Towards gauge coupling unification in minimal SU(5) at 3-loop accuracy

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Outline

- Georgi-Glashow SU(5)
- A minimal extension
- Neutrino masses
- Unification
- Why 3-loop accuracy ?
- Results

Georgi Glashow SU(5)

VOLUME 32, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1974

Unity of All Elementary-Particle Forces

Howard Georgi* and S. L. Glashow
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 10 January 1974)

Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5).

We present a series of hypotheses and speculations leading inescapably to the conclusion that SU(5) is the gauge group of the world—that all elementary particle forces (strong, weak, and electromagnetic) are different manifestations of the same fundamental interaction involving a single coupling strength, the fine-structure constant. Our hypotheses may be wrong and our speculations idle, but the uniqueness and simplicity of our scheme are reasons enough that it be taken seriously.

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Georgi Glashow SU(5)

- Particle content and SM embedding:

- Gauge sector: $$24_V = \underbrace{(1,1,0)_V}_{B} \oplus \underbrace{(1,3,0)_V}_{W} \oplus \underbrace{(8,1,0)_V}_{G} \oplus \underbrace{(3,2,-\tfrac{5}{6})_V}_{X} \oplus \underbrace{(\overline{3},2,+\tfrac{5}{6})_V}_{\overline{X}}$$

- SM fermions: $$\overline{5}_F = \underbrace{(\overline{3},1,+\tfrac{1}{3})_F}_{d^c} \oplus \underbrace{(1,2,-\tfrac{1}{2})_F}_{\ell}$$

$$10_F = \underbrace{(\overline{3},1,-\tfrac{2}{3})_F}_{u^c} \oplus \underbrace{(3,2,+\tfrac{1}{6})_F}_{q} \oplus \underbrace{(1,1,+1)_F}_{e^c}$$

- Scalar sector: $$5_H = \underbrace{(3,1,-\tfrac{1}{3})_H}_{\mathcal{T}} \oplus \underbrace{(1,2,+\tfrac{1}{2})_H}_{h}$$

$$24_H = \underbrace{(1,1,0)_H}_{S_H} \oplus \underbrace{(1,3,0)_H}_{T_H} \oplus \underbrace{(8,1,0)_H}_{O_H} \oplus \underbrace{(3,2,-\tfrac{5}{6})_H}_{X_H} \oplus \underbrace{(\overline{3},2,+\tfrac{5}{6})_H}_{\overline{X}_H}$$

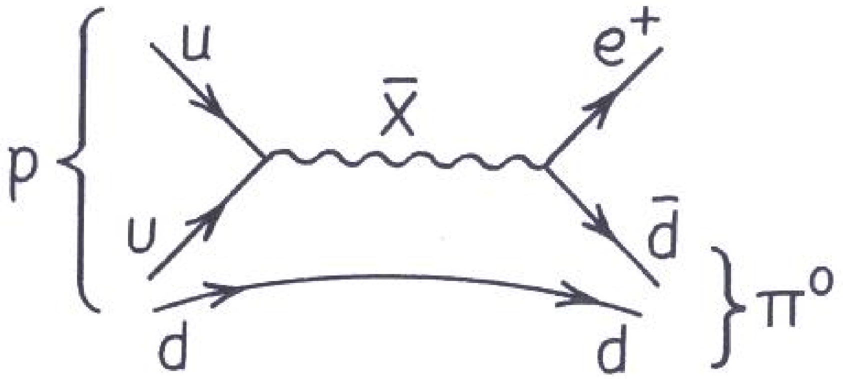
Georgi Glashow SU(5)

- Particle content and SM embedding: $24_V, \overline{5}_F, 10_F, 5_H, 24_H$

  - SU(5) breaking

$$SU(5) \xrightarrow[M_X]{\langle 24_H \rangle} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow[M_Z]{\langle 5_H \rangle} SU(3)_C \otimes U(1)_Q$$

