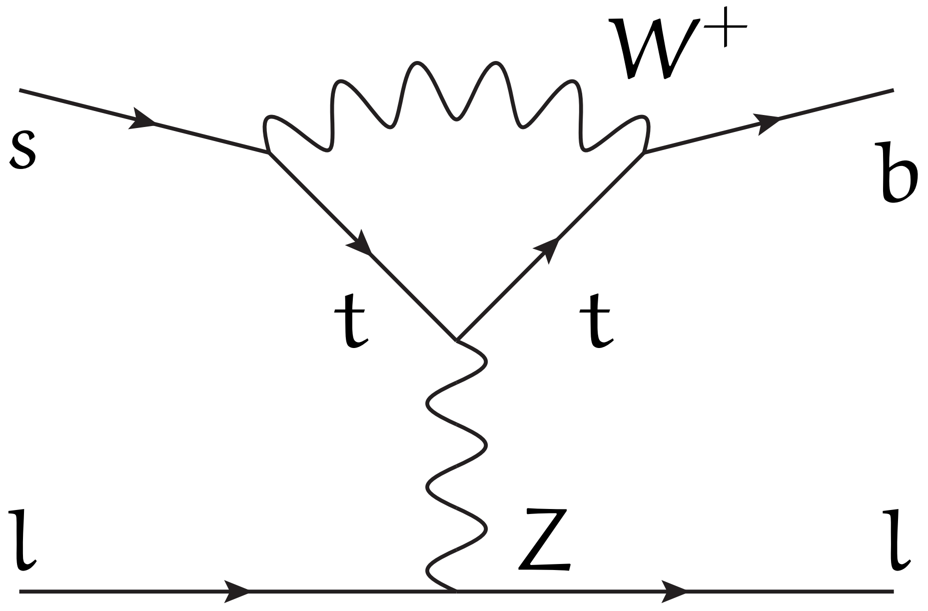
$B_{s(d)} \to \mu^+ \mu^-$ and Electroweak Interactions

Probing the Standard Model and New Physics at
Low and High Energies
Portoroz
15 April 2013

Based on work with
Christoph Bobeth, Emanuel Stamou &
Joachim Brod, Sandro Casagrande

Martin Gorbahn
University of Liverpool

$B_s \to \mu^+ \mu^-$



B_s is pseudoscalar – no photon penguin

$$Q_A = (\bar{b}_L \gamma_\mu q_L)(\bar{l} \gamma_\mu \gamma_5 l)$$

Dominant operator (SM) Wilson

helicity suppression $\left(\propto \frac{m_l^2}{M_B^2}\right)$

Effective Lagrangian in the SM (NP + chirality flipped):

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} (C_S Q_S + C_P Q_P + C_A Q_A) + \text{h.c.}$$

Scalar operators: $Q_S = m_b (\bar{b}_R q_L)(\bar{l} l)$ $\quad Q_P = m_b (\bar{b}_R q_L)(\bar{l} \gamma_5 l)$

Alternative normalisation [Misiak `11]:

$$\mathcal{L}_{\text{eff}} = G_F^2 M_W^2 \, V_{tb}^* V_{ts} \, (C_A Q_A + C_S Q_S + C_P Q_P) + \text{h.c.}$$

Theory Status

Standard Model: Scalar Operators are highly suppressed

C_A is known at NLO in QCD [Buras, Buchalla; Misiak, Urban `99]

$C_A(m_t / M_W)^{NLO} = 1.0113\, C_A(m_t / M_W)^{LO}$
– for QCD MS-bar $m_t = m_t(m_t)$

The matrix-element <Q_A> is given through the precisely known decay constant f_{Bs} (=225(3)MeV [Dowdall `13])

Soft photon corrections [Buras, Guadagnoli, Isidori `12] and time integration [de Bruyn, Fleischer et. al. `12] depend on experiment
→ do not put in theory prediction – extract from experiment

Electroweak Corrections

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} C_A Q_A + \text{h.c.}$$

$G_F\,\alpha/\sin^2\theta_W$ does not renormalise under QCD: can be factored out for QCD calculation

Only $G_F\,\alpha/\sin^2\theta_W\,C_A(m_t/M_W)$ invariant under electroweak scheme change

This combination should always give the same result if we use the same input (G_F, α, M_Z, M_t, M_H) up to higher order corrections

Renormalisation of G_F

We identify G_F with the measured muon lifetime and its theory prediction $G_\mu = G_\mu^{(0)} + G_\mu^{(1)} + \ldots$

G_F can be combined or factored out of the Wilson coefficient

$$\mathcal{H}_{\text{eff}} = \tilde{C}\, Q = G_F C\, Q$$

Since G_F is now an observable and C dimensionless the vacuum expectation dependence cancels in C:

$$C^{(0)} = \frac{\tilde{C}^{(0)}}{G_\mu^{(0)}}, \qquad C^{(EW)} = \frac{\tilde{C}^{(EW)}}{G_\mu^{(0)}} - \frac{\tilde{C}^{(0)} G_\mu^{(EW)}}{\left(G_\mu^{(0)}\right)^2}$$

but other parameters are not so easily fixed from experiment

Electroweak Scheme Uncertainties

$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} C_A\left(\frac{m_t}{M_W}\right) Q_A + \text{h.c.}$$

[Buras, et. al. `12]

	MS-bar	OS	unct. $B_s\ \mu^+\mu^-$
$\sin \theta_W$	0,231	0,223	4 %
m_t(QCD-MS-bar)	163,2 GeV	164,5 GeV	1 %

Tadpole dependence

Electroweak scheme shift larger then present pure theory error

[Buras, Guadagnoli, Isidori `12]: Follow [Brod, MG, Stamou] and use MS-bar θ_W plus renormalise the masses on-shell (hybrid)

Box contributions contribute differently to $l^+ l^-$ modes than to $\nu\nu$.
Are both box and penguin corrections tiny in the hybrid scheme?

Renormalisation Schemes

Calculate in the MS-bar scheme
using tadpole counterterms to produce gauge independence for intermediate results
fit g_1, g_2, v, λ, m_t from data (G_F, α, M_Z, M_t, M_H)

Use OS scheme: Determine M_W including 1-loop corrections from input – then $\sin^2\theta_W = 1 - M_W^2 / M_Z^2$

Add finite $\sin\theta_W$, m_t and M_W counterterms to $C_A^{(EW)}$

NLO predictions should agree up to residual scheme uncertainties if we use the same input data.

