

MINIMAL LEPTON FLAVOR VIOLATING REALIZATIONS OF MINIMAL SEESAW MODELS

A. Degee

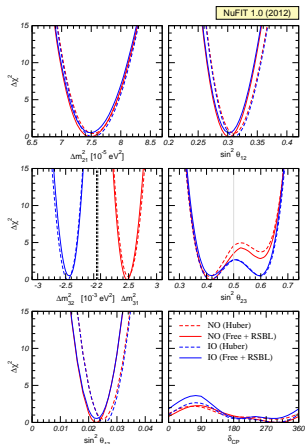
in collaboration with D. Aristizabal Sierra and J.F. Kamenik

based on JHEP 1207 (2012) 135

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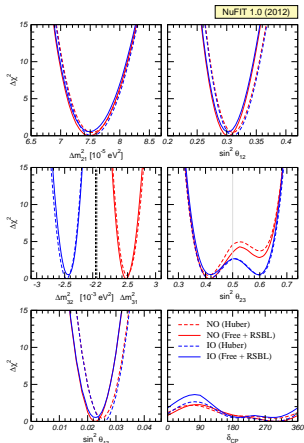
April 17, Portoroz 2013

Evidence of neutrino oscillations



M. C. Gonzalez-Garcia, Michele Maltoni, Jordi Salvado, Thomas Schwetze (2011) [JHEP 1212 (2012) 123], [arXiv :1209.3023]

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For normal spectrum :

Parameter	2σ
Δm_{12}^2 [10 ⁻⁵ eV ²]	7.27 - 8.01
$ \Delta m_{31}^2 $ [10 ⁻³ eV ²]	2.34 - 2.69
$\sin^2 \theta_{12}$	0.29 - 0.35
$\sin^2 \theta_{23}$	0.41 - 0.62
$\sin^2 \theta_{13}$	0.019 - 0.033
δ	0-2 π

Neutrino flavor oscillations

→ **nondegenerate
neutrino masses
and mixings**

→ **lepton flavor violation
in the neutral sector**

Lepton flavor violating (LFV) signals

Neutrino masses \rightarrow Charged LFV processes

No signal for LFV in the charged lepton sector has been observed yet.

No definitive model for neutrino masses \rightarrow no definitive predictions for LFV processes.

- Detection \rightarrow new physics scale \sim TeV
- Predictions \rightarrow LFV signals determined by low-energy neutrino data

Model + flavor structure :

We study the lepton flavor violation (LFV) related phenomenology of the type-I seesaw assuming minimal flavor violation (MFV).

Outline

1 Generalities

- Seesaw mechanism
- MFV and $U(1)_R$ symmetry

2 Models

- First model
- Second model

3 Results

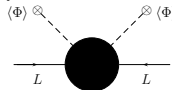
- $l_i \rightarrow l_j \gamma$
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4 Conclusion

Seesaw mechanism

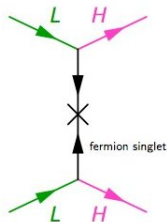
Majorana neutrino ($\bar{\nu} = \nu$) mass term violates lepton number with

the dimension 5 effective operator $\mathbf{Y}_{ij} \frac{L_i \Phi \Phi L_j}{M}$:



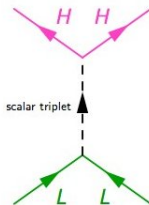
Weinberg, 1979

3 tree-level realizations :



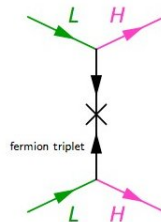
Type I

P.Minkowski (1997) ; R.N. Mohapatra (1980)...



Type II

J.Schechter and J.W.F.Valle (1980)...



Type III

R.Foot, H.Lew, X.G.He and G.C.Joshi (1989).

Type-I seesaw

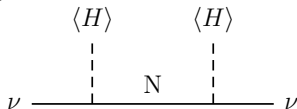
SM + n $SU(2)_L$ singlets neutrinos N_R with heavy Majorana masses.

$$-\mathcal{L} = \bar{l}_L \lambda^* N \tilde{H} + \frac{1}{2} \bar{N} C M_R \bar{N}^T + H.C..$$

The $(3 + n) \times (3 + n)$ neutral fermion mass matrix :

$$M = \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix}, \quad m_D = v\lambda.$$

The light neutrino mass matrix in the seesaw limit $m_D \ll M_R$:



$$m_\nu^{\text{eff}} = m_D^T (M_R^{-1}) m_D.$$

LFV signals and type-I seesaw

Neutrino mass ($m_\nu = v^2 \lambda M_R^{-1} \lambda^T$) constraints require either :