  - B and L number violating currents: proton is unstable



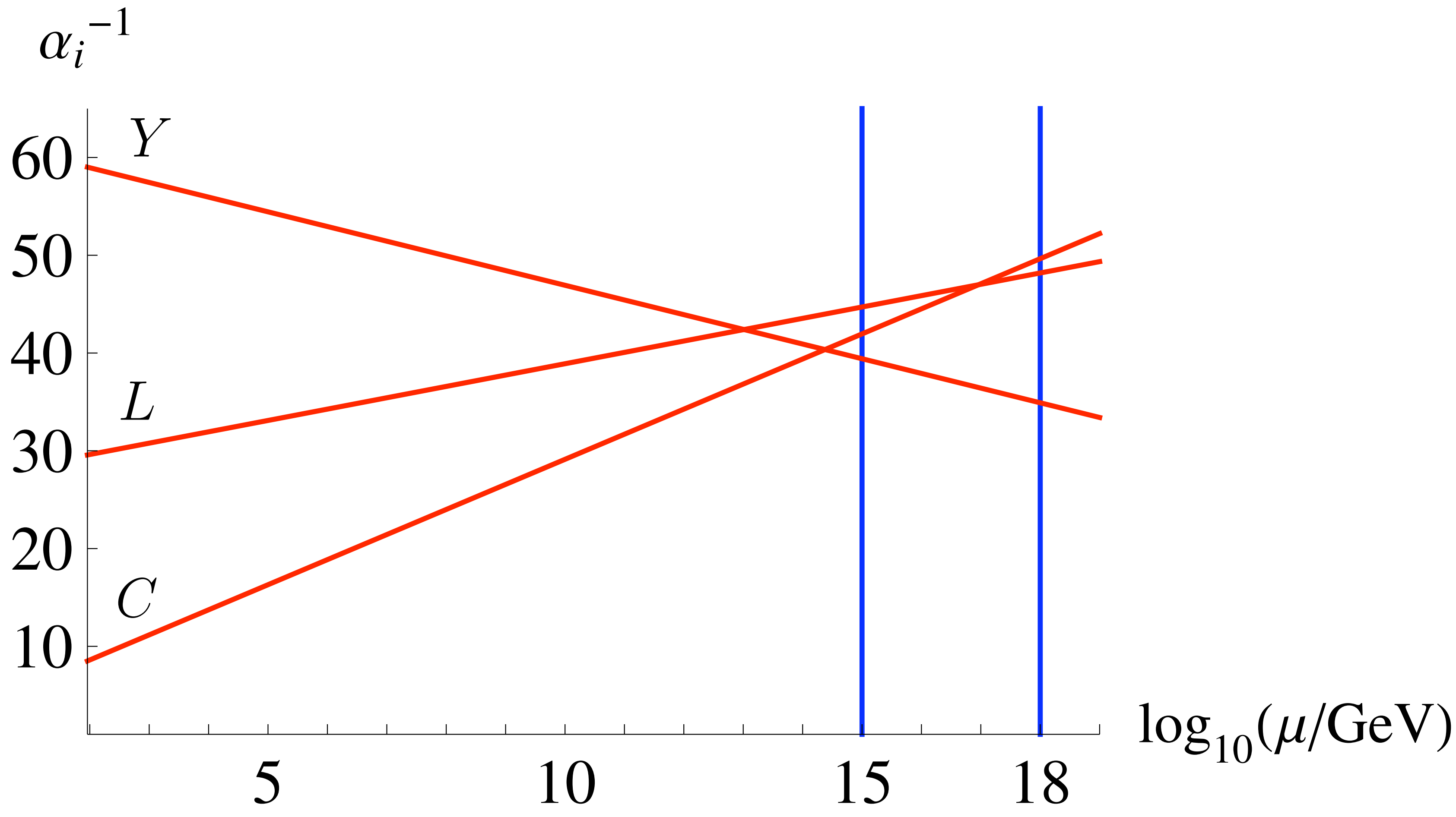
$$\tau_p^{\text{exp}} > 10^{34}\ \text{yr}$$ [Super-Kamiokande (2012)]

$$\tau_p^{\text{th}} \approx \alpha_G^{-2} \frac{M_X^4}{m_p^5}$$

$$\Longrightarrow \qquad M_X \gtrsim 10^{15.5}\ \text{GeV}$$

Why is the GG SU(5) ruled out ?

- Gauge couplings do not unify (even after including thresholds)



Why is the GG SU(5) ruled out ?