Matching Correction for C_A

There are sizeable shifts going from 1-loop to 2-loop

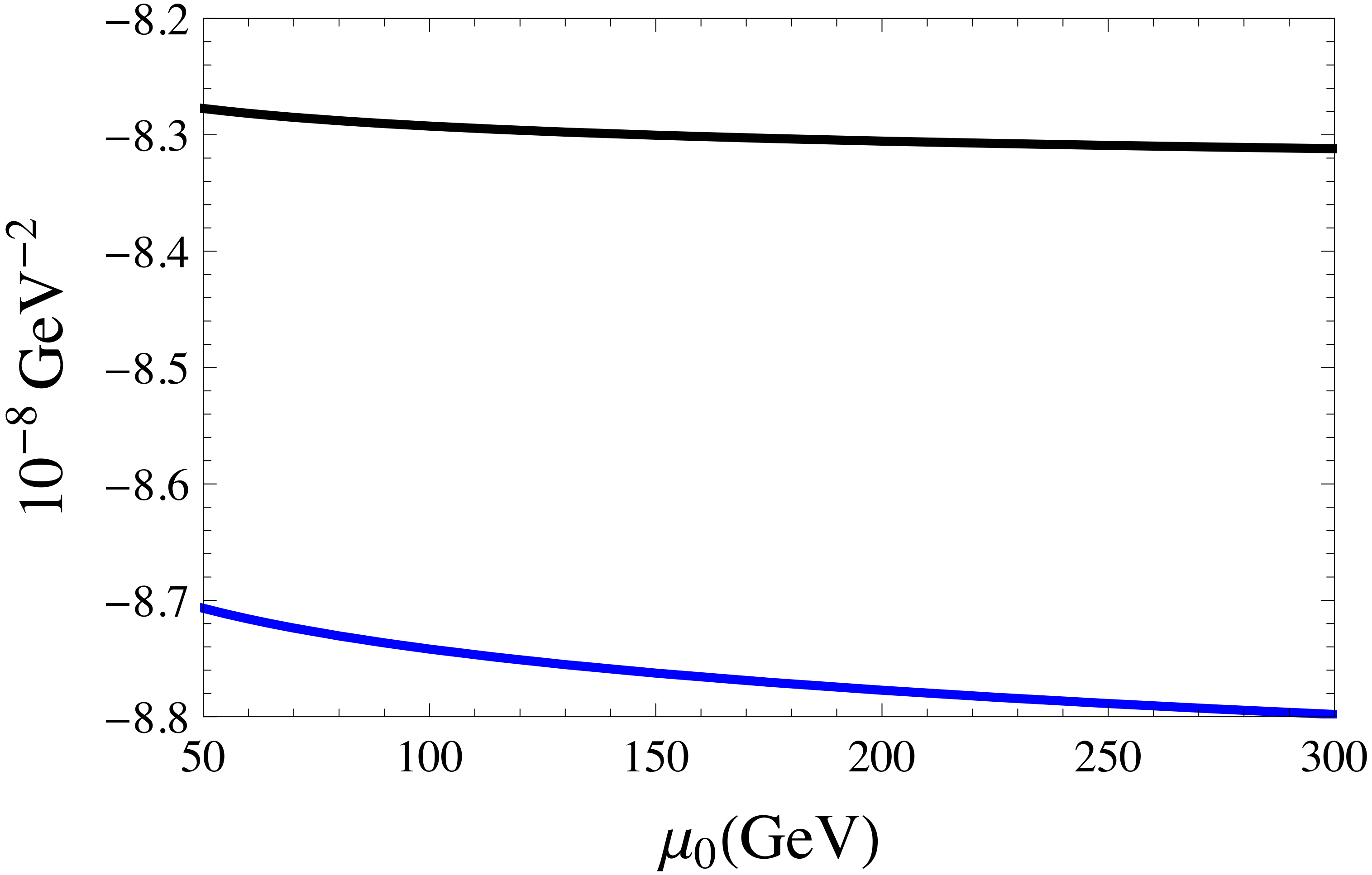
$$2^{-1/2} G_F \, \alpha \, \pi / \sin^2\theta_W \, C_A$$

