Rare B decays at the NNLO in QCD

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1. Introduction

2. $\bar{B} \to X_s \gamma$:

   2a. (Photonic dipole)–(four quark) vertex interference at the NNLO for $m_c = 0$

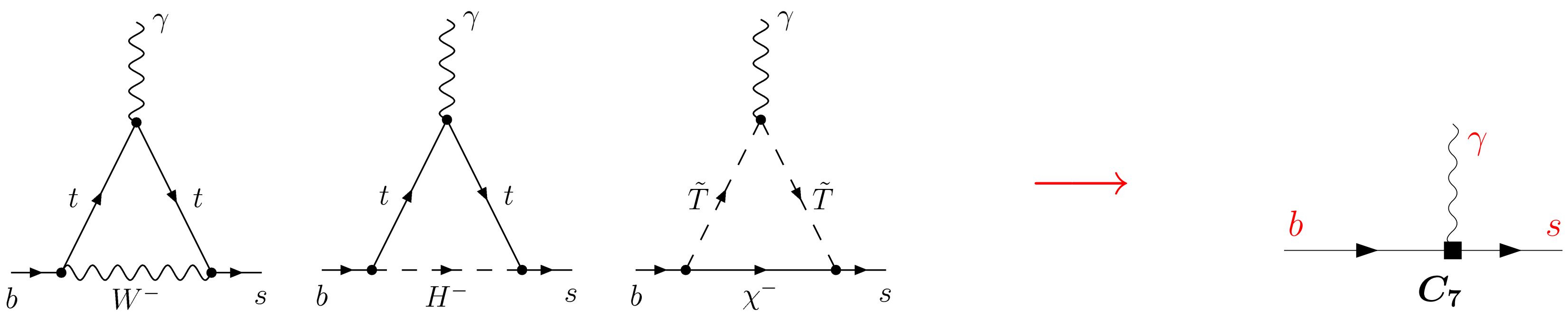
   2b. Estimating the uncertainties

   2c. Update of the SM prediction

3. $B_s \to \mu^+ \mu^-$: electroweak-scale matching at the NNLO

4. Summary

Information on electroweak-scale physics in the $b \to s\gamma$ transition is encoded in an effective low-energy local interaction:



$$b \in \bar{B} \equiv (\bar{B}^0 \text{ or } B^-)$$

The inclusive $\bar{B} \to X_s\gamma$ decay rate is well approximated by the corresponding perturbative decay rate of the b-quark:

$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^p\gamma)_{E_\gamma > E_0} + \left(\begin{array}{c}\text{non-perturbative effects} \\ \sim (2 \pm 5)\% \\ \text{Benzke et al., arXiv:1003.5012}\end{array}\right)$$

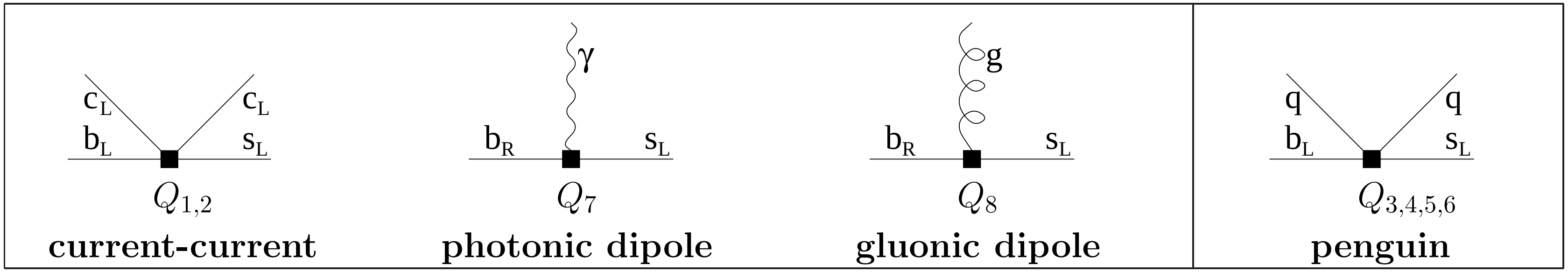
provided E_0 is large $(E_0 \sim m_b/2)$
but not too close to the endpoint $(m_b - 2E_0 \gg \Lambda_{\rm QCD})$.

Conventionally, $E_0 = 1.6\,\text{GeV} \simeq m_b/3$ is chosen.

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak interaction Lagrangian:

$$L_{\text{weak}} \sim \Sigma\, C_i(\mu)\, Q_i$$

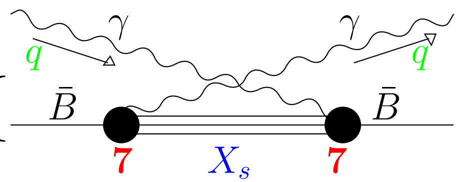
8 operators matter in the SM when the higher-order EW and/or CKM-suppressed effects are neglected:



$$\Gamma(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = |C_7|^2\, \Gamma_{77}(E_0) + (\text{other})$$

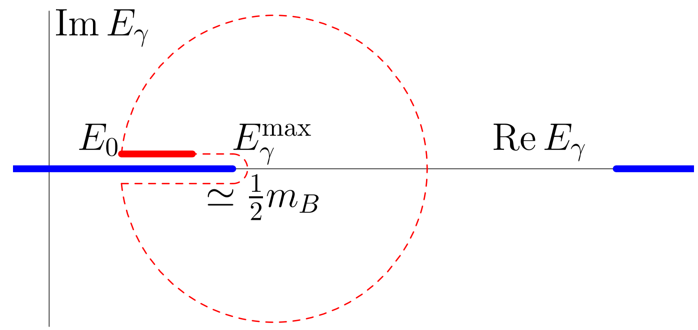
Optical theorem:

$$\frac{d\Gamma_{77}}{dE_\gamma} \sim \text{Im}\{\ \ \}$$



$$\equiv \text{Im}\, A$$

Integrating the amplitude A over E_γ:



OPE on the ring $\Rightarrow$ Non-perturbative corrections to $\Gamma_{77}(E_0)$ form a series in $\frac{\Lambda_{\text{QCD}}}{m_b}$ and α_s that begins with

$$\frac{\mu_\pi^2}{m_b^2},\ \frac{\mu_G^2}{m_b^2},\ \frac{\rho_D^3}{m_b^3},\ \frac{\rho_{LS}^3}{m_b^3},\ldots;\ \frac{\alpha_s \mu_\pi^2}{(m_b - 2E_0)^2},\ \frac{\alpha_s \mu_G^2}{m_b(m_b - 2E_0)};\ldots,$$

where $\mu_\pi, \mu_G, \rho_D, \rho_{LS} = \mathcal{O}(\Lambda_{\text{QCD}})$ are extracted from the semileptonic $\bar{B} \to X_c e \bar{\nu}$ spectra and the B–$B^\star$ mass difference.

The relevant perturbative quantity:

$$\frac{\Gamma[b\to X_s\gamma]_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2\;\Gamma[b\to X_u e\bar{\nu}]} = \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{\rm em}}{\pi} \sum_{i,j} C_i\, C_j\, K_{ij}$$

Expansions of the Wilson coefficients and K_{ij}:

$$C_i(\mu) = C_i^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)}(\mu) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 C_i^{(2)}(\mu) + \ldots$$

