Quark and Lepton Flavor Symmetry and the 126 GeV Higgs Boson

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What has the Higgs Discovery taught us?

Last year (2012) was the year of the Higgs Boson. We now know that there is a particle at 126 GeV which looks very much like the one Higgs boson of the standard model (SM). What has it taught us?

There are two possibilities:
(1) The SM is it, and we can just clean up the details and go home.
(2) There is new physics lurking, but it should naturally give us the 126 GeV particle as observed!

Examples of (2):

(2a) In supersymmetry (SUSY), there are two Higgs doublets. In general, the lightest neutral physical scalar is not a linear combination equaling that of the SM, unless the SUSY breaking scale is rather high. This may still be the case, as many people hope.

(2b) A second scalar doublet (η^+, η^0) with an exactly conserved odd Z_2 [Deshpande/Ma(1978)] is good, because it does not mix with the SM Higgs and is a possible dark-matter candidate.

In 2006 I proposed the scotogenic (from the Greek scotos meaning darkness) model of radiative neutrino mass, using the addition of N_i which are also odd under Z_2 together with (η^+, η^0), thereby linking neutrino mass to the existence of dark matter. Two months later, Barbieri/Hall/Rychkov(2006) proposed (η^+, η^0) by itself (called the inert Higgs doublet) to make the SM Higgs boson itself very heavy. [This idea is of course now proven to be wrong!] Let $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$, then η_R could be (cold) dark matter [Ma(2006)], or N_i of order 10 keV could be (warm) dark matter [Ma(2012)].

Flavor Structure of Quarks and Leptons

A long standing puzzle of particle physics is the flavor structure of quarks and leptons. In the SM, $\mathcal{M}_u$ and $\mathcal{M}_d$ are arbitrary. Once they are diagonalized:

$$U_L^{(u,d)} \mathcal{M}_{u,d} U_R^{(u,d)} = \mathcal{M}_{u,d}^{diag},$$

their mixing matrix

$$V_{CKM} = {U_L^{(u)}}^\dagger U_L^{(d)}$$

is observed and found to be nearly diagonal.

This alignment cries out for an explanation and the best hope is by way of a flavor symmetry, However, this means that the Yukawa interactions $\bar{d}_{iL} d_{jR} \phi_k^0$ and $\bar{u}_{iL} u_{jR} \bar{\phi}_k^0$ must be extended to include more than just one Higgs doublet. The immediate consequence of this is the appearance of tree-level flavor changing neutral-current (FCNC) interactions. Experimentally, these are known to be very small, e.g. $\Delta m_{K^0}/m_{K^0} = 7.0 \times 10^{-15}$. The usual solution to this conundrum is to retain just one Higgs doublet and make the Yukawa interactions nonrenormalizable by adding singlet flavons which carry the flavor symmetry. The mass scale of flavons is usually taken to be unreachably high, such as that of grand unification, i.e. 10^{16} GeV. Even if this is true, we will never find out experimentally.

In models with two or more Higgs doublets, the linear combination corresponding to the SM Higgs is generally not a mass eigenstate. Therefore, given that the 126 GeV particle at the LHC looks very much like the SM Higgs, we should look for a flavor symmetry such that it explains the V_{CKM} alignment and gives us the SM Higgs, at least approximately.

S_3 Model (2004)

In 2004, Chen/Frigerio/Ma proposed a flavor model of quarks and leptons based on the non-Abelian discrete symmetry S_3: 6 elements with irreducible $\underline{1}, \underline{1}', \underline{2}$ representations. It may be generated by the 2 noncommuting matrices

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix},$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. In this basis, if $(\Phi_1, \Phi_2) \sim \underline{2}$, then $(\Phi_2^\dagger, \Phi_1^\dagger) \sim \underline{2}$.

