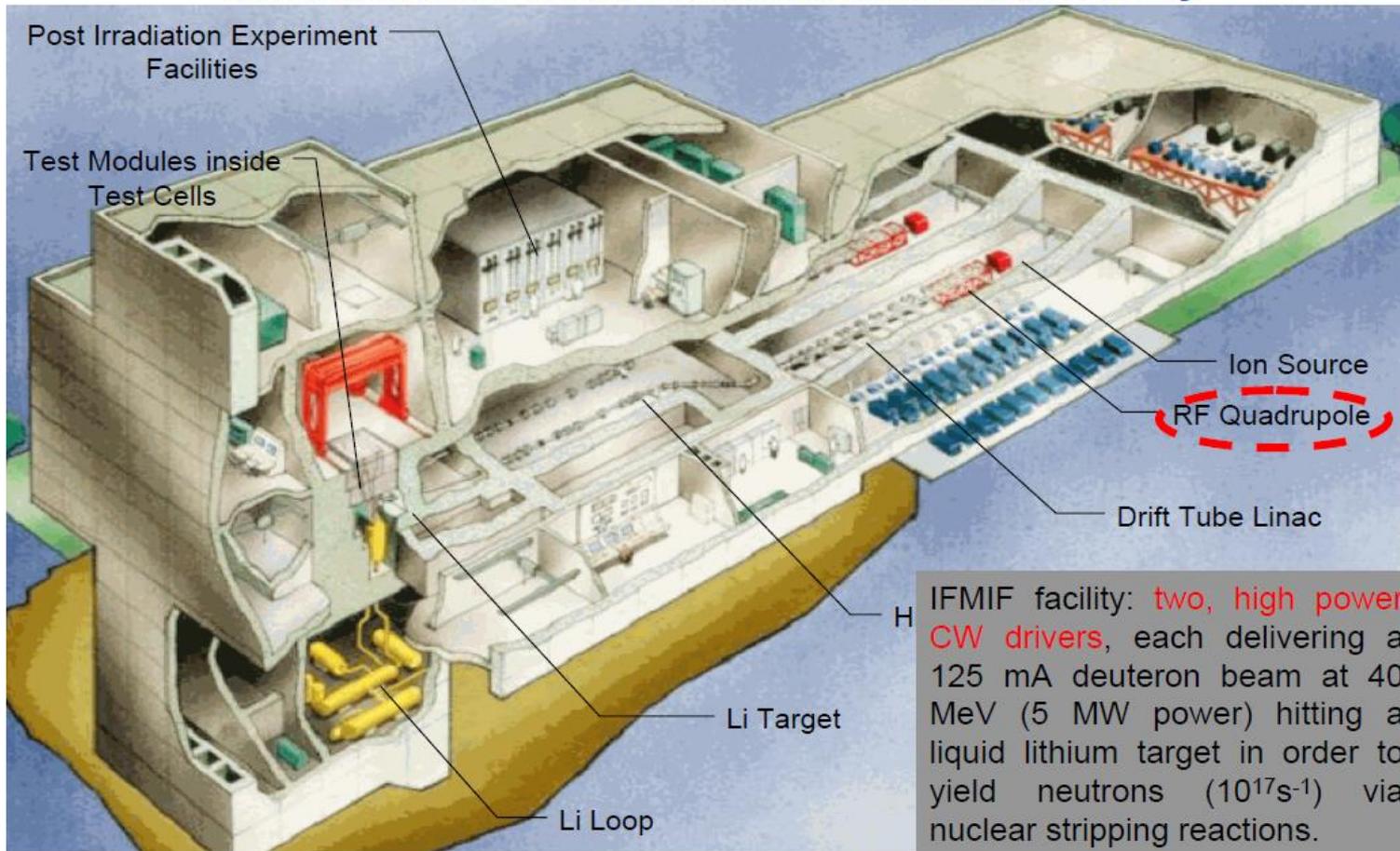
Parameter	Unit	Value
Average beam power	MW	5
Number of target stations		1
Number of instruments in construction budget		22
Maximum number of instruments		44
Number of beam ports		50
Number of moderators		2
Separation of ports in degrees	°	5
Proton kinetic energy	GeV	2.5
Average macro-pulse current	mA	50
Macro-pulse length	ms	2.86
Pulse repetition rate	Hz	14
Maximum accelerating cavity surface field	MV/m	40
Maximum linac length (without 100 m upgrade space)	m	482.5
Annual operating period	h	5200
Reliability	%	95

Table 2: *High Level* parameters, April 18, 2011.



IFMIF “Artist View”

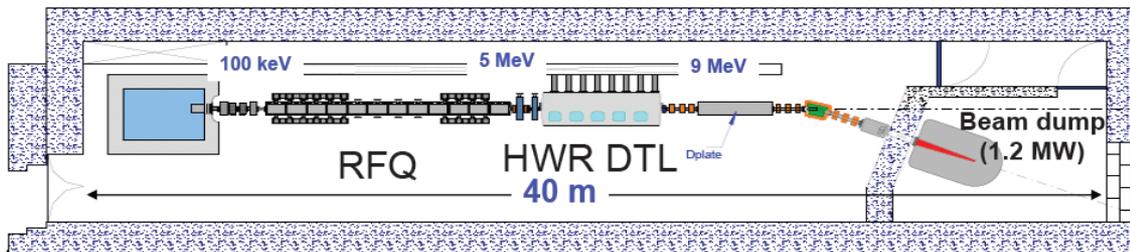
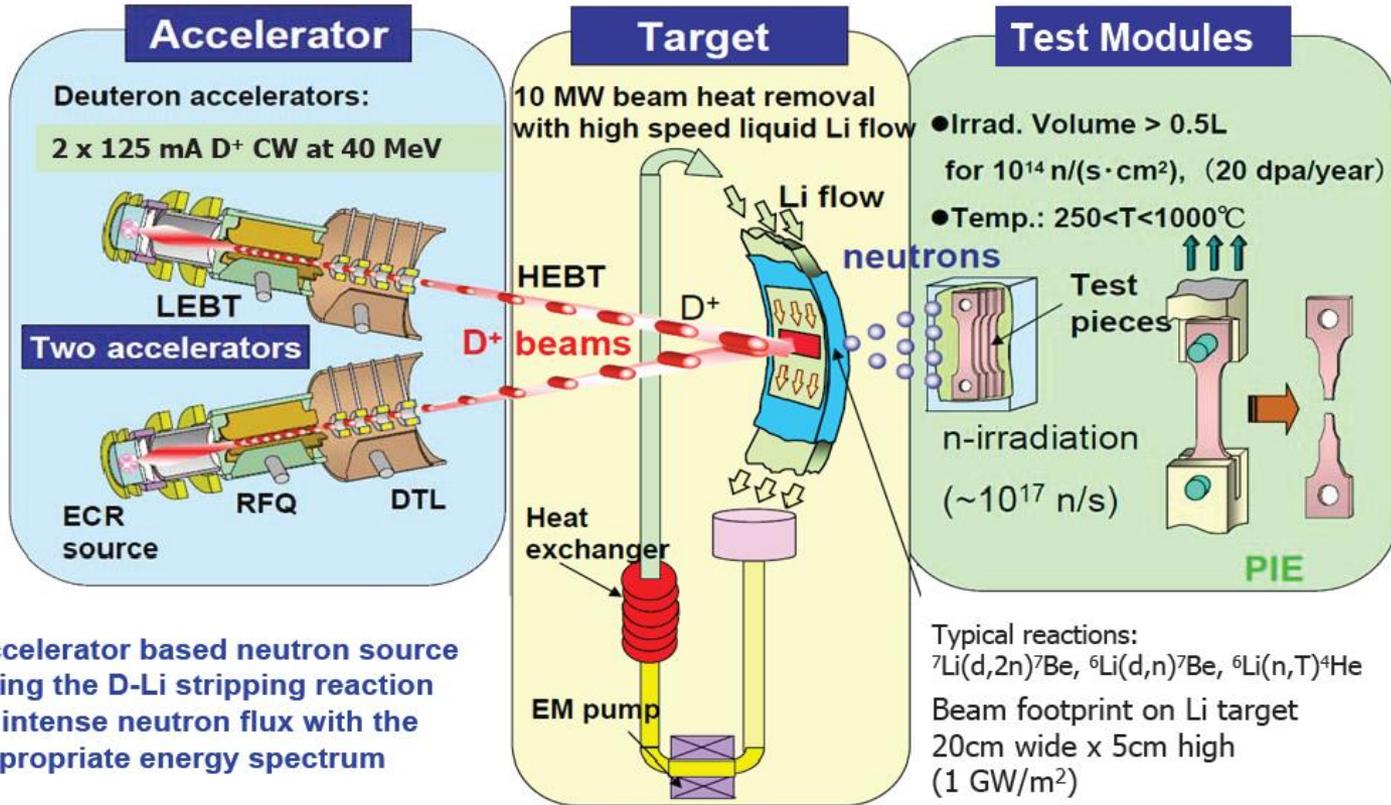
International Fusion Material Irradiation Facility



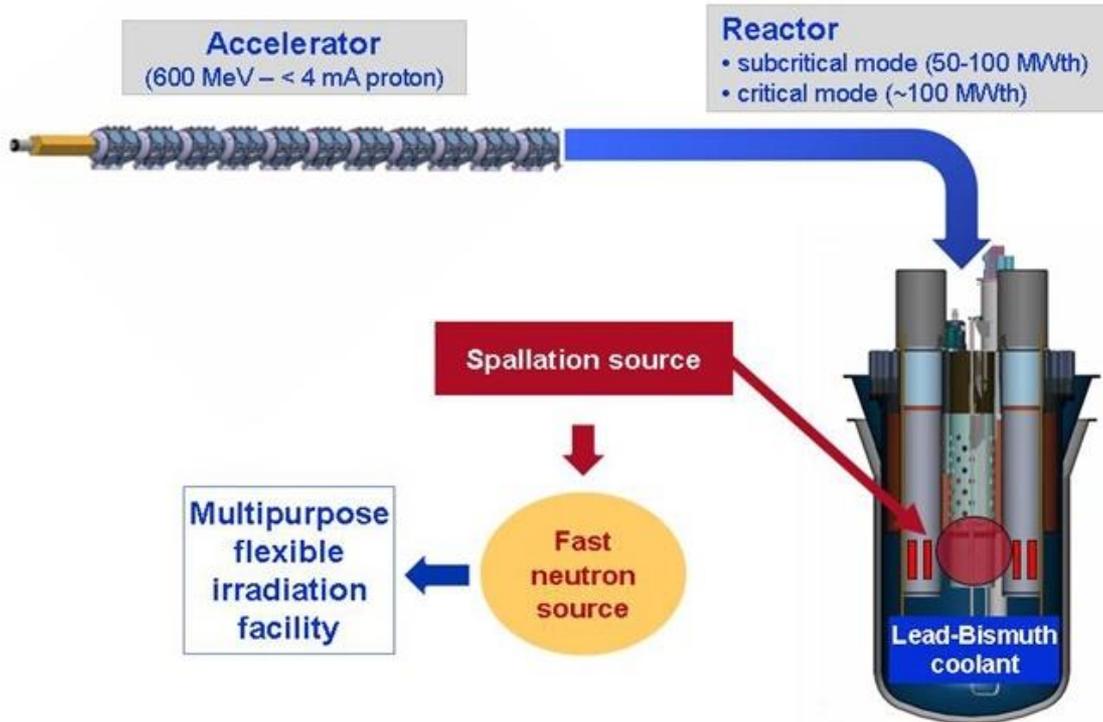
IFMIF facility: **two, high power CW drivers**, each delivering a 125 mA deuteron beam at 40 MeV (5 MW power) hitting a liquid lithium target in order to yield neutrons (10^{17}s^{-1}) via nuclear stripping reactions.

Test under strong neutron fluxes of materials to be used in ITER

IFMIF principles



IFMIF-EVEDA in construction
(CEA, INFN, CIEMAT, JAEA):
9 MeV 125 mA CW deuteron linac (9-m RFQ + SC cavities)



A linac coupled to a spallation source provides the missing neutrons to maintain the reaction in a **subcritical reactor**.

Use of **alternative fuel cycles** (thorium), no safety concerns (subcritical).

Can be coupled with a **transmutation facility**, the accelerator can process long-life nuclear waste and transform it into shorter lifetime waste.

The **MYRRHA Project** (Multi-purpose hYbrid Research Reactor for High-tech Applications) based at Mol (BE) is the European way to an ADS demonstrator: in the ESFRI list, partly supported by EU and by the Belgian government, looking for other international partners.

ADS demonstrator + fast neutron irradiation facility

Advantages/disadvantages of accelerator driven systems as compared to conventional reactors:

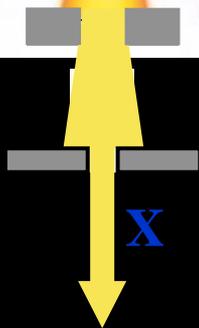
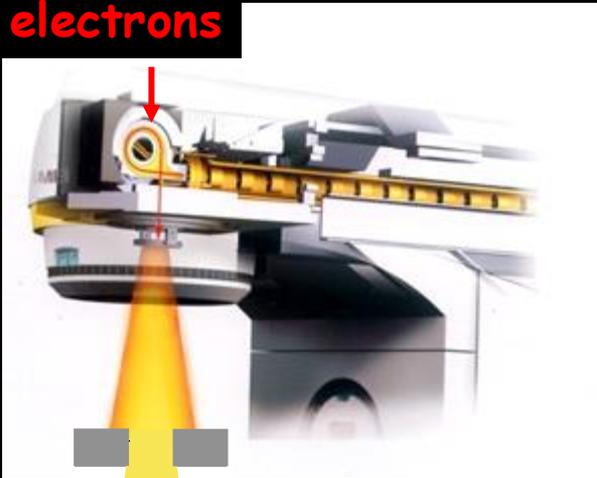
Pros	Cons
Safety: subcritical reaction, allows for immediate switch-off	High reliability (\rightarrow cost) required for the accelerator, to protect structures from thermal shocks
Possibility to operate below criticality opens the way to new reactor concepts	Reduction in net plant power efficiency due to power consumption of accelerator
Simple reactor control by modulating accelerator current	Increased complexity (and cost)

	Goal	Examples	Accelerator
Material processing (electrons)	Improve polymer resins inducing cross-linking of polymer chains → higher stress resistance	Heat-shrinkable films for food packaging, tires and cable insul. Gemstone irradiation	Electrons, 100 keV-10 MeV
Sterilization	Kill microroganisms	Sterilization of medical products Food processing (public acceptance!)	Electrons, ~10 MeV
Wastewater treatment	Distruction of organic compounds	Russia, Korea, USA, Brazil	Electrons, ~10 MeV
Non-destructive testing	Detect discontinuities in a material (cracks, etc.)	Inspection of pipelines, ships, bridges, etc. (depth + variable energy)	Electrons for X-rays, 1-15 MeV, portable (9 GHz)
Cargo inspection	Screening of trucks or containers for illegal objects	Many ports, customs, etc.	Electrons for X-rays, 3-6 MeV
Ion implantation	Alter near-surface properties of semiconductors (doping)	Semiconductor industry (arsenic, boron, indium, phosphorus,...)	Ions, from low to high energy (5 MeV)
PET isotope production	Production of radiotracers for Positron Emission Tomography	Linacs are smaller and have less res. activation than cyclotrons	Protons, 7 MeV
Neutron testing	Neutron generation for non-destructive inspection	Inspection of materials, cargo, etc.	Protons, 1-10 MeV

More than 20'000 industrial linacs in operation in the world.