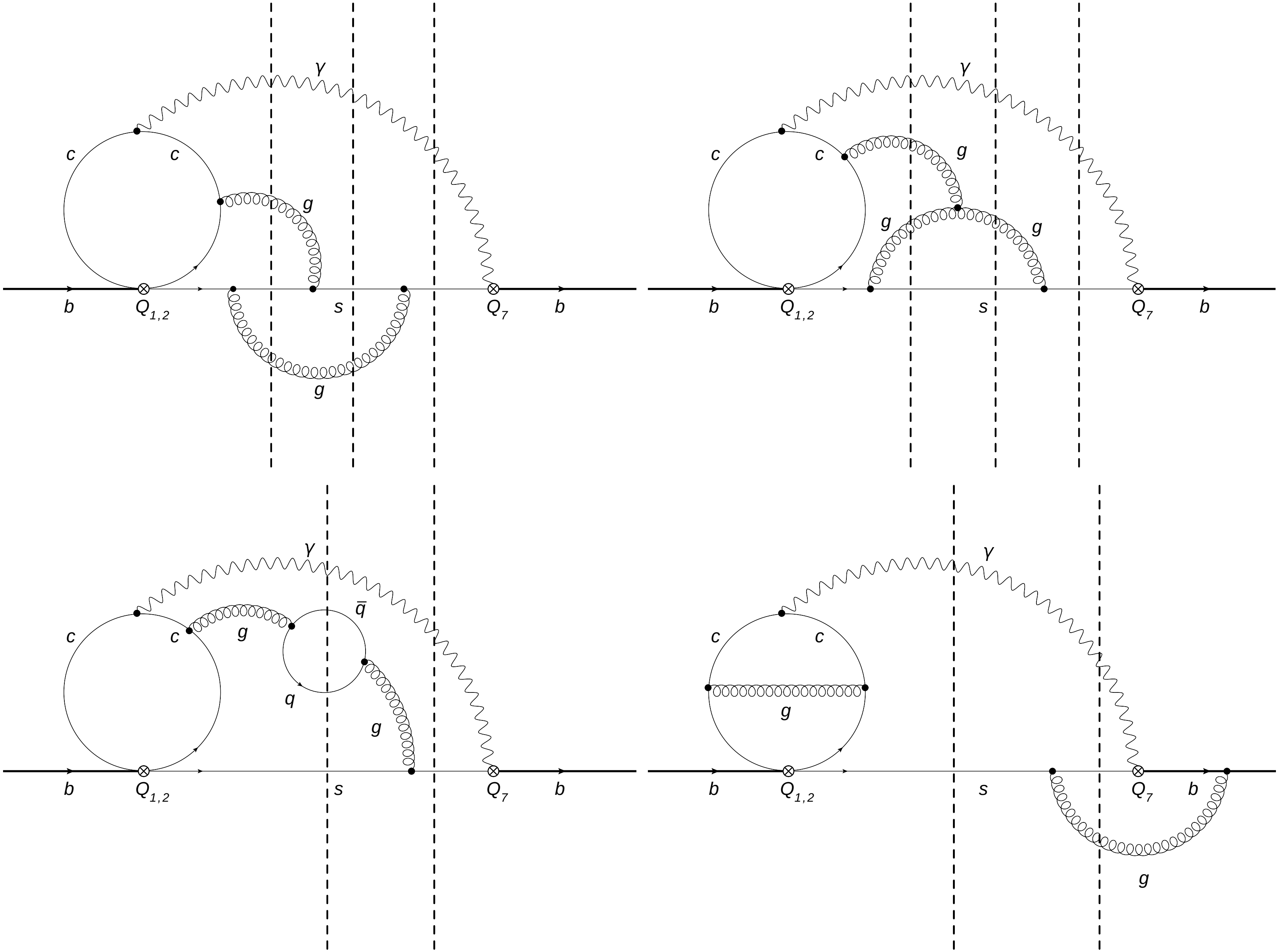
$$K_{ij} = K_{ij}^{(0)} + \frac{\alpha_s(\mu_b)}{4\pi} K_{ij}^{(1)} + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 K_{ij}^{(2)} + \ldots \qquad \mu_b \sim \frac{m_b}{2}$$

Most important at the NNLO: $K_{77}^{(2)}$, $K_{27}^{(2)}$ **and** $K_{17}^{(2)}$.

They depend on $\frac{\mu_b}{m_b}$, $\frac{E_0}{m_b}$ **and** $r = \frac{m_c}{m_b}$.

Evaluation of $K_{27}^{(2)}$ and $K_{17}^{(2)}$ for $m_c = 0$:

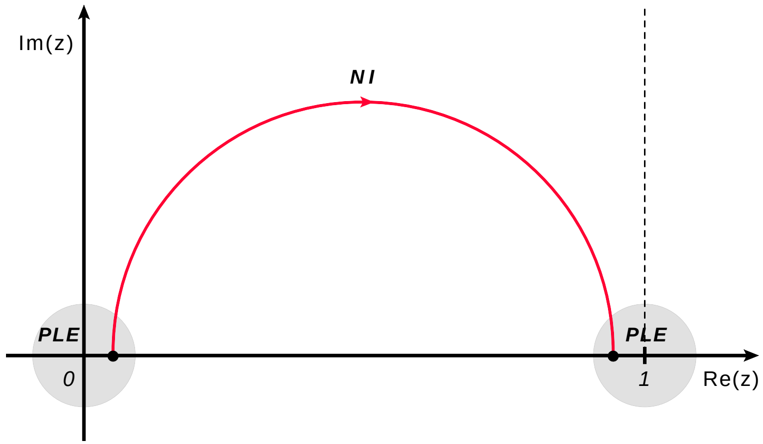
[**M. Czakon, P. Fiedler, T. Huber, M. Misiak, T. Schutzmeier, M. Steinhauser, arXiv:1305.nnnn**]



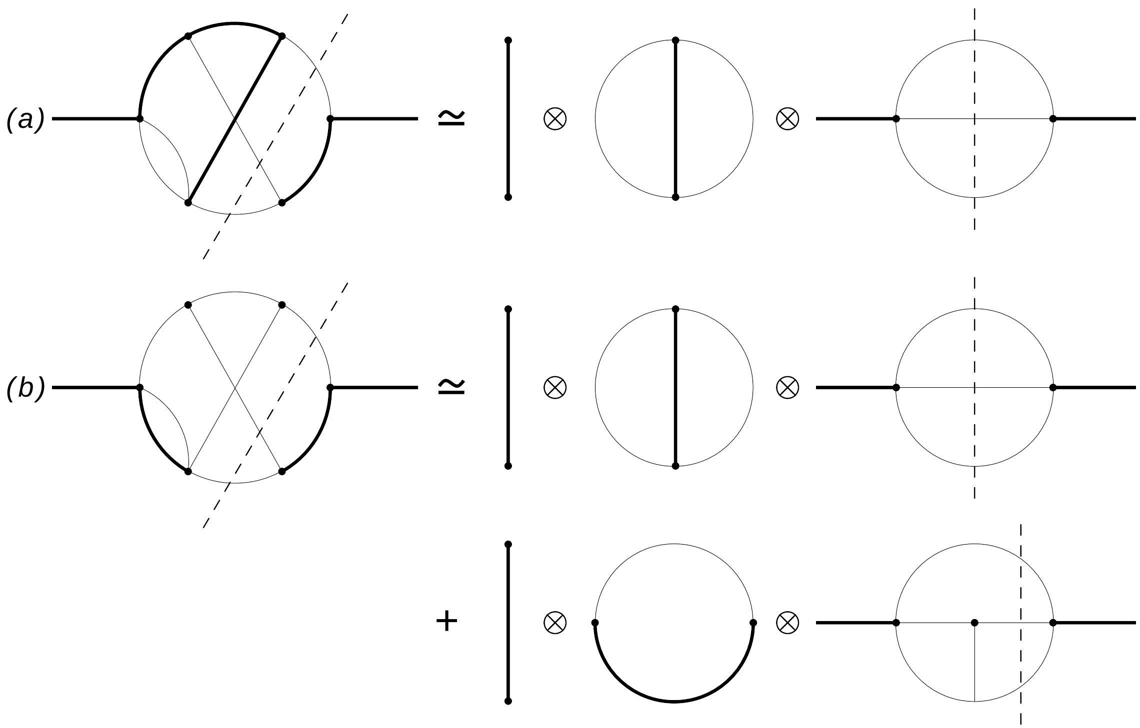
Master integrals and differential equations:

	n_D	n_{OS}	n_{eff}	$n_{massless}$
2-particle cuts	**292**	**92**	**143**	**9**
3-particle cuts	**267**	**54**	**110**	**11**
4-particle cuts	**292**	**17**	**37**	**7**