- Gauge couplings do not unify (even after including thresholds)
- Neutrinos are (practically) massless

$$\mathcal{L}_Y = \underbrace{Y_1 \bar{5}_F 10_F 5^*_H}_{M_D = M_E^T} + \underbrace{Y_2 10_F 10_F 5_H}_{M_U} + \frac{1}{\Lambda}\left[Y_3 \bar{5}_F \bar{5}_F 5_H 5_H + \ldots \right]$$

$$m_\nu \sim Y_3 \frac{v^2}{\Lambda} \lesssim 10^{-4}\ \text{eV} \quad \longleftrightarrow \quad m_{\nu_3} \gtrsim \sqrt{\Delta m^2_{\text{atm}}} \sim 0.05\ \text{eV}$$

for $\Lambda \approx 100 \times M_G \approx 10^{17}$ GeV (b-tau + perturbativity)

Add a fermionic 24_F

- Solves both the problems at once*

[Bajc, Senjanovic (2006)]
[Bajc, Nemevsek, Senjanovic (2007)]

$$24_F = \underbrace{(1,1,0)_F}_{S_F} \oplus \underbrace{(1,3,0)_F}_{T_F} \oplus \underbrace{(8,1,0)_F}_{O_F} \oplus \underbrace{(3,2,-\tfrac{5}{6})_F}_{X_F} \oplus \underbrace{(\overline{3},2,+\tfrac{5}{6})_F}_{\overline{X}_F}$$

- Neutrino masses through seesaw

- RGEs are modified

*Minimal extension of GG SU(5) is not unique: e.g. add a 15_H

[Dorsner, Fileviez Perez (2005)]

[Dorsner, Fileviez Perez, Gonzalez Felipe (2005)]

Neutrino masses

- New Yukawa terms with 24_F

$$\delta\mathcal{L} = L_i \left(y_T^i T_F + y_S^i S_F \right) H + m_T T_F T_F + m_S S_F S_F + \text{h.c.}$$

- Mixed type-III + type-I seesaw

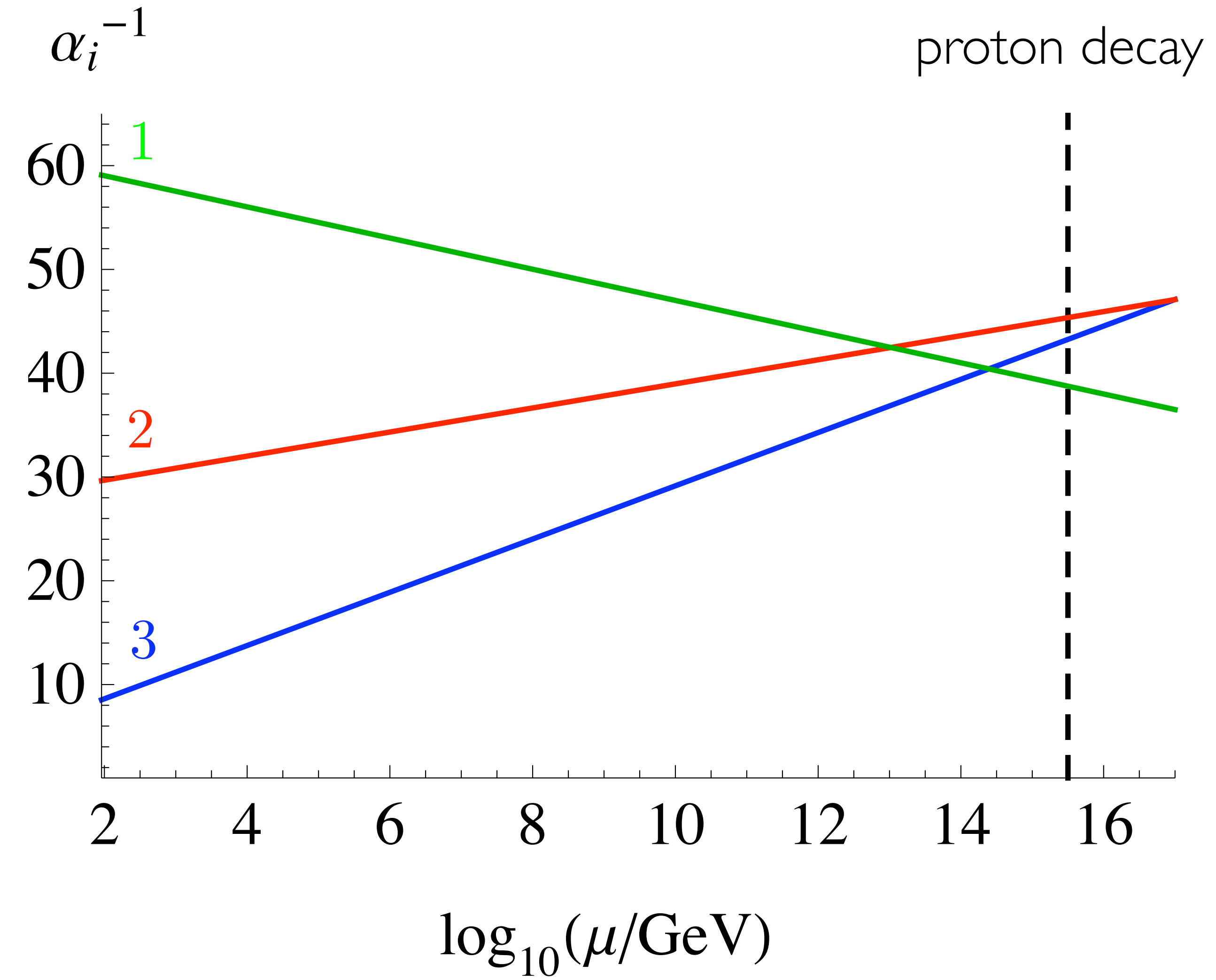
- Two massive neutrinos: $(m_\nu)^{ij} = v^2 \left(\frac{y_T^i y_T^j}{m_T} + \frac{y_S^i y_S^j}{m_S} \right)$