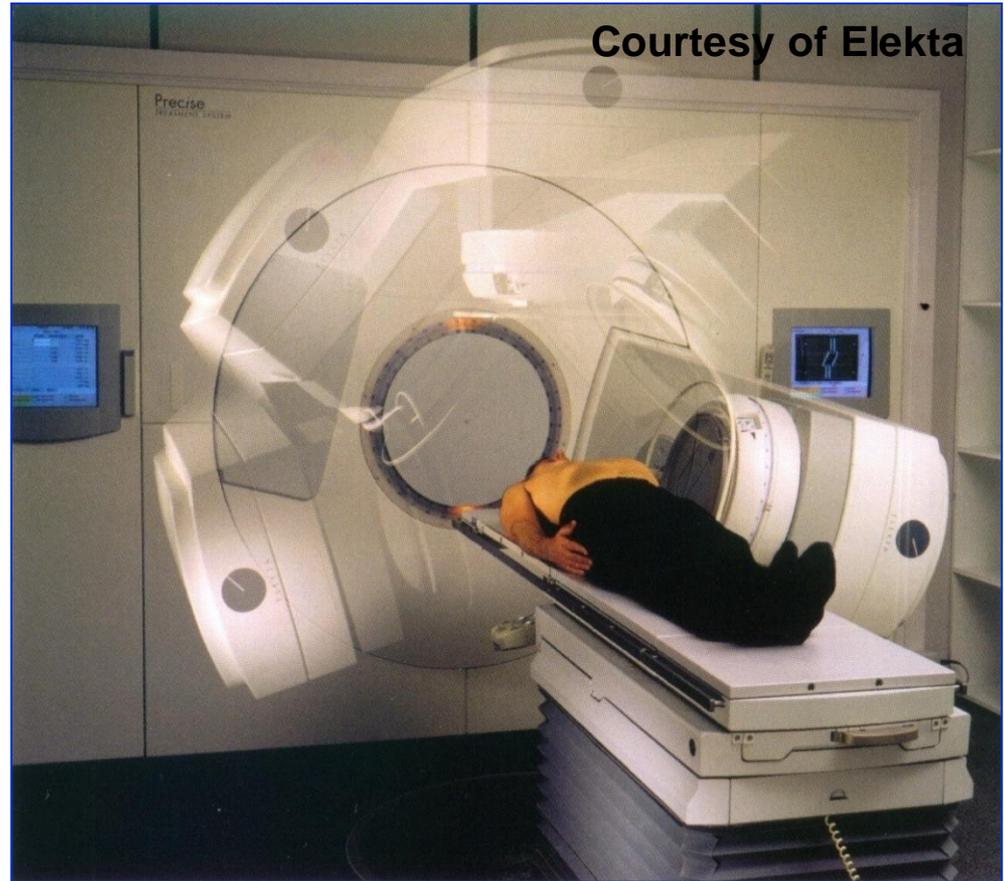
A large fraction is made of small electrostatic machines for ion implantation (10'000 units).⁴⁹

electrons



**Linac for electrons
@3 GHz
5-20 MeV**
(usually short side-coupled structures)

>7000 electron linacs in the world for radiotherapy



Courtesy of Elekta

**20 000 patients per year every
10 million inhabitants**

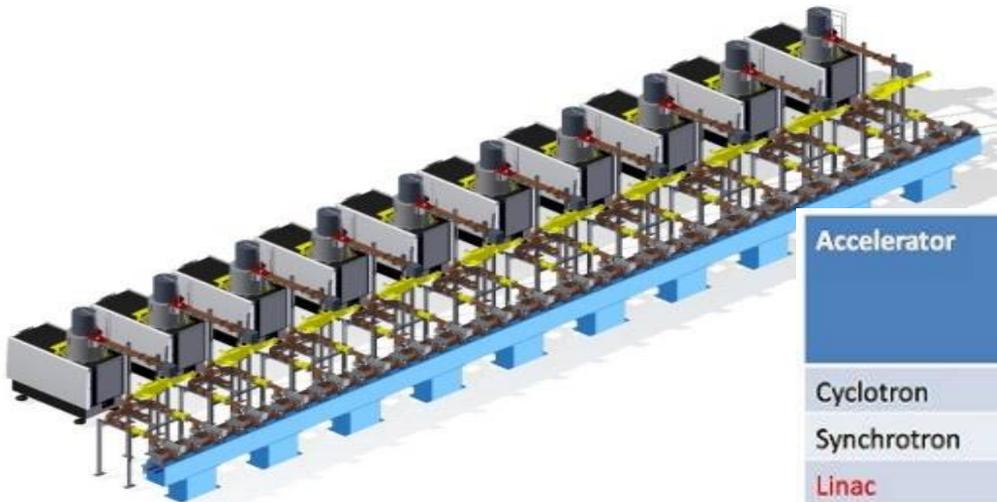
Compact proton linacs for hadrontherapy

LIGHT (Linac for Image Guided Hadron Therapy) project developed by the ADAM company. Collaboration with CERN (related to TERA).

230 MeV energy in 20 m

Recently built and successfully tested a prototype unit (2 modules).

Modular, compact, variable energy.
Conventional proton linac as injector.



LIGHT Parameters	Value
Typical LIGHT output proton beam energy	230 MeV
Typical LIGHT input beam energy	30 MeV
Typical number of acceleration module assemblies	10
Electronic beam energy variation range up to maximum energy	70%
Time required to change beam energy	2-3 msec
Pulse repetition rate	200 Hz
Proton Linac typical length	20 metres

Accelerator	Beam always present during treatments	Energy variation by electronic methods	Time needed for varying the energy
Cyclotron	YES	NO	80-100 ms (*)
Synchrotron	NO	YES	1-2 seconds
Linac	YES	YES	2-3 milliseconds (**)

(*) With movable absorbers

(**) The energy is changed by adjusting the RF power to the modules

Linear Accelerators - part 2

- 
1. Beam dynamics fundamentals
 2. Linac architecture
 3. The Radio-Frequency Quadrupole

4 - Fundamentals of linac beam dynamics

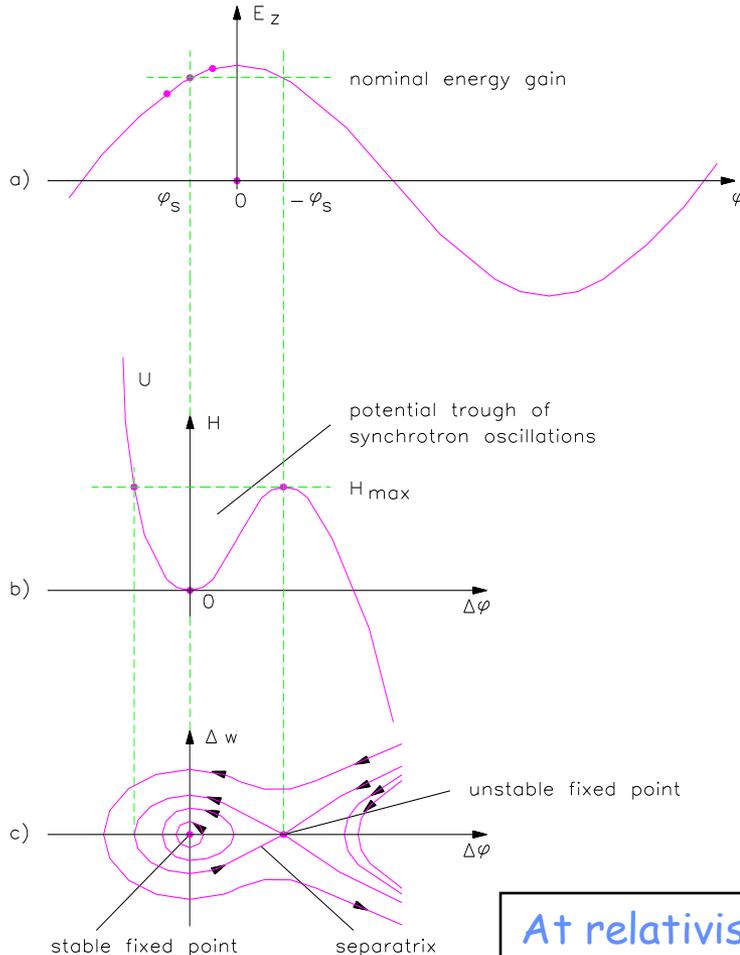


A linear accelerator requires an accurate beam optics design, in order to:

- ❑ Minimize emittance growth (remember Liouville: emittance can only increase!);
- ❑ Minimize beam loss, to: a) avoid activation of the accelerator (of the linac and of the following machine!) and b) reduce the requirements on the ion source.

Note that the operating regimes in linacs can be very different, between

- μA peak currents for heavy ion linacs ($\sim 10^5$ particles / bunch)
- mA peak currents for proton linacs ($\sim 10^8$ particles / bunch)
- 100's mA peak currents for high-power proton linacs ($\sim 10^{11}$ particles / bunch)
- A's peak currents for electron linacs



- Ions are accelerated around a (negative = linac definition) synchronous phase.
- Particles around the synchronous one perform oscillations in the longitudinal phase space.
- Frequency of small oscillations:

$$\omega_l^2 = \omega_0^2 \frac{qE_0 T \sin(-\varphi)\lambda}{2\pi mc^2 \beta\gamma^3}$$

- Tends to zero for relativistic particles $\gamma \gg 1$.
- Note phase damping of oscillations:

$$\Delta\varphi = \frac{const}{(\beta\gamma)^{3/4}} \quad \Delta W = const \times (\beta\gamma)^{3/4}$$

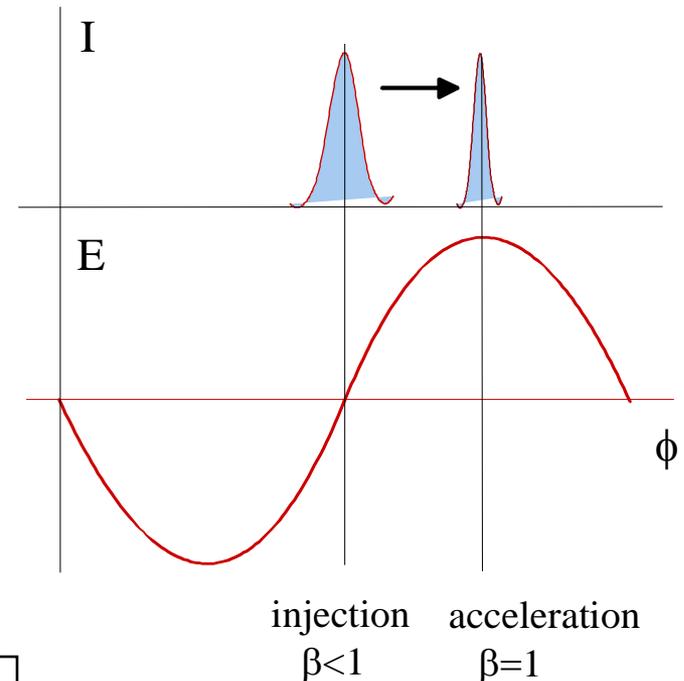
At relativistic velocities phase oscillations stop, and the beam is compressed in phase around the initial phase. The crest of the wave can be used for acceleration.

- Electrons at $v=c$ remain at the injection phase.
- Electrons at $v < c$ injected into a TW structure designed for $v=c$ will move from injection phase φ_0 to an asymptotic phase φ , which depends only on gradient and β_0 at injection.
- The beam can be injected with an offset in phase, to reach the crest of the wave at $\beta=1$
- **Capture condition**, relating E_0 and β_0 :

$$\frac{2\pi}{\lambda_g} \frac{mc^2}{qE_0} \left[\sqrt{\frac{1-\beta_0}{1+\beta_0}} \right] = 1$$

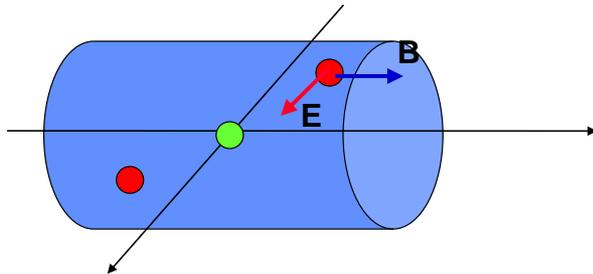
Example: $\lambda=10\text{cm} \rightarrow W_{\text{in}}=150 \text{ keV}$ for $E_0=8 \text{ MV/m}$.

$$\sin \varphi = \sin \varphi_0 + \frac{2\pi}{\lambda_g} \frac{mc^2}{qE_0} \left[\sqrt{\frac{1-\beta_0}{1+\beta_0}} - \sqrt{\frac{1-\beta}{1+\beta}} \right]$$



In high current linacs, a bunching and pre-acceleration sections up to 4-10 MeV prepares the injection in the TW structure (that occurs already on the crest)

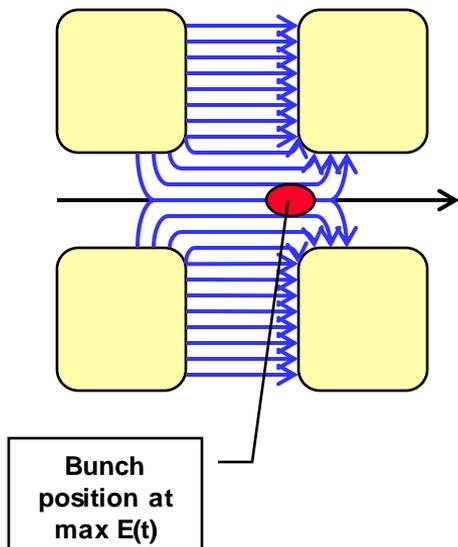
- Large numbers of particles per bunch ($\sim 10^{10}$).
- Coulomb repulsion between particles (space charge) plays an important role and is the main limitation to the maximum current in a linac.
- But **space charge forces $\sim 1/\gamma^2$ disappear at relativistic velocity**



Force on a particle inside a long bunch with density $n(r)$ traveling at velocity v :

$$E_r = \frac{e}{2\pi\epsilon} \int_0^r n(r) r dr \quad B_\phi = \frac{\mu}{2\pi} \frac{ev}{r} \int_0^r n(r) r dr$$

$$F = e(E_r - vB_\phi) = eE_r \left(1 - \frac{v^2}{c^2}\right) = eE_r (1 - \beta^2) = \frac{eE_r}{\gamma^2}$$



- RF defocusing experienced by particles crossing a gap on a longitudinally stable phase. Increasing field means that the defocusing effect going out of the gap is stronger than the focusing effect going in.
- In the rest frame of the particle, only electrostatic forces → no stable points (maximum or minimum) → radial defocusing.
- Lorentz transformation and calculation of radial momentum impulse per period (from electric and magnetic field contribution in the laboratory frame):

$$\Delta p_r = -\frac{\pi e E_0 T L r \sin \varphi}{c \beta^2 \gamma^2 \lambda}$$

- **Transverse defocusing $\sim 1/\gamma^2$ disappears at relativistic velocity** (transverse magnetic force cancels the transverse RF electric force).