Matching Correction for C_A

There are sizeable shifts going from 1-loop to 2-loop

largest shift in the on-shell scheme

$2^{-1/2} G_F \alpha \pi/\sin^2\theta_W C_A$

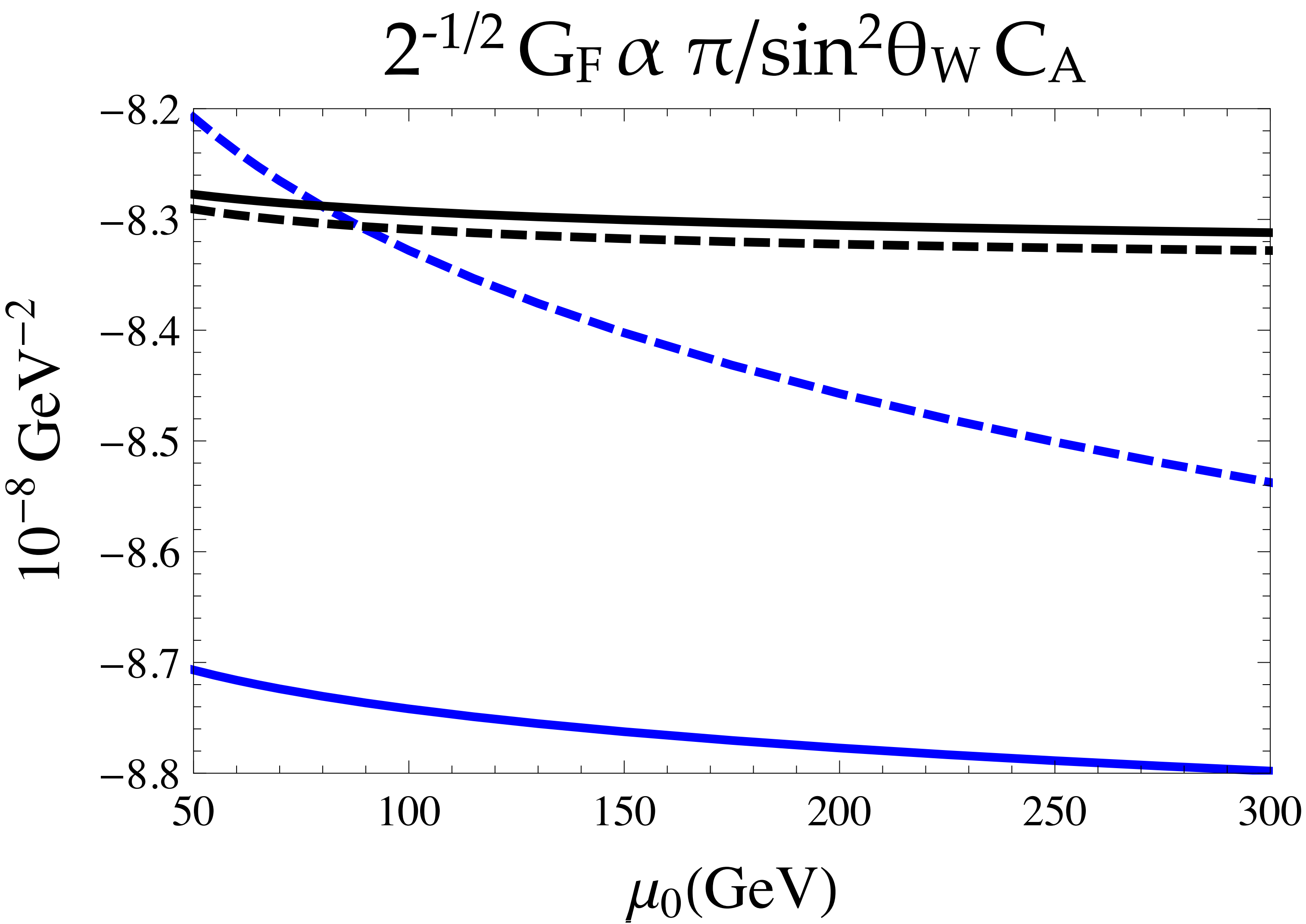


Matching Correction for C_A

There are sizeable shifts going from 1-loop to 2-loop

largest shift in the on-shell scheme

large scale dependence for the MS-bar scheme



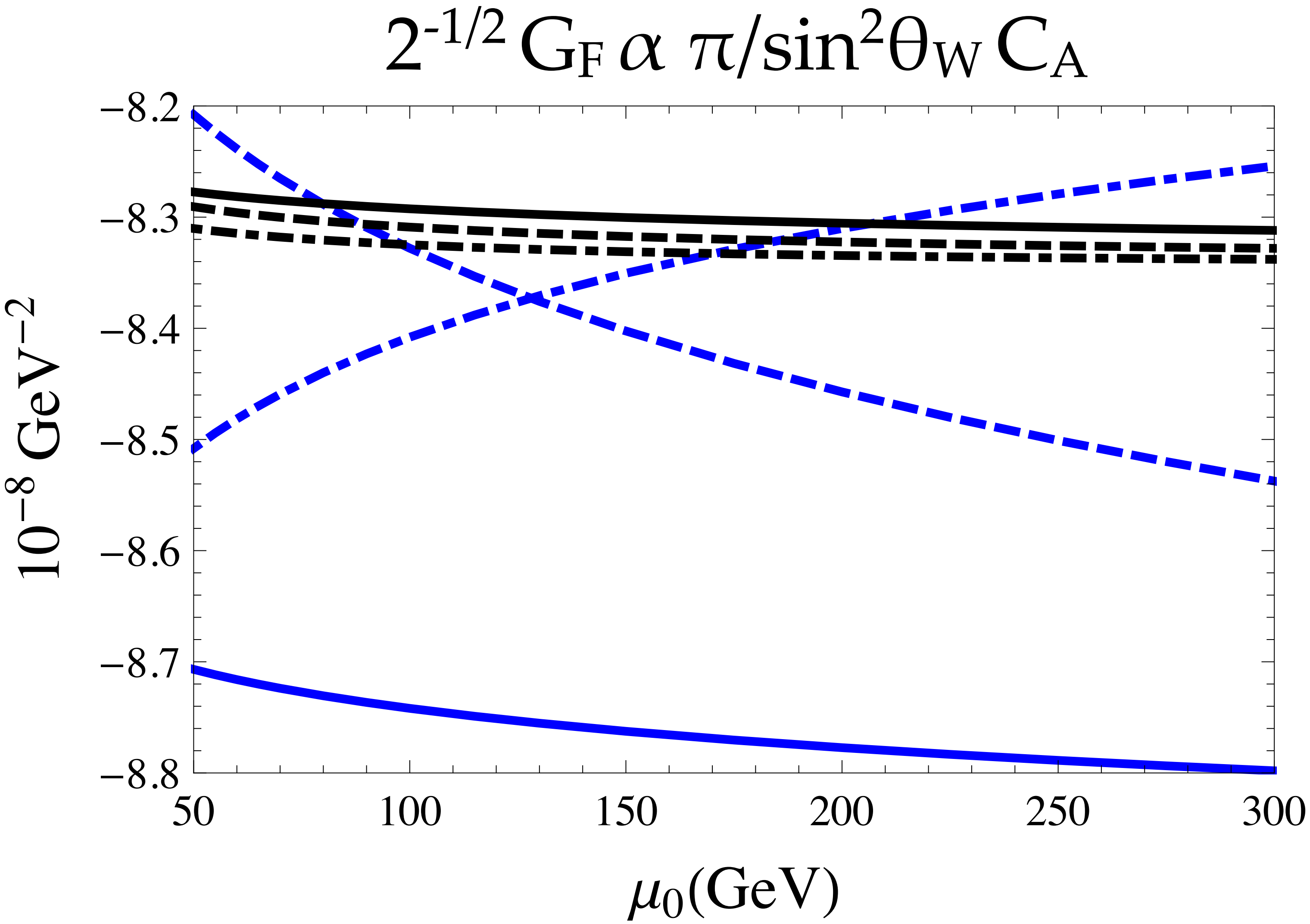
Matching Correction for C_A

There are sizeable shifts going from 1-loop to 2-loop

largest shift in the on-shell scheme

large scale dependence for the MS-bar scheme

and the hybrid scheme



