Hadronic effects in the exclusive $b \to s$ transitions

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$B \to K^{(*)} \ell^+\ell^-$, the effective Hamiltonian

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)\Big|_{\mu\sim m_b}$$

- "direct" $b \to s\ell\ell$, $b \to s\gamma$ operators:

$$O_{9(10)} = \frac{\alpha_{em}}{4\pi}[\bar{s}_L\gamma_\mu b_L]\ell\gamma^\mu(\gamma_5)\ell, \quad C_9(m_b) \simeq 4.4, \quad C_{10}(m_b) \simeq= -4.7,$$

$$O_{7\gamma} = -\frac{em_b}{8\pi^2}[\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b]F^{\mu\nu}, \quad C_7(m_b) \simeq -0.3$$

- quark-gluon operators, combined with quark e.m. current :

$$O_1^{(c)} = [\bar{s}_L\gamma_\rho c_L][\bar{c}_L\gamma^\rho b_L], \qquad C_1(m_b) \simeq 1.1$$
$$O_2^{(c)} = [\bar{c}_L\gamma_\rho c_L][\bar{s}_L\gamma^\rho b_L], \qquad C_2(m_b) \simeq -0.25$$

$$O_{8g} = -\frac{m_b}{8\pi^2}\bar{s}\sigma_{\mu\nu}(1+\gamma_5)bG^{\mu\nu}, \quad C_8(m_b) \simeq 0.2$$

O_{3-6} - quark-penguin operators , $\quad C_{3,4,5,6} < 0.03$

- the $\sim V_{ub}V_{us}^*$ part neglected

$B \to K^{(*)}\ell^+\ell^-$ decay amplitude

$$A(B \to K^{(*)}\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i \langle K^{(*)}\ell^+\ell^- \mid O_i \mid B\rangle$$

- hadronic matrix elements:

$$A(B \to K^{(*)}\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_{em}}{2\pi} \Bigg[(\bar{\ell}\gamma^\rho\gamma_5\ell)\, C_{10} \langle K^{(*)}|\bar{s}\gamma_\rho(1-\gamma_5)b|B\rangle$$
$$+(\bar{\ell}\gamma^\rho\ell)\Bigg(C_9 \langle K^{(*)}|\bar{s}\gamma_\rho b|B\rangle + C_7 \frac{2m_b}{q^2} q^\nu \langle K^{(*)}|\bar{s}i\sigma_{\nu\rho}(1+\gamma_5)b|B\rangle$$
$$+\frac{8\pi^2}{q^2} \sum_{i=1,2,\dots,6,8} C_i\, \mathcal{H}_i^\rho \Bigg)\Bigg]$$

- include $B \to K^{(*)}$ form factors and nonlocal hadronic matrix elements

$$\boxed{\mathcal{H}_i^\rho(q,p) = \langle K^{(*)}(p)|i\int d^4x\, e^{iqx}\, T\{j^\rho_{em}(x), O_i(0)\}|B(p+q)\rangle},$$

- hereafter, consider the kaon final state, $B \to K\ell^+\ell^-$

Status of $B \to K$ form factors in QCD

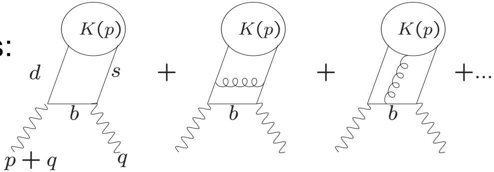
- lattice QCD at $q^2 \geq 15$ GeV^2, recent $n_f = 3$ results:
  [Fermilab-MILC 1211.1390 [hep-lat]; Cambridge-(MILC) 1101.2726[hep-ph]]

- QCD light-cone sum rules (LCSR), at $q^2 \leq 15$ GeV^2:

  correlation function = hadronic sum ⇒ $\langle K|\, j\,|B\rangle$

  ⇑ {OPE, light-cone DAs} ⇓ {quark-hadron duality}

- LCSR with kaon DAs:



- soft-gluon (low virtuality) and hard-gluon effects enter separate terms of OPE
- alternative version of LCSR: B-meson DAs (HQET) and kaon interpolating current (LCSR in SCET)

LCSR results

- $q^2 \leq 12 - 15\ \text{GeV}^2$ accessible,
  complementing the lattice FF's
- LCSR with kaon DAs, the recent update
  [A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]
- employing z-parameterization:

$$F(q^2) = \frac{F(0)}{1 - q^2/m^2_{B_s(J^P)}} \left\{ 1 + b_1 \left(z(q^2, t_0) - z(0, t_0) + \frac{1}{2}\left[z(q^2, t_0)^2 - z(0, t_0)^2 \right] \right) \right\},$$

$B \to K$ form factor	$F^i_{BK}(0)$	b^i_1	$B_s(J^P)$	input at $q^2 < 12\ \text{GeV}^2$
f^+_{BK}	$0.34^{+0.05}_{-0.02}$	$-2.1^{+0.9}_{-1.6}$	$B_s^*(1^-)$	
f^0_{BK}	$0.34^{+0.05}_{-0.02}$	$-4.3^{+0.8}_{-0.9}$	no pole	LCSR
f^T_{BK}	$0.39^{+0.05}_{-0.03}$	$-2.2^{+1.0}_{-2.00}$	$B_s^*(1^-)$	with K DA's

Hadronic input in $B \to K\ell\ell$

$$A(B \to K\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{\pi} V_{tb} V_{ts}^* \Bigg[\bar{\ell}\gamma_\mu \ell \, p^\mu \Bigg(C_9 f_{BK}^+(q^2)$$

$$+ \frac{2(m_b + m_s)}{m_B + m_K} C_7^{eff} f_{BK}^T(q^2) + \sum_{i=1,2,\dots,6,8} C_i \, \mathcal{H}_i^{(BK)}(q^2) \Bigg) + \bar{\ell}\gamma_\mu\gamma_5 \ell \, p^\mu C_{10} f_{BK}^+(q^2) \Bigg]$$

- the leading short-distance contributions determined by $B \to K$ form factors calculable in QCD
- remaining nonlocal matrix elements:

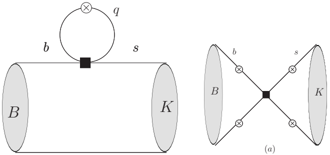
  $$\mathcal{H}_i^{(BK)}(q^2) \sim \langle K(p)| i \int d^4x \, e^{iqx} \, T\{ j^\rho_{em}(x), O_i(0)\} |B(p+q)\rangle$$

  $$j^\rho_{em} = \sum_{q=u,d,s,c,b} Q_q \bar{q}\gamma^\rho q \,, \qquad \text{the hierarchy } O_i = O^{(c)}_{1,2},\ O_{8g}, O^{(q)}_{3,4,5,6}, O^{(u)}_{1,2}$$

  $\Rightarrow$ corrections to fundamental short-distance coeff.:
  $C_9 \to C_9 + \Delta C_9^{(BK,i)}(q^2)$ (q^2- and process-dependent)
- have to be estimated one by one

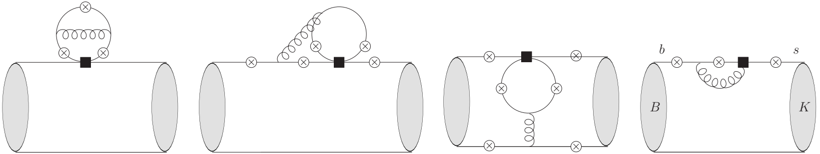
Are the nonlocal matrix elements calculable in QCD?