The equilibrium between external focusing force and internal defocusing forces defines the **frequency of beam oscillations**.

Oscillations are characterized in terms of **phase advance per focusing period σ_t** or **phase advance per unit length k_t** .

Ph. advance = Ext. quad focusing - RF defocusing - space charge

$$k_t^2 = \left(\frac{\sigma_t}{N\beta\lambda} \right)^2 = \left(\frac{qGl}{2mc\beta\gamma} \right)^2 - \frac{\pi q E_0 T \sin(-\varphi)}{mc^2 \lambda \beta^3 \gamma^3} - \frac{3q I \lambda (1-f)}{8\pi\epsilon_0 r_0^3 mc^3 \beta^2 \gamma^3}$$

q =charge
 G =quad gradient
 l =length foc. element
 f =bunch form factor
 r_0 =bunch radius
 λ =wavelength
 ...

Approximate expression valid for:

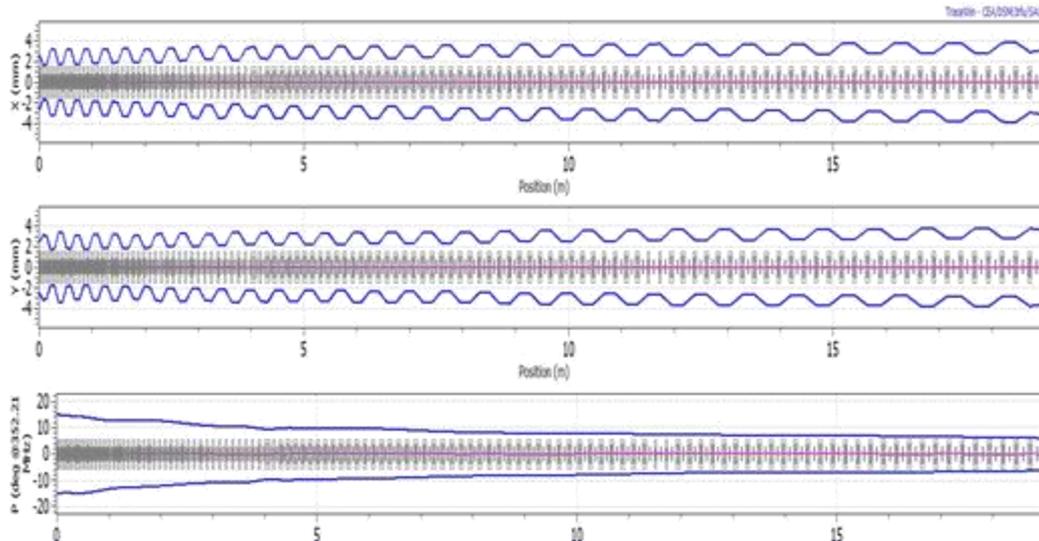
FODO lattice, smooth focusing approximation, space charge of a uniform 3D ellipsoidal bunch.

A “low-energy” linac is dominated by space charge and RF defocusing forces !!

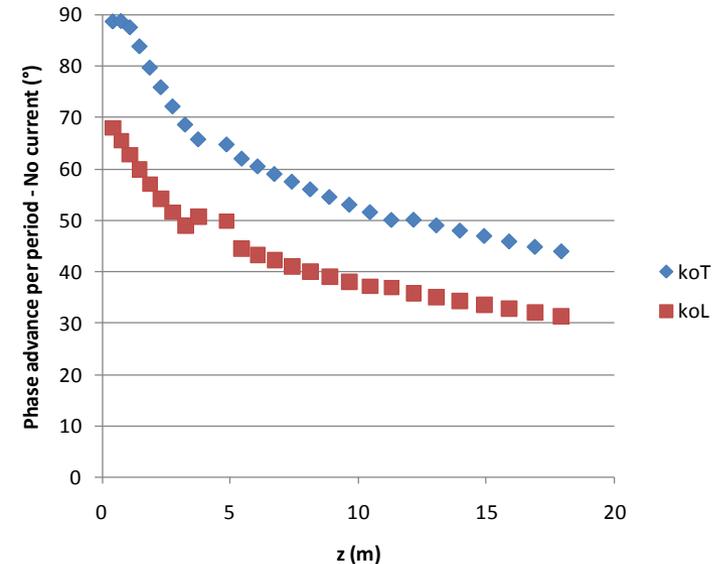
Phase advance per period must stay in reasonable limits (30-80 deg), phase advance per unit length must be continuous (smooth variations) → at low β , we need a strong focusing term to compensate for the defocusing, but the limited space limits the achievable G and I → needs to use short focusing periods $N\beta\lambda$.

Note that the RF defocusing term $\propto f$ sets a higher limit to the basic linac frequency (whereas for shunt impedance considerations we should aim to the highest possible frequency, $Z \propto \sqrt{f}$).

Beam optics of the Linac4 Drift Tube Linac (DTL): 3 to 50 MeV, 19 m, 108 focusing quadrupoles (permanent magnets).



Oscillations of the beam envelope (coordinates of the outermost particle) along the DTL (x, y, phase)



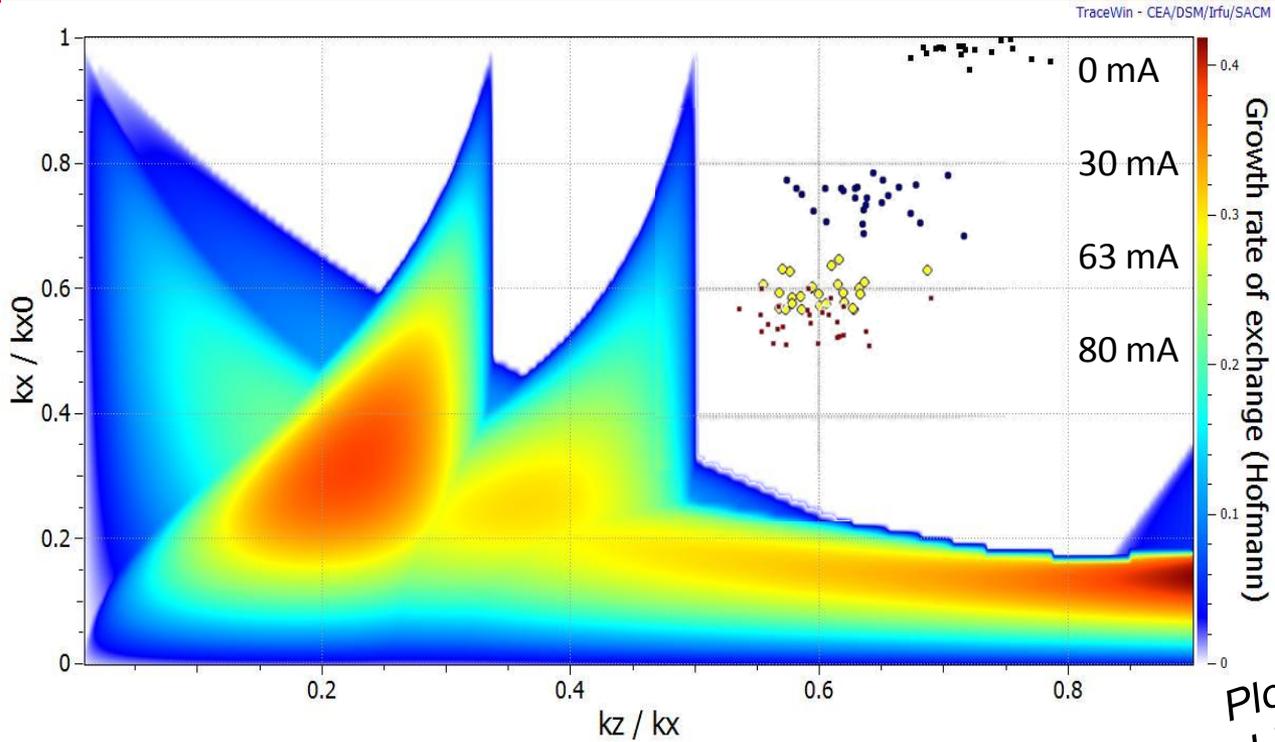
Corresponding phase advance per period

Design prescriptions:

- Transverse phase advance at zero current always less than 90° .
- Smooth variation of the phase advance.
- Avoid resonances (see next slide).

Instabilities in linacs - the Hoffman plot

Ratio between transverse phase advance and zero current phase advance (a measure of space charge)



Ratio between longitudinal and transverse phase advance

The "tune diagram" of a linac

Plot courtesy of J.B. Lallement / CERN

Linac4 DTL: the operating point(s) for all possible current levels are far from the resonances between transverse and longitudinal oscillations which are enhanced by space charge.

Effect of the resonances: emittance exchange, transverse emittance growth, migration of particles into the beam halo → particularly dangerous for high intensity machines (beam loss).

Focusing usually provided by quadrupoles.

Need to keep the **phase advance in the good range**, with an approximately constant phase advance per unit length → The **length of the focusing periods has to change** along the linac, going gradually from **short periods** in the initial part (to compensate for high space charge and RF defocusing) to **longer periods** at high energy.

For Protons (high beam current and high space charge), distance between two quadrupoles (=1/2 of a FODO focusing period):

- $\beta\lambda$ in the DTL, from ~70mm (3 MeV, 352 MHz) to ~250mm (40 MeV),
- can be increased to 4-10 $\beta\lambda$ at higher energy (>40 MeV).
- longer focusing periods require special dynamics (example: the IH linac).

For Electrons (no space charge, no RF defocusing):

focusing periods up to several meters, depending on the required beam conditions. Focusing is mainly required to control the emittance.

What did we learn?

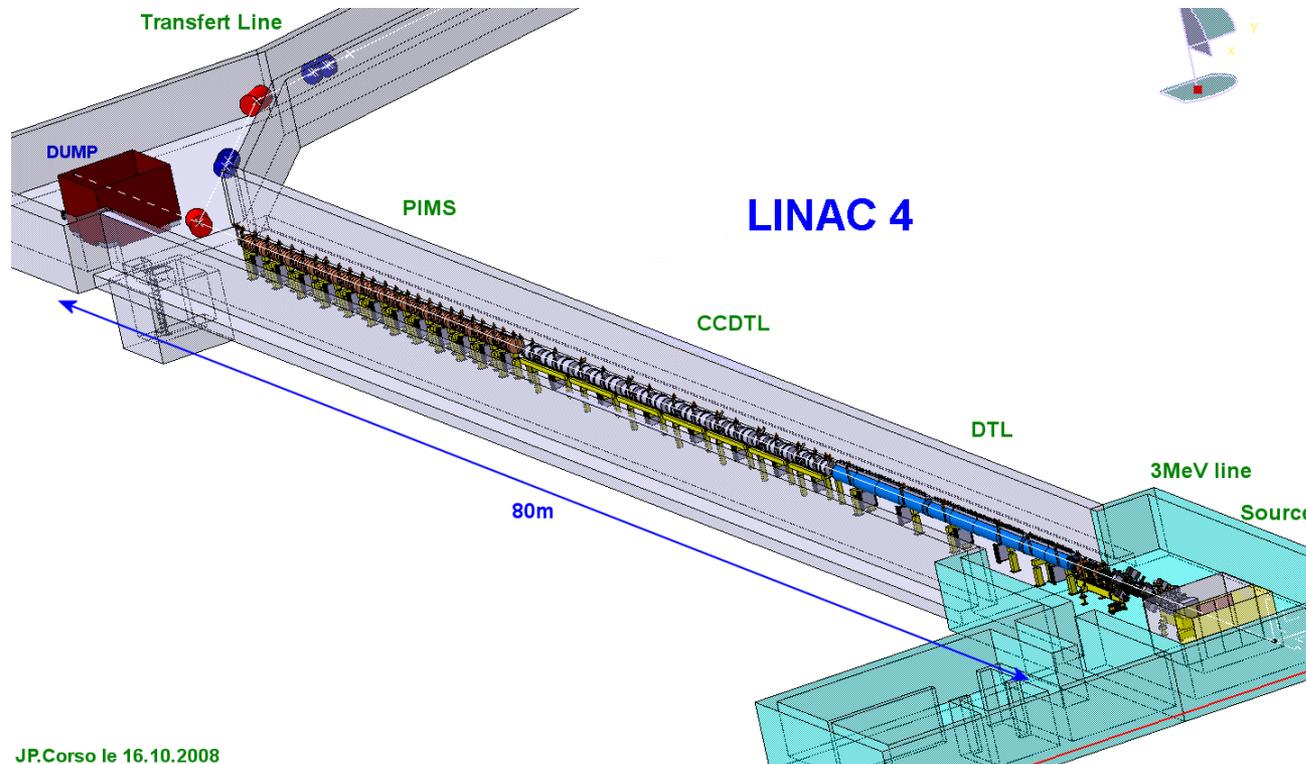
1. Transverse beam dynamics in linacs is dominated by space charge and RF defocusing forces.
2. In order to keep the transverse phase advance within reasonable limits, focusing has to be strong (large focusing gradients, short focusing periods) at low energy, and can then be relaxed at higher energy.
3. A usual linac is made of a sequence of structures, matched to the beam velocity, and where the length of the focusing period increases with energy.
4. The very low energy section remains a special problem → next lecture

5. Linac architecture



EXAMPLE: the **Linac4 project at CERN**. H⁻, 160 MeV energy, 352 MHz.
 A 3 MeV injector + 22 multi-cell standing wave accelerating structures of 3 types

DTL, 3-50 MeV: every cell is different, focusing quadrupoles in each drift tube, 0-mode
CCDTL, 50-100 MeV: sequences of 2 identical cells, quadrupoles every 3 cells, 0 and $\pi/2$ mode
PIMS, 100-160 MeV: sequences of 7 identical cells, quadrupoles every 7 cells, $\pi/2$ mode



Two basic principles to remember:

1. As beta increases, phase error between cells of identical length becomes small \rightarrow we can have short sequences of identical cells (lower construction costs).
2. As beta increases, the distance between focusing elements can increase.

approximate scaling laws for linear accelerators:

⇒ RF defocusing (ion linacs)	~ frequency
⇒ Cell length ($=\beta\lambda/2$)	~ (frequency) ⁻¹
⇒ Peak electric field	~ (frequency) ^{1/2}
⇒ Shunt impedance (power efficiency)	~ (frequency) ^{1/2}
⇒ Accelerating structure dimensions	~ (frequency) ⁻¹
⇒ Machining tolerances	~ (frequency) ⁻¹

- Higher frequencies are economically convenient (shorter, less RF power, higher gradients possible) but the limitation comes from mechanical precision required in construction (tight tolerances are expensive!) and beam dynamics for ion linacs.
- The main limitation to the initial frequency (RFQ) comes from RF defocusing ($\sim 1/(\lambda\beta^2\gamma^2)$ - 402 MHz is the maximum achievable so far for currents in the range of tens of mA's.
- High-energy linacs have one or more frequency jumps (start 200-400 MHz, first jump to 400-800 MHz, possible a 3rd jump to 600-1200 MHz): compromise between focusing, cost and size.