- $M_R \gg \Lambda_{EW}$ (if $\lambda \sim 1$, $M_R \sim 10^{14}$ GeV to have $m_\nu \sim 10^{-1}$ eV)
- $\lambda \ll 1$ (if $M_R \sim 1$ TeV, $\lambda \sim 10^{-6}$ to have $m_\nu \sim 10^{-1}$ eV)

The simple seesaw neutrino model does not induce experimentally observable rates for lepton flavor violating processes, such as $\mu \rightarrow e\gamma$:

$$\Gamma(\mu \rightarrow e\gamma) \propto \lambda^2 \frac{m_\mu^5}{M_R^4} \text{ is very suppressed.}$$

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Minimal flavor violation hypothesis (MFV)

The **global flavor symmetry group** of the SM without the Yukawa couplings is :

$$G_F = U(3)_{Q_L} \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E.$$

In MFV model, G_F is taken to be a symmetry of the full Lagrangian **promoting the Yukawa mass matrices to spurion fields** with definite group transformations.

MFV hypothesis :

- The Yukawa couplings are the only sources of flavour violation.
- Spurions are entirely determined by low energy data.

R. S. Chivukula and H. Georgi, *Phys* (1987) ; L. J. Hall and L. Randall (1990) ; G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia (2002) [*Nucl.Phys. B*645 (2002) 155-187].

Quark sector : Y_U and Y_D sources of FV \rightarrow MFV uniquely implemented.

Lepton sector : Y_e only source of FV \rightarrow LFV effects can be rotated away.

MFV seesaw and $U(1)_R$ symmetry

The kinetic Lagrangian of the SM extended with n RH neutrinos exhibits the global :

$$G = U(3)_E \times U(3)_L \times U(n)_N = U(1)_L \times U(1)_Y \times \mathbf{U}(1)_R \times G_F,$$

where E, L, N denotes triplets in flavor space.

G_F determines the flavor symmetry which is explicitly broken in the Yukawa sector.

$U(1)_{Y,L}$ = global phase rephasing of hypercharge and lepton number.

$U(1)_R$ = global rephasing of lepton sector.

A. Strumia (2002)[Nucl.Phys. B645 (2002) 155-187]; R. Alonso, G. Isidori, L. Merlo, L. A. Munoz and E. Nardi [JHEP 1106 :037,2011]

→ arbitrary R charge assignment → 2 classes of generic models.

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First type-I seesaw model with $U(1)_R$ symmetry

Type-I seesaw model with 2 right-handed neutrinos N_1, N_2 and R assignments :

Charge	N_1	N_2	l	e
L	1	1	1	1
R	1	1	0	0

$$\Delta R = 0 : \Delta L = 0, \quad M = 0, \quad m_\nu = 0.$$

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$$-\mathcal{L} = \epsilon \bar{l} \lambda^* N \tilde{H} + \frac{1}{2} \epsilon^2 \mu N^T C Y_N N + H.C.,$$

$$\Delta R = 0 : \Delta L = 0, \quad M = 0, \quad m_\nu = 0.$$

$$\Delta R \neq 0 : \Delta L \neq 0, \quad M = \begin{pmatrix} 0 & \epsilon \nu \lambda \\ \epsilon \nu \lambda & \epsilon^2 \mu Y_N \nu \lambda \end{pmatrix}, \quad m_\nu^{eff} = -\frac{\nu^2}{\mu} \lambda \cdot Y_N^{-1} \cdot \lambda^T$$

$$(\mathcal{O}(Y_N, \lambda) \sim 1, \quad m_\nu \sim 10^{-1} \text{eV} \quad \rightarrow \quad \mu \sim 10^{14} \text{GeV})$$

First type-I seesaw model with $U(1)_R$ symmetry

$$\hat{M}_N = \epsilon^2 \mu \hat{Y}_N$$

is decoupled from μ as long as $\epsilon \ll 1$, (e.g. : $\epsilon \sim 10^{-5}$, $M_N \sim 1\text{TeV}$)

First type—/ seesaw model with $U(1)_R$ symmetry

$$\hat{M}_N = \epsilon^2 \mu \hat{Y}_N$$

is decoupled from μ as long as $\epsilon \ll 1$, (e.g. : $\epsilon \sim 10^{-5}$, $M_N \sim 1\text{TeV}$)

Does $O(M_N) \sim \text{TeV}$ imply sizable LFV?