- LO, factorizable and weak annihilation

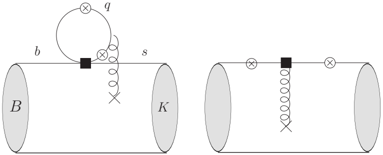


⊗ -virtual photon

- NLO, nonfactorizable ...



- soft (low virtuality) gluons, nonfactorizable



...

Use of OPE and effective theories

- at low q^2 (large recoil of $K^{(*)}$):

- $\mathcal{H}_i$ in $O(\alpha_s)$ obtained from QCD factoriz. (HQET/SCET) $(m_b \to \infty, E_{K^{(*)}} \sim m_b)$
- "nonspectator" contributions $\Rightarrow$ $B \to K^{(*)}$ FF's, "spectator " contributions factorized; nonpert. inputs B and $K^{(*)}$ DAs
  [M.Beneke, Th.Feldmann, D.Seidel (2001)], ...
  the talk by S. Jaeger at this workshop
- the magnitude of soft gluon effects ?? (power-suppressed in OPE)

New approach to nonlocal hadronic matrix elements

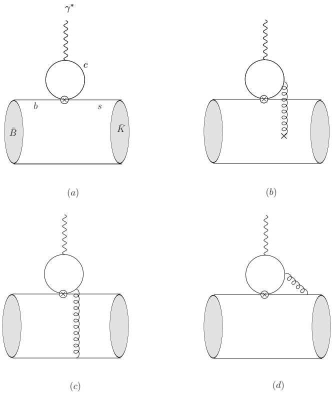
- charm-loop effects in $B \to K^{(*)}\ell\ell$
  [A.K., Th. Mannel, A. Pivovarov and Yu-M. Wang, JHEP (2010)]

- complete analysis of $B \to K\ell\ell$
  [A.K., Th. Mannel and Yu-M. Wang, JHEP (2013)]

Charm loops in $B \to K^{(*)}\ell^+\ell^-$

the outline:

- consider $\mathcal{H}_i^{(BK^{(*)})}(q^2)$ at spacelike $q^2 \ll 4m_c^2$, $(i = 1, 2, O_{1,2}^c)$
- apply OPE to the virtual $\bar{c}c$-loop
- factorizable part: loop $\otimes$FF
- "soft" gluon emission yields a nonlocal operator $\tilde{\mathcal{O}} \sim \bar{s}Gb$
- each extra gluon $\Rightarrow \sim \frac{\Lambda^2_{QCD}}{4m_c^2 - q^2}$
- use LCSR to calculate $\langle K^{(*)}|\tilde{\mathcal{O}}|B\rangle$



- perturbative ("hard") gluons: principally accessible with LCSR, but technically complicated multiloop calculations, remedy: use QCDF estimates *[M. Beneke,T. Feldmann,D. Seidel (2001)]*

Accessing the timelike q^2 region

- analyticity of the hadronic matrix element in q^2, $\oplus$ unitarity $\Rightarrow$ hadronic dispersion relation:

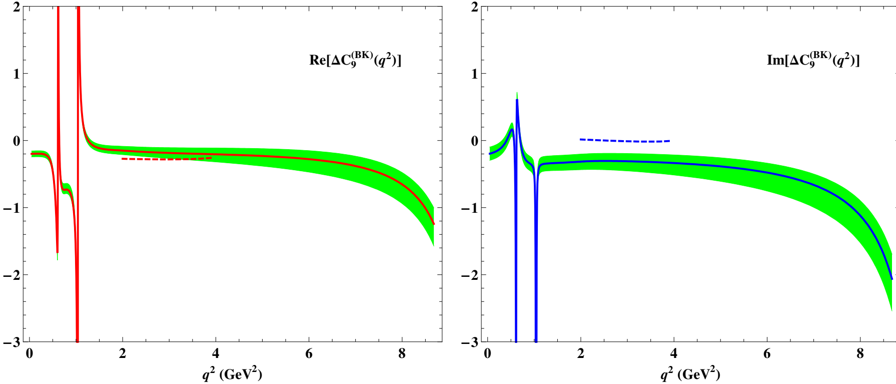
$$\mathcal{H}_i^{(BK)}(q^2) = \mathcal{H}_i^{(BK)}(0) + q^2 \Big[\sum_{\psi=J/\psi,\psi(2S),..} \frac{f_\psi A_{B\psi K}}{m_\psi^2 (m_\psi^2 - q^2 - i m_\psi \Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s - q^2 - i\epsilon)} \Big]$$