Advantages of Superconductivity:

- Much smaller RF system (only beam power) → prefer low current/long pulse
- Larger aperture (lower beam loss).
- Lower operating costs (electricity consumption).
- Higher gradients (thanks to cleaning procedures)

Disadvantages of Superconductivity:

- Need cryogenic system (in pulsed machines, size dominated by static loss → prefer low repetition frequency or CW to minimize filling time/beam time).
- In proton linacs, need cold/warm transitions to accommodate quadrupoles → becomes more expensive at low energy (short focusing periods).
- Individual gradients difficult to predict (large spread) → for protons, need large safety margin in gradient at low energy.



Conclusions:

1. Superconductivity gives a large advantage in cost at high energy (protons)/ high duty cycle.
2. At low proton energy / low duty cycle superconducting sections are more expensive. 67



The CERN Accelerator School

Electron linac architecture



courtesy of R. Muñoz, ALBA-CELLS

3

Buncher
22 cell - standing wave $\pi/2$
Bunch compression
Energy Gain: 16 MeV

4

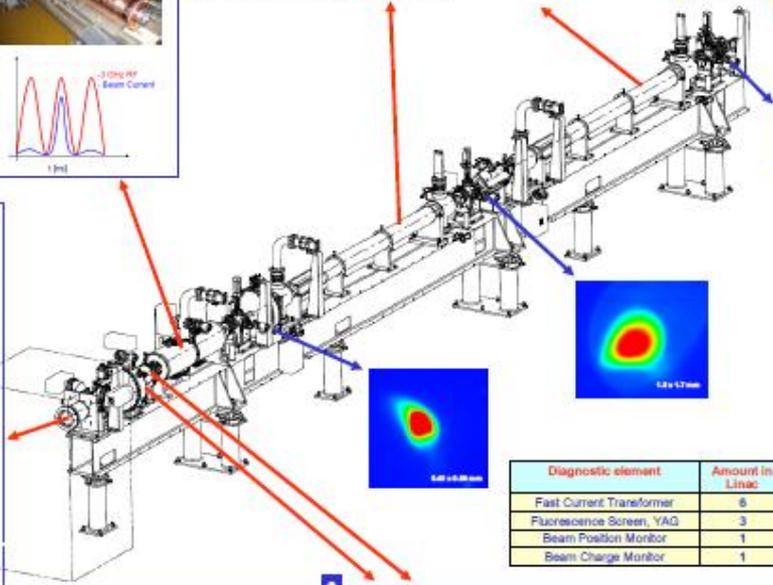
2 Accelerating Sections
96 cell - $2/3 \pi$ Travelling Wave.
Constant gradient: 10-15 MV/m
Beam at crest
Energy gain: 55 MeV

5

Beam at Linac exit: $E = 110 \text{ MeV}$
 $\Delta E = 0.35 \%$

1

Thermionic cathode
50 keV electrons
DC-beam



RF power to cavities
2 Klystrons TH2100
Pulsed at 3 GHz
37 MW peak

Diagnostic element	Amount in Linac
Fast Current Transformer	8
Fluorescence Screen, YAG	3
Beam Position Monitor	1
Beam Charge Monitor	1

LINAC INJECTION MODES

Single Bunch Mode (SBM)
Number of bunches per injection: 1-16
Time interval between bunches: 6-256 ns

Multi-Bunch Mode (MBM)
Number of bunches per injection: 18 - 512
Time interval between bunches: fixed, 2 ns

2

2 Pre-bunchers:
500 MHz and 3 GHz
Bunch compression and energy spread reduction

500 MHz pill-box cavity

EXAMPLE:
injector linac of the ALBA Synchrotron Light Facility (Barcelona):

100 MeV electron linac supplied by Thales in 2008. Produces a beam up to 4 nC/bunch in either single or multi-bunch mode at repetition rate up to 5 Hz. Normalized beam emittance below $30 \pi \text{ mm mrad}$.

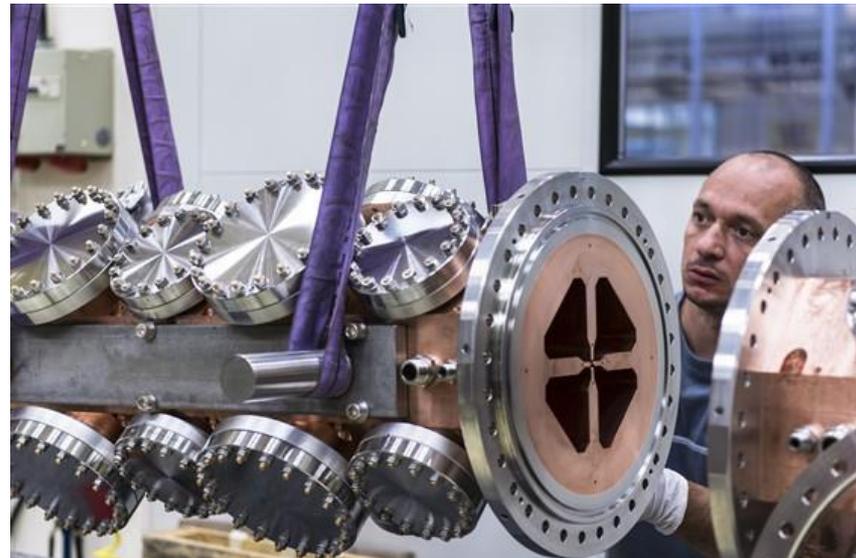
Injector + sequence of identical multi-cell traveling wave accelerating structures.



What did we learn?

1. A proton linac is made of different sections optimized for a particular range of beta. Each section is characterized by its RF frequency, by the periodicity of the RF structure and by the focusing distance. The frequency of the first section (RFQ) is limited by beam dynamics (RF defocusing). In the following sections, the frequency can be increased (by multiples of the basic frequency), to gain in efficiency.
2. Superconductivity gives a clear advantage in cost to linacs operating at high duty cycle.
3. An electron linac is made of an injector where the beam is generated, bunched and brought to the speed of light (usually by a standing-wave buncher), followed by a sequence of identical traveling-wave structures.

6. The Radio-Frequency Quadrupole



Low energy →

for protons,

between ~ 50 keV (source extraction) and ~ 3 MeV (limit for an effective use of the DTL)

→ range $\beta = 0.01 - 0.10$

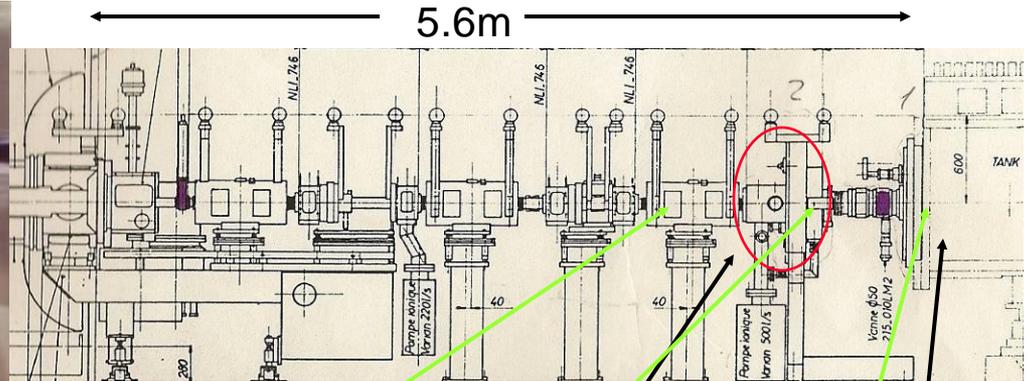
Why it is a problem?

1. We have seen that at low energy we need strong focusing (strong space charge!), but the short cell length ($\sim\beta\lambda$) limits the length of quadrupoles, for ex.
 $\beta\lambda(1\text{MeV},352\text{MHz}) = 3.9\text{cm}$
2. in this region the beam needs to be bunched → standard bunching systems are quite ineffective (~50% beam loss...).
3. At low energy, the usual accelerating structures have low efficiency (low shunt impedance).

The classical solution:

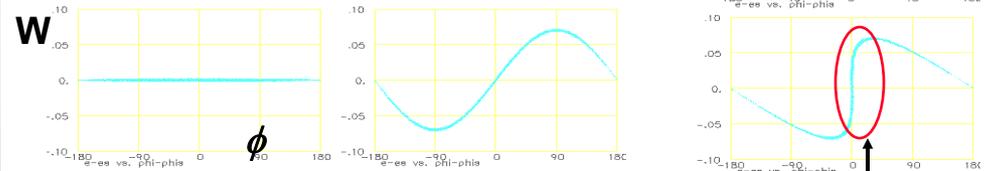
1. Increase as much as possible the extraction voltage from the source → huge HV installations, up to the maximum of some 800 kV.
2. Add a bunching section (1 or 2 cavities) after the source extraction.
3. Start the first accelerating structure (usually a Drift Tube Linac) from the minimum possible energy.

The classical solution: HV column + LEBT + bunching



Double harmonic buncher (200-400 MHz)

DTL



Principle of single-harmonic bunching

Useful beam (inside DTL acceptance)

Drawbacks:

- Large and expensive HV column
- Reliability (800 kV...)
- Bunching efficiency (~50%)
- Long line with inefficient magnetic focusing ($\propto \beta$)
- Difficult DTL at low energy (short tubes and quads)
- Large emittances for high currents

RFQ compared to the old pre-injectors



The old pre-injector at CERN (1976):
Source+
Cockroft Walton
+line+bunching



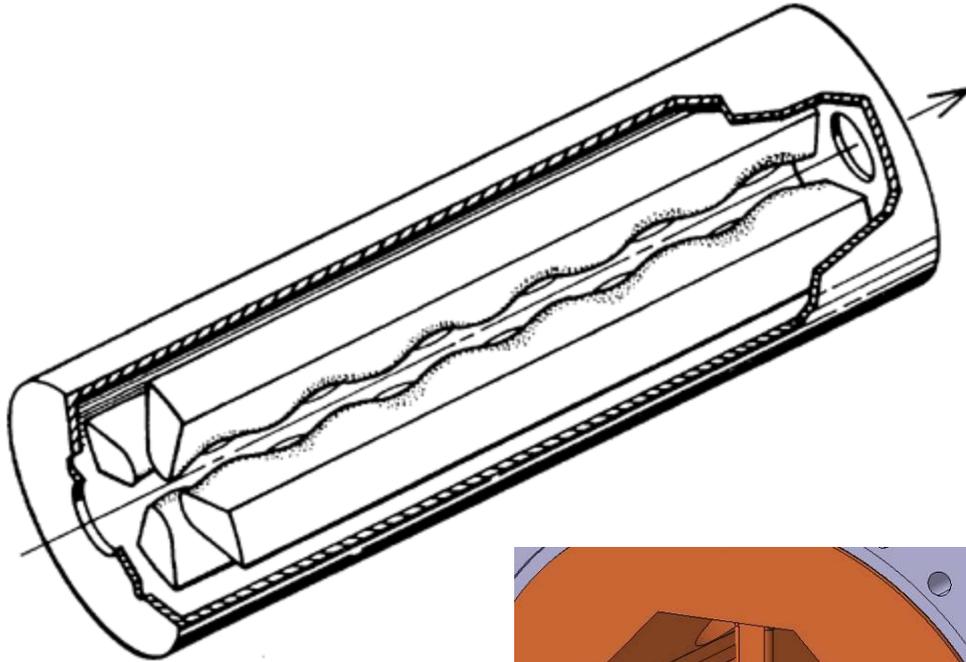
The new
RFQ2 pre-
injector at
CERN (1993):
Source+LEBT
+RFQ

← 3.2m →

The driving force for the development of something new for the low-energy section was the research in URSS and USA on **high-current proton accelerators**. The idea is to break the limitation to current coming from *space charge in the beam transport* and from *bunching losses*.

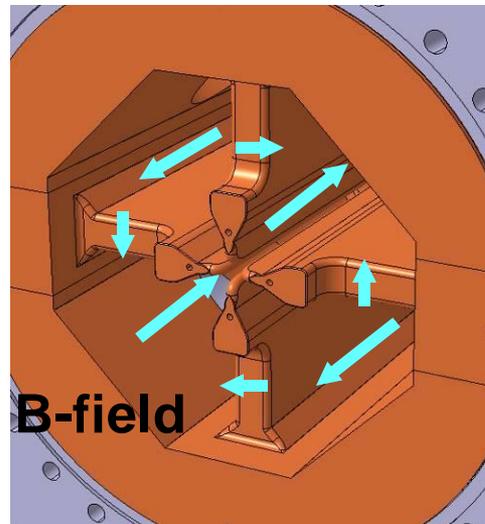
- 1960's: Early works of I. Kapchinski at ITEP (Moscow): idea to *use at low energy an electric quadrupole focusing channel, excited at RF frequency, and modulated to add a longitudinal field component providing adiabatic bunching and acceleration*.
- 1969: an RF resonator is designed around Kapchinski's electrodes by V. Tepliakov (IHEP). **First paper** on the RFQ by Kapchinski and Teplyakov (in Russian). First **experimental RFQ** in Russia (1974).
- 1977: the idea arrives at **Los Alamos** (USA), introduced by a Czech refugee.
- 1977-1980: the Los Alamos team is enthusiastic about the idea (for their Fusion Material Irradiation), makes some improvements to the original Kapchinski structure and develops a new resonator design. The **first complete RFQ** is built at Los Alamos and successfully operated (for a few hours...) in 1980.
- 1980's: the RFQ principle spreads around the world, more RFQs are built in the USA and in Europe (1st CERN RFQ: 1984). Long and difficult **learning curve** (RFQs are not simple devices...).
- 1985-1995 : **reliable RFQ designs** exist and progressively replace the old pre-injectors in most accelerator laboratories (CERN: 1993). Different design and applications are proposed all over the world.
- 1995-now : new RFQs are designed and built for extreme applications, like **very high intensity** (CW, high current). Start to design compact RFQs.