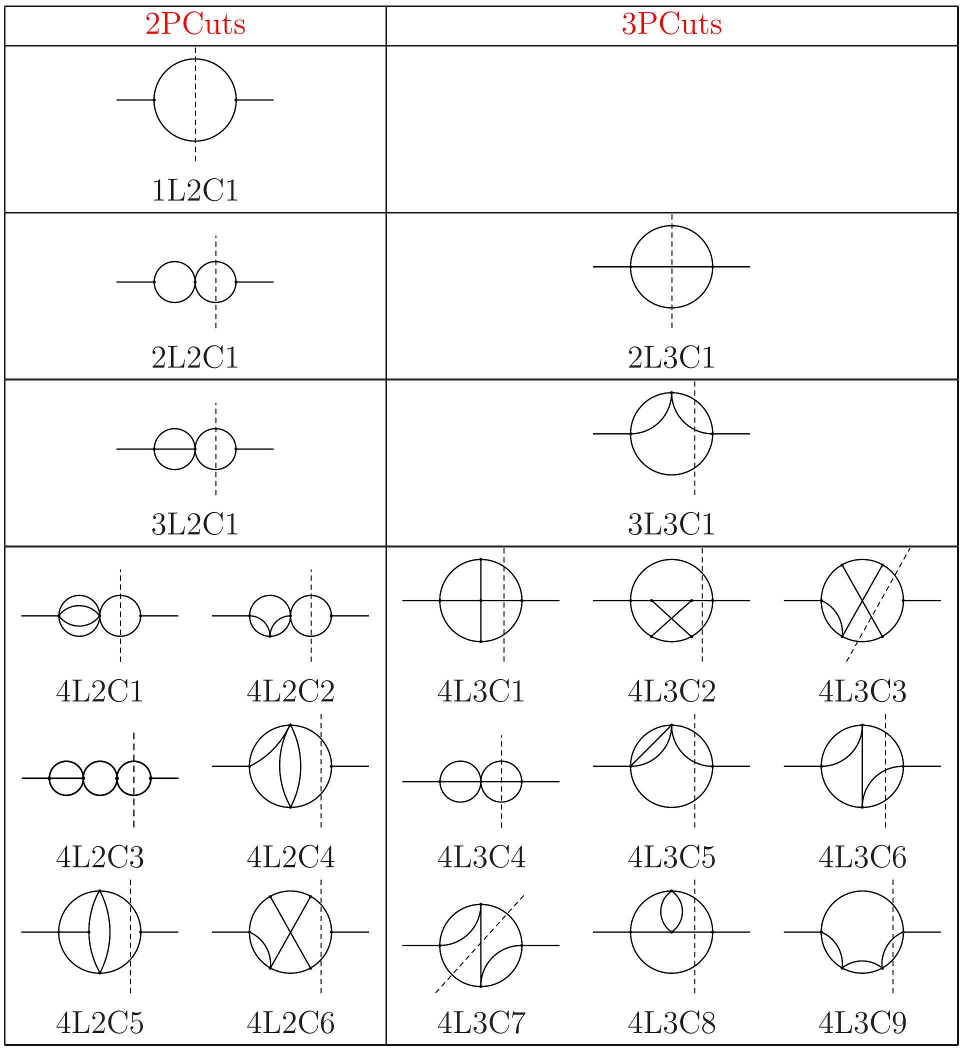
$$\frac{d}{dz} I_i(z) = \sum_j R_{ij}(z) I_j(z), \qquad z = \frac{p^2}{m_b^2}.$$

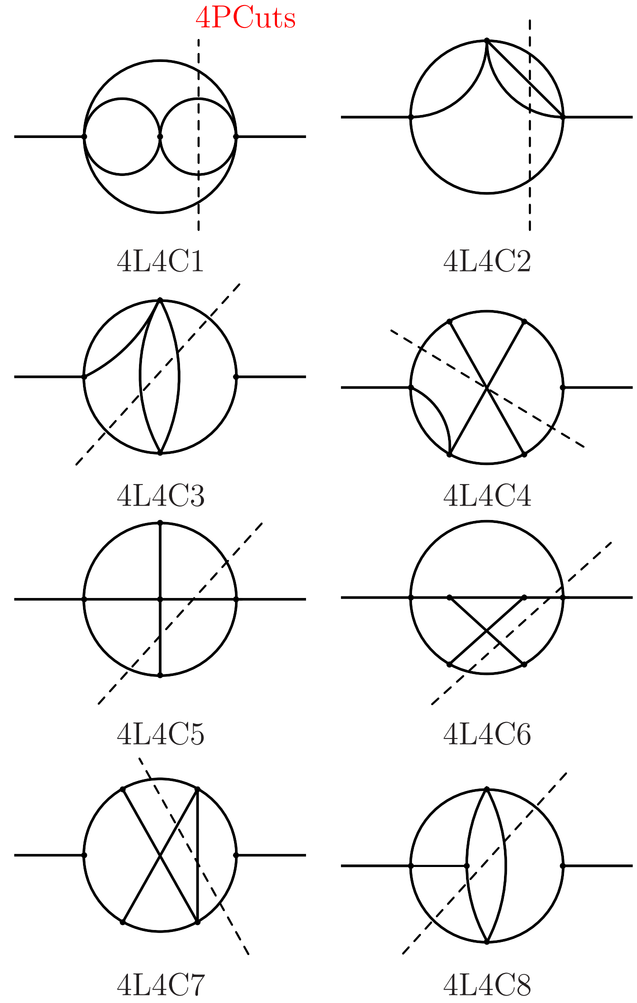


Boundary conditions in the vicinity of $z = 0$:



Massless integrals for the boundary conditions:





The final results:

$$K_{27}^{(2)}(r,E_0) = A_2 + F_2(r,E_0) + 3f_q(r,E_0) + f_b(r) + f_c(r) + \frac{8}{3}\phi_{27}^{(1)}(r,E_0)\ln r + \left[(4L_c - x_m)\,r\frac{d}{dr} + x_m E_0\frac{d}{dE_0}\right] f_{NLO}(r,E_0) + \frac{416}{81}x_m + \left(\frac{10}{3}K_{27}^{(1)} - \frac{2}{3}K_{47}^{(1)} - \frac{208}{81}K_{77}^{(1)} - \frac{35}{27}K_{78}^{(1)} - \frac{254}{81}\right)L_b - \frac{5948}{729}L_b^2,$$

$$K_{17}^{(2)}(r,E_0) = -\frac{1}{6}K_{27}^{(2)}(r,E_0) + A_1 + F_1(r,E_0) + \left(\frac{94}{81} - \frac{3}{2}K_{27}^{(1)} - \frac{3}{4}K_{78}^{(1)}\right)L_b - \frac{34}{27}L_b^2,$$

where $F_i(0,0) \equiv 0$, $A_1 \simeq 22.605$, $A_2 \simeq -37.314$ **from the current calculation.**

Impact of the unknown $F_i(r,E_0)$ on the branching ratio:

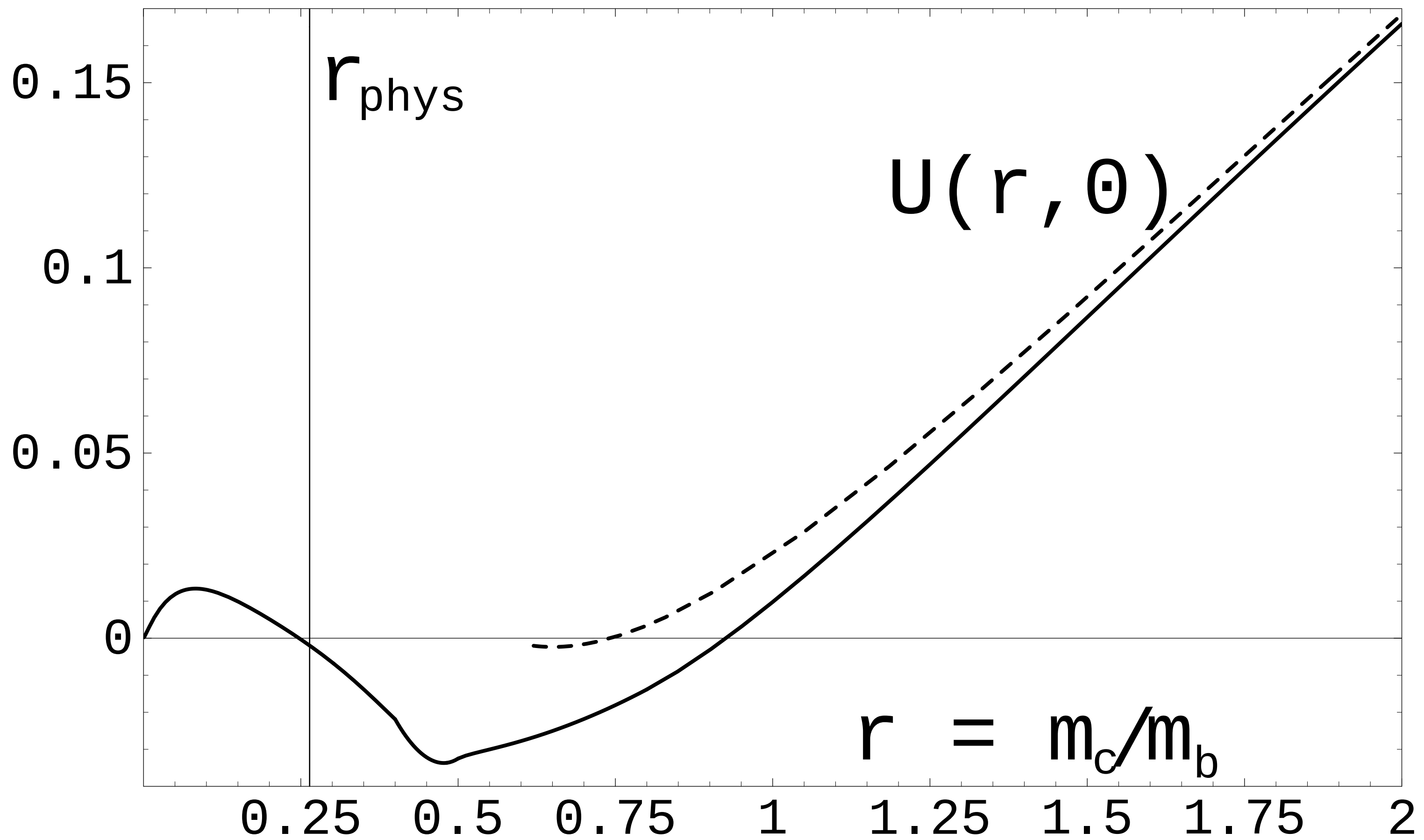
$$\frac{\Delta\mathcal{B}}{\mathcal{B}} \simeq U(r,E_0) \equiv \left[C_7^{(0)\text{eff}}(\mu_b)\right]^{-1}\left\{C_1^{(0)}(\mu_b)F_1(r,E_0) + \left(C_2^{(0)}(\mu_b) - \tfrac{1}{6}C_1^{(0)}(\mu_b)\right)F_2(r,E_0)\right\}.$$

$U(0,0) = 0$,

$U(r,E_0)$ **at large** r: **[MM, M. Steinhauser, hep-ph/0609241, arXiv:1005.1173].**

Impact of the unknown $U(r, E_0)$ on the branching ratio:

$\frac{\Delta\mathcal{B}}{\mathcal{B}} \simeq U(r, E_0)$, $U(0,0) = 0$, **asymptotic behaviour of** $U(r, E_0)$ **at large** r **is known.**



Dashed line: $U(r,0) = 0.023 + 0.116\,\ln r + 0.135\,\ln^2 r + \mathcal{O}\left(\frac{1}{2r}\right)$

Solid line: $-0.251 + 0.038\,f_q(r,0) + 0.016\,f_{NLO}(r,0) + 0.019\,r\frac{d}{dr}f_{NLO}(r,0)$

Estimate at $r_{\rm phys}$: $(0 \pm 3)\%$

Incorporating other perturbative contributions evaluated after September 2006:

1. Four-loop mixing (current-current) $\rightarrow$ (gluonic dipole)
   M. Czakon, U. Haisch and M. Misiak, JHEP 0703 (2007) 008 [hep-ph/0612329]

2. Diagrams with massive quark loops on the gluon lines
   R. Boughezal, M. Czakon and T. Schutzmeier, JHEP 0709 (2007) 072 [arXiv:0707.3090]
   H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, Phys. Lett. B 647 (2007) 173 [hep-ph/0611123]
   T. Ewerth, Phys. Lett. B 669 (2008) 167 [arXiv:0805.3911]

3. Complete interference (photonic dipole)–(gluonic dipole)
   H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola,
   Phys. Rev. D 82 (2010) 074006 [arXiv:1005.5587]

4. New BLM corrections to contributions from 3-body and 4-body final states for interferences not involving the photonic dipole:
   A. Ferroglia and U. Haisch, Phys. Rev. D 82 (2010) 094012 [arXiv:1009.2144]
   M. Misiak and M. Poradziński, Phys. Rev. D 83 (2011) 014024 [arXiv:1009.5685]

5. LO contributions from $b \to s\gamma q\bar{q}$, $(q = u, d, s)$ from the four quark operators ("penguin" ones or CKM-suppressed ones).
   M. Kamiński, M. Misiak and M. Poradziński, Phys. Rev. D 86 (2012) 094004 [arXiv:1209.0965]

Taking into account new non-perturbative analyses:

M. Benzke, S. J. Lee, M. Neubert and G. Paz, JHEP 1008 (2010) 099 [arXiv:1003.5012]
T. Ewerth, P. Gambino and S. Nandi, Nucl. Phys. B 830 (2010) 278 [arXiv:0911.2175]

Updating the parameters:

P. Gambino, C. Schwanda, in preparation

Update of the SM prediction (preliminary)

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm SM}_{E_\gamma>1.6~{\rm GeV}} = (3.13 \pm 0.22) \times 10^{-4}$$

Contributions to the total TH uncertainty (summed in quadrature):

5% non-perturbative, **3%** from the unknown $U(r, E_0)$

3% higher order $\mathcal{O}(\alpha_s^3)$, **2.2%** parametric

Experimental world average (HFAG, 2.08.2012):

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm EXP}_{E_\gamma>1.6~{\rm GeV}} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$$

Experiment agrees with the SM at better than $\sim 1\sigma$ **level. Uncertainties: TH** $\sim 7\%$, **EXP** $\sim 6.5\%$.

$B_s \to \mu^+\mu^-$ — flavour physics highlight of the LHC

- **It is a strongly suppressed, loop-generated process in the SM. Its CP-averaged time-integrated branching ratio (with final-state photon bremsstrahlung included) reads:**

$$\overline{\mathcal{B}}_{\rm SM} = (3.67 \pm 0.22) \times 10^{-9}$$

before including the NNLO QCD and the full NLO EW matching corrections.

talk of Robert Fleischer yesterday:
$(3.56 \pm 0.18) \times 10^{-9}$

- **It is very sensitive to new physics even in models with Minimal Flavour Violation (MFV). Enhancements by orders of magnitude are possible even when constraints from all the other measurements are taken into account.**

- **It has a clear experimental signature: PEAK in the dimuon invariant mass.**

- **First evidence (3.5σ) for its observation has been recently announced by the LHCb Collaboration (arXiv:1211.2674):**

$$\overline{\mathcal{B}}_{\rm exp} = \left(3.2^{+1.5}_{-1.2}\right) \times 10^{-9}.$$

Error budget for the CP-averaged and time-integrated branching ratio

last update: December 2012

$(\kappa^{-1} = 1 - \tau_{B_s}\Delta\Gamma_s/2)$

K. de Bruyn *et al.*,
Phys. Rev. Lett. 109 (2012) 041801.

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\rm SM} = \frac{G_F^4 M_W^4 m_\mu^2 M_{B_s}}{8\pi^5} \times$$

$$\times \underbrace{|V_{tb}^* V_{ts}|^2}_{\pm 3.5\%} \underbrace{\kappa \tau_{B_s}}_{\pm 1.1\%} \left\{ \underbrace{f_{B_s}^2}_{\pm(3.6 \div 7.9)\%} \left[\underbrace{Y_0\left(\frac{m_t^2}{M_W^2}\right) + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_{\rm em})}_{\pm 1.6\% (\text{for } \overline{m}_t(\overline{m}_t) = 165(1)\ \text{GeV})} + \underbrace{\mathcal{O}(\alpha_s^2)\Big]^2 + \mathcal{O}(\alpha_{\rm em})}_{\pm 3\% (\text{to be removed})} \right\}$$