Unification patterns

- Running btw M_Z and M_G :

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \frac{a_i}{\pi} \log(\mu/\mu_0)$$

SU(5)	Threshold	$(\Delta a_1, \Delta a_2, \Delta a_3)$
5_H	$\mathcal{T}$	$(\frac{1}{60}, 0, \frac{1}{24})$
24_H	T_H	$(0, \frac{1}{12}, 0)$
24_F	T_F	$(0, \frac{1}{3}, 0)$
24_H	O_H	$(0, 0, \frac{1}{8})$
24_F	O_F	$(0, 0, \frac{1}{2})$
24_F	X_F	$(\frac{5}{6}, \frac{1}{2}, \frac{1}{3})$



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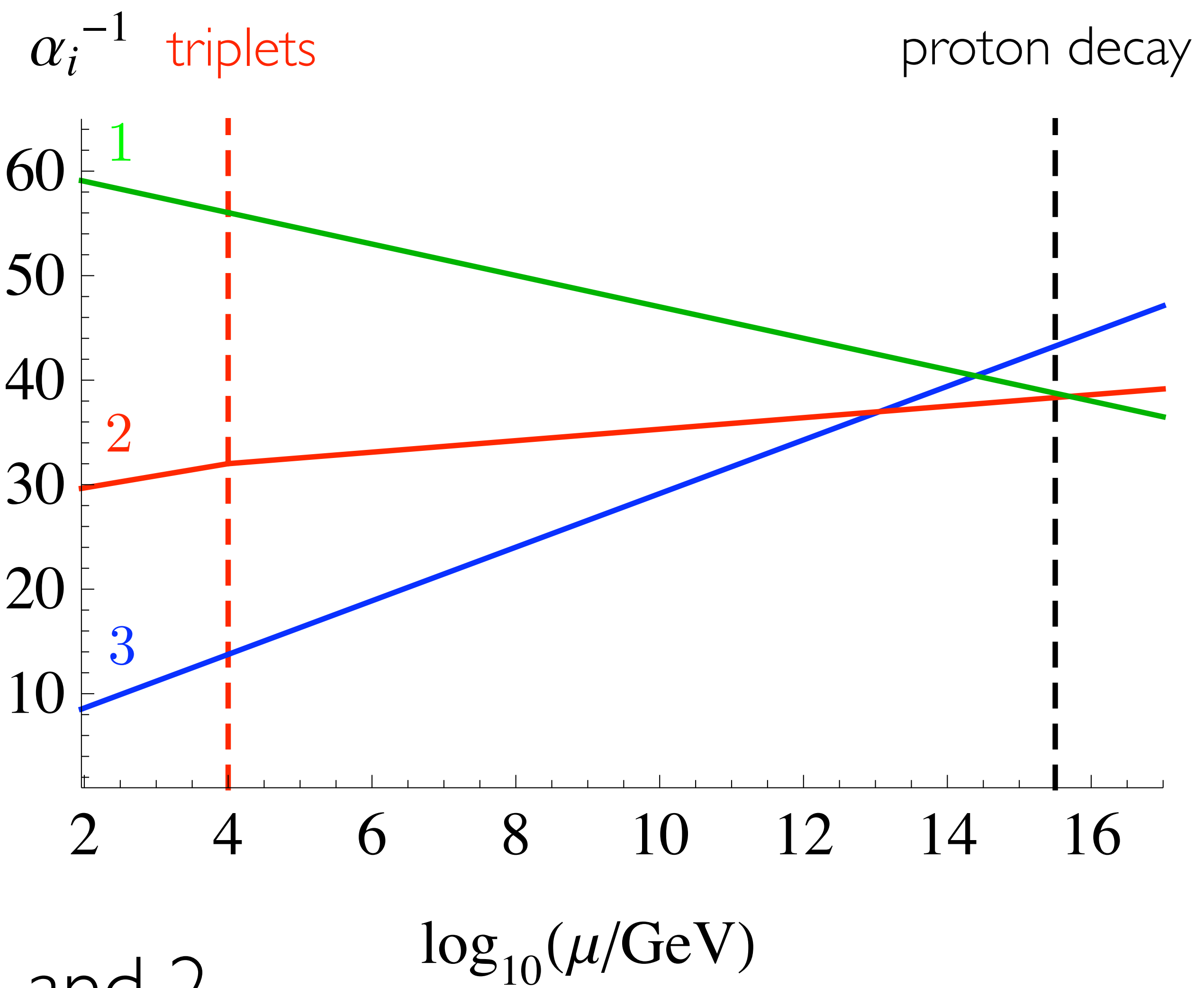