The 4-vane RFQ



The RFQ will result in cylinder containing the 4 vanes, which are connected (large RF currents!) to the cylinder along their length.

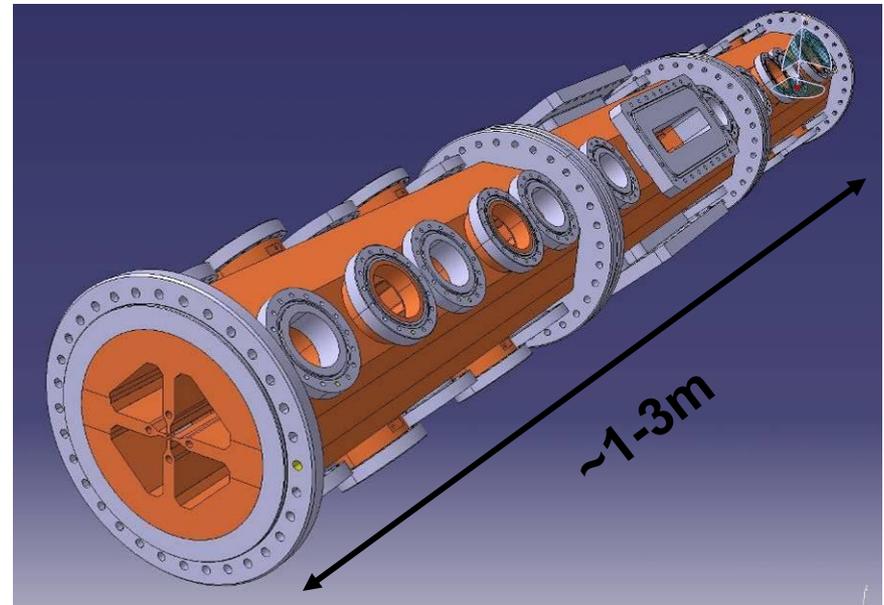
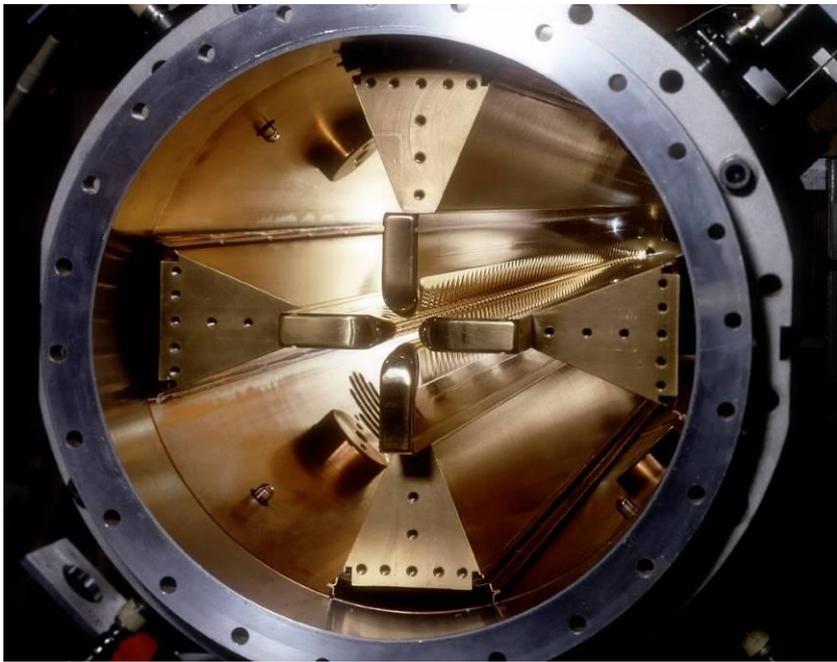
Field excitation via a loop or an iris in one (or more) quadrants



A critical feature of this type of RFQs are the end cells: The magnetic field flowing longitudinally in the 4 “quadrants” has to close its path and pass from one quadrant to the next via some openings at the end of the vanes, tuned at the RFQ frequency!

The Radio Frequency Quadrupole (RFQ)

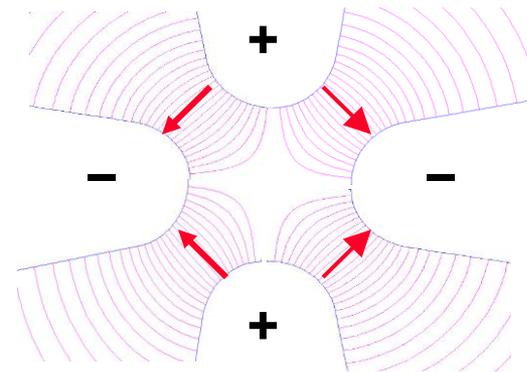
RFQ = Electric quadrupole focusing channel + bunching + acceleration



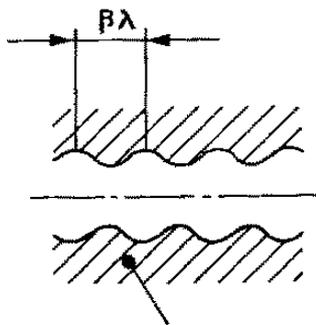
New and performing accelerator.

Compact and critical structure, where beam dynamics, RF and mechanical aspects are closely interconnected.

1. Four electrodes (called **vanes**) between which we excite an RF Quadrupole mode → **Electric focusing channel**, alternating gradient with the period of the RF. Note that electric focusing does not depend on the velocity (ideal at low β !)

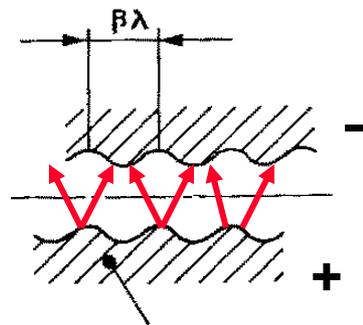


2. The vanes have a **longitudinal modulation** with period = $\beta\lambda$ → this creates a longitudinal component of the electric field. The modulation corresponds exactly to a series of RF gaps and can provide acceleration.



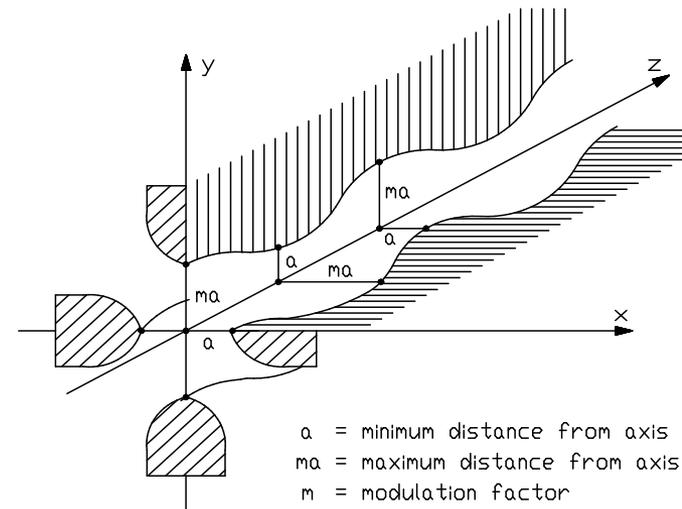
Modulated vane

Opposite vanes (180°)



Modulated vane

Adjacent vanes (90°)

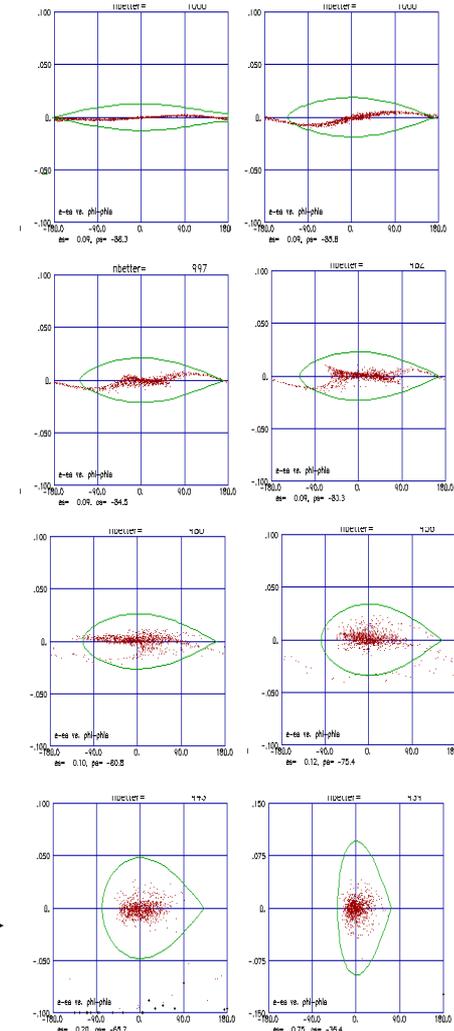


3. The modulation period (distance between maxima) can be slightly adjusted to change the phase of the beam inside the RFQ cells, and the amplitude of the modulation can be changed to change the accelerating gradient → we can start at -90° phase (linac) with some bunching cells, progressively bunch the beam (adiabatic bunching channel), and only in the last cells switch on the acceleration.

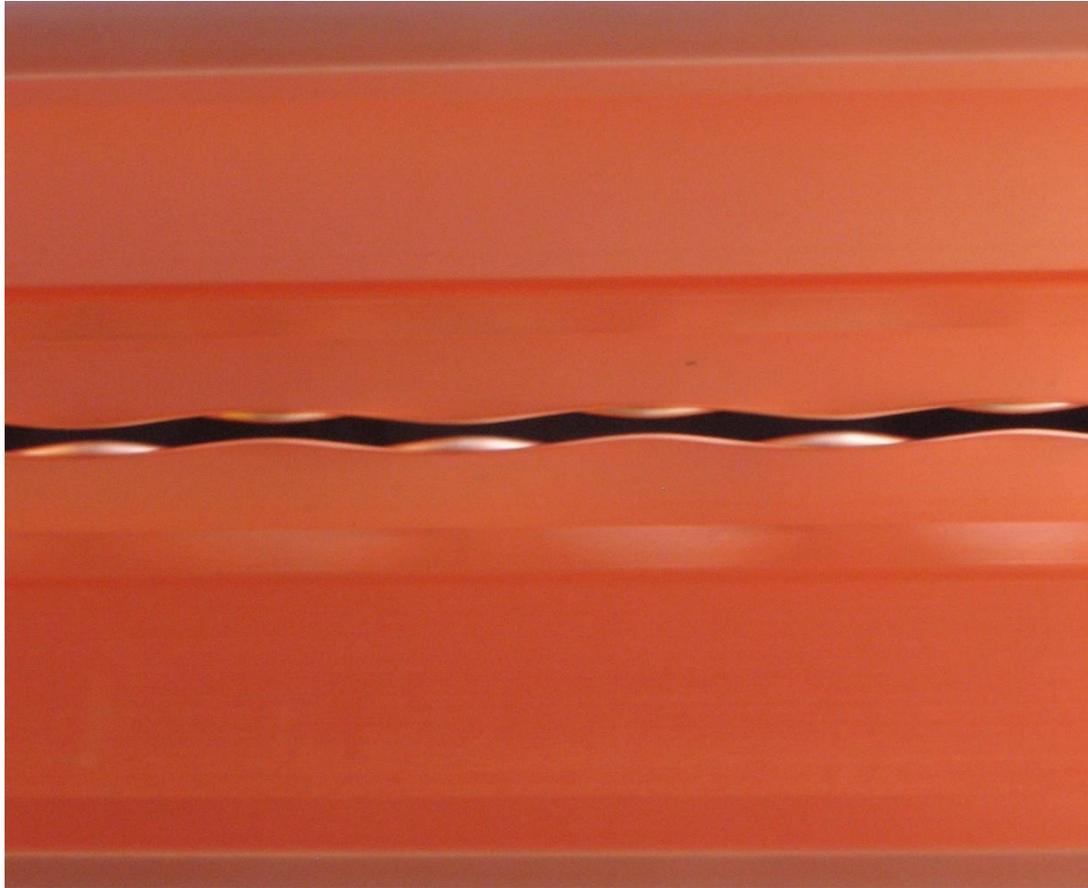
☞ An RFQ has 3 basic functions:

1. Adiabatically bunching of the beam.
2. Focusing, on electric quadrupole.
3. Accelerating.

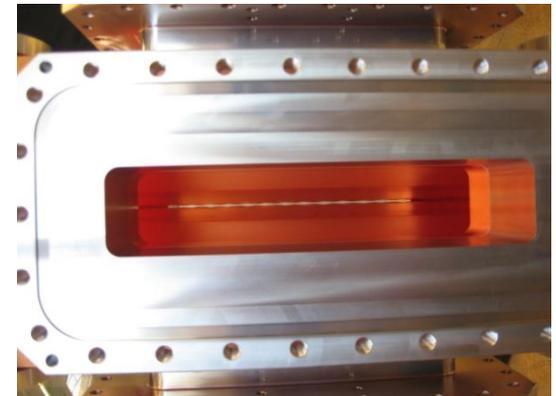
Longitudinal beam profile of a proton beam along the CERN RFQ2: from a continuous beam to a bunched accelerated beam in 300 cells. →



Peeping into an RFQ...



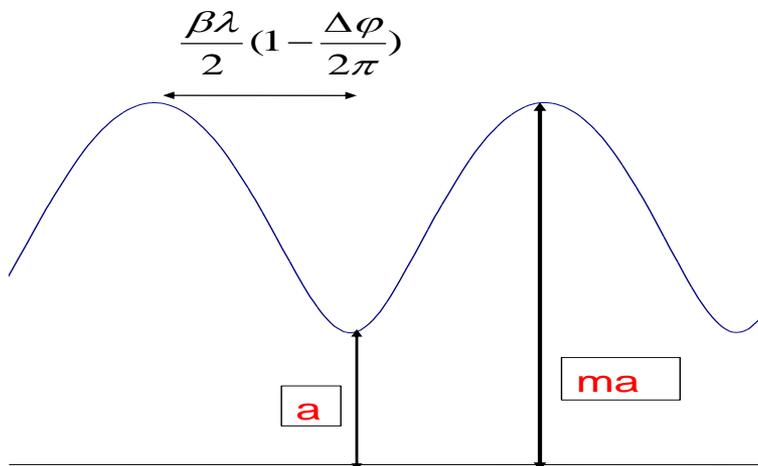
Looking from the RF port into the new CERN RFQ (Linac4, 2011)



An RFQ is made of a sequence of cells (length $\beta\lambda \rightarrow$ in 1 m we can have > 100 cells) where the beam dynamics designer can vary 3 parameters for each cell:

1. **Aperture** a (defines the focusing strength)
2. **Modulation factor** m (defines the longitudinal component)
3. The **beam phase** ϕ , phase difference between bunch center and RF wave (defines the bunching and/or accelerating action).