Matching Correction for C_A

There are sizeable shifts going from 1-loop to 2-loop

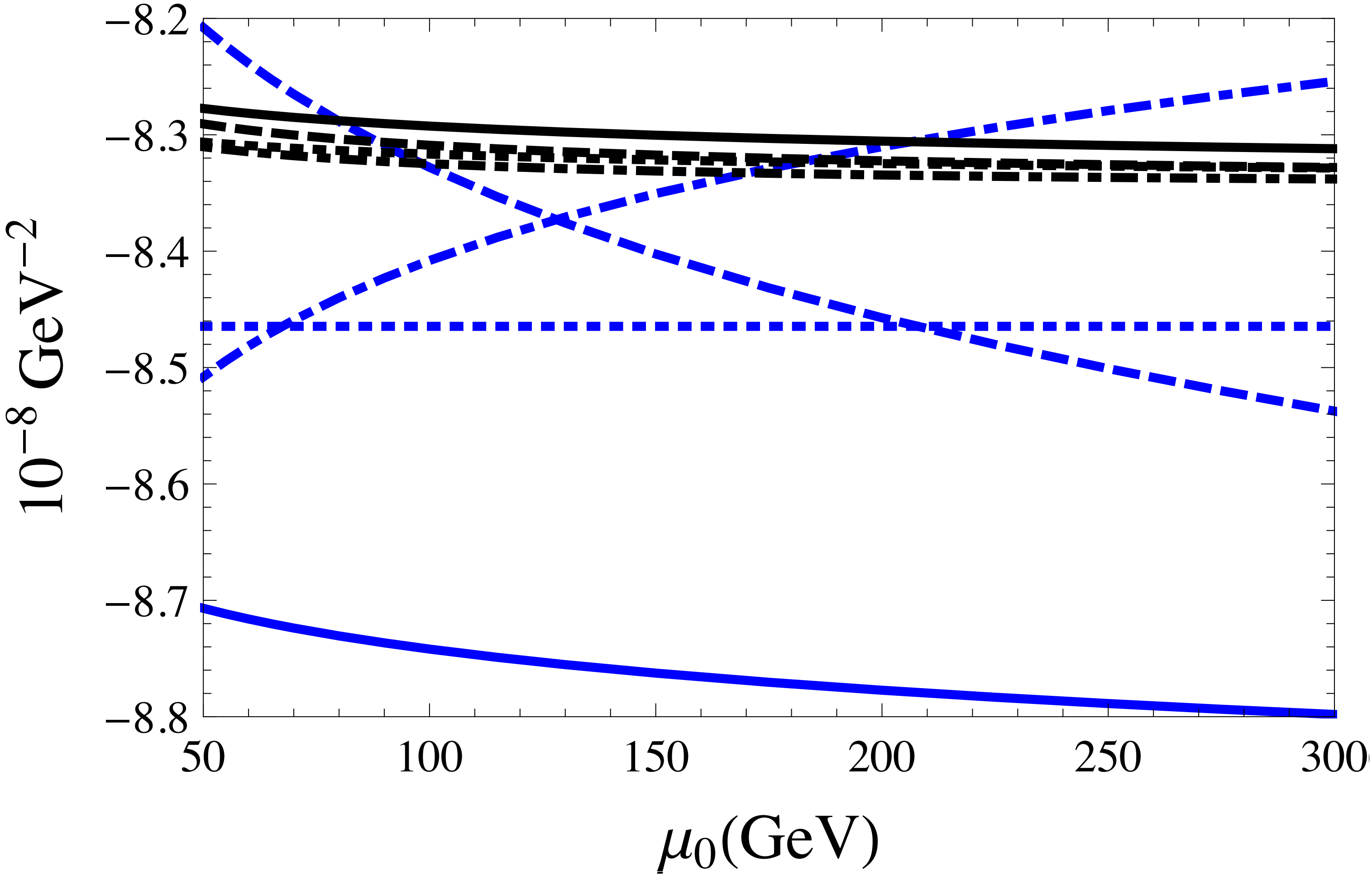
largest shift in the on-shell scheme

large scale dependence for the MS-bar scheme

and the hybrid scheme

$G_F^2 M_W^2$ removes `artificial´ scale (and parameter dependence)

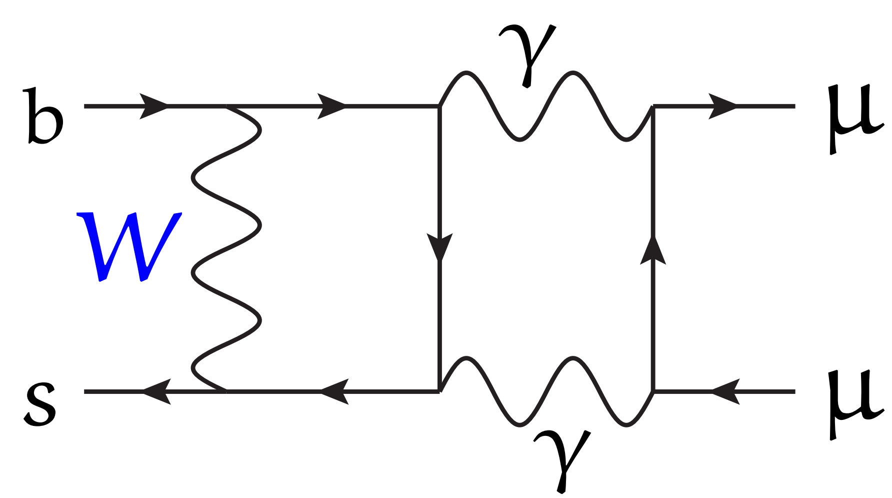
$2^{-1/2} G_F \alpha \pi/\sin^2\theta_W C_A$



2-loop electroweak corrections reduce the modulus of the Wilson coefficient

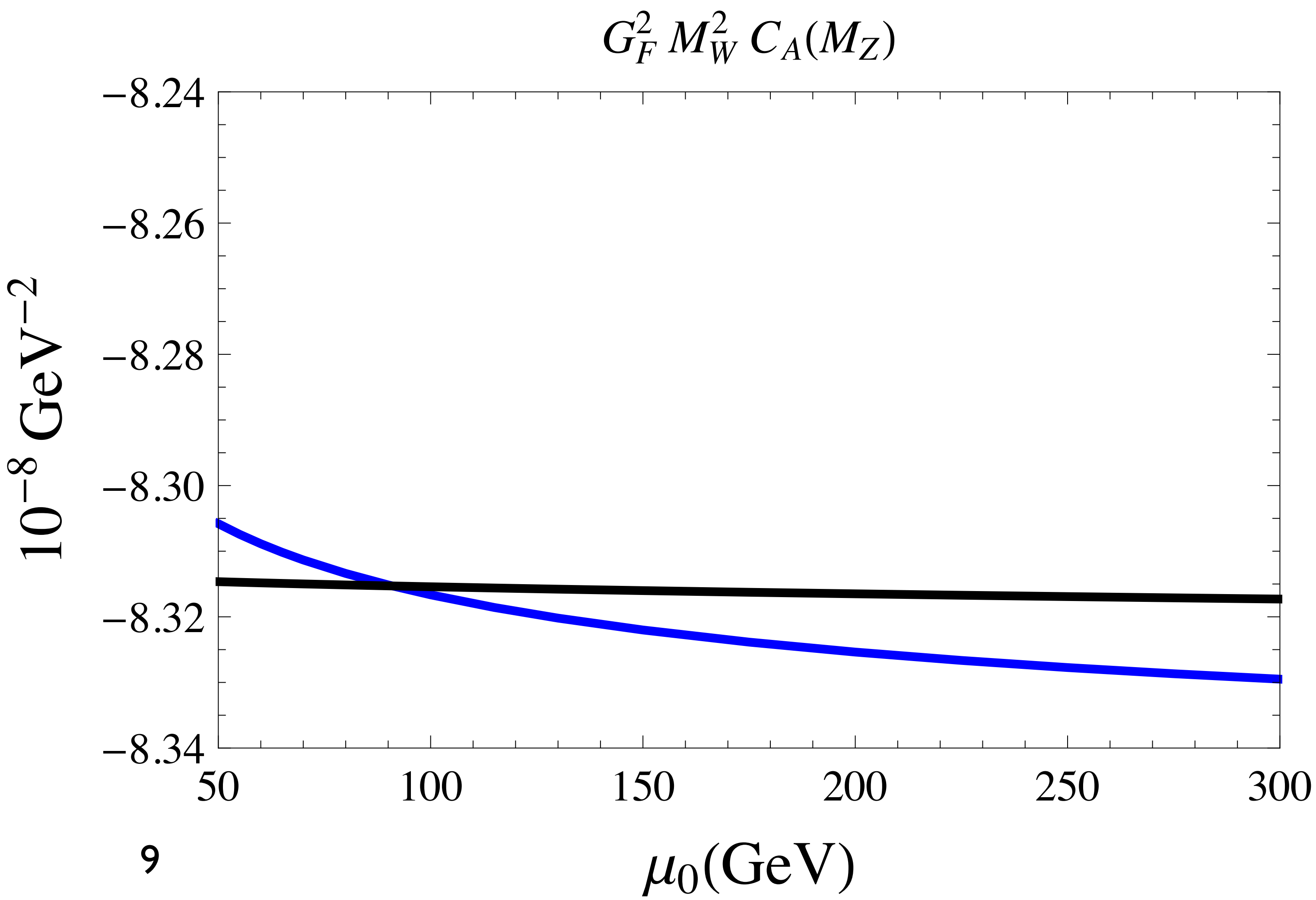
Renormalisation Group Equation

Log enhanced QED corrections known [Bobeth, Gambino, MG, Haisch `03; Huber et. al. `05, Misiak `11]