- the residues $|A_{B\psi K}|$ and $|f_\psi|$ determined by $BR(B \to \psi K)$, $BR(\psi \to \ell^+ \ell^-)$
- FSI phase in $A(B \to \psi K)$, (Im part in $(p+q)^2$) will appear , dual to perturbative gluon effects

All we wanted to know about $B \to K\ell^+\ell^-$

- all operators $O_{1,2}$, O_{8g}, O_{3-6} included
- quark-loop soft-gluon effects at $q^2 < 0$ calculated
- soft-gluon emission from gluonic penguin operator (new LCSR calculation)
- hard-gluon effects estimated employing QCDF (partly cross-checked with LCSRs)
- LO weak annihilation (small effect)
- dispersion relation includes $V = \rho, \omega, \phi$ in addition to $V = J/\psi, \psi'$; the parameters fitted with $q^2 < 0$ calculation and with measured $BR(B \to VK)$.

$\Delta C_9(q^2)$ below J/ψ region



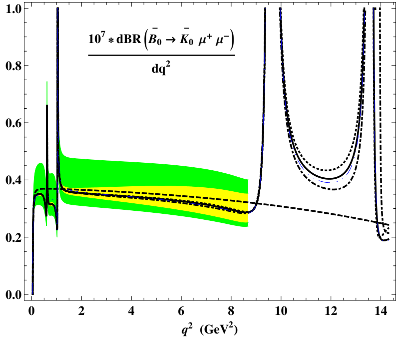
the red (blue) solid curve corresponds to the Re (Im) part obtained from the hadronic dispersion relation, fitted to the QCD calculation at q2 < 0 (central input, default parametrization). The shaded areas indicate the uncertainties. The dashed curves correspond to the prediction of QCDF.

$d\mathrm{BR}(B \to K\mu^+\mu^-)/dq^2$ and bins

solid (dotted) lines - central input, default (alternative) parametrization for the dispersion integrals.

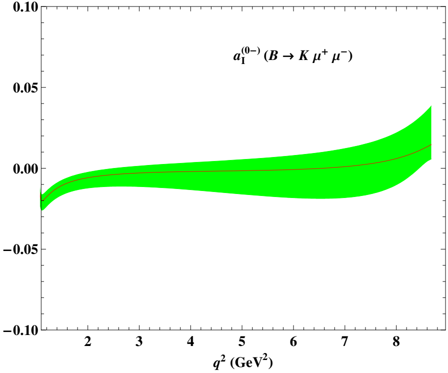
long-dashed line -the width calculated without nonlocal hadronic effects.

The green (yellow) shaded area indicates the uncertainties including (excluding) the one from the $B \to K$ FF normalization.



$[q^2_{min}, q^2_{max}]$	Belle	CDF	LHCb	LHCb	this work
[0.05, 2.0]	$0.81^{+0.18}_{-0.16} \pm 0.05$	$0.33 \pm 0.10 \pm 0.02$	$0.21^{+0.27}_{-0.23}$	$0.56 \pm 0.05 \pm 0.03$	$0.71^{+0.22}_{-0.08}$
[2.0, 4.3]	$0.46^{+0.14}_{-0.12} \pm 0.03$	$0.77 \pm 0.14 \pm 0.05$	$0.07^{+0.25}_{-0.21}$	$0.57 \pm 0.05 \pm 0.02$	$0.80^{+0.27}_{-0.11}$
[4.3, 8.68]	$1.00^{+0.19}_{-0.08} \pm 0.06$	$1.05 \pm 0.17 \pm 0.07$	1.2 ± 0.3	$1.00 \pm 0.07 \pm 0.04$	$1.39^{+0.53}_{-0.22}$
[1.0, 6.0]	$1.36^{+0.23}_{-0.21} \pm 0.08$	$1.29 \pm 0.18 \pm 0.08$	$0.65^{+0.45}_{-0.35}$	$1.21 \pm 0.09 \pm 0.07$	$1.76^{+0.60}_{-0.23}$