+ 1 more parameter that is common to all cells or can be changed only smoothly:
the **RF voltage** V .



a = minimum aperture

m = modulation factor (ratio bw. max and min aperture)

cell length/ $\beta\lambda$ = changing the length of the cell with respect to the optimum length for a given beta will change the RF phase seen by the beam.

In order to define the 3-dimensional shape of the RFQ electrodes, Kapchinski introduced an analytical expression for the fields in an RFQ channel :

- The region between the vanes is small w.r.t. the wavelength → static approximation, we can use the formulae for static fields.
- The potential in the intervane region is then a solution of the Laplace equation, which in cylindrical coordinates can be solved by a series of Bessel functions.
- Kapchinski's idea: of all the terms in the series, take only the 2 that are interesting for us (*the transverse quadrupole term + a longitudinal focusing and accelerating term*) and try to **build some electrodes** that give only those 2 terms.

$$V(r, \vartheta, z) = A_0 r^2 \cos 2\theta + A_{10} I_0(kr) \cos kz$$

$$k = 2\pi/\beta\lambda$$

Transverse
quadrupole term

"Longitudinal"
term

- an RFQ cell is defined by the 2 parameters, A_0 and A_{10} (plus the phase)
- the 3 dimensional profile of an RFQ electrode must correspond to an equipotential surface of $V(r, \theta, z)$

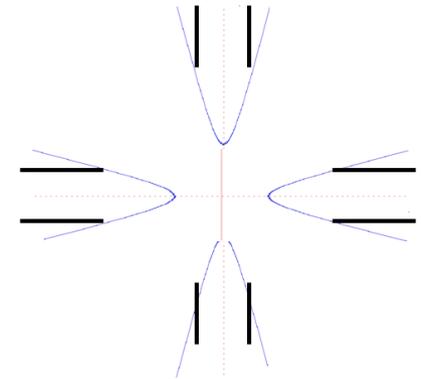
$$V(r, \vartheta, z) = A_0 r^2 \cos 2\theta + A_{10} I_0(kr) \cos kz$$

→ The electrodes have to follow equipotential surfaces of this equation

The equipotential surfaces giving the 2-term RFQ potential are **hyperbolic surfaces with a longitudinal sinusoidal modulation**.

→ The vanes in the 1st generation of RFQs were perfect truncated hyperbolae.

V=voltage applied between 2 adjacent vanes



The constants A_0 , A_{10} depend on the geometry, and can be related to the modulation factors and to the intervane voltage V :

$$A_0 = \frac{V_0}{2a^2} \frac{I_0(ka) + I_0(kma)}{m^2 I_0(ka) + I_0(kma)} \quad A_{10} = \frac{V_0}{2} \frac{m^2 - 1}{m^2 I_0(ka) + I_0(kma)}$$

But truncated hyperbolic surfaces are difficult to machine, while modern field calculation codes allow to use vane profiles that cannot be analyzed analytically.

→ after the first generation of RFQs, the designers are now using simplified vane profiles with **constant curvature radius** or simplified surfaces → introduction of **multipoles**, can be calculated⁸² and kept within acceptable limits.

→ **Transverse focusing coefficient**

$$B = \left(\frac{q}{m_0} \right) \left(\frac{V}{a} \right) \left(\frac{1}{f^2} \right) \frac{1}{a} \left(\frac{I_o(ka) + I_o(mka)}{m^2 I_o(ka) + I_o(mka)} \right)$$

limited by sparking

Transverse field distortion due to modulation (=1 for un-modulated electrodes)

→ **Longitudinal bunching and accelerating field**

$$E_0 T = \frac{m^2 - 1}{m^2 I_o(ka) + I_o(mka)} \cdot V \frac{2}{\beta \cdot \lambda} \frac{\pi}{4}$$

Accelerating efficiency: fraction of the field deviated in the longitudinal direction (=0 for un-modulated electrodes)

cell length

Transverse focusing B is the external focusing contribution to phase advance (see linac lecture)

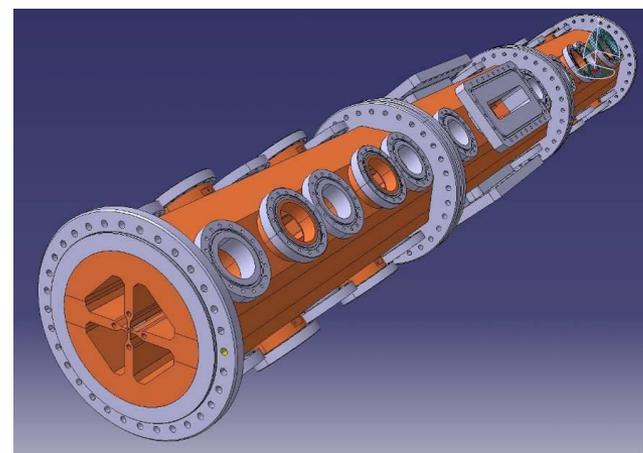
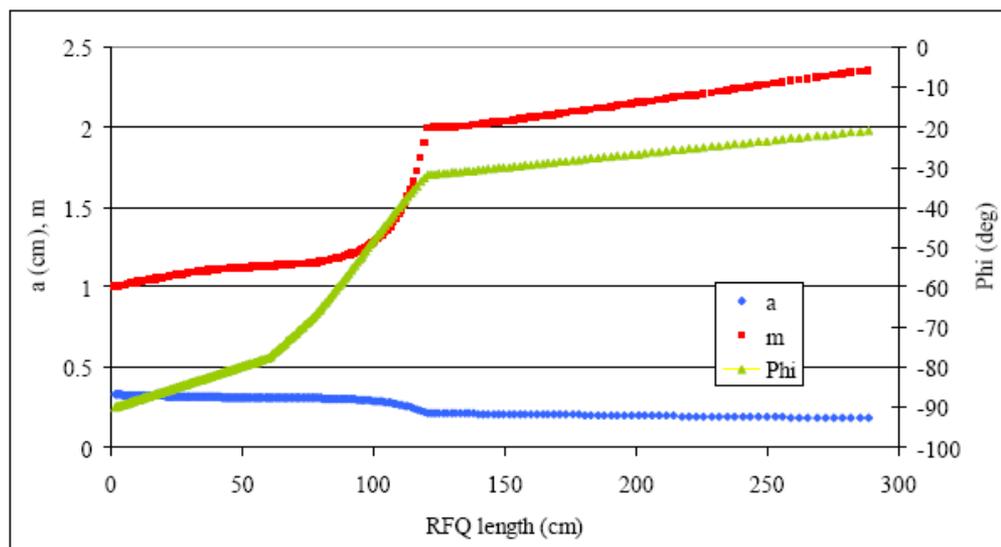
$$\sigma = \sqrt{\frac{B^2}{8\pi^2} - \frac{\pi q E_0 T \sin(\varphi) \lambda}{m c^2 \beta \gamma^3} - \frac{3 Z_0 q I \lambda^3 (1 - f(p))}{8 \pi m c^2 \gamma^3 r^2 b}} \quad 83$$

Example of an RFQ Beam Dynamics design

The new CERN Linac4 RFQ:

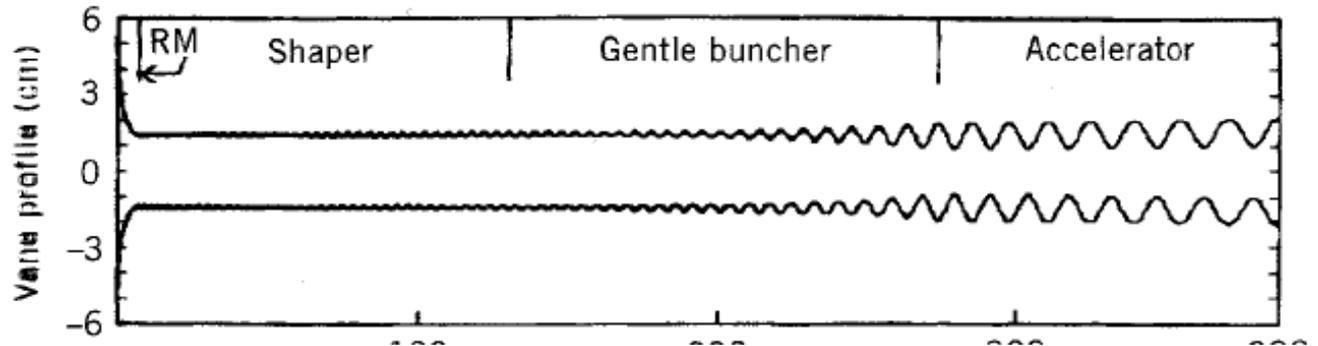
352 MHz, 45 keV to 3 MeV, 303 cells, 3 m length, 70 mA beam current

Beam transmission 95 % (calculated)



The first ~200 cells are used for adiabatic bunching of the beam: the synchronous phase is slowly increased from -90 to -20 deg → bunching with low beam loss!

RFQ sections

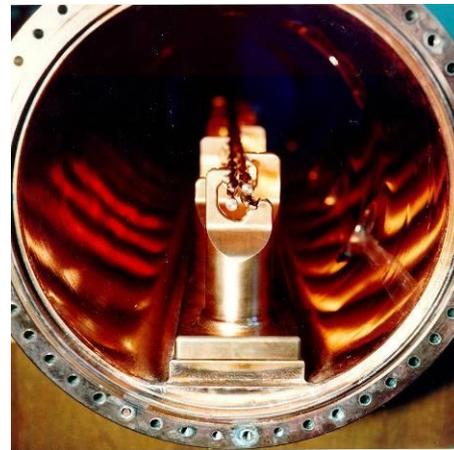
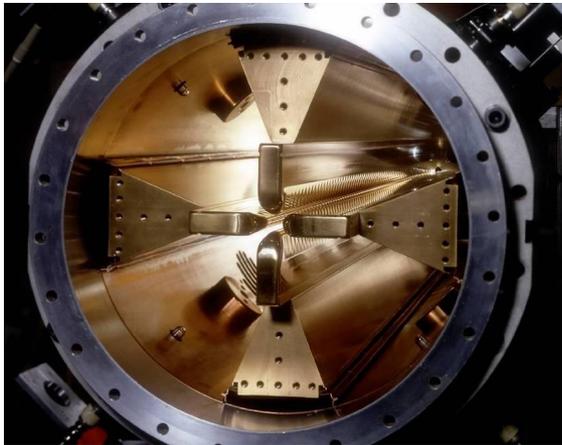


Radial matching to adapt the beam to a time-varying focusing system		
		aperture smoothly brought to the average value
shaping to give the beam a longitudinal structure		
Taper phase to $-80, -60$ deg	start modulation	aperture such that focusing is constant
bunching to bunch and begin acceleration		
Taper phase to $-30, -20$ deg	modulation to max	aperture such that focusing is constant
acceleration to bring the beam to the final energy.		
Constant phase	Constant modulation	Constant aperture
output matching to adapt the beam to the downstream user's need.		

Problem:

How to produce on the electrodes the quadrupole RF field?

2 main families of resonators: 4-vane and 4-rod structures



plus some more exotic options
(split-ring, double-H, etc.)



Remark:

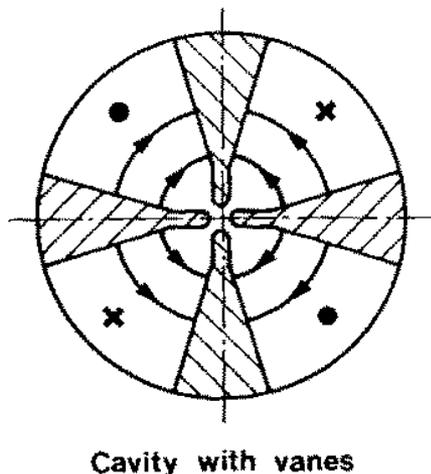
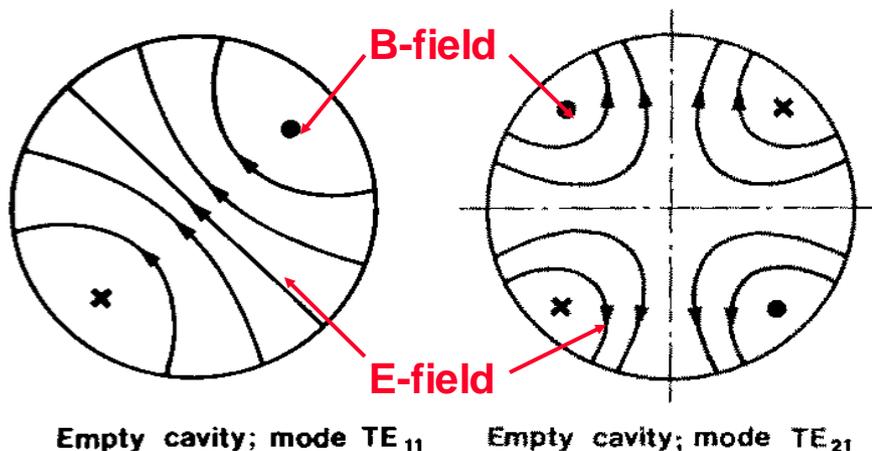
what is the ideal frequency for an RFQ?

Cell length $\beta\lambda/2$ at injection should be mechanically achievable, of the order of few mm.

For heavy ions,
 $\beta \sim 10^{-4} - 10^{-3}$
corresponding to
 $f \sim 10 - 100$ MHz

For protons,
 $\beta \sim 10^{-2}$ makes higher frequencies possible, but beam dynamics (focusing $\sim f^{-2}$) and technology limit to
 $f \sim 200 - 400$ MHz

The "4-vane" RFQ



Basic idea:

An empty cylindrical cavity can be excited on **different modes**.

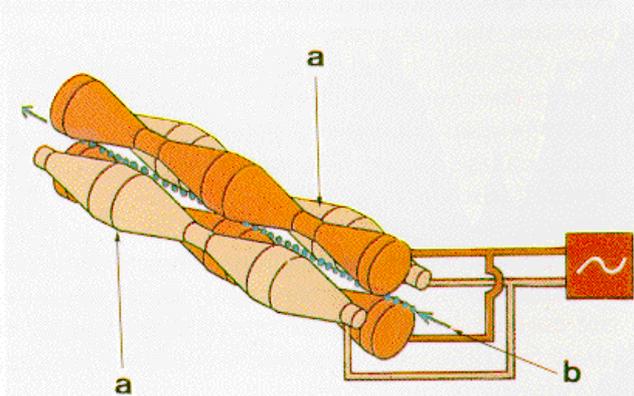
Some of these modes have only **transverse electric field** (the TE modes), and in particular going up in frequency one can find a "quadrupole" mode, the TE_{210} .

The introduction of 4 electrodes (the vanes) can then "load" the TE_{210} mode, with 2 effects:

- Concentrate the electric field on the axis, increasing the efficiency.
- Lower the frequency of the TE_{210} mode, separating it from the other modes of the cylinder.