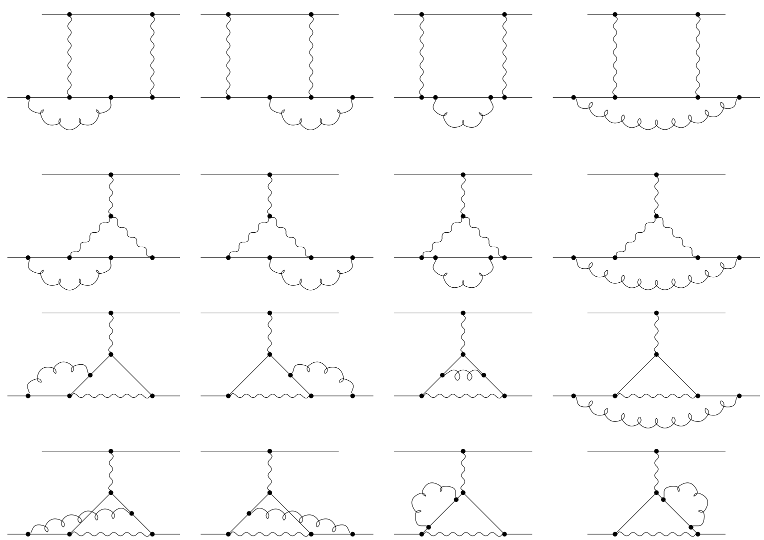
$$= \begin{cases} (3.67 \pm \underbrace{0.22}_{6.1\%}) \times 10^{-9} & \text{for } f_{B_s} = 225.0(4.0)\ \text{MeV} \quad [\text{HPQCD, arXiv:1110.4150}] \\ (4.23 \pm \underbrace{0.39}_{9.3\%}) \times 10^{-9} & \text{for } f_{B_s} = 242.0(9.5)\ \text{MeV} \quad [\text{FNAL/MILC, arXiv:1112.3051}] \end{cases}$$

The $\mathcal{O}(\alpha_s)$ corrections enhance the branching ratio by around $+2.2\%$ when $\overline{m}_t(\overline{m}_t)$ is used at the leading order.

G. Buchalla and A.J. Buras, Nucl. Phys. B 400 (1993) 225,
MM and J. Urban, Phys. Lett. B 451 (1999) 161,
G. Buchalla and A.J. Buras, Nucl. Phys. B 548 (1999) 309.

Logarithmically $\left(\ln\left(m_t^2/m_b^2\right)\right)$ enhanced electromagnetic corrections and the known electroweak corrections suppress the branching ratio by around -1.6%.

G. Buchalla, A. J. Buras, Phys. Rev. D 57 (1998) 216,
C. Bobeth, P. Gambino, M. Gorbahn, U. Haisch, JHEP 0404 (2004) 071,
T. Huber, E. Lunghi, MM, D. Wyler, Nucl. Phys. B 740 (2006) 105,
A. J. Buras, J. Girrbach, D. Guadagnoli, G. Isidori, Eur. Phys. J C72 (2012) 2172.



Another observable:

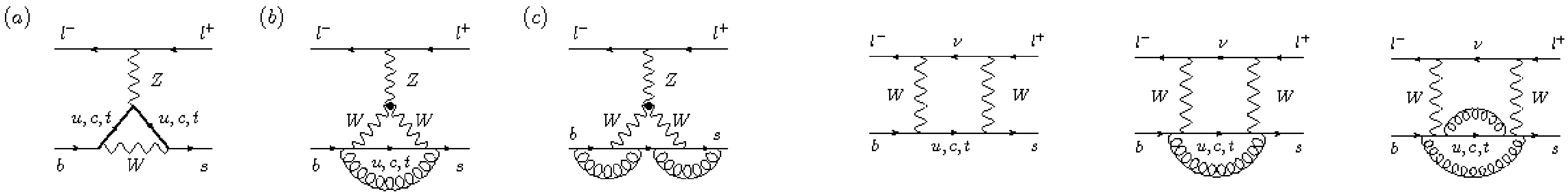
(with different NP sensitivity)

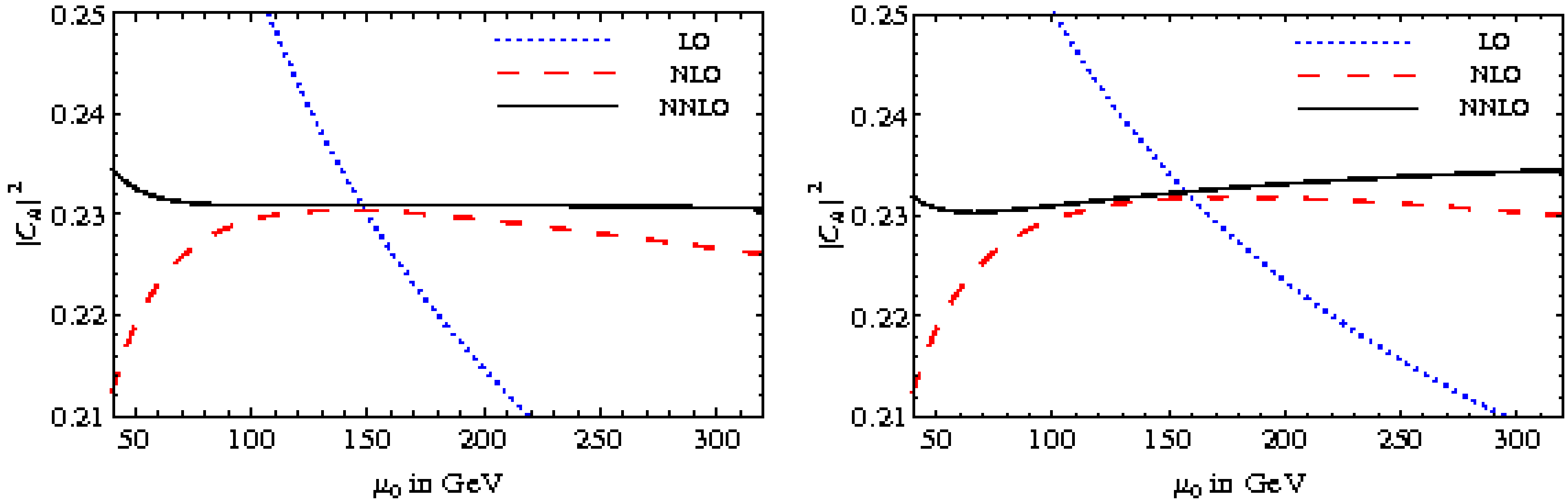
$$\frac{\mathcal{B}(B_s \to \mu^+\mu^-)}{\Delta M_{B_s} \tau_{B_s}}$$

A. J. Buras, Phys. Lett. B 566 (2003) 115..

Evaluation of the NNLO QCD matching corrections

[T. Hermann, M. Misiak and M. Steinhauser, to be published]





(a) (b)

Conclusion: Combination with the NLO EW calculation is necessary

[C. Bobeth, J. Brod, S. Casagrande, M. Gorbahn, E. Stamou, to be published]

Summary

- The dominant NNLO corrections to $\mathcal{B}(\bar{B} \to X_s \gamma)$ are now known not only in the large m_c limit, but also at $m_c = 0$.

- The updated SM prediction is $\mathcal{B}(\bar{B} \to X_s \gamma) = (3.13 \pm 0.22) \times 10^{-4}$ for $E_0 = 1.6 GeV$

- Combining the recently calculated NNLO QCD and NLO EW corrections to $\mathcal{B}(\bar{B}_s \to \mu^+ \mu^-)$ will allow for a significant reduction of the residual perturbative uncertainties.