Quark and lepton assignments under S_3:

$L_e = (\nu_e, e), \ Q_1 = (u, d), \ \Phi_3 = (\phi_3^0, \phi_3^-) \sim \underline{1},$

$e^c, \mu^c, u^c, c^c, d^c, s^c \sim \underline{1}, \quad \tau^c, t^c, b^c \sim \underline{1}',$

$(L_\mu, L_\tau), (Q_2, Q_3), (\Phi_1, \Phi_2) \sim \underline{2}.$

Yukawa invariants: $\underline{2} \times \underline{1} \times \underline{2}, \quad \underline{2} \times \underline{1}' \times \underline{2}, \quad \underline{1} \times \underline{1} \times \underline{1},$

thus

$$\mathcal{M}_{u,d} = \begin{pmatrix} g_3^u v_3^* & g_4^u v_3^* & 0 \\ 0 & g_1^u v_1^* & -g_2^u v_1^* \\ 0 & g_1^u v_2^* & g_2^u v_2^* \end{pmatrix}, \begin{pmatrix} g_3^d v_3 & g_4^d v_3 & 0 \\ 0 & g_1^d v_2 & -g_2^d v_2 \\ 0 & g_1^d v_1 & g_2^d v_1 \end{pmatrix}.$$

Note that (v_1, v_2) in $\mathcal{M}_d$ is replaced by (v_2^*, v_1^*) in $\mathcal{M}_u$.
This is important in getting a realistic V_{CKM}.

Let $v_3 = 0$ and $v_1 = v_2$ (i.e. $S_3 \to Z_2$), then $\mathcal{M}_{u,d}$ are both rotated by $\pi/4$, so their mismatch is zero, i.e. perfect alignment with $\theta_{23} = 0$. Hence this residual symmetry is a good explanation of why V_{CKM} is almost diagonal. Its breaking occurs when $v_3 \neq 0$ and $v_1 \neq v_2$, which may be assumed to be small naturally.

In the lepton sector, $\mathcal{M}_l$ is just like $\mathcal{M}_d$, but $\mathcal{M}_\nu$ may be chosen to be diagonal if it is Majorana. Hence $\nu_\mu - \nu_\tau$ mixing is predicted to be maximal, i.e. $\theta_{23} = \pi/4$, in agreement with experiment.

Update (2013)

The original 2004 model mainly dealt with the lepton sector and predicted very small θ_{13}, in disagreement with present data. However, it neglected $e-\mu$ mixing which is generally present, so the model is still viable.

Here the quark sector is studied instead.

[Ma/Melic(2013)] Consider first only the 2 heavy quark families with (Φ_1, Φ_2). Let

$$V_{12} = \mu_1^2(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2) - \mu_2^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) +$$

$$\frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2)^2+\frac{1}{2}\lambda_2(\Phi_1^\dagger\Phi_1-\Phi_2^\dagger\Phi_2)^2+\lambda_3(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1).$$

This is invariant under S_3 except for the soft μ_2^2 term which breaks S_3 to Z_2 $(\Phi_1 \leftrightarrow \Phi_2)$. The Z_2 symmetry enforces $\langle \phi_1^0 \rangle = \langle \phi_2^0 \rangle = v = 123$ GeV, resulting in the mass eigenstates:

$$h^0 = \phi_{1R} + \phi_{2R}, \quad m^2 = 2(2\lambda_1 + \lambda_3)v^2,$$
$$H^0 = \phi_{1R} - \phi_{2R}, \quad m^2 = 2\mu_2^2 + 2(2\lambda_2 - \lambda_3)v^2,$$
$$A = \phi_{1I} - \phi_{2I}, \quad m^2 = 2\mu_2^2,$$
$$H^\pm = (\phi_1^\pm - \phi_2^\pm)/\sqrt{2}, \quad m^2 = 2\mu_2^2 - 2\lambda_3 v^2.$$

At this level, h^0 is even under Z_2 and is naturally identified with the SM Higgs. The other scalars are odd under Z_2. Note that if $\mu_2^2 = 0$, then A would be massless.