- $\Delta a_2 > \Delta a_1$ to delay the meeting of 1 and 2

Unification patterns

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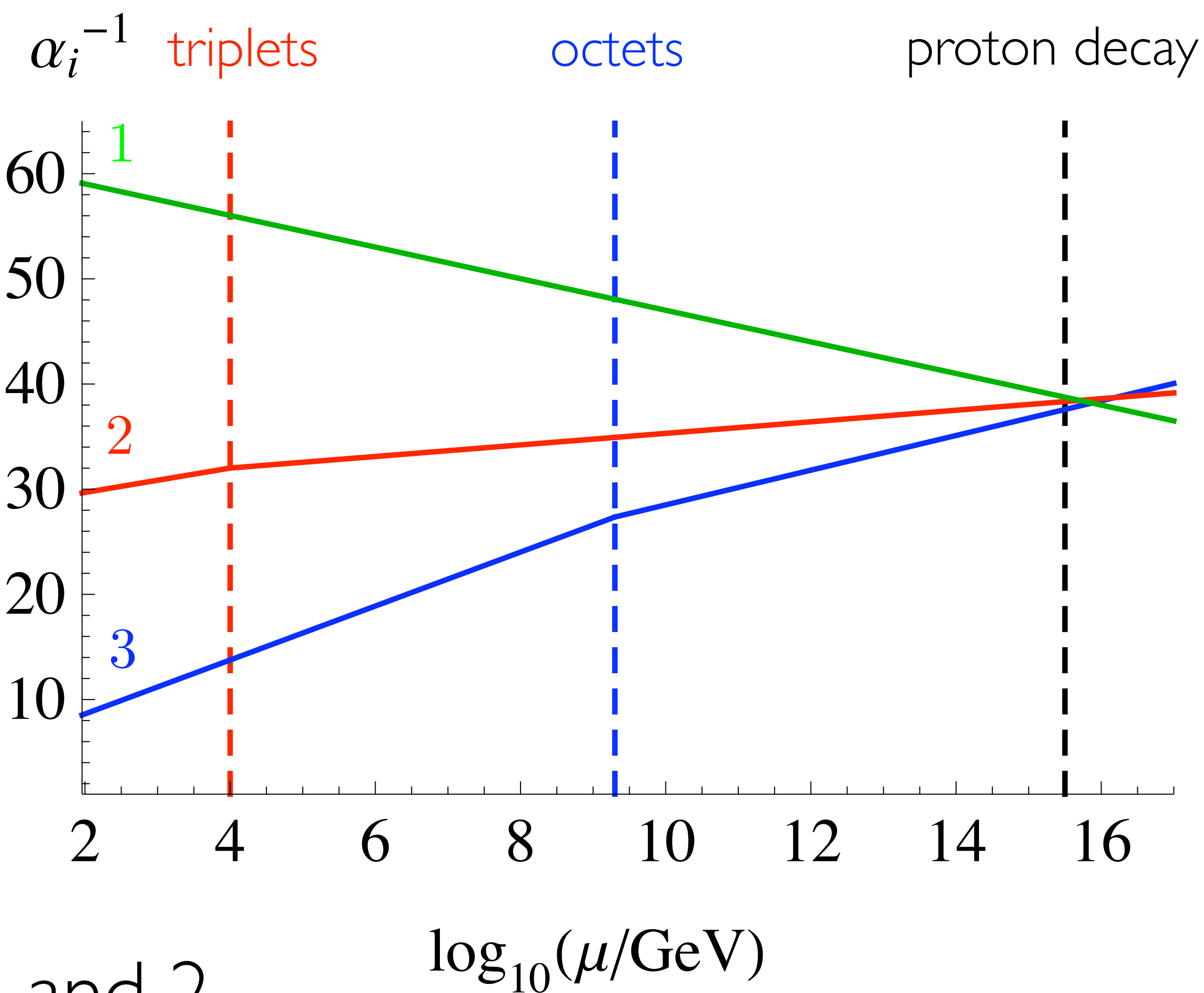
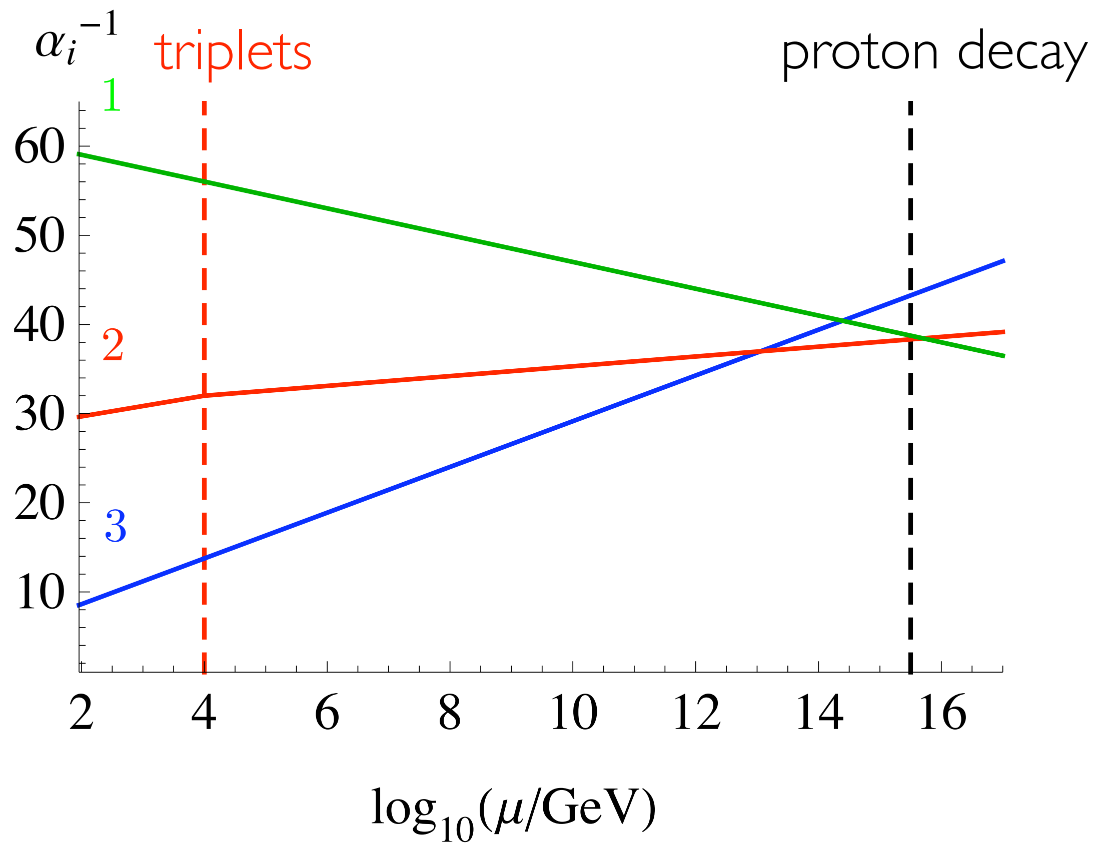


- $\Delta a_2 > \Delta a_1$ to delay the meeting of 1 and 2
- $\Delta a_3 > \Delta a_1, \Delta a_2$ for the convergence of 3 with 1 and 2
- unification patterns require $m_T \ll m_O \ll M_G$

How heavy the triplets can be ?