The Yukawa couplings

$$\tilde{\lambda} = \epsilon \lambda$$

So,

$$\text{Br}(\mu \rightarrow e\gamma) \sim \frac{\alpha}{1024\pi^4} \frac{m_\mu^5}{\Gamma_{\text{tot}}^\mu} \frac{\tilde{\lambda}^4}{M_N^4}$$

Same R charge for N_1 and N_2 doesn't produce sizable LFV effect :

$$\text{Br}(\mu \rightarrow e\gamma) \sim 10^{-54}$$

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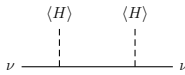
Second type-I seesaw model with $U(1)_R$ symmetry

Type-I seesaw with 2 right-handed neutrinos N_1 and N_2 and R-charge assignments :

Charge	N_1	N_2	l	e
L	1	1	1	1
R	1	-1	1	1

$$-\mathcal{L} = \bar{l}\lambda^* N \tilde{H} + \frac{1}{2} N^T C M_N N + H.C.,$$

$$\lambda^* = \begin{pmatrix} \lambda_{11}^* & 0 \\ \lambda_{21}^* & 0 \\ \lambda_{31}^* & 0 \end{pmatrix}, \quad M_N = \begin{pmatrix} 0 & & \\ & M & \\ & & 0 \end{pmatrix}.$$

$\Delta R = 0$: $\Delta L \neq 0 \rightarrow$  but $m_\nu = 0$

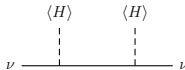
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$$-\mathcal{L} = \bar{l}\lambda^* N \tilde{H} + \frac{1}{2} N^T C M_N N + H.C.,$$

$$\lambda^* = \begin{pmatrix} \lambda_{11}^* & \epsilon_\lambda \lambda_{12}^* \\ \lambda_{21}^* & \epsilon_\lambda \lambda_{22}^* \\ \lambda_{31}^* & \epsilon_\lambda \lambda_{32}^* \end{pmatrix}, \quad M_N = \begin{pmatrix} M_{11}\epsilon_N & M \\ M & M_{22}\epsilon_N \end{pmatrix}.$$

$\Delta R = 0$: $\Delta L \neq 0 \rightarrow$  but $m_\nu = 0$

$\Delta R \neq 0$: $\Delta L \neq 0$, $m_\nu \propto \frac{v^2 \epsilon_\lambda}{M} (\vec{\lambda}_1 \otimes \vec{\lambda}_2 + \vec{\lambda}_2 \otimes \vec{\lambda}_1)$,

$$M_{N_{1,2}} = M \mp \frac{M_{11} + M_{22}}{2} \epsilon_N$$

Second type— I seesaw model with $U(1)_R$ symmetry

light m_ν due to $\epsilon_\lambda \ll 1 \rightarrow \lambda$ large and $O(M) \sim \text{TeV}$.

Potentially large LFV.

MLFV realized due to the structure of m_ν^{eff} :

$$\lambda_{i1} = |\lambda_1|(\sqrt{1 + \rho}U_{i3}^* + \sqrt{1 - \rho}U_{i2}^*)$$

$$\rho = \frac{\sqrt{1+r} - \sqrt{r}}{\sqrt{1+r} + \sqrt{r}}, \quad r = \frac{\Delta m_{12}^2}{\Delta m_{23}^2}$$

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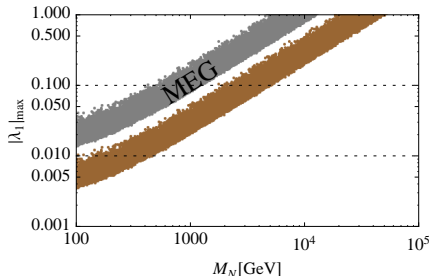
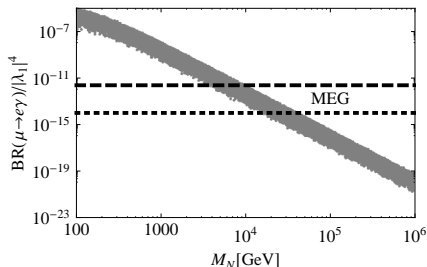
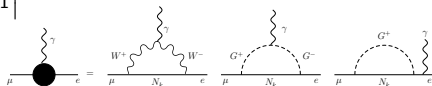
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Br($\mu \rightarrow e\gamma$) in the normal spectrum (NS)

$$\text{BR}(l_i \rightarrow l_j \gamma) \sim \frac{\alpha}{1024\pi^4} \frac{m_i^5}{M^4} \frac{|\lambda_1|^4}{\Gamma_{\text{Tot}}^{l_i}} \left| \hat{\lambda}_{i1} \hat{\lambda}_{j1}^* \right|^2$$

Ilakovac and Pilaftis (1994)



- $\text{Br}(\mu \rightarrow e\gamma) \leq 2.4 \times 10^{-12}$ (current) - 10^{-14} (future) at 90% level.

