

Isospin $B \rightarrow K^{(*)} l+l$ in and beyond the SM

- a) Generic remarks $B \rightarrow K^{(*)} l+l$ and isospin
- b) What drives isospin violation
- c) Isospin asymmetries in and beyond SM
- d) Remarks on low recoil region (high q^2)

CP³ - Origins

Particle Physics & Origin of Mass

The Higgs Centre
for Theoretical Physics

Roman Zwicky

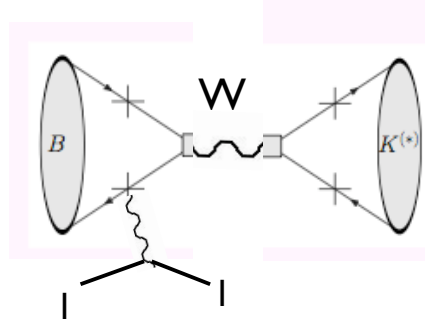
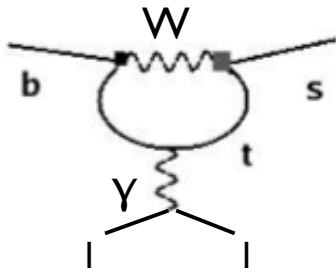
Higgs-centre for theoretical physics -- Edinburgh University
in collaboration James Lyon & (Maria Dimou)

Challenges in Flavour physics --- 15-18 April 2013 Portoroz

Note: corresponds to arXiv.1304.479 (appeared after talk)
later includes $B \rightarrow \rho\gamma/\ell\ell$ isospin analysis as well
as some further aspects on phenomenology aspects.

Why $B \rightarrow K^{(*)} l+l$? And what it is.

- 1) It's an FCNC ($b \rightarrow s$ -transition); thus loop suppressed in SM
- 2) It's measured at experimental facilities (currently LHCb future KEK2 past: Belle/BaBar/CDF)



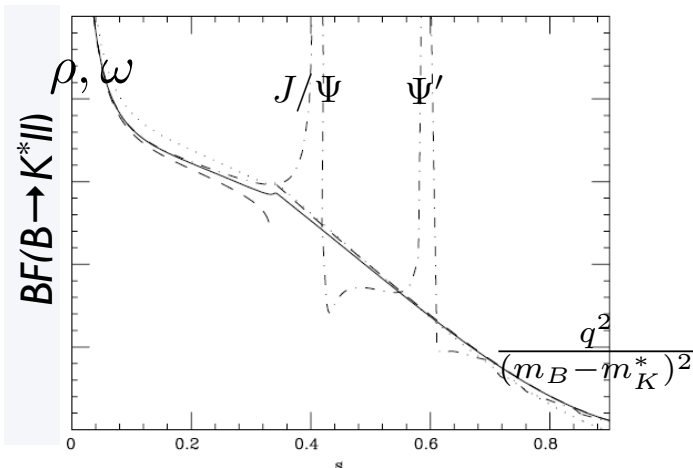
Wilson coefficient (UV-physics SM & **BSM?**) operator (IR-physics)

$$H_{\text{eff}} = \sum_i C^i(\mu_F) O^i(\mu_F)$$

$$\mathcal{A} = \langle Vll | H_{\text{eff}} | B \rangle = \sum_i C_i(m_b) \langle Vll | O_i(m_b) | B \rangle$$

Amplitude

non-perturbative



low q^2 (large recoil) $E_{K^*} \gg \Lambda_{\text{QCD}}$

- \Rightarrow light-cone dynamics
- QCD-factorization/SCET & LCSR -- form factor LCSR

high q^2 (low recoil) $E_{K^*} \approx \Lambda_{\text{QCD}}$

- OPE (*Grinstein, Pirjol'04, Beylich et al'11*) $1/m_b \sqrt{q^2}$
- form factors Lattice

this talk
comments end

Definition of isospin asymmetries

Lyon & RZ'13

- Experimental definition (Recall: \mathbf{q}^2 lepton pair momentum squared)

$$\frac{dA_I^{\bar{0}-}}{dq^2} \equiv \frac{d\Gamma[\bar{B}^0 \rightarrow \bar{K}^{*0} l^+ l^-]/dq^2 - d\Gamma[B^- \rightarrow K^{*-} l^+ l^-]/dq^2}{d\Gamma[\bar{B}^0 \rightarrow \bar{K}^{*0} l^+ l^-]/dq^2 + d\Gamma[B^- \rightarrow K^{*-} l^+ l^-]/dq^2}$$

$K^* \gamma, Kll$ analogous

- In terms of K^* -helicity $(0, \pm)$ & $(\bar{l}l)_{V,A}$

$$\frac{dA_I^{\bar{0}-}}{dq^2} [B \rightarrow K^* l^+ l^-] = \frac{\sum_{i=\{0, \pm\}} \text{Re}[h_i^{V,0}(q^2) \Delta_i^{V,d-u}(q^2)]}{\sum_{i=\{0, \pm\}} [|h_i^{V,0}(q^2)|^2 + |h_i^A(q^2)|^2]} + \mathcal{O}([\Delta_i^{V,d-u}(q^2)]^2, m_l)$$

$$\Delta_i^{V,d-u}(q^2) \equiv (h_i^{V,d}(q^2) - h_i^{V,u}(q^2))$$

- above isospin linear effect -- interference with isospin neutral part
 - a) compute SM asymmetry
 - b) extend the basis to include most generic isospin sensitive dimension 6 operators (N.B. do not extend SM isospin neutral part; as “know” to be small by rate)

How do we extend the basis?