BACKUP SLIDES

Perturbative expansion of the Wilson coefficients:

$$C_i(\mu) = C_i^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)}(\mu) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 C_i^{(2)}(\mu) + \ldots$$

Branching ratio:

$$\mathcal{B}(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = \mathcal{B}(\bar{B} \to X_c e \bar{\nu})_{\rm exp} \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\rm em}}{\pi\, C} \left[\underset{\textbf{pert.}}{P(E_0)} + \underset{\textbf{non-pert.}}{N(E_0)} \right]$$

$$\frac{\Gamma[b \to X_s \gamma]_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2\, \Gamma[b \to X_u e \bar{\nu}]} = \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\rm em}}{\pi} P(E_0), \qquad C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma[\bar{B} \to X_c e \bar{\nu}]}{\Gamma[\bar{B} \to X_u e \bar{\nu}]}$$

$$P(E_0) = \sum_{i,j} C_i\, C_j\, K_{ij}$$

Perturbative expansion of K_{ij}:

$$K_{ij} = K_{ij}^{(0)} + \frac{\alpha_s(\mu_b)}{4\pi} K_{ij}^{(1)} + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 K_{ij}^{(2)} + \ldots \qquad \mu_b \sim \frac{m_b}{2}$$

Perturbative expansion of $P(E_0)$:

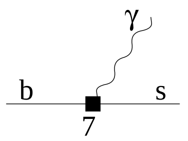
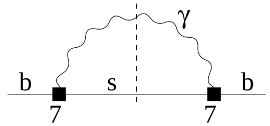
$$P = P^{(0)} + \frac{\alpha_s}{4\pi}\left(P_1^{(1)} + P_2^{(1)}(r)\right) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 \left(P_1^{(2)} + P_2^{(2)}(r) + P_3^{(2)}(r)\right)$$

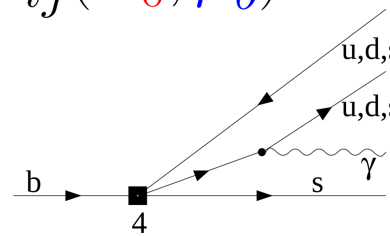
$$P_1^{(1)}, P_3^{(2)} \sim C_i^{(0)} C_j^{(1)}, \qquad P_2^{(1)}, P_2^{(2)} \sim C_i^{(0)} C_j^{(0)}, \qquad P_1^{(2)} \sim \left(C_i^{(0)} C_j^{(2)},\ C_i^{(1)} C_j^{(1)}\right)$$

$r = \frac{m_c}{m_b}$ **Most important at the NNLO: $K_{77}^{(2)}$, $K_{27}^{(2)}$ and $K_{17}^{(2)}$.**

Perturbative evaluation of $\Gamma(b \to X_s^{\rm p}\gamma)$ at $\mu_b \sim \frac{m_b}{2}$.

$$\Gamma(b \to X_s^{\rm p}\gamma)_{E_\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{\rm em}}{32\pi^4} \left|V_{ts}^* V_{tb}\right|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

LO: $G_{77} = 1$ $\left|\right.$  $\left.\right|^2$ $\Leftrightarrow$ 

Other LO are small, e.g.,: 

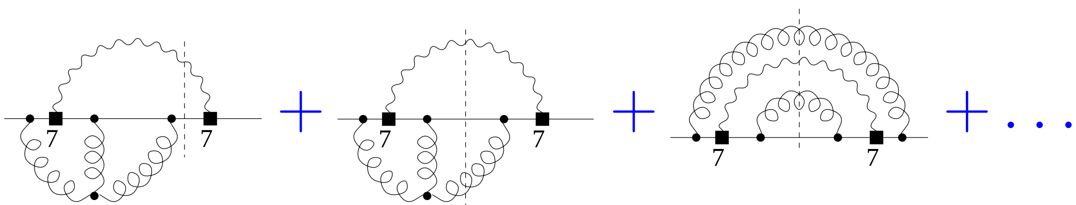
[Kamiński, Poradziński, MM, 2012]

NLO: **1996: Quasi-complete** G_{ij} { [Greub, Hurth, Wyler, 1996] [Ali, Greub, 1991-1995]

2002: Complete$^{(*)}$ G_{ij} { [Buras, Czarnecki, Urban, MM, 2002] [Pott, 1995]

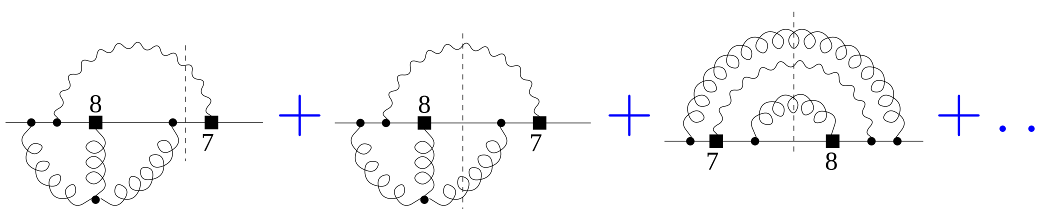
$^{(*)}$**Up to** $b \to s q \bar{q} \gamma$ **channel contributions involving diagrams similar to the above LO one.**
They get suppressed by $\alpha_s C_{3,4,5,6}$ **and phase-space for** $E_0 \sim m_b/3$.

NNLO: **We are still on the way to the quasi-complete case:**

G_{77} **is fully known:** 

$+ \ldots$

{ [Blokland et al., 2005] [Melnikov, Mitov, 2005] [Asatrian et al., 2006-2007]

G_{78} **is fully known:** 

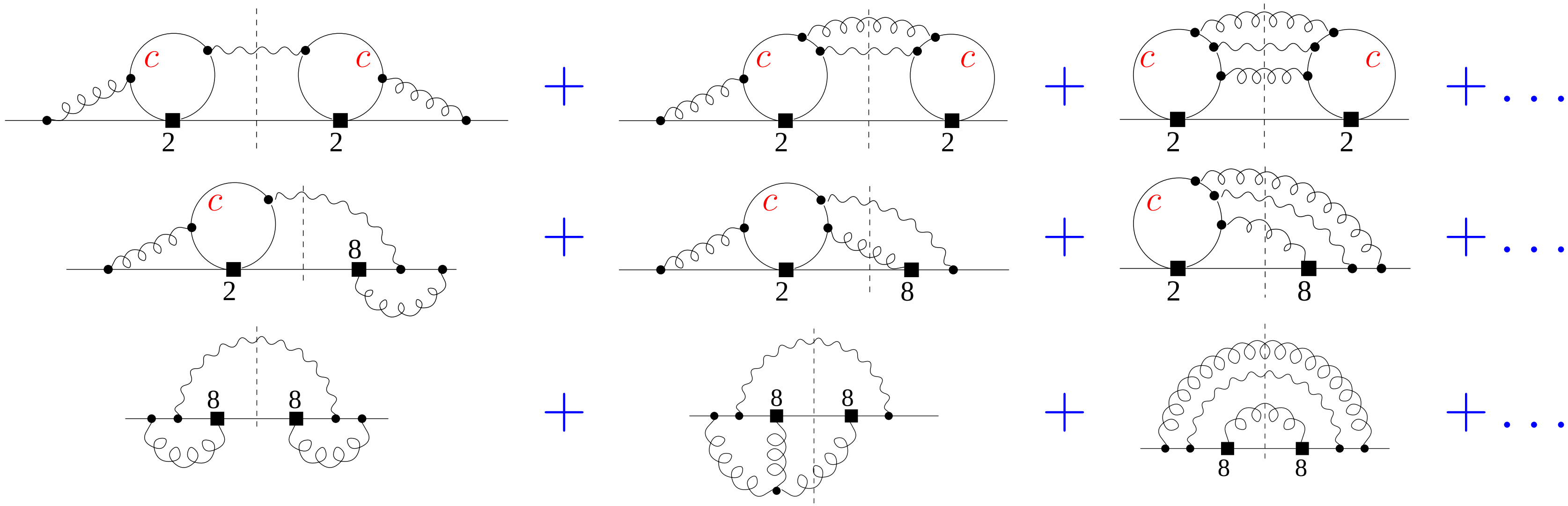
$+ \ldots$

[Asatrian et al., arXiv:1005.5587]

G_{22}**:**
(and analogous G_{11} & G_{12})

G_{28}**:**
(and analogous G_{18})

G_{88}**:**



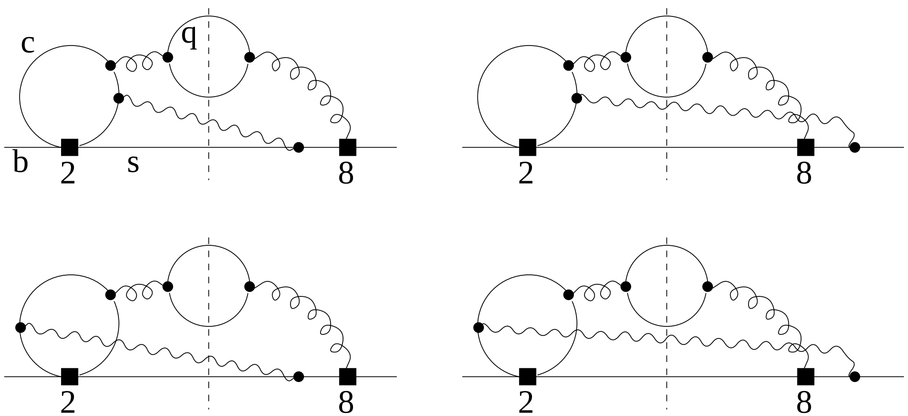
Two-particle cuts are known (just $|\text{NLO}|^2$).