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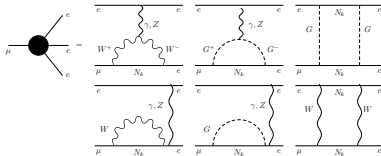
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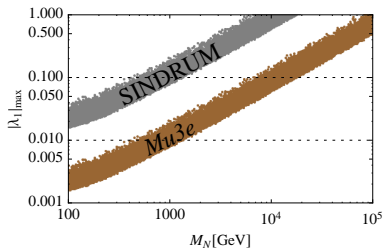
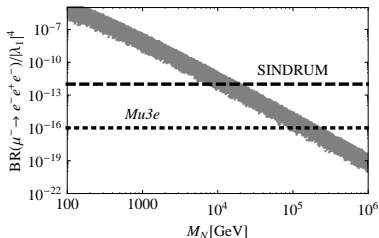
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Br($\mu^- \rightarrow e^- e^+ e^-$) in the normal spectrum (NS)



Ilakovac and Pilaftsis (1994)



- Bounds from SINDRUM : $\text{Br}(\mu^- \rightarrow e^+ e^+ e^-) \leq 10^{-12}$ at 90% level.
- Mu3e/PSI expects 10^{-16} .

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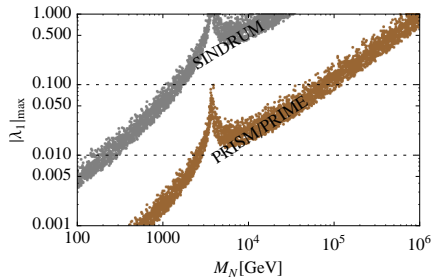
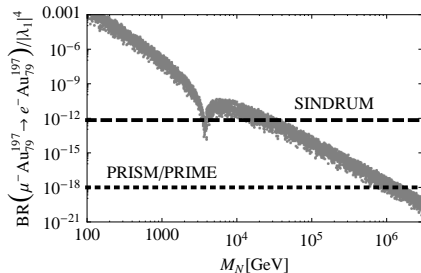
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4 Conclusion

Br(μ -e conversion) in the NS for Au



- Bound on $\mu - e$ conversion from SINDRUM on Ti is 4.3×10^{-12} at 90% level. [C. Dohmen et al. \[SINDRUM II Collaboration.\] \(1993\)](#)
- Bound on $\mu - e$ conversion from SINDRUM on Au is 7×10^{-13} at 90% level. [W. Bertl et al. \[SINDRUM II Collaboration\] \(2006\)](#)
- PRISM/PRIME expects 10^{-16} - 10^{-18} . [C. Ankenbrandt et al.\[physics/0611124\].](#)

Conclusion

We have studied the **implications of a slightly broken $U(1)_R$ symmetry in the minimal Type-I seesaw model.**

Two classes of generic models can be identified, models in which :

- $R(N_1) = R(N_2)$:
 - charged lepton flavor violating decay branching ratios are negligible.
- $R(N_1) = -R(N_2)$:
 - the intrinsic structure leads to a **flavor structure completely determined by low-energy neutrino observables.**
 - **sizable $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and $\mu - e$ processes.**

Thank you !

Neutrino data

For normal spectrum :

Parameter	Best fit $\pm 1\sigma$	2σ	3σ
$\Delta m_{12}^2 [10^{-5} eV^2]$	$7.59^{+0.20}_{-0.18}$	7.24 - 7.99	7.09 - 8.19
$ \Delta m_{31}^2 [10^{-3} eV^2]$	2.45 ± 0.09	2.28 - 2.64	2.18 - 2.73
$\text{Sin}^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28 - 0.35	0.27 - 0.36
$\text{Sin}^2\theta_{23}$	0.51 ± 0.06	0.41 - 0.61	0.39 - 0.64
$\text{Sin}^2\theta_{13}$	$0.01^{+0.009}_{-0.006}$	≤ 0.027	≤ 0.035