What “drives” sizeable isospin asymmetries?

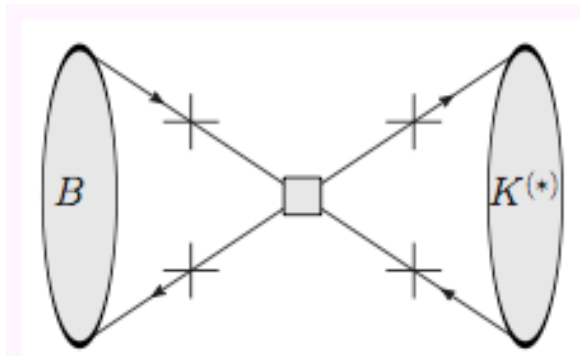
- **Not QCD** as effects known to be small: $m(K^{*-})/m(K^{*0}) = 0.995$

- Recall:
$$\mathcal{A} = \langle Vll | H_{\text{eff}} | B \rangle = \sum_i C_i(m_b) \langle Vll | O_i(m_b) | B \rangle$$

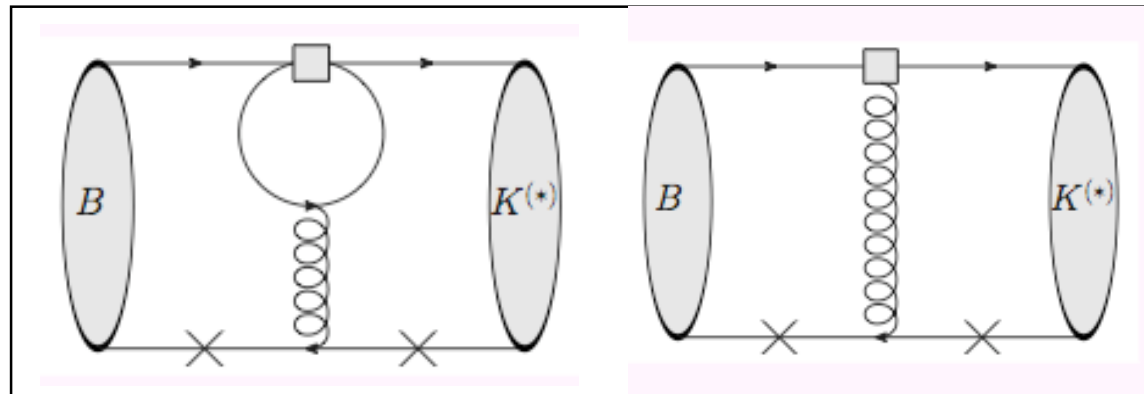
Weak Annihilation
(WA)

Quark-loop spectator
scattering (QLSS)

Chromomagnetic-operator
(O_8)



spectator is not a spectator
(sensitive to UV isospin violation)



resemble each other
(e.g. - if loop massive shrinks to a point & mix under RG)

Answer title: IR **QED**-effects & **BSM UV isospin violation** manifested in WA

- very specific operators \Rightarrow answers the question: “**why isospin asymmetries?**”

A rough overview of what we did.

- **WA:** 1) extend (*Khodj.&Wyler, Ali&Braun'95*) to $q^2 \neq 0$ within Light-cone sum rules
2) introduce most general dimension 6 H^{eff} at $O(\alpha_s^0)$

$$\mathcal{H}^{\text{WA},q} = -\frac{G_F}{\sqrt{2}} \lambda_t \sum_{i=1}^{10} a_i^q O_i^{\text{WA}}$$

$$O_9^{\text{WA}} \equiv \bar{q} \sigma_{\mu\nu} b \bar{s} \sigma^{\mu\nu} q \quad O_{10}^{\text{WA}} \equiv \bar{q} \sigma_{\mu\nu} \gamma_5 b \bar{s} \sigma^{\mu\nu} q$$

$$\begin{aligned} O_1^{\text{WA}} &\equiv \bar{q} b \bar{s} q & O_2^{\text{WA}} &\equiv \bar{q} \gamma_5 b \bar{s} q & O_3^{\text{WA}} &\equiv \bar{q} b \bar{s} \gamma_5 q & O_4^{\text{WA}} &\equiv \bar{q} \gamma_5 b \bar{s} \gamma_5 q \\ O_5^{\text{WA}} &\equiv \bar{q} \gamma_\mu b \bar{s} \gamma^\mu q & O_6^{\text{WA}} &\equiv \bar{q} \gamma_\mu \gamma_5 b \bar{s} \gamma^\mu q & O_7^{\text{WA}} &\equiv \bar{q} \gamma_\mu b \bar{s} \gamma^\mu \gamma_5 q & O_8^{\text{WA}} &\equiv \bar{q} \gamma_\mu \gamma_5 b \bar{s} \gamma^\mu \gamma_5 q \end{aligned}$$