Three- and four-particle cuts are known in the BLM approximation only: [Ligeti, Luke, Manohar, Wise, 1999], [Ferroglia, Haisch, arXiv:1009.2144], [Poradziński, MM, arXiv:1009.5685].

NLO+(NNLO BLM) corrections are not big $(+3.8\%)$**.**

Example:

Evaluation of the $(n > 2)$**-particle cut contributions to** G_{28} **in the Brodsky-Lepage-Mackienzie (BLM) approximation ("naive nonabelianization", large-**β_0 **approximation)** [Poradziński, MM, arXiv:1009.5685]:



q **– massless quark,**

N_q **– number of massless flavours (equals to 3 in practice because masses of** u, d, s **are neglected).**

Replacement in the final result:

$$-\tfrac{2}{3}N_q \quad \longrightarrow \quad \beta_0 = 11 - \tfrac{2}{3}(N_q + 2).$$

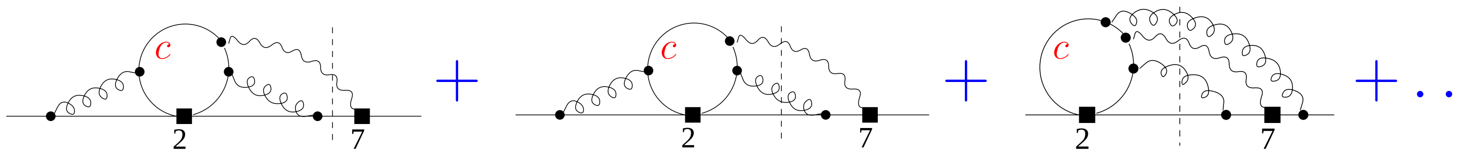
The diagrams have been evaluated using the method of Smith and Voloshin [hep-ph/9405204].

Non-BLM contributions to G_{ij} **from quark loops on the gluon lines are quasi-completely known.**

[Boughezal, Czakon, Schutzmeier, 2007], [Asatrian, Ewerth, Gabrielyan, Greub, 2007], [Ewerth, 2008].

The most troublesome NNLO contribution to G_{ij}:

G_{27}:
(and analogous G_{17})



$m_c = 0$: Czakon, Fiedler, Huber, Misiak, Schutzmeier Steinhauser, to be published]

163 massive 4-loop on-shell master integrals (with cuts).

The $m_c \gg m_b/2$ limit is known [Steinhauser, MM, 2006].

The BLM approximation is known for arbitrary m_c: [Bieri, Greub, Steinhauser, 2003], [Ligeti, Luke, Manohar, Wise, 1999].

Towards G_{27} at the NNLO for arbitrary m_c.

[M. Czakon, R.N. Lee, A. Rehman, M. Steinhauser, A.V. Smirnov, V.A. Smirnov, MM] **in progress.**

1. **Generation of diagrams and performing the Dirac algebra to express everything in terms of four-loop two-scale scalar integrals with unitarity cuts.**

2. **Reduction to master integrals with the help of Integration By Parts (IBP).**

   Available C++ codes:

FIRE	[A.V. Smirnov, arXiv:0807.3243] (public in the *Mathematica* version only),
REDUZE	[C. Studerus, arXiv:0912.2546],
DiaGen/IdSolver	[M. Czakon, unpublished (2004)].

   The IBP for 2-particle cuts has just been completed with the help of FIRE: ~ 0.5 TB RAM has been used ~ 1 month at CERN and KIT.

   Number of master integrals: around 500.

3. **Extending the set of master integrals I_n so that it closes under differentiation with respect to $z = m_c^2/m_b^2$. This way one obtains a system of differential equations**

$$\frac{d}{dz} I_n = \Sigma_k \, w_{nk}(z,\epsilon) \, I_k, \qquad\qquad (*)$$

where w_{nk} are rational functions of their arguments.

4. **Calculating boundary conditions for** $(*)$ **using automatized asymptotic expansions at $m_c \gg m_b$.**

5. **Calculating three-loop single-scale master integrals for the boundary conditions using dimensional recurrence relations** [R.N. Lee, arXiv:0911.0252].

6. **Solving the system** $(*)$ **numerically** [A.C. Hindmarsch, `http://www.netlib.org/odepack`] **along an ellipse in the complex z plane. Doing so along several different ellipses allows us to estimate the numerical error.**

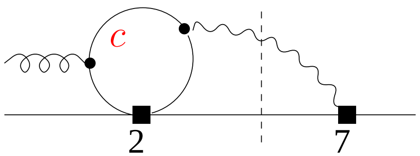
This algorithm has already been successfully applied for diagrams with (massless and massive) quark loops on the gluon lines where $18 + 47 + 38 = 103$ master integrals were present.

[R. Boughezal, M. Czakon, T. Schutzmeier, arXiv:0707.3090]

Non-perturbative contributions from the photonic dipole operator alone ("77" term) are well controlled for $E_0 = 1.6$ GeV:

$\mathcal{O}\left(\frac{\alpha_s^n \Lambda}{m_b}\right)_{n=0,1,2,\ldots}$ vanish, $\mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right)$ [Bigi, Blok, Shifman, Uraltsev, Vainshtein, 1992], [Falk, Luke, Savage, 1993], $\mathcal{O}\left(\frac{\Lambda^3}{m_b^3}\right)$ [Bauer, 1997], $\mathcal{O}\left(\frac{\alpha_s \Lambda^2}{m_b^2}\right)$ [Ewerth, Gambino, Nandi, 2009].

The dominant non-perturbative uncertainty originates from the "27" interference term:



$$\frac{\Delta\mathcal{B}}{\mathcal{B}} = -\frac{6C_2 - C_1}{54C_7}\left[\frac{\lambda_2}{m_c^2} + \sum_n b_n \mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\left(\frac{m_b\Lambda}{m_c^2}\right)^n\right)\right]$$

$\lambda_2 \simeq 0.12\,\mathrm{GeV}^2$

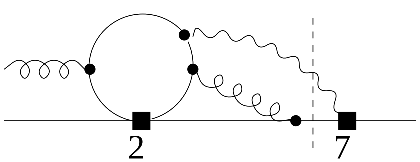
from B–B^* mass splitting

The coefficients b_n decrease fast with n.

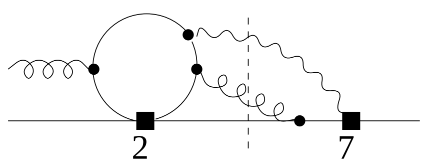
[Voloshin, 1996], [Khodjamirian, Rückl, Stoll, Wyler, 1997]
[Grant, Morgan, Nussinov, Peccei, 1997]
[Ligeti, Randall, Wise, 1997], [Buchalla, Isidori, Rey, 1997]

New claims by Benzke, Lee, Neubert and Paz in arXiv:1003.5012:

One cannot really expand in $m_b\Lambda/m_c^2$. All such corrections should be treated as Λ/m_b ones and estimated using models of subleading shape functions. Dominant contributions to the estimated $\pm 5\%$ non-perturbative uncertainty in $\mathcal{B}$ are found this way, with the help of alternating-sign shape functions that undergo weaker suppression at large gluon momenta.



correction to the above



phase-space suppressed

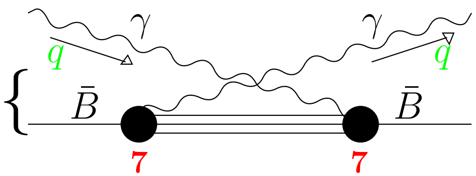
$\mathcal{O}\left(\frac{\alpha_s \Lambda}{m_b}\right)$ **Main worry in hep-ph/0609232, and reason for the $\pm 5\%$ non-perturbative uncertainty.**

The “hard” contribution to $\bar{B} \to X_s\gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b)\langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + \ldots \right|^2$

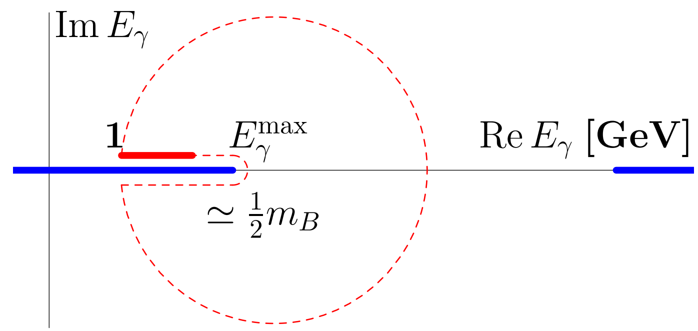
The “77” term in this sum is purely “hard”. It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

Im{  } $\equiv$ Im A

When the photons are soft enough, $m_{X_s}^2 = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2 \Rightarrow$ **Short-distance dominance** $\Rightarrow$ **OPE.**
However, the $\bar{B} \to X_s\gamma$ **photon spectrum is dominated by hard photons** $E_\gamma \sim m_b/2$.