Seesaw model conserving L : inverse seesaw

Models in which **lepton flavour (LF) violating scale (λ) can be disconnected from the lepton number (LN) violating one (M_R) :**

- we assume $\Delta L = 0$, $M_N \sim 100 \text{ GeV} - 100 \text{ TeV}$ and $\lambda \sim 10^{-2} - 1$
 $\rightarrow \text{Br}(\mu \rightarrow e\gamma) \sim 10^{-12}$ (experimental limit) but $m_\nu \sim 0$;
- we assume $\Delta L \neq 0$ slightly by a small perturbation ϵ
 $\rightarrow m_\nu \propto \epsilon$ rather than MR^{-1} and $\text{Br}(\mu \rightarrow e\gamma) \sim 10^{-12}$.

LN and LF

Distinguish :

- **lepton number (LN)** violating interactions : LN is $U(1)$ symmetry such as $L(\text{leptons}) = 1 \rightarrow$ **Majorana breaks LN.**
- **lepton flavor (LF)** violating interactions : LF is an $SU(3)$ symmetry, mixing different flavors \rightarrow **Yukawa breaks LF.**

MFV in the quark and lepton sectors

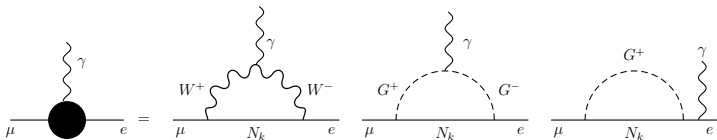
- **Quark sector** in SM : in absence of masses :

$$G_{F,quarks} = U(3)^3 = SU(3)^3 \times U(1)^3.$$

MFV hypothesis :

- All breaking of G_F must transform as the Yukawa interactions.
 - In the mass eigenstate basis, **all mixings are parametrized by CKM.**
-
- **Lepton sector** in SM : since $m_\nu = 0$, MLFV cannot be uniquely implemented in the lepton sector.
→ depends on the new physics responsible for neutrino masses.

Radiative lepton flavor violating decays : $l_i \rightarrow l_j \gamma$



For $m_j \ll m_i$:

$$\text{Br}(l_i \rightarrow l_j \gamma) = \frac{\alpha \alpha_W^2}{256 \pi^2} \frac{m_i^5}{M_W^4} \frac{1}{\Gamma_{Tot}^{l_i}} |(\lambda G_\gamma \lambda^\dagger)_{ij}|^2$$

Parametrization in the second model and bounds

For the normal spectrum (NS) :

$$\lambda_{i1} = |\vec{\lambda}_1|(\sqrt{1+\rho}U_{i3}^* + \sqrt{1-\rho}U_{i2}^*)$$

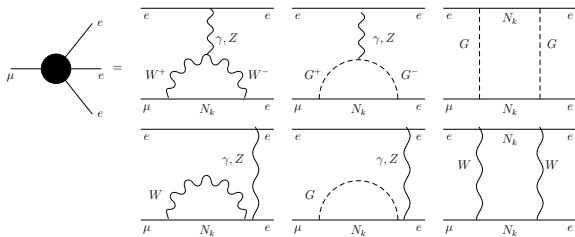
$$\rho = \frac{\sqrt{1+r} - \sqrt{r}}{\sqrt{1+r} + \sqrt{r}}, \quad r = \frac{\Delta m_{12}^2}{\Delta m_{23}^2}$$

M. B. Gavela, T. Hambye, D. Hernandez and P. Hernandez (2009) : arXiv :0906.1461

Bounds from MEG :

- $\text{Br}(\mu \rightarrow e\gamma) \leq 2.4 \times 10^{-12}$ (current) - 10^{-14} (future) at 90% level.
- $\text{Br}(\tau \rightarrow e\gamma) \leq 3.3 \times 10^{-8}$ (current) - $10^{-?}$ (future) at 90% level.
- $\text{Br}(\tau \rightarrow \mu\gamma) \leq 4.4 \times 10^{-8}$ (current) - $10^{-?}$ (future) at 90% level.