10(20) operators

- **QLSS:** extend (*Feldmann & Matias '02*) to include most general dimension 6 H^{eff} at $O(\alpha_s^0)$
We resort to QCDF as LCSR involves 2-loops and complicated analytic structure ..

$$\mathcal{H}^{\text{QLSS}} = -\frac{G_F}{\sqrt{2}} \lambda_t \sum_{x,\chi,f} s_{x\chi}^f Q_{x\chi}^{4f}, \quad x = 1, 2, \quad \chi = L, R, \quad f = SU(3)_F, c, b$$

10 operators ($m_q=0$)

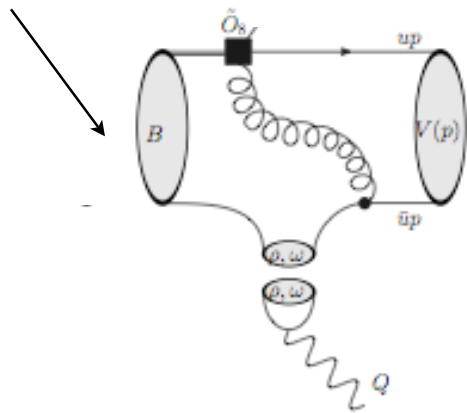
$$Q_{1L(R)}^{4f} \equiv \bar{f} t^a \gamma_\mu f \bar{s}_{L(R)} t^a \gamma^\mu b, \quad Q_{2L(R)}^{4f} \equiv \bar{f} t^a \sigma_{\mu\nu} f \bar{s}_{L(R)} t^a \sigma_{\mu\nu} b, \quad Q_{xL(R)}^{4SU(3)_F} \equiv \sum_{q=u,d,s} Q_{xL(R)}^{4q}$$

N.B. $10(20)_{\text{WA}}$ & $10_{\text{QLSS}(m_q=0)}$ are not orthogonal but linearly independent

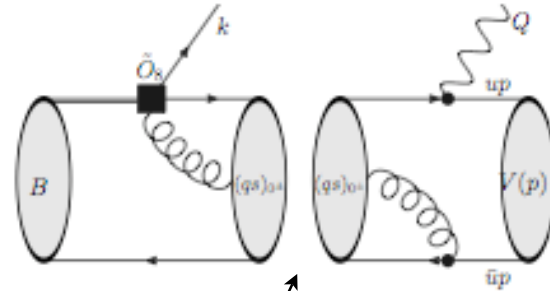
- **O₈:** earlier work (*Dimou, Lyon & RZ '12*) BSM: flipped chirality \Rightarrow trivial

Hadronic contributions & strong phases

- e.g. ρ, ω -thresholds when photon emitted from light-quark -- seen O_8 , WA not in (QLSS as LO QCDF)



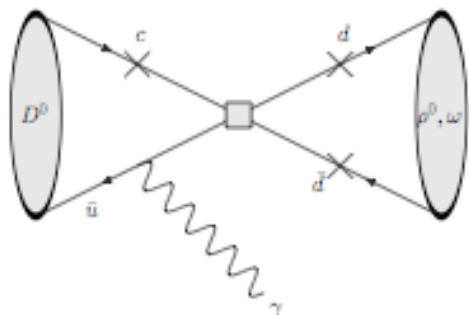
strong phases



- Multihadron state $(\bar{s}q)_{0\pm}$ q-number and momentum squared m_B^2

Quick comment (un)related topic: $A_{CP}[D^0 \rightarrow \rho^0 \gamma]$

- Strong phase of O_8 and WA (dominant) interfere to CP-violation in $D^0 \rightarrow \rho^0 \gamma$



$\times \text{Im}[C_8]O_8$

Lyon & RZ 12

$LD e^{i\delta(\text{strong})} \times \text{Im}[C_8]O_7$

RG-mixing

First: Isidori & Kamenik 12

main difference: IK: LD not specified depends sizeable strong phase

LZ: LD=WA no strong phase at leading order – strong phase through O_8

Fun with dispersion relations

- Main tool for sum rules (besides LC-OPE) is the construction of a dispersion relation:

$$g_i(p_B^2, \dots) = \int_{\Gamma} \frac{ds g_i(s, \dots)}{s - p_B^2 - i0}$$

Cauchy's thm

- 1) Γ : path encircles singularities $C_{p_B^2}$
- 2) Γ : chosen s.t. relates hadronic states

"usually"

Physical Region

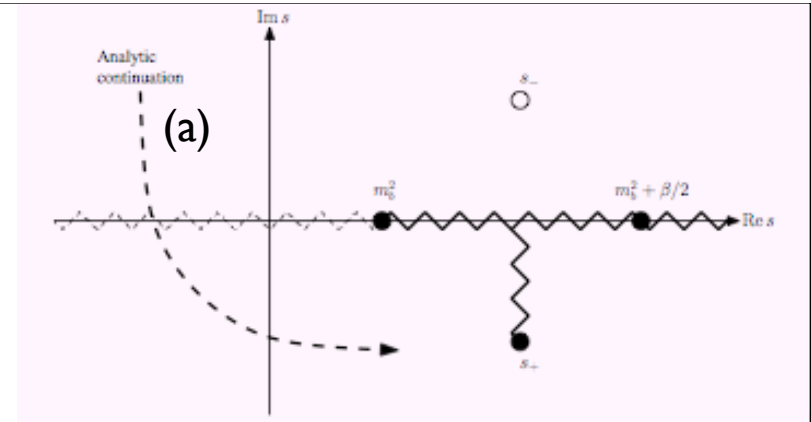
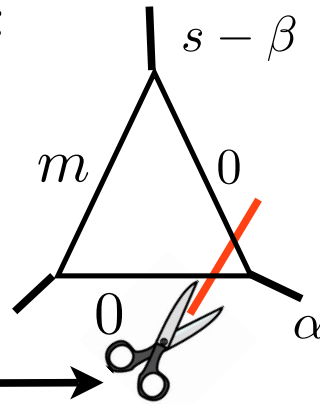
m_b^2

- Investigate singularities Landau equations: leading Landau singularities s_{\pm} of a three point function appearing in the computation has got two complex solutions.
- Are they on the physical Riemann sheet (PRS)?
For real singularities its relatively straightforward to answer not for complex ones!
- Found 4 ways to show/convince ourselves that one is present on PRS
 - 1) Kallen-Wightman paper '59 analytic properties three pts fcts (axiomatic approach)
 - 2) 6-dimensional projective geometry (did not do finally)
 - 3) deformation from non-complex case (tricky in case at hand)
 - 4) "invented method" using Feynman parameter integral (next slide)

Passarino-Veltman reduction C_0 :

➤ study analytic continuation

ρ/ω -thresholds &
 $(\bar{s}q)_{0\pm}$ -mutiparticle threshold s



- ❶ Real line: $C_0(s) = C_0^F(s) \equiv \int_0^1 dx \int_0^{1-x} dy ((1-x-y)(xs+y(s-\beta)-m^2) \dots + xy\alpha + i0)^{-1}$
- ❷ Real α, β, m C_0^F no singularities upper half plane ➤ valid analytic continuation & $\mathbf{s_-} \notin \text{PRS}$
- ❸ Lower half plane -- analytic continuation via (a) -- unphysical branch cut $s < m^2$

❹ Principle: impose continuity across real line $s < m^2$ ➤ eliminate branch cut

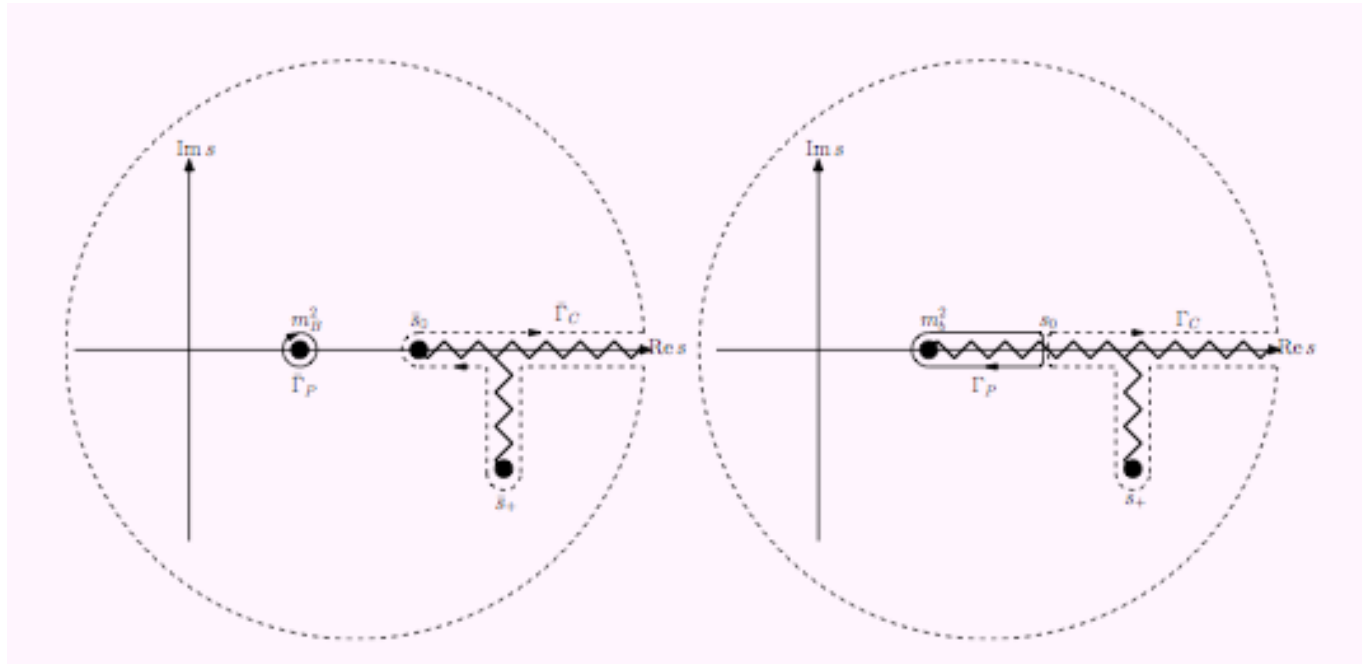
$$\text{Im}[s] \neq 0 : [C_0^F(s^*)]^* = C_0^F(s) \text{ Reflection principle}$$

$$C_0(s) = \begin{cases} C_0^F(s) & \text{Im}[s] > 0 \\ C_0^F(s^*)^* + C_0^{\text{rem}}(s) & \text{Im}[s] < 0 \end{cases} \quad C_0^{\text{rem}}(s) = \text{Im}[C_0^F(s)] \quad \text{Im}[s] = 0$$

is continuous and thus the unique analytic continuation! N.B. $[C_0(s^*)]^* \neq C_0[s]$

- ❺ Inspect C_0^{rem} note $\mathbf{s_+} \in \text{PRS}!!$ Know how to choose path appropriately

very briefly the analytic structure in full QCD and in partonic QCD



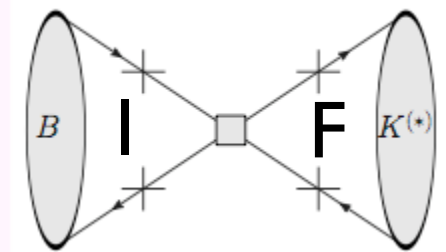
- leading Landau singularities (anomalous thresholds) are not related to insertion of a hadronic state(s).
- Essence for sum rules is that the branch cut from s_+ is above continuum states and therefore will be exponentially Borel suppressed

end of technical excursion

Selection rules

- **General:** a) $B[0^-] \rightarrow K[0^-](\gamma^*[1^-] \rightarrow U[1^-]) \Rightarrow$ p-wave; i.e. $l = 1$,
induced by parity conserving interaction ... $(1 - \gamma_5)s$
- b) other way around for K^* (0-helicity = longitudinal polarisation)
- **WA:** more stringent selection rules ground of Lorentz-invariance etc
(at least at the level of the factorisable contribution)

		Twist	Operator O_n^{WA}									
			1	2	3	4	5	6	7	8	9	10
$B \rightarrow K$	cov. (α_s^0)		\times	\times	\times		\times	\times	\times		\times	\times
	χ -even (ϕ_K)	2								I,F		
	χ -odd $(\phi_{P,\sigma})$	3				I,F						
	cov. $(\alpha_s^n, n > 0)$		\checkmark	\times	\times	\checkmark	\checkmark	\times	\times	\checkmark	\checkmark	\times
$B \rightarrow K^*$	cov. (α_s^0)		\times		\times					\times	\times	
	χ -even $(g_\perp^{(v)}, g_\perp^{(a)})$	3					I,F	I,F				
	χ -odd (ϕ_\perp)	2		F		F					I	I
	χ -odd $(h_\parallel^{(l)}, h_\parallel^{(s)})$	3		F								I
	cov. $(\alpha_s^n, n > 0)$		\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark



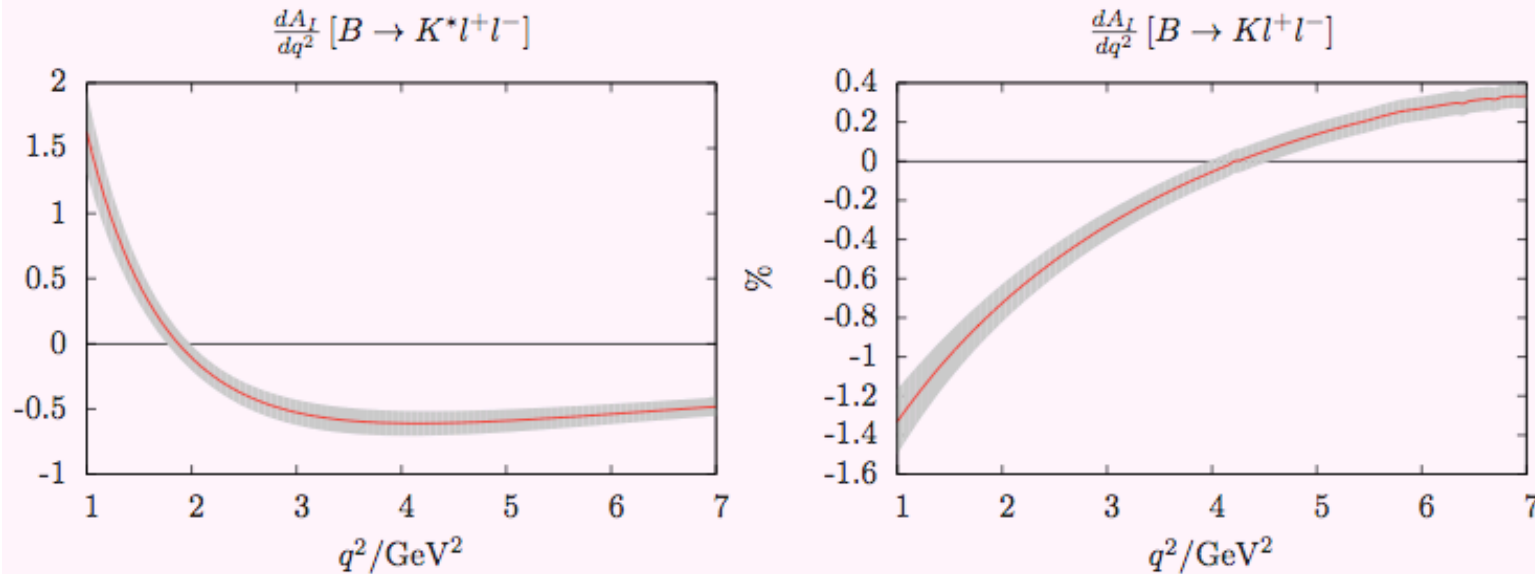
I(initial) & F(inal) state radiation

“the K-meson contribution corresponds to the longitudinal part of K^* -meson”

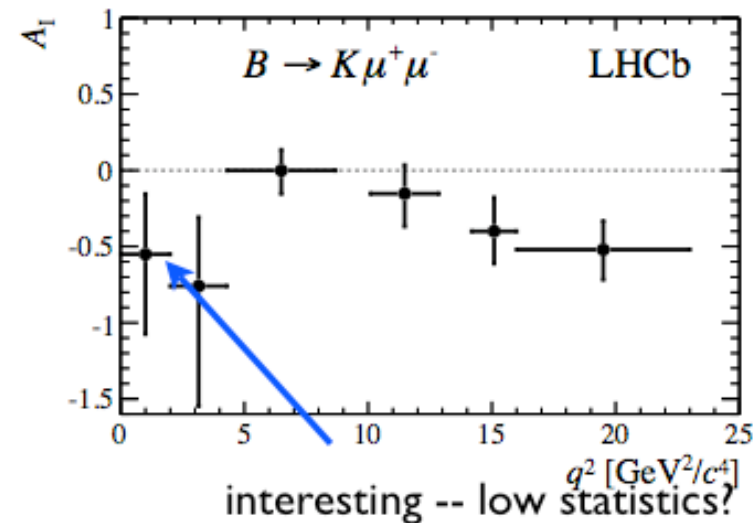
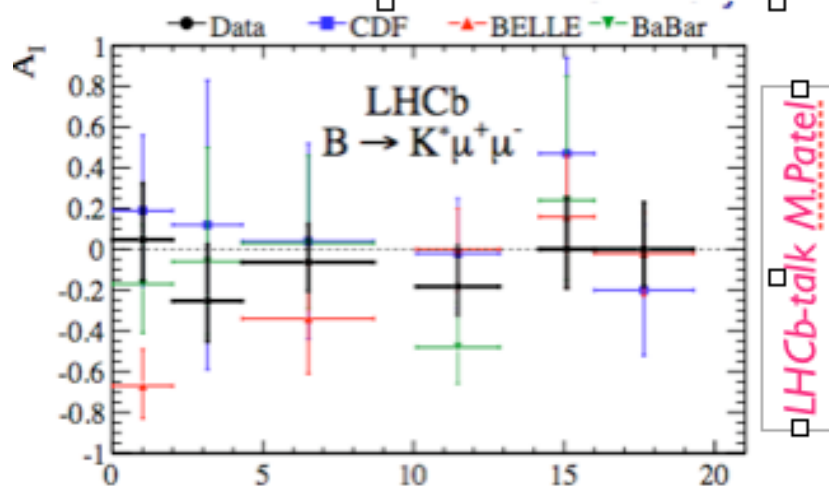
true leading twist in the SM where V-A imposes $a_6 = -a_8$; V+A not true

Isospin asymmetries in the SM

- Are small for $B \rightarrow K^{(*)} l l$ -- accidental sizeable tree-level WC double Cabibbo suppressed



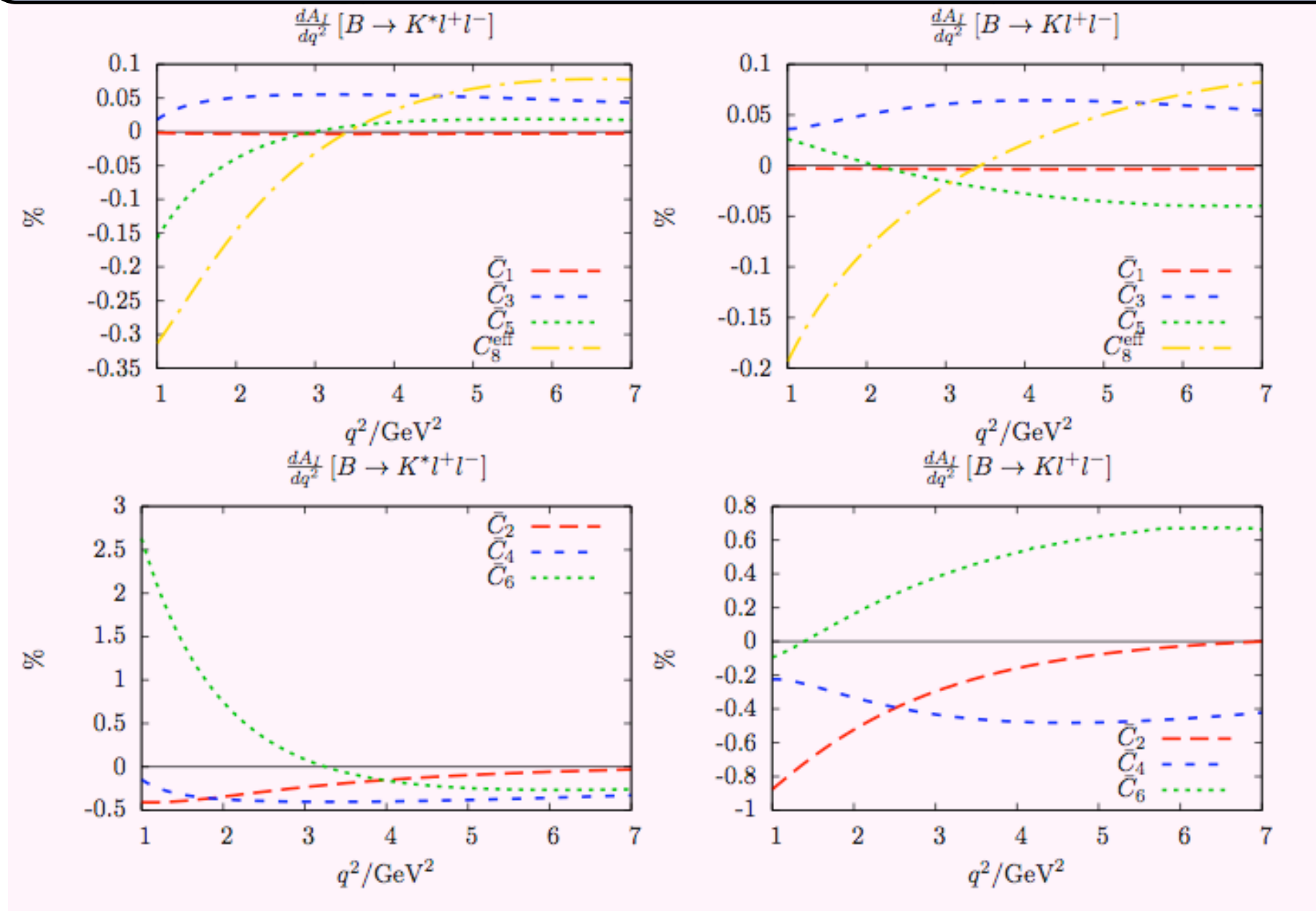
lower and upper boundaries: ρ, ω & $J/\psi, \psi(2s)$ resonances



- Larger for $B \rightarrow K^{(*)} \gamma$

$$a_I^{\bar{0}-}(K^* \gamma)_{LZ} = 7\%, \quad a_I^{\bar{0}-}(K^* \gamma)_{\text{HFAG}} = 5.2(2.6)\%$$

The breakdown in the SM...

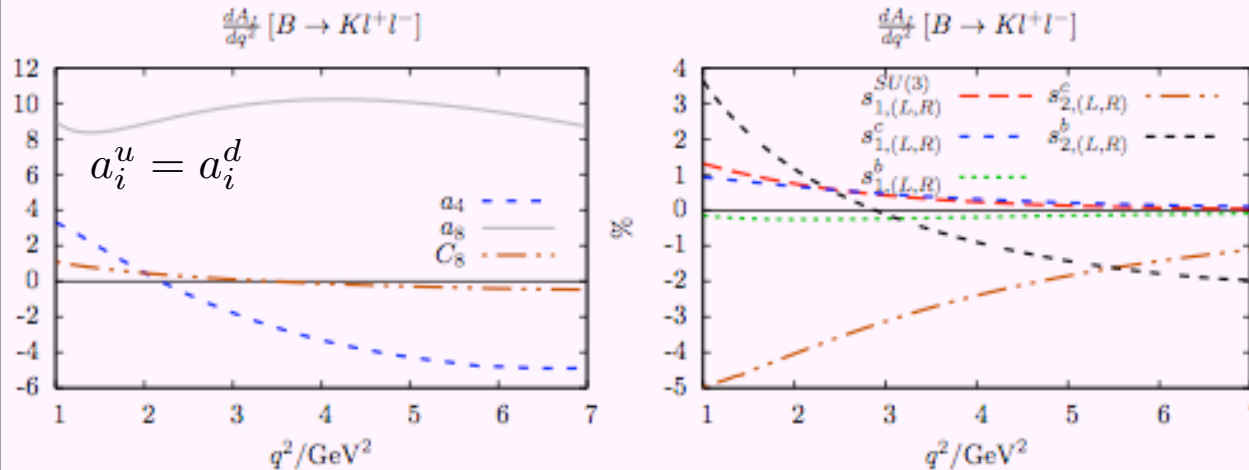


Isospin asymmetries BSM

- After selection rules:
still many operators!

	$C_8^{(l)}$	WA	QLSS	total
K^*	2[1]	12[3]	10[3]	24[7]
K	1[1]	4[3]	5[3]	10[7]

$a_{2,4,5,6,9,10}^q$ all no $i=2, f=\text{SU}(3)$
 $a_{4,8}^q$ idem no $\chi = A$



- K^* even more “.. by the laws of probability cancellation ought to be the rule rather than the exception.”
- Or a new principle for flavour physics)
- q^2 -spectrum ought to help
can't be unlucky all the way

N.B. all BSMWCs set to unity (paper $a_i^q=0.1$)

- Are there **constraints?**
 - a) non-leptonic decays (large hadronic uncertainties) to be seen: some constraint not all as selection rules differ isospin & non-leptonic decays might marry to constrain $bsqq^*$
 - b) isospin asymmetry in $B \rightarrow K^* \gamma$ of course

Take $a_i \neq 0$ others zero then to be within 2σ of $a_I^{0-}(K^* \gamma)_{\text{HFAG}} = 5.2(2.6)\%$.

$$0 > a_2 > -3 \cdot 10^{-1}, \quad 0 > a_4 > -3 \cdot 10^{-1}, \quad 0 < a_5 < 5 \cdot 10^{-1}$$

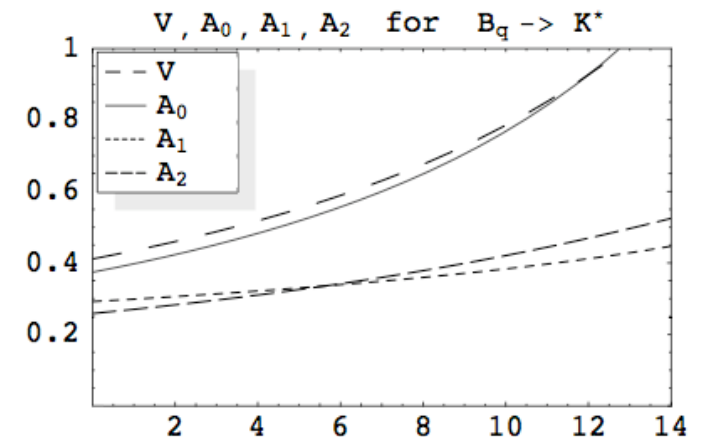
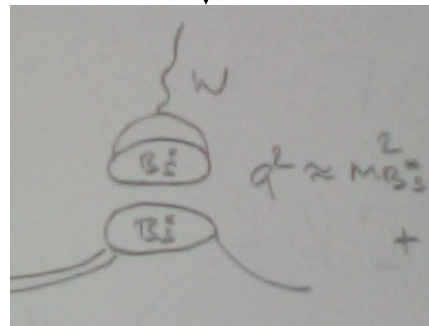
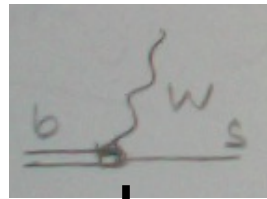
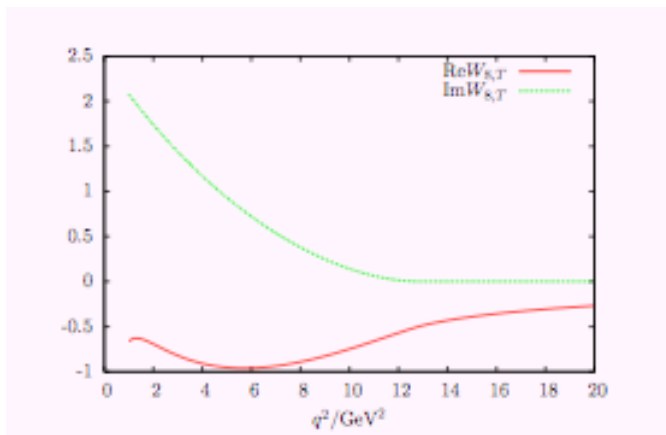
$$0 > a_6 > -7 \cdot 10^{-1}, \quad 0 < a_9 < 6 \cdot 10^{-2}, \quad 0 < a_{10} < 6 \cdot 10^{-2}.$$

indicative constraints

Comments on high q^2

1) Generic remarks

- Isospin violation: - through photon \triangleright enhanced through photon pole low q^2
 \triangleright isospin asymmetry has to decrease (module conspiracy)
 at high q^2 as rate dominated by Z-penguins and boxes (e.g. $C_9^{\text{eff.}}$)
- On top of that isospin transitions (IT) compete with penguin form factors who show increase, at high q^2 , due to nearby t-channel $B_s^*[I^+]$ poles and alike whereas IT have no such enhancements (at least at leading order)



- ## 2) OPE language: form factors dim 3 operators isospin violation dim 5,6

• *Beylich, Buchalla, Feldmann'11*

... Epilogue

- Isospin asymmetry driven by QED-effects & Weak Annihilation
- SM isospin accidentally small as large WC is doubly Cabibbo suppressed
- Difficult to see how isospin asymmetry can be large at high q^2
Isospin symmetric WC as well as matrix element (form factors) raise
- BSM many operators contribute
 - Is by the laws of probability cancellation the rule rather than the exception
 - This is where q^2 -spectrum ought to help - can't be unlucky everywhere in q^2
- Theoretical improvement
 - SM: compute WA at $O(\alpha_s)$ -- presumably difficult -- important charm $D \rightarrow V\gamma$
 - BSM: compute QLSS with LCSR

thanks for your attention!