Study residual scale dependence for the $G_F^2 M_W^2$ normalised results

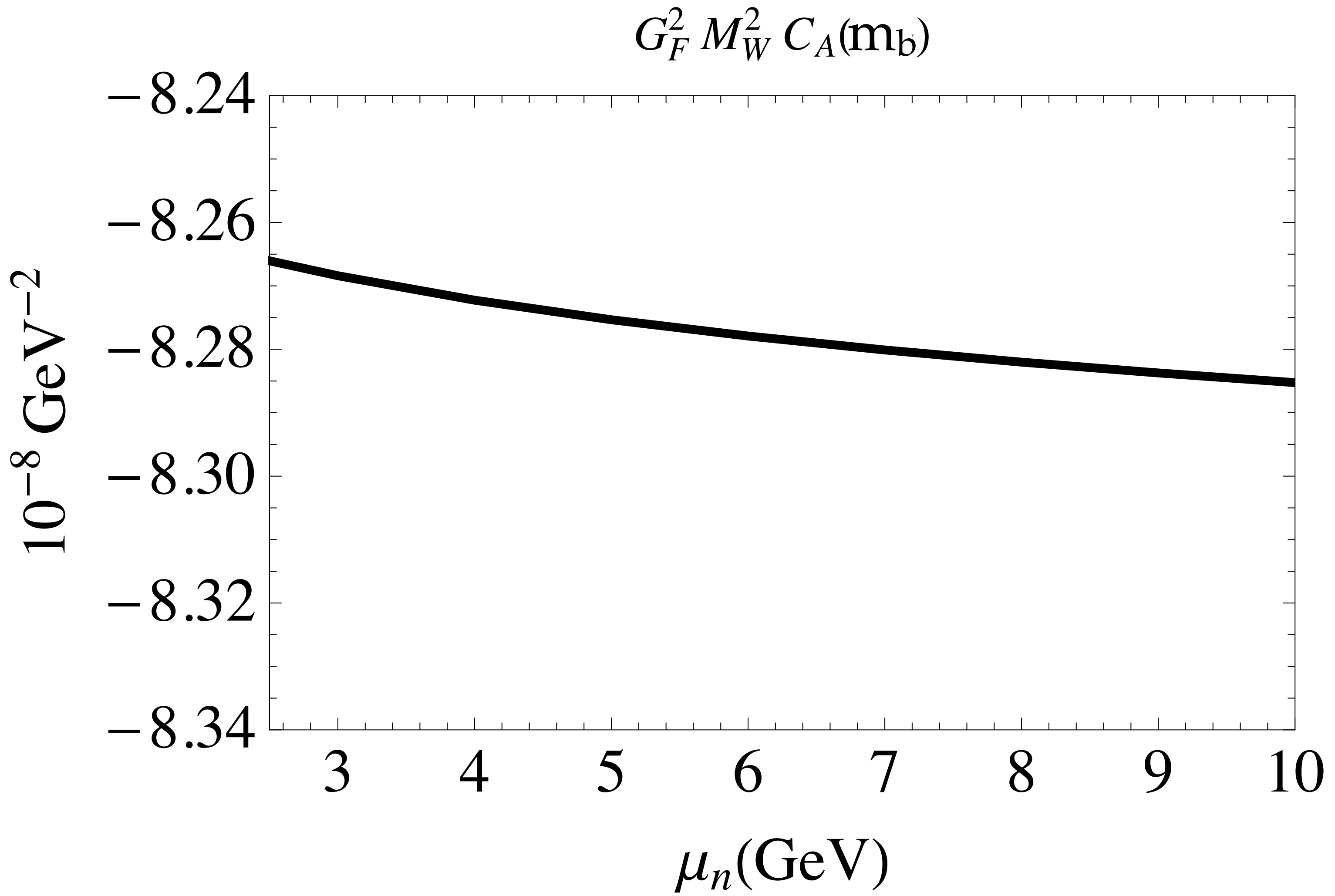
$G_F^2 M_W^2 C(\mu_0)$ is scale dependent, while $U(M_Z, \mu_0)\, G_F^2 M_W^2 C(\mu_0)$ is only residually scale dependent.



Wilson Coefficient at m_b

The log enhanced QED corrections further reduce the modulus of the Wilson coefficient

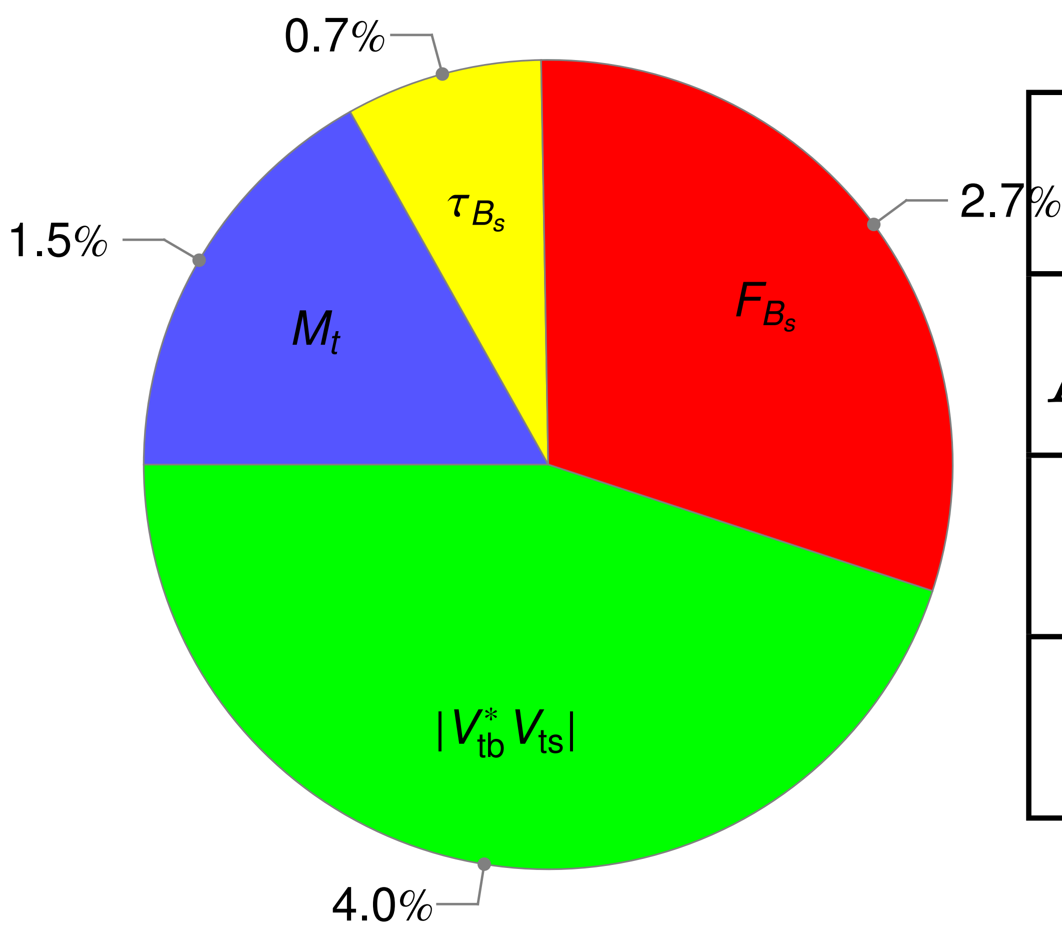
Varying μ_b in $U(\mu_b, m_t)\, G_F^2 M_W^2\, C(m_t)$ gives a measure of uncertainty regarding the contributions of virtual QED corrections at m_b.