$$l_i^- \rightarrow l_j^- l_j^- l_j^+$$



$$T(l_i^- \rightarrow l_j^- l_j^- l_j^+) = T_{\gamma\text{-penguin}} + T_{Z\text{-penguin}} + T_{\text{Boxes}}$$

$$\begin{aligned}
\text{BR}(l_i^- \rightarrow l_j^+ l_j^- l_j^+) &= \frac{\alpha_W^4}{24576\pi^3} \frac{m_i^5}{M_W^4} \frac{1}{\Gamma_{\text{Total}}^{l_i}} \\
&\left\{ 2 \left| \frac{1}{2} F_{\text{Box}}^{l_i 3l_j} + F_Z^{l_i l_j} - 2s_W^2 (F_Z^{l_i l_j} - F_\gamma^{l_i l_j}) \right|^2 \right. \\
&+ 4s_W^4 \left| F_Z^{l_i l_j} - F_\gamma^{l_i l_j} \right|^2 + 16s_W^2 \text{Re} \left[\left(F_Z^{l_i l_j} + \frac{1}{2} F_{\text{Box}}^{l_i 3l_j} \right) G_\gamma^{l_i l_j*} \right] \\
&\left. - 48s_W^4 \text{Re} \left[\left(F_Z^{l_i l_j} - F_\gamma^{l_i l_j} \right) G_\gamma^{l_i l_j*} \right] + 32s_W^4 |G_\gamma^{l_i l_j}|^2 \left(\ln \frac{m_i^2}{m_j^2} - \frac{11}{4} \right) \right\}
\end{aligned}$$

$\mu - e$ conversion in nuclei

Currently the best bound comes from the search for coherent $\mu^- \rightarrow e^-$ conversions in the field of a nucleus, $\mu^- N \rightarrow e^- N$.

$$\text{Br}(\mu^- \rightarrow e\gamma) = \frac{\Gamma_{\text{conversion}}}{\Gamma_{\text{capture}}} = \frac{\Gamma(\mu^- N \rightarrow e^- N)}{\Gamma(\mu^- N \rightarrow \nu N)}$$

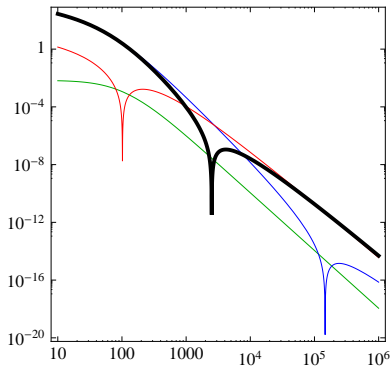
$$\Gamma_{\text{conversion}} = 2G_F^2 |A_R^* D + (2g_{LV}^{(u)} + g_{LV}^{(d)})V^{(p)} + (g_{LV}^{(u)} + 2g_{LV}^{(d)})V^{(n)}|^2$$

$$A_R = \frac{\sqrt{2}}{8G_F M_W^2} \frac{\alpha_W}{8\pi} e G_\gamma^{\mu e}$$

$$g_{LV}^{(u)} = \frac{\sqrt{2}\alpha_W^2}{8G_F M_W^2} \left((F_Z^{\mu e} + 2F_{\text{Box}}^{\mu 3e(1)}) - 4Q_u S_W^2 (F_Z^{\mu e} - F_\gamma^{\mu e}) \right)$$

$$g_{LV}^{(d)} = -\frac{\sqrt{2}\alpha_W^2}{8G_F M_W^2} \left((F_Z^{\mu e} + \frac{1}{2}F_{\text{Box}}^{\mu 3e(1)}) + 4Q_d S_W^2 (F_Z^{\mu e} - F_\gamma^{\mu e}) \right)$$

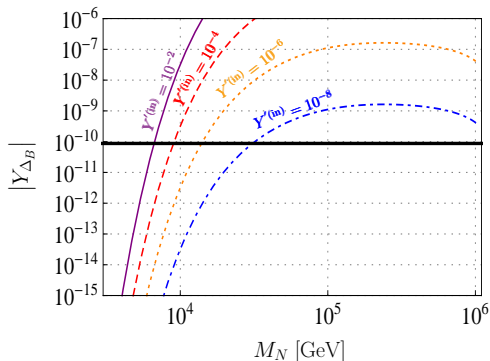
Plot $\mu - e$ conversion



Primordial lepton asymmetries

Sizable charged lepton flavor violating effects and generating/erasing a B-L asymmetry are mutually exclusive.

For $|\lambda_1| = 10^{-5}$:



$$\begin{aligned} Y_{\Delta_B} &= \frac{n_B - n_{\bar{B}}}{s} \\ &= (8.75 \pm 0.23) \times 10^{-11} \end{aligned}$$

Moreover, we've studied the consequences of these setups for models of high scale baryogenesis.

- The phenomenological requirements of **sizable charged lepton flavor violating effects and the generation of a B-L asymmetry (or of not erasing a preexisting one)** are therefore mutually exclusive.