The $c-t$ and $s-b$ mass matrices are both of the form

$$\mathcal{M} = \begin{pmatrix} f_1 v & -f_2 v \\ f_1 v & f_2 v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} f_1\sqrt{2}v & 0 \\ 0 & f_2\sqrt{2}v \end{pmatrix}.$$

Consequently, the physical s, b quarks couple to h^0 according to $(m_s/2v)\bar{s}s + (m_b/2v)\bar{b}b$ as in the SM. The other scalar couplings are given by

$$\mathcal{L}_Y = \frac{m_s}{\sqrt{2}v}\left[H^+\bar{t}_L + \left(\frac{H^0 + iA}{\sqrt{2}}\right)\bar{b}_L\right] s_R$$

$$+ \frac{m_b}{\sqrt{2}v}\left[H^+\bar{c}_L + \left(\frac{H^0 + iA}{\sqrt{2}}\right)\bar{s}_L\right] b_R + H.c.,$$

which maintains the Z_2 symmetry with t, b odd and c, s even. This forbids $b \to s\gamma$ but allows $B_s - \bar{B}_s$ mixing. The coefficient of the $(\bar{s}_L b_R)^2$ operator is

$$\frac{m_b^2}{4v^2}\left(\frac{1}{m_H^2} - \frac{1}{m_A^2}\right).$$

The coefficient of the $(\bar{s}_L b_R)(\bar{s}_R b_L)$ operator is

$$\frac{m_s m_b}{4v^2}\left(\frac{1}{m_H^2} + \frac{1}{m_A^2}\right).$$

The hadronic matrix element of the former (latter) gives $-23.87 \times 10^{-6}\ \mathrm{GeV}^3$ and $1.20 \times 10^{-6}\ \mathrm{GeV}^3$.

The experimental value $\Delta m_{B_s} = 1.164 \pm 0.005 \times 10^{-11}$ GeV agrees with the Standard-Model prediction to within 10%, so we obtain

$$\left| -23.87 \left(\frac{1}{m_H^2} - \frac{1}{m_A^2} \right) + 1.20 \left(\frac{1}{m_H^2} + \frac{1}{m_A^2} \right) \right| < 1.16,$$

where $m_{H,A}$ are in units of TeV.

If $m_H = m_A$, then $m_{H,A} > 1.44$ TeV.

If $m_H = 1$ TeV, then $1.03 < m_A < 1.08$ TeV.

If $m_H = 0.7$ TeV, then $0.73 < m_A < 0.75$ TeV.

Add Φ_3 with $\langle\phi_3^0\rangle = v_3 << v$, then $\mathcal{M}_{d,u}$ are diagonalized on the left by

$$V_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c' & -s' \\ 0 & s' & c' \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where $s'/c' = v_2/v_1$, and

$$V_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & s' & -c' \\ 0 & c' & s' \end{pmatrix} \begin{pmatrix} c_u & -s_u e^{i\delta} & 0 \\ s_u e^{-i\delta} & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Hence $V_{CKM} = V_u^\dagger V_d =$

$$\begin{pmatrix} c_u c_d + c'' s_u s_d e^{i\delta} & -c_u s_d + c'' s_u c_d e^{i\delta} & s'' s_u e^{i\delta} \\ -s_u c_d e^{-i\delta} + c'' c_u s_d & s_u s_d e^{-i\delta} + c'' c_u c_d & s'' c_u \\ -s'' s_d & -s'' c_d & c'' \end{pmatrix},$$

where $s''/c'' = (c'^2 - s'^2)/2s'c'$. Using the 2012 PDG values, we obtain

$$s'' = 0.04135,\ \ s_u = 0.08489,\ \ s_d = 0.20983,$$

with $\cos\delta = -5.47 \times 10^{-3}$, and
$J_{CP} = s_u c_u s_d c_d (s'')^2 c'' \sin\delta = 2.96 \times 10^{-5}$.

This scheme does not predict any precise value of the measured parameters, but it does provide an understanding of why $(s'')^2, (s_u)^2, (s_d)^2$ are small.