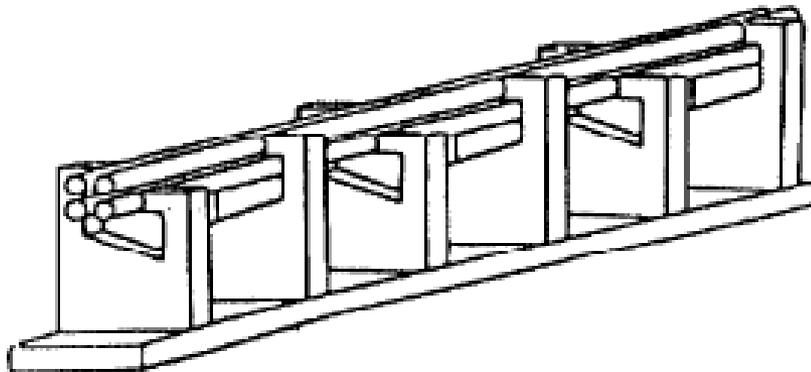
Unfortunately, the dipole mode TE_{110} is lowered as well, and remains as a perturbing mode in this type of RFQs.

The 4-rod RFQ



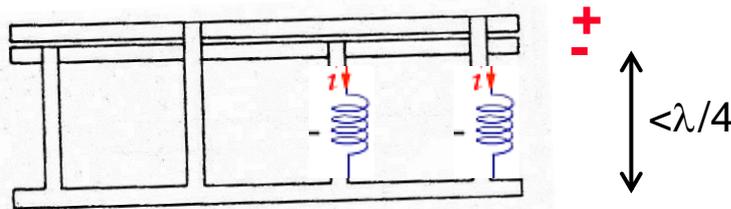
An alternative solution is to machine the modulation not on the tip of an electrode, but on a set of rods (machining on a lathe, old design) or on some small “vanelets”.

The 4 electrodes are then brought to the correct quadrupole potential by an arrangement of quarter-wavelength transmission lines. The set-up is then inserted into a cylindrical tank.



Cost-effective solution, becomes critical at high frequencies → dimensions become small and current densities go up.

Power densities are higher than in the 4-vane → more problems for high power applications.



Commonly used for heavy ions and protons at low frequency – low duty cycle ($f < 200$ MHz). Under development versions for high duty

Two main mechanical problems:

1. The need to achieve the tight tolerances in vane machining and positioning required by beam dynamics and RF.



Machining of a vane for the new CERN RFQ (linac4)

RF and beam dynamics both require **tight tolerances** in the position of the electrodes (Linac4 RFQ: $<30 \mu\text{m}$).

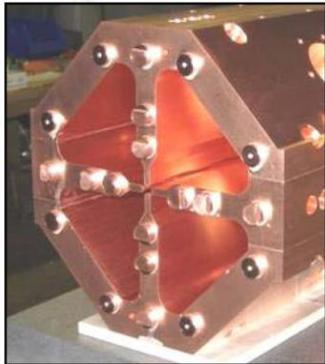
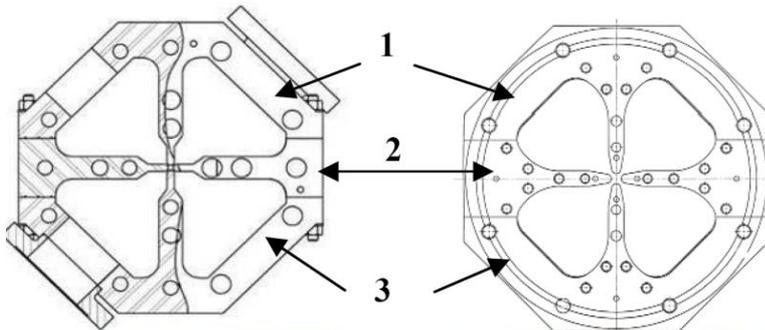
RF: presence of dipole and/or longitudinal components.

Beam dynamics: introduction of multipoles (Linac4 RFQ average aperture $r_0 = 3.3 \text{ mm}$, 1% of aperture is $\sim 30 \mu\text{m}$). Minimum aperture $a = 1.8 \text{ mm} !!$

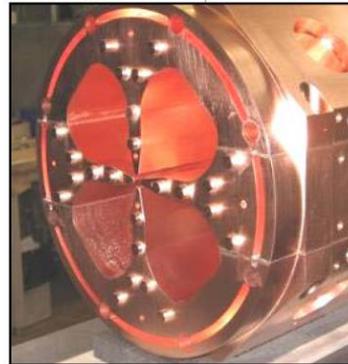
Linac4 RFQ Mechanical Tolerances	Value	Units
Machining error	± 20	μm
Vane modulation error	± 20	μm
Vane tilt over 1 m	± 100	μm
Vane positioning error (displacement h+V)	± 30	μm
Vane thickness error	± 10	μm
Electrode gap (contiguous modules)	100 ± 15	μm
Section tilt over 1 m	± 30	μm
Electromagnetic field error	± 1	%

2. The need to assemble a LEGO[®] of several components (tanks, vanes or rods, supports, etc.) that have to fit together keeping the tolerances and providing a good quality RF contact (large currents flowing!).

4-vane, high frequency: **furnace brazing** of 4 copper elements

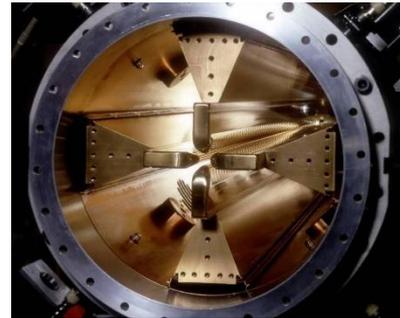


TRASCO, LNL, Italy



IPHI, CEA-CNRS, France

4-vane, low frequency: *EB welding or bolting of copper or copper plated elements*



RFQ1 and RFQ2, CERN

SPIRAL2, CEA-CNRS, France



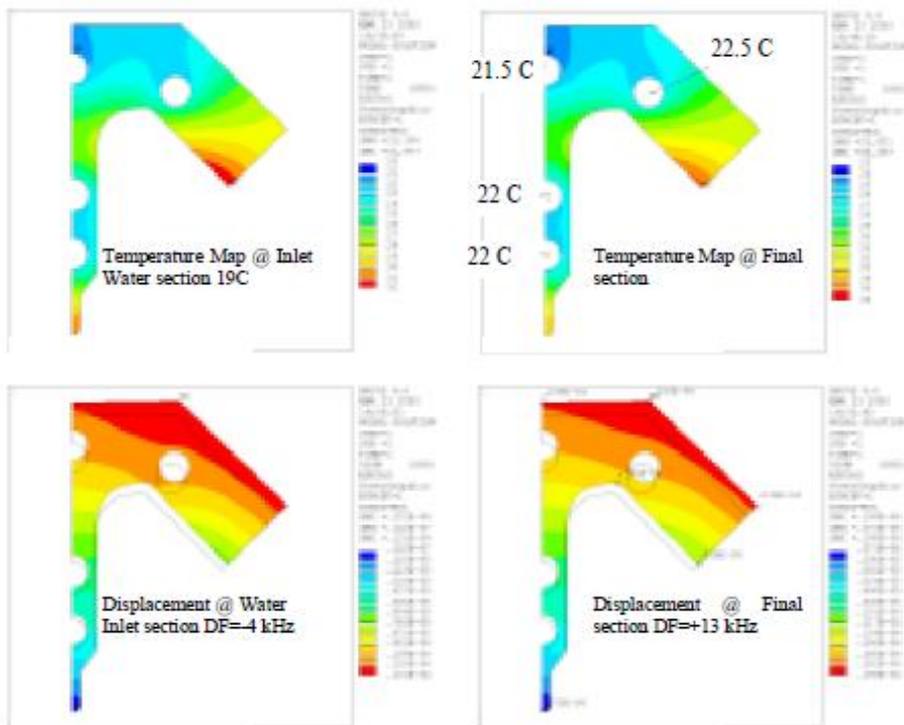


Fig. 6: Top: temperature maps at begin (left) and at the end (right) of one RFQ section. Bottom: deformation maps and frequency shifts.

Example: thermal study of the TRASCO RFQ (CW, 352 MHz, 1 kW/cm) – courtesy of LNL, Legnaro

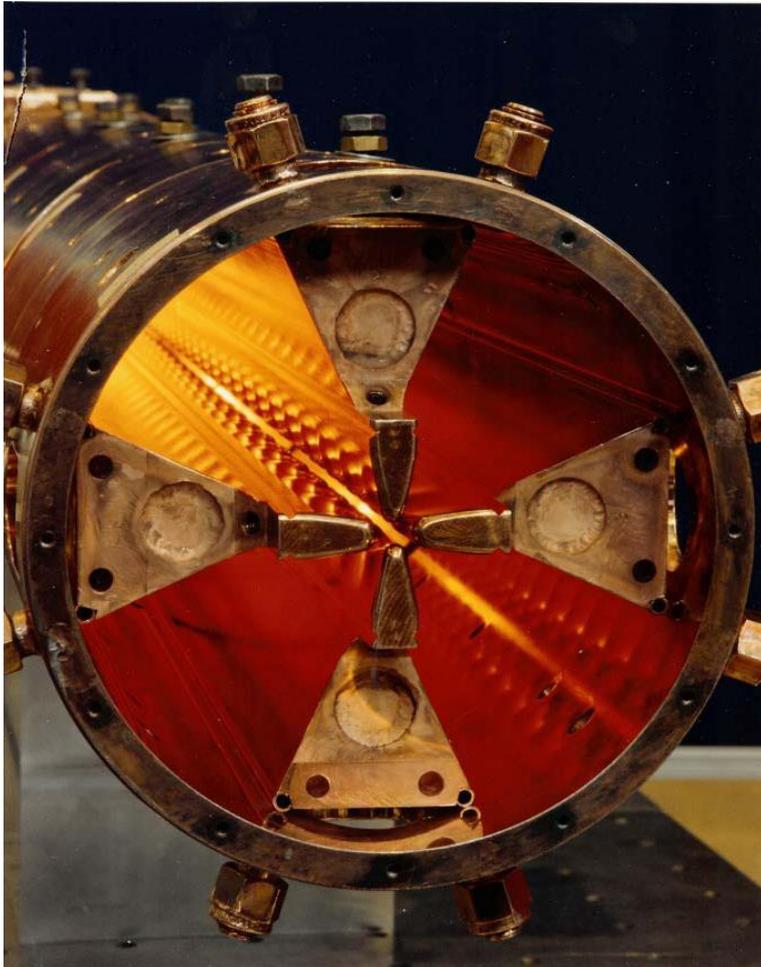
1. High (beam) power RFQs need to dissipate large amounts of RF power in small volumes (vanes are “thin” to maximize shunt impedance).
2. Thermal deformations can lead to large voltage variations and to beam loss.



Need to carefully design and dimension the cooling channels to keep High (beam) power RFQs need to dissipate large amounts of RF power in small volumes (vanes are “thin” to maximize shunt impedance).

1. Thermal deformations can lead to large voltage variations and to beam loss.

The 1st 4-vane RFQ

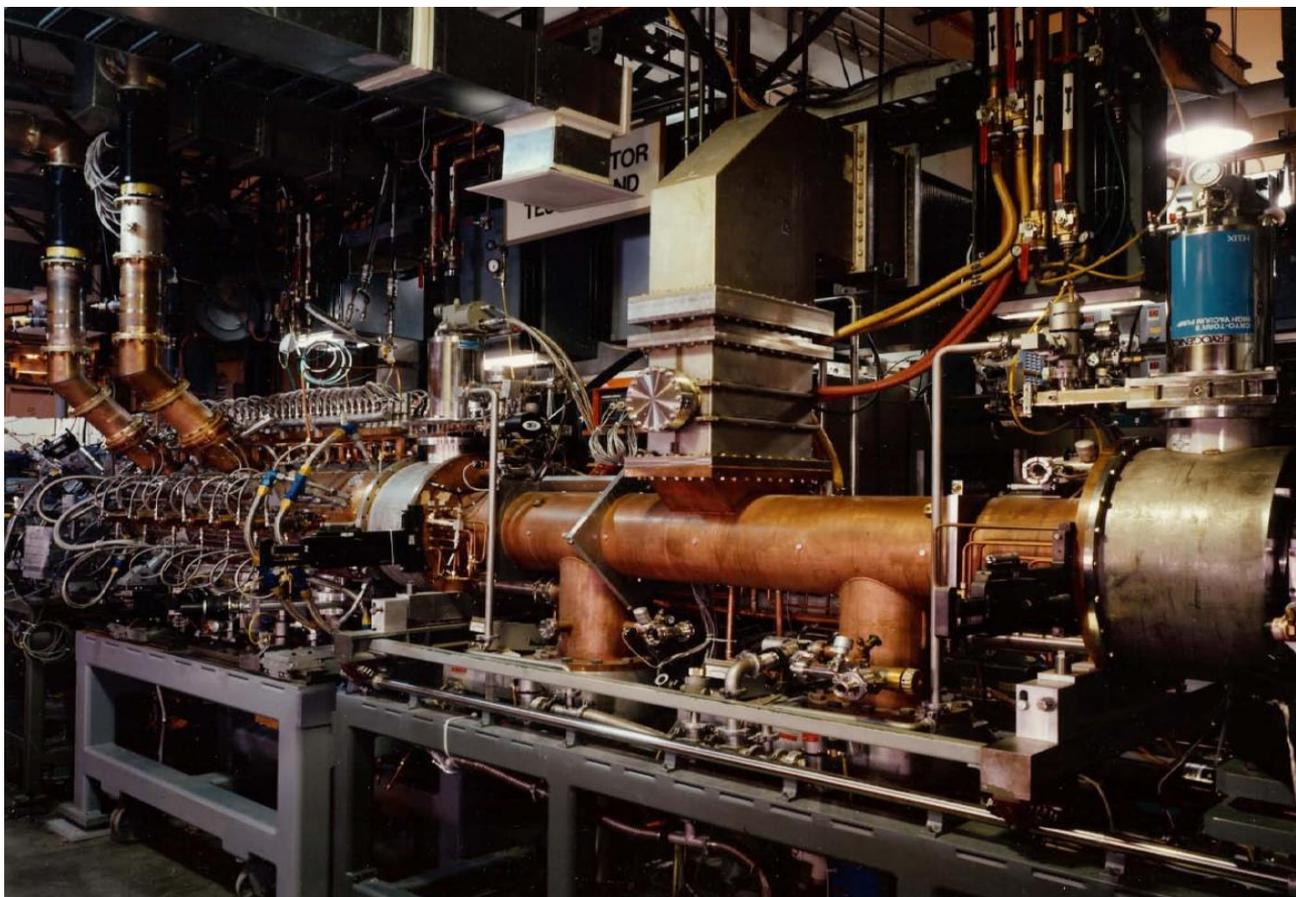


Proof of Principle (POP) RFQ, Los Alamos
1980 – the 1st vane-type RFQ
100 KeV - 650 KeV, 30 mA , 425 MHz