- RGEs can constrain max value of $m_3 = \left(m_{T_F}^4 m_{T_H}\right)^{1/5}$
  - maximize the mass of the extra thresholds with $\Delta a_1 > \Delta a_2$
  - m_3^{max} - M_G correlation depends on the convergence of α_1 and α_2 : precision observable !



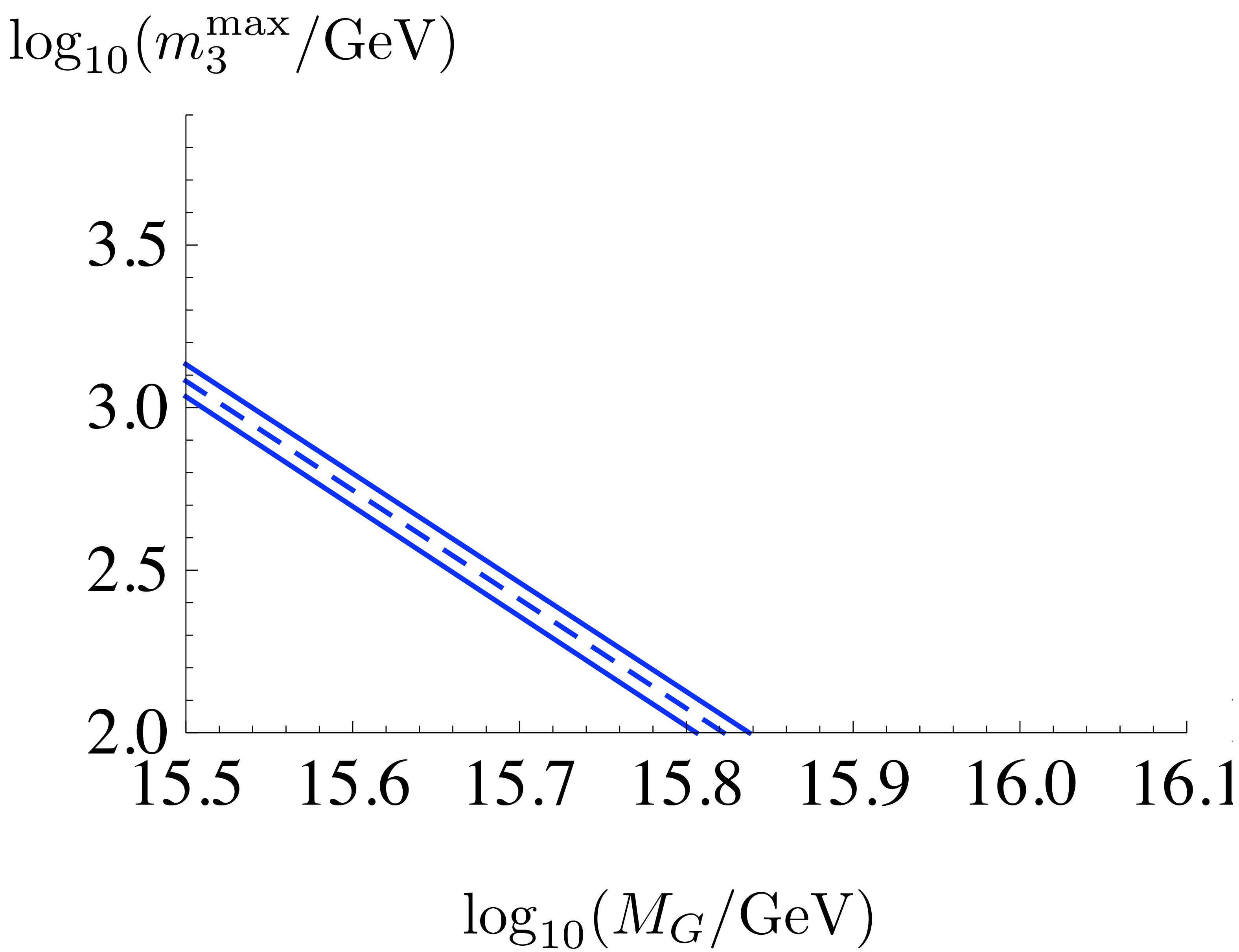
$$\frac{\Delta\alpha_1}{\alpha_1}(M_Z) \approx 0.02\%$$