To obtain $v_1 \neq v_2$, the Z_2 symmetry must be broken: add $\mu_3^2(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2)$. This changes h^0. However, in the limit of large $\mu_2^2 > 0$,

$$h^0 - h^0_{SM} \simeq \frac{(\lambda_1 - \lambda_2 + \lambda_3)(v_1^2 - v_2^2)}{2\mu_2^2}\ H^0,$$

Thus this model has a symmetry limit $(v_1 = v_2)$ which predicts that $h^0 = h^0_{SM}$. $[(v_1^2 - v_2^2)/4v^2 = 0.0207.]$

Adding Φ_3 means the addition of 5 quartic terms invariant under S_3. One term $\Phi_3^\dagger(\Phi_1\Phi_2^\dagger\Phi_1 + \Phi_2\Phi_1^\dagger\Phi_2)$ may be eliminated by imposing an extra Z_2 symmetry under which Φ_3 and $(u,d)_L$ are odd, and all others even. This Z_2 symmetry is then allowed to be broken softly by the term $\mu_4^2\Phi_3^\dagger(\Phi_1 + \Phi_2)$.
As a result, for large $m_3^2 > 0$, $v_3 \simeq -\mu_4^2(v_1 + v_2)/m_3^2$. Hence ϕ_{3R} mixes with $(v_1\phi_{1R} + v_2\phi_{2R})/\sqrt{v_1^2 + v_2^2}$ by $v_3/\sqrt{v_1^2 + v_2^2}$. This means that

$$h^0 - h^0_{SM} \simeq \frac{v_3 m_h^2}{2v m_3^2}\,\phi_{3R}.$$

The exchange of h^0 could induce $K^0 - \bar{K}^0$ mixing, but its contribution is negligible compared to the direct exchange of ϕ_3^0 which has the effective interaction

$$\frac{s_d^2 c_d^2 m_d m_s}{v_3^2 m_3^2} (\bar{d}_L s_R)(\bar{d}_R s_L).$$

Allowing this to be 20% of the experimental measurement $\Delta m_K = 3.483 \pm 0.006 \times 10^{-15}$ GeV, $v_3 m_3 > 6 \times 10^4$ GeV^2 is obtained.

For example, if $v_3 = 10$ GeV, then $m_3 > 6$ TeV.

Experimental Signatures?

The scalar spectrum of this model has only one light Higgs boson h^0 which coincides with the SM Higgs to a very good approximation. As for the other two scalar doublets, they are much heavier. The linear combination $\Phi_1 - \Phi_2$ is constrained by $B_s - \bar{B}_s$ mixing to be heavier than about 0.7 TeV, whereas Φ_3 is constrained by $K^0 - \bar{K}^0$ mixing to be heavier than about 6 TeV if $v_3 = 10$ GeV. With these masses, all rare processes involving only quarks but not leptons such as $b \to s\gamma$ are negligible. However, the $s - b$ sector is connected to the

$\mu - \tau$ sector:

$$\mathcal{L}_Y = \frac{m_\mu}{\sqrt{2}v}\left[H^+\bar{\nu}_{\tau L} + \left(\frac{H^0 + iA}{\sqrt{2}}\right)\bar{\tau}_L\right]\mu_R$$

$$+ \frac{m_\tau}{\sqrt{2}v}\left[H^+\bar{\nu}_{\mu L} + \left(\frac{H^0 + iA}{\sqrt{2}}\right)\bar{\mu}_L\right]\tau_R + H.c.,$$

This means that the decay $b \to s\tau^-\mu^+$ proceeds through the exchange of $H^0 + iA$ with a possible branching fraction of 10^{-7}, but $b \to s\tau^+\mu^-$ will not be seen. This is a possible unique signature of this model. Currently, $B(B^+ \to K^+\tau^\pm\mu^\mp) < 7.7 \times 10^{-5}$.

Conclusion

In conclusion, h^0 of our S_3 model is very nearly identical to the SM Higgs, because of the residual symmetry Z_2 from S_3, and the extra Z_2 symmetry for Φ_3 and $(u,d)_L$.

We have thus successfully constructed a flavor model of quarks which explains the observed pattern of masses and mixing, as well as having $h^0 \simeq h^0_{SM}$ The unique prediction of this model is $b \to s\tau^-\mu^+$ which may perhaps be testable at Super KEKB.