Once $A(E_\gamma)$ **is considered as a function of arbitrary complex** E_γ, ImA **turns out to be proportional to the discontinuity of** A **at the physical cut. Consequently,**

$$\int_{1\,\mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma \; \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma \; A(E_\gamma).$$



Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ **is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives**

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1-2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0)|Q^{(j)}_{\mathrm{local\ operator}}|\bar{B}(\vec{p}=0)\rangle.$$

Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b.

At $(\Lambda/m_b)^0$: $\langle \bar{B}(\vec{p})|\bar{b}\gamma^\mu b|\bar{B}(\vec{p})\rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s\gamma) = \Gamma(b \to X_s^{\mathrm{parton}}\gamma) + \mathcal{O}(\Lambda/m_b)$.

At $(\Lambda/m_b)^1$: **Nothing! All the possible operators vanish by the equations of motion.**

At $(\Lambda/m_b)^2$: $\langle \bar{B}(\vec{p})|\bar{h}D^\mu D_\mu h|\bar{B}(\vec{p})\rangle = -2m_B\lambda_1$, $\quad \lambda_1 = (-0.27 \pm 0.04)\mathrm{GeV}^2$ **from** $\bar{B} \to X\ell^-\nu$ **spectrum.**
$\langle \bar{B}(\vec{p})|\bar{h}\sigma^{\mu\nu}G_{\mu\nu}h|\bar{B}(\vec{p})\rangle = 6m_B\lambda_2$, $\quad \lambda_2 \simeq \frac{1}{4}\left(m_{B^*}^2 - m_B^2\right) \simeq 0.12\,\mathrm{GeV}^2$.

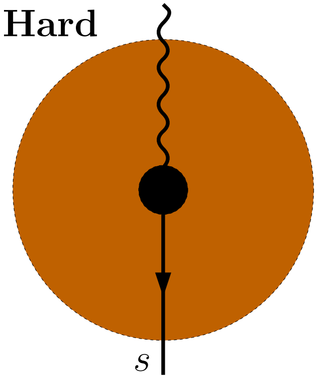
The HQET heavy-quark field $h(x)$ **is defined by** $h(x) = \frac{1}{2}(1+\not{v})b(x)\exp(im_b\, v\cdot x)$ **with** $v = p/m_B$.

Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6\,\mathrm{GeV})$ [see MM, arXiv:0911.1651]

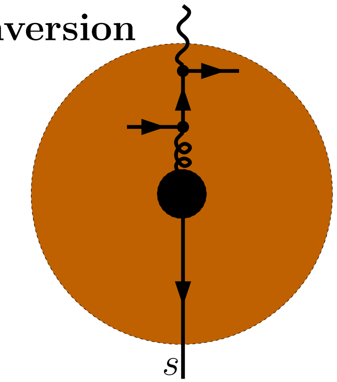
A. Without long-distance charm loops:

1. **Hard**



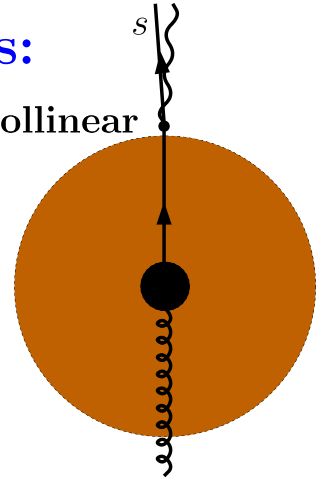
Dominant, well-controlled.

2. **Conversion**



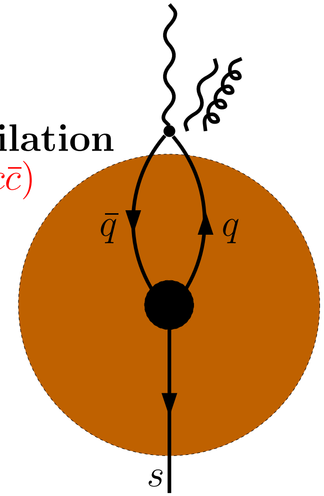
$\mathcal{O}(\alpha_s \Lambda / m_b)$, $(-1.6 \pm 1.2)\%$.
[Benzke, Lee, Neubert, Paz, 2010]

3. **Collinear**



$\sim -0.2\%$ or $(+0.8 \pm 1.1)\%$.
[Kapustin,Ligeti,Politzer, 1995]
[Benzke, Lee, Neubert, Paz, 2010]

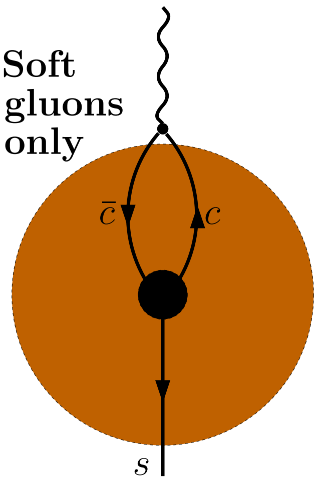
4. **Annihilation** $(q\bar{q} \neq c\bar{c})$



Exp. π^0, η, η', ω subtracted.
Perturbatively $\sim 0.1\%$.

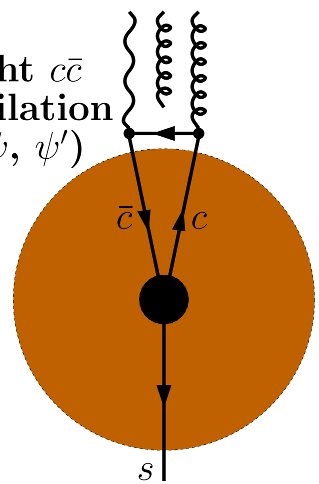
B. With long-distance charm loops:

5. **Soft gluons only**



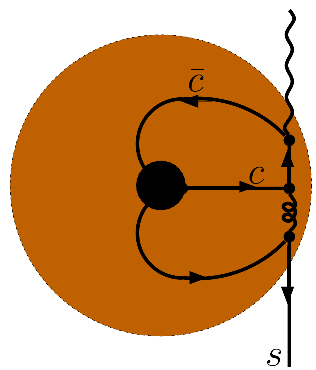
$\mathcal{O}(\Lambda^2/m_c^2)$, $\sim +3.1\%$.
[Voloshin, 1996], [...],
[Buchalla, Isidori, Rey, 1997]
[Benzke, Lee, Neubert, Paz, 2010]: add $(+1.1 \pm 2.9)\%$

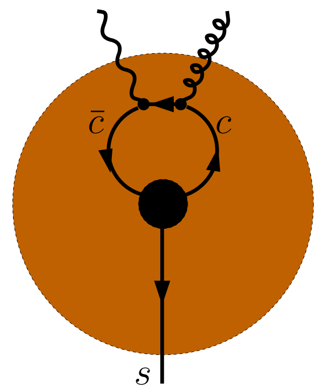
6. **Boosted light $c\bar{c}$ state annihilation (e.g. η_c, J/ψ, ψ')**



Exp. J/ψ subtracted ($< 1\%$).
Perturbatively (including hard): $\sim +3.6\%$.

7. **Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state**





$\mathcal{O}(\alpha_s (\Lambda/M)^2)$ $\mathcal{O}(\alpha_s \Lambda/M)$

$M \sim 2m_c, 2E_\gamma, m_b$.

e.g. $\mathcal{B}[B^- \to D_{sJ}(2457)^- \, D^*(2007)^0] \simeq 1.2\%$,
$\mathcal{B}[B^0 \to D^*(2010)^+ \, \bar{D}^*(2007)^0 K^-] \simeq 1.2\%$.