Isospin asymmetry: $\bar{B}^0 \to \bar{K}^0 \ell^+\ell^-$ vs $B^- \to K^-\ell^+\ell^-$



- integrated over $1.0 < q^2 < 6.0$ GeV^2.

Belle (2009)	BaBar (2012)	LHCb (2012)	this work
$-0.41^{+0.25}_{-0.20} \pm 0.07$	$-0.41 \pm 0.25 \pm 0.01$	$-0.35^{+0.23}_{-0.27}$	$(-0.4)\% \div (-0.3)\%$

Towards complete analysis of $B \to K^*\ell^+\ell^-$

- the main challenge: $B \to K^*$ form factors:

only quenched lattice QCD,
LCSR with K^* DAs with $\Gamma^*_K = 0$ (sort of "quenched") [Ball,Zwicky(2004)]
LCSR with B DA's have a large uncertainty;
[AK,Mannel,Pivovarov,Wang(2010)]

- plans to extend the LCSR approach to

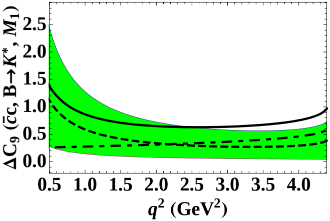
$B \to K\pi\ell\ell$ form factors (embedding K^* and other resonances)

- $\Delta C_9/C_9$ for $B \to K^*\ell^+\ell^-$ are generally larger at small q^2

(due to $1/q^2$ multiplying $\mathcal{H}_i^{BK^*}$)

- illustration: ΔC_9 in $B \to K^*\ell\ell$

(only charm loop $\oplus$ soft-gluon effect)



a lot of work ahead to assess the hadronic input in $B \to K^*\ell\ell$

BACKUP SLIDES

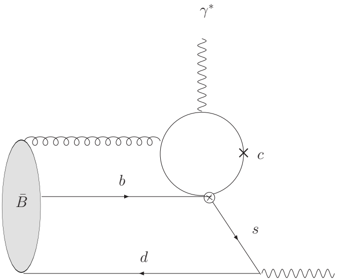
LCSR for the soft-gluon hadronic matrix element

- soft gluon emission from charm loop reduced to an effective nonlocal operator

$$\widetilde{\mathcal{O}}_\mu(q) = \int d\omega\, I_{\mu\rho\alpha\beta}(q, m_c, \omega) \bar{s}_L \gamma^\rho \delta[\omega - \frac{(in_+\mathcal{D})}{2}] \widetilde{G}_{\alpha\beta} b_L \ ,$$

- the correlation function:

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = i \int d^4y e^{ip\cdot y}$$
$$\langle 0| T\{ j^K_\nu(y) \widetilde{\mathcal{O}}_\mu(q)\}|B(p+q)\rangle \,,$$



- hadronic dispersion relation in the kaon channel

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = \frac{i f_K p_\nu}{m_K^2 - p^2}[(p\cdot q) q_\mu - q^2 p_\mu] \tilde{A}(q^2) + \int_{s_h}^{\infty} ds\, \frac{\tilde{\rho}_{\mu\nu}(s,q^2)}{s - p^2}$$

- $\Delta C_9^{(BK)}(q^2 < 0)$