$$\frac{\Delta\alpha_2}{\alpha_2}(M_Z) \approx 0.06\%$$

$$\frac{\Delta\alpha_3}{\alpha_3}(M_Z) \approx 0.6\%$$

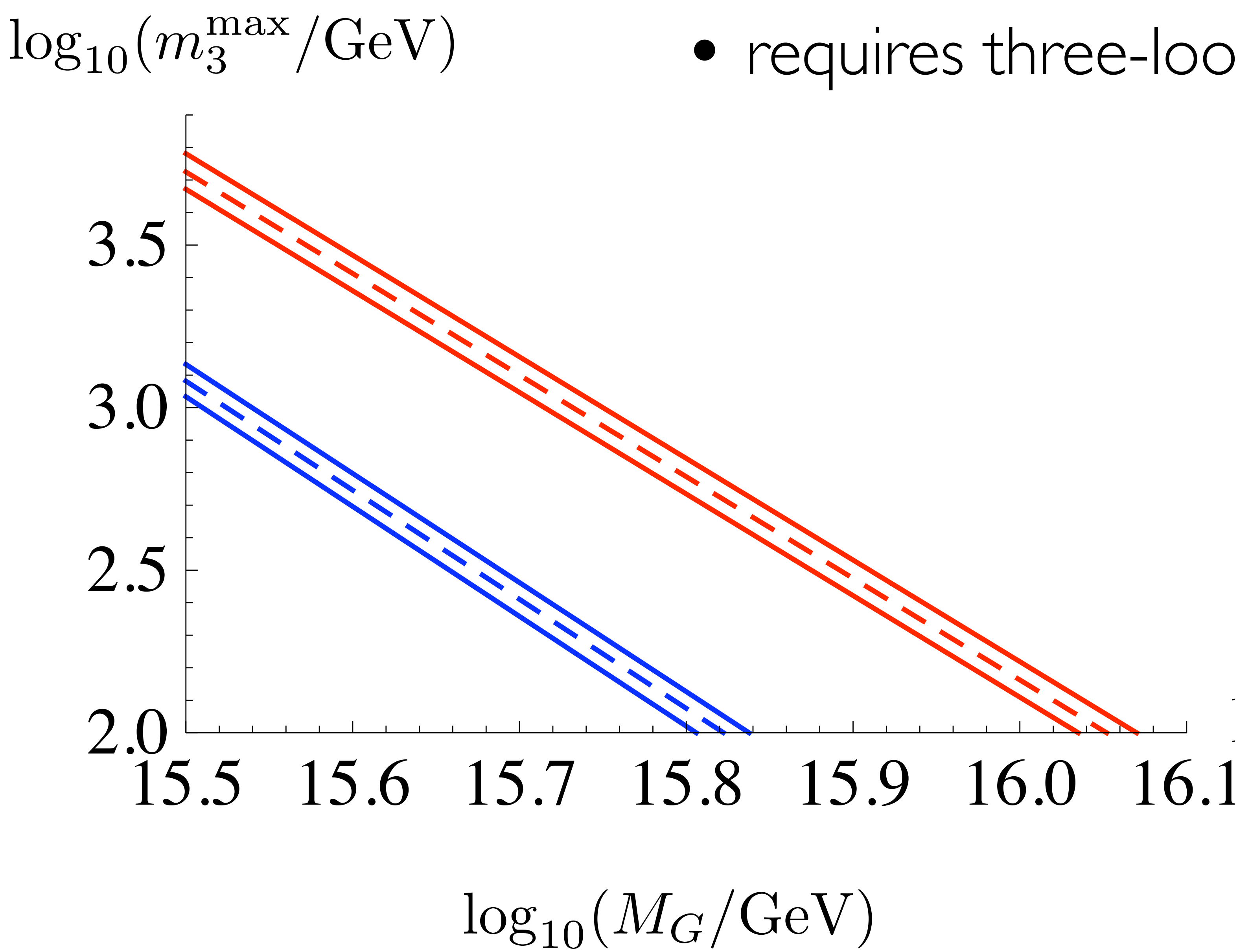
m_3^{max} - M_G correlation

- one-loop



m_3^{max} - M_G correlation