Preliminary Theory Prediction

Electroweak corrections shift BFGK value 3.25 $10^{-9} \rightarrow$ 3.16 10^{-9}

$$\mathcal{B}^{(0)}_{B_s} = 3.16 \times 10^{-9} \left(\frac{M_t}{173.2\mathrm{GeV}}\right)^{3.07} \left(\frac{F_{B_s}}{225\mathrm{MeV}}\right)^2 \left(\frac{\tau_{B_s}}{1.500\mathrm{ps}}\right) \left|\frac{V^*_{tb}V_{ts}}{0.0405}\right|^2$$



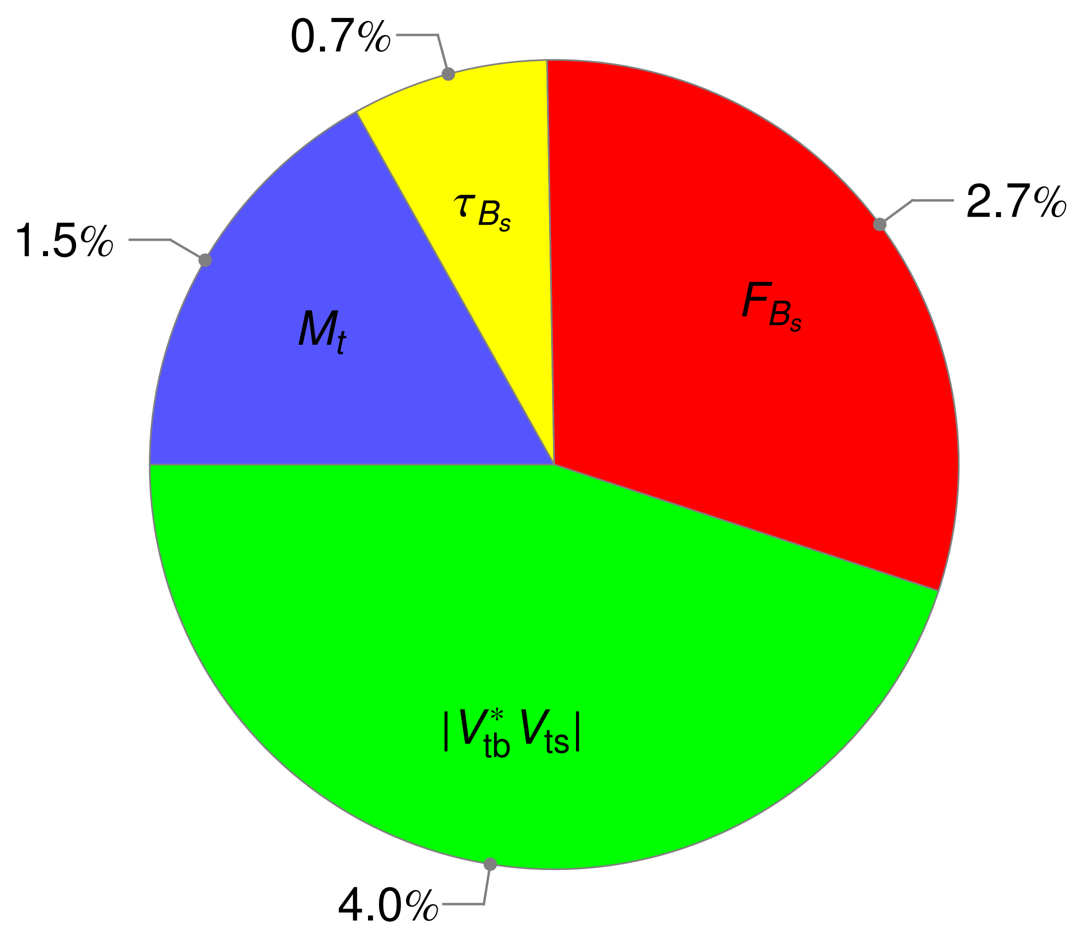
	$Br(B_s \rightarrow \mu^+ \mu^-)$
ATLAS	< 2.2 10^{-8}
CMS	< 7.7 10^{-9}
LHCb	3.2 $^{+1.4}_{-1.2}$ (stat) $^{+0.5}_{-0.3}$ (syst) 10^{-9}

[Buras, Fleischer, Girrbach, Knegjens `13]

Preliminary Theory Prediction

Electroweak corrections shift BFGK value 3.25 10^{-9} → 3.16 10^{-9}

$$\mathcal{B}^{(0)}_{B_s} = 3.16 \times 10^{-9} \left(\frac{M_t}{173.2\mathrm{GeV}}\right)^{3.07} \left(\frac{F_{B_s}}{225\mathrm{MeV}}\right)^{2} \left(\frac{\tau_{B_s}}{1.500\mathrm{ps}}\right) \left|\frac{V^*_{tb}V_{ts}}{0.0405}\right|^2$$