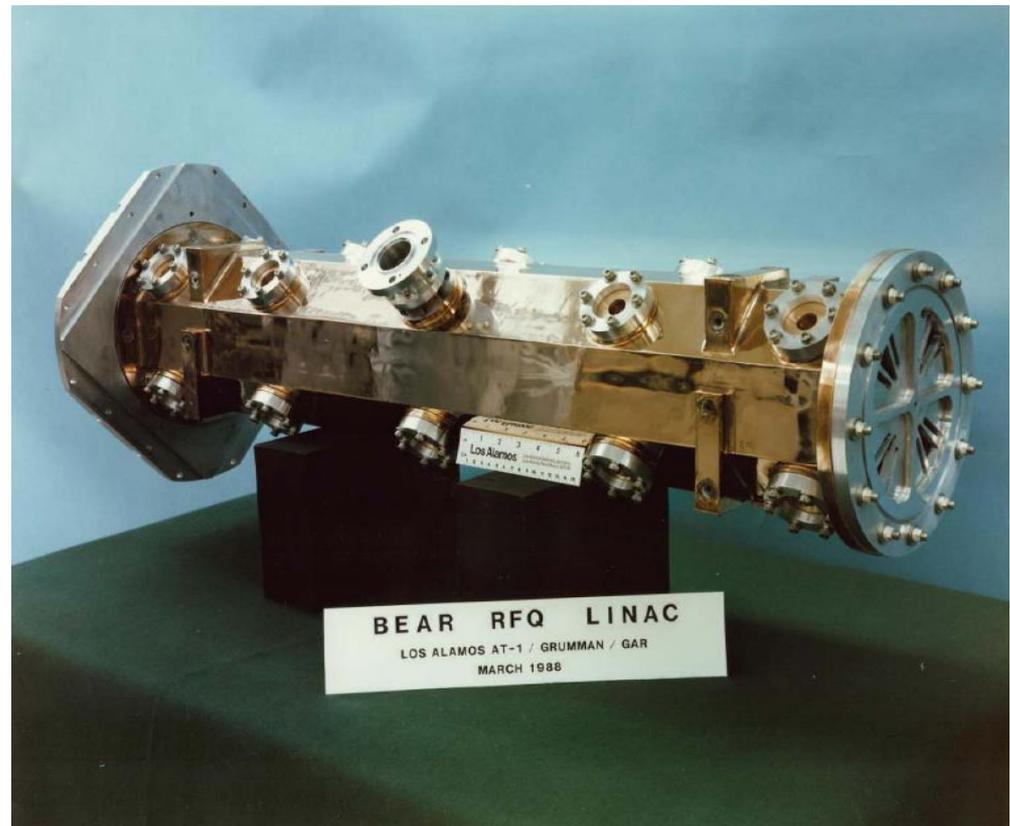


Examples of RFQs - 1

“Star Wars” RFQ (now de-classified), 1983, LANL
2 MeV, 100 mA, ~5% duty, H-minus, 425 MHz
Cu plated carbon steel vanes and cavity, manifold coupled
Demonstrated very small emittance H-minus beams



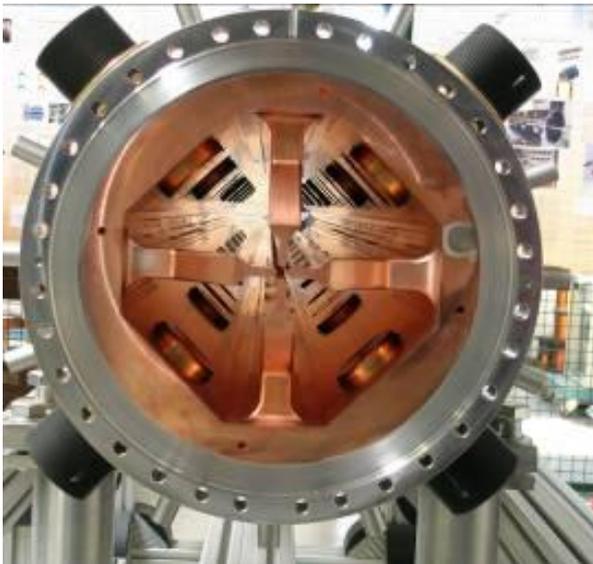
“BEAR” RFQ (beam experiment aboard a rocket)(partly classified) 1989
30 KeV – 1 MeV, 20 mA, <1% duty H-minus
425 MHz, solid-state RF system
Cu plated Al quadrants, joined by electroforming, 55 kg
Operated in sub-orbital flight with a “neutral” beam, LANL



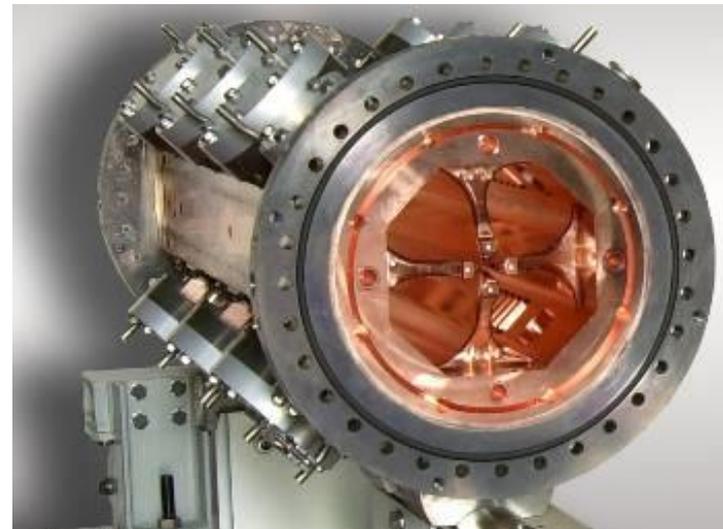
Examples of RFQs - 3

High frequency (352 MHz), high duty cycle (CW)
for ADS studies and other applications.

2 RFQs in construction in Europe:



TRASCO@LegnaroINFN



IPHI@Saclay.CEA



Al prototype and the final installation of the superconducting RFQ at LNL, Italy



Superconducting RFQs:

Only one operating Superconducting RFQs built so far in the world (INFN Legnaro, Italy).

The modulation is extremely difficult to realise in Nb → a superconducting RFQ is limited to few cells at low frequency → heavy ions.

LNL superconducting RFQ: 2 separate structures, 1.4 m and 0.8 m, 41 and 13 cells

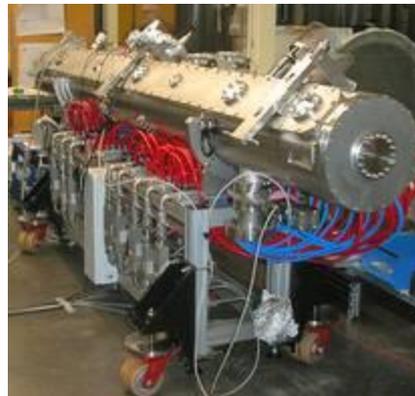
On proton RFQs with high intensity, the unavoidable beam loss during the bunching process would be very dangerous for a superconducting structure.

Examples of RFQ - 5

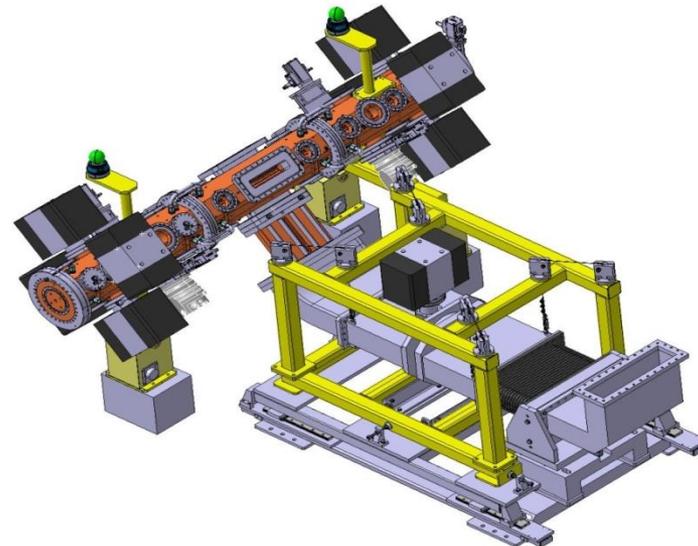
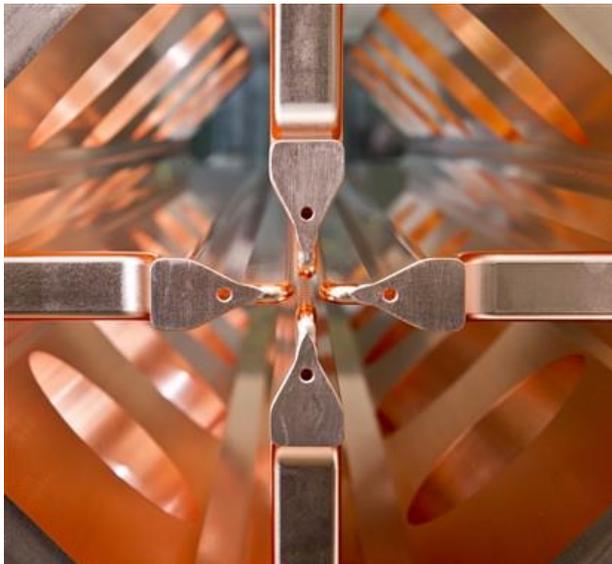


Medium frequency (176 MHz), high duty cycle (CW), 4-rod design for high-intensity deuteron and proton acceleration.

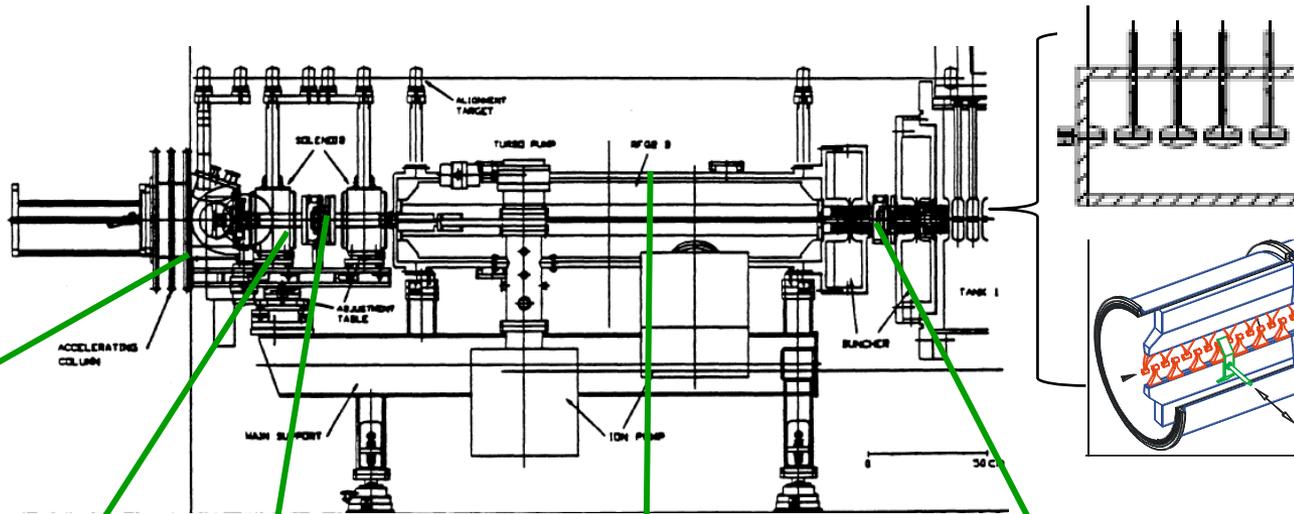
The SARAF RFQ, built by NTG for the Soreq Nuclear Research Center in Israel.



The CERN Linac4 RFQ
Delivered its first beam in March 2013
45 keV – 3 MeV, 3 m
80 mA H-, max. 10% duty cycle



Low-energy proton linacs



Ion source

HV extraction, 20-100 kV

Low Energy Beam Transport (LEBT), 2 solenoid or electrostatic

RFQ, 1 MeV to 5 MeV depending on application

Medium Energy Beam Transport (MEBT), match to following structure with space for diagnostics etc.

DTL, 3 MeV to 80 MeV, high current, high (>200 MHz) frequency

Interdigital structures (IH, CH) 3 MeV to 80 MeV, low current, lower (>350 MHz) frequency

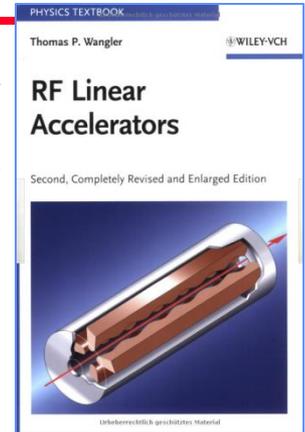
Pulsed beam (1-100 Hz)

10 MeV in 5 – 10 meters

future challenge: go to 10 MeV in 3 – 4 meters for a Compact Proton Linac for Medical and Industrial Applications

1. Reference Books:

- T. Wangler, Principles of RF Linear Accelerators (Wiley, New York, 1998). 
- P. Lapostolle, A. Septier (editors), Linear Accelerators (Amsterdam, North Holland, 1970).
- I.M. Kapchinskii, Theory of resonance linear accelerators (Harwood, Chur, 1985).
- K. Wille, The physics of particle accelerators (Oxford Press, Oxford, 2001).



2. General Introductions to linear accelerators

- M. Puglisi, The Linear Accelerator, in E. Persico, E. Ferrari, S.E. Segré, Principles of Particle Accelerators (W.A. Benjamin, New York, 1968).
- P. Lapostolle, Proton Linear Accelerators: A theoretical and Historical Introduction, LA-11601-MS, 1989.
- P. Lapostolle, M. Weiss, Formulae and Procedures useful for the Design of Linear Accelerators, CERN-PS-2000-001 (DR), 2000.
- P. Lapostolle, R. Jameson, Linear Accelerators, in Encyclopaedia of Applied Physics (VCH Publishers, New York, 1991).

3. CAS Schools

- S. Turner (ed.), CAS School: Cyclotrons, Linacs and their applications, CERN 96-02 (1996).
- M. Weiss, Introduction to RF Linear Accelerators, in CAS School: Fifth General Accelerator Physics Course, CERN-94-01 (1994), p. 913.
- N. Pichoff, Introduction to RF Linear Accelerators, in CAS School: Basic Course on General Accelerator Physics, CERN-2005-04 (2005).
- M. Vretenar, Differences between electron and ion linacs, in CAS School: Small Accelerators, CERN-2006-012.
- M. Vretenar, Low-beta Structures, in CAS RF School, Ebeltoft 2010 (in publication).