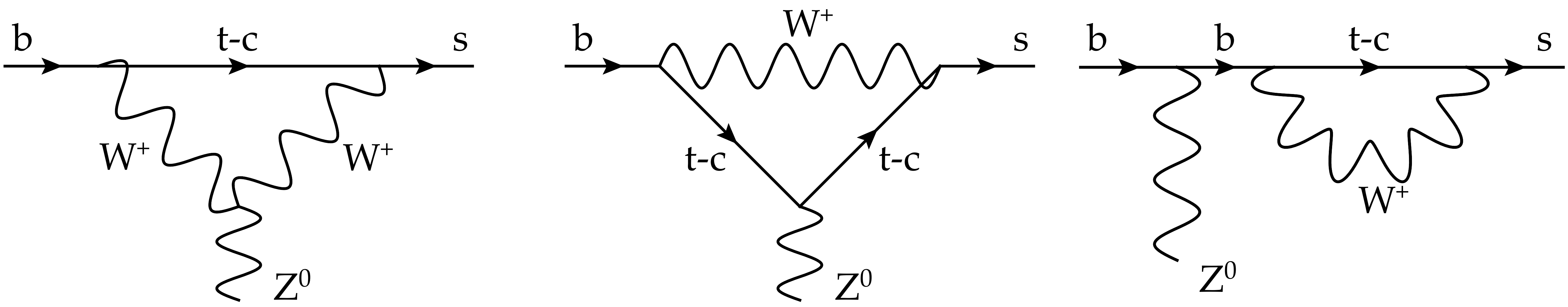
[Buras, Fleischer, Girrbach, Knegjens `13]

Pie chart is now free of electroweak scheme uncertainties

But: These numerics use multi-loop OS to OCD MS-bar shift for m_t → QCD Matching (talk by Mikolaj)

1% uncertainty from QED corrections

Standard Model and Beyond



$$\sum_i V^*_{is} V_{id} F(x_i) = V^*_{tb} V_{ts} (F(x_t) - F(x_c)) \stackrel{x_t \to \infty}{=} V^*_{tb} V_{ts} x_t$$

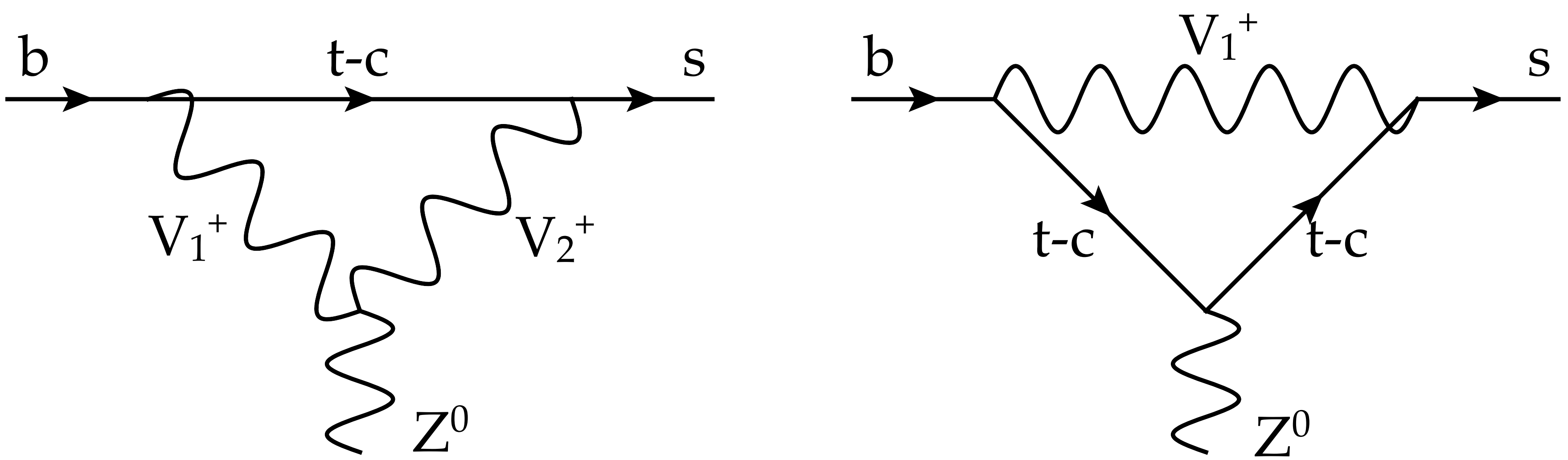
To get a finite result $M_W = M_Z \cos(\theta_w)$ has to hold

Involves the Z-Boson coupling to
1, up and 2, down-quarks and 3 W-Bosons
as well as 4, W-Boson couplings to quarks

Can we study all standard model extensions at once?

Generic Extensions

Let us consider a theories with arbitrary number of $W^{\pm}$



$$\mathcal{L}_3 = \sum_{f_1 f_2 v_1 \sigma} g^{\sigma}_{v_1 \bar{f}_1 f_2} V_{v_1,\mu} \bar{f}_1 \gamma^{\mu} P_{\sigma} f_2 + \sum_{v_1 v_2 v_3} g_{v_1 v_2 v_3} \Big[V_1, V_2, V_3 \Big]$$

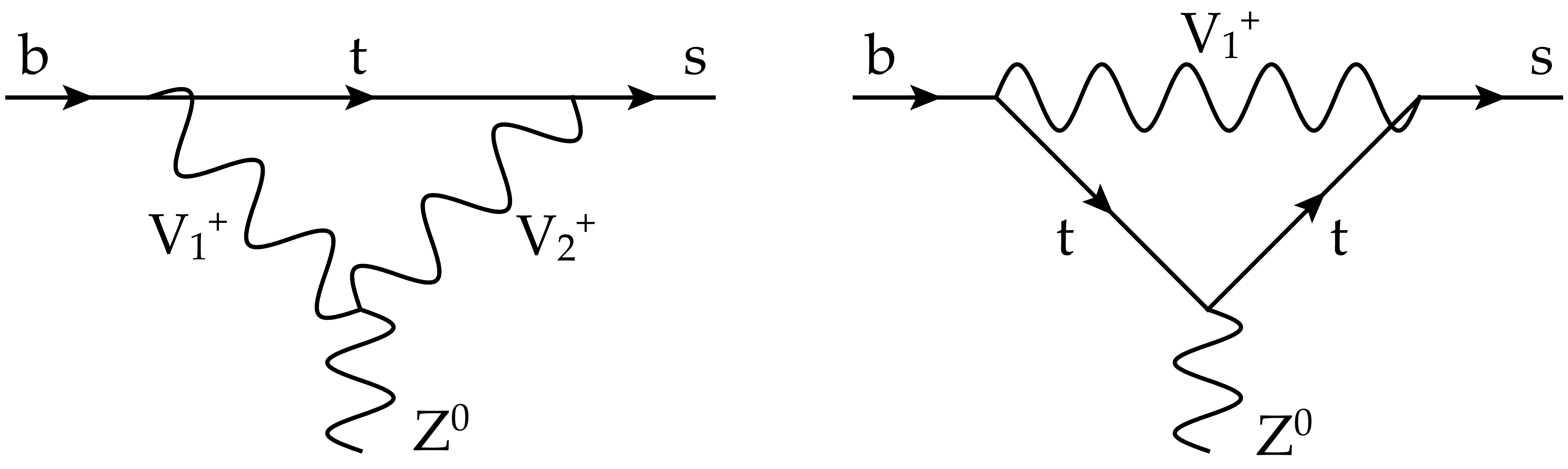
$$\Big[V_1, V_2, V_3\Big] = \tfrac{i}{6}\Big(V_{1,\mu} V_{2,\nu}\, \partial^{[\mu} V_3^{\nu]} + V_{3,\mu} V_{1,\nu}\, \partial^{[\mu} V_2^{\nu]} + V_{2,\mu} V_{3,\nu}\, \partial^{[\mu} V_1^{\nu]}\Big)$$

We assume a perturbative unitary theory

(vector-bosons are R_{ξ}-gauge fixed)

Fixes Goldstone-Boson interactions

Generic Result



$$\sum_{f_1 v_1 v_2} k^{\sigma}_{f_1 v_1 v_2} C_0\left(m_{f_1}, M_{v_1}, M_{v_2}\right) + \tilde{k}^{\sigma}_{f_1 v_1 v_2}\left(\tilde{C}_0\left(m_{f_1}, M_{v_1}, M_{v_2}\right) + \tfrac{1}{2}\right) + k'^{\sigma}_{f_1 v_1 v_2} +$$

$$\sum_{f_1 f_2 v_1} k^{\sigma}_{f_1 f_2 v_1} C_0\left(m_{f_1}, m_{f_2}, M_{v_1}\right) + \tilde{k}^{\sigma}_{f_1 f_2 v_1}\left(\tilde{C}_0\left(m_{f_1}, m_{f_2}, M_{v_1}\right) - \tfrac{1}{2}\right) + k'^{\sigma}_{f_1 f_2 v_1}$$

C_0 is a finite loop function – only $\tilde{C}_0$ is divergent $\sum_{f_1 v_1 v_2} \tilde{k}^{\sigma}_{f_1 v_1 v_2} + \sum_{f_1 f_2 v_1} \tilde{k}^{\sigma}_{f_1 f_2 v_1} = 0$

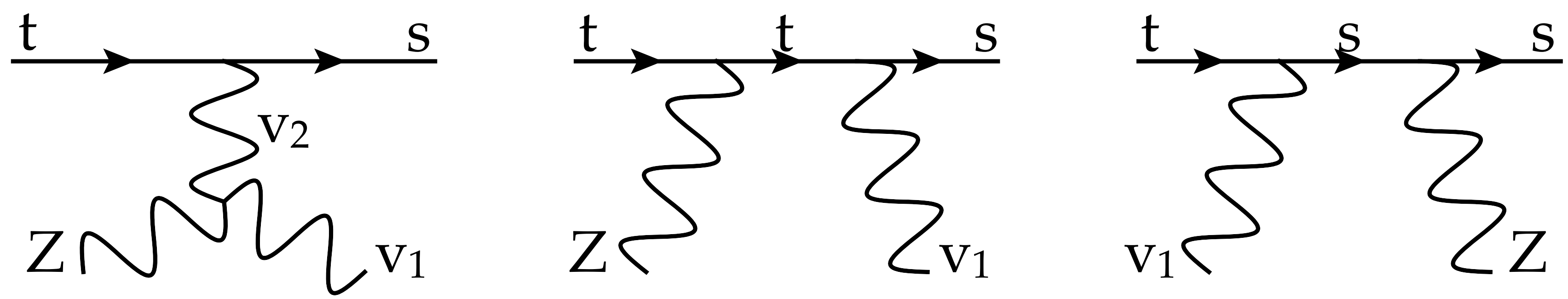
$$\sum_{v_1}\left(-\sum_{v_2}\frac{M^2_{v_1}+M^2_{v_2}-M^2_Z}{4M^2_{v_1}M^2_{v_2}} g_{Z v_1^+ v_2^-} g^{\sigma}_{v_1^- \bar{b} t} g^{\sigma}_{v_2^+ \bar{t} s} - \frac{1}{2M^2_{v_1}} g^{\sigma}_{Z\bar{s}s} g^{\sigma}_{v_1^- \bar{b} t} g^{\sigma}_{v_1^+ \bar{t} s}\right.$$

$$\left. + \frac{1}{2M^2_{v_1}} g^{\bar{\sigma}}_{Z\bar{t}t} g^{\sigma}_{v_1^- \bar{b} t} g^{\sigma}_{v_1^+ \bar{t} s}\right) = 0$$

(using GIM & Universality)

Renormalisation

Let us consider a theories with arbitrary number of $W^{\pm}$



Unitarity leads to the following constraints on the couplings

$$g^{\sigma}_{v_1^+ \bar{t} s}\, g^{\sigma}_{Z \bar{t} t} \rightarrow \sum_{v_2} g_{Z v_1^+ v_2^-}\, g^{\sigma}_{v_2^+ \bar{t} s} + g^{\sigma}_{v_1^+ \bar{t} s}\, g^{\sigma}_{Z \bar{s} s}$$

plus the one $\propto m_t$:

$$g^{\sigma}_{v_1^+ \bar{t} s}\, g^{\bar{\sigma}}_{Z \bar{t} t} \rightarrow \sum_{v_2} \frac{M_{v_1}^2 - M_Z^2}{2 M_{v_2}^2}\, g_{Z v_1^+ v_2^-}\, g^{\sigma}_{v_2^+ \bar{t} s} + \frac{1}{2} g^{\sigma}_{v_1^+ \bar{t} s} \left(g^{\sigma}_{Z \bar{s} s} + g^{\sigma}_{Z \bar{t} t} \right)$$

Which imply the finiteness of the Z-Penguin through:

$$g^{\sigma}_{v_1^+ \bar{t} s}\, g^{\bar{\sigma}}_{Z \bar{t} t} \rightarrow \sum_{v_2} \frac{M_{v_1}^2 + M_{v_2}^2 - M_Z^2}{2 M_{v_2}^2}\, g_{Z v_1^+ v_2^-}\, g^{\sigma}_{v_2^+ \bar{t} s} + g^{\sigma}_{v_1^+ \bar{t} s}\, g^{\sigma}_{Z \bar{s} s}$$

generalisation of the SM $M_W = M_Z \cos(\theta_w)$ renormalisation

Renormalised Result

Applying the unitarity constraints on the full result yields

$$\sum_{\nu_1\nu_2} g_{Z\nu_1^+\nu_2^-} g^L_{\nu_1^-\bar{b}t} g^L_{\nu_2^+\bar{t}s} F_1(m_t, M_{\nu_1}, M_{\nu_2}) + \sum_{\nu_1} g^L_{Z\bar{s}s} g^L_{\nu_1^-\bar{b}t} g^L_{\nu_2^+\bar{t}s} F_0(m_t, M_{\nu_1})$$

a finite result for the (left-handed) Z-Penguin

The result can be extended to include an arbitrary number of new scalar-bosons and fermions

[Brod, MG, Casagrande]

Can be used to classify and study new physics contributions to Wilson Coefficients

Could be useful to combine indirect and direct search results

Conclusions

No more electroweak scheme ambiguities in $B_q \to \mu^+ \mu^-$

Corrections small w.r.t. experimental error,
large w.r.t. to other theory errors

$B_q \to \mu^+ \mu^-$ can provide a precision probe of scalar, but also axial vector coupling interactions

Classification of loop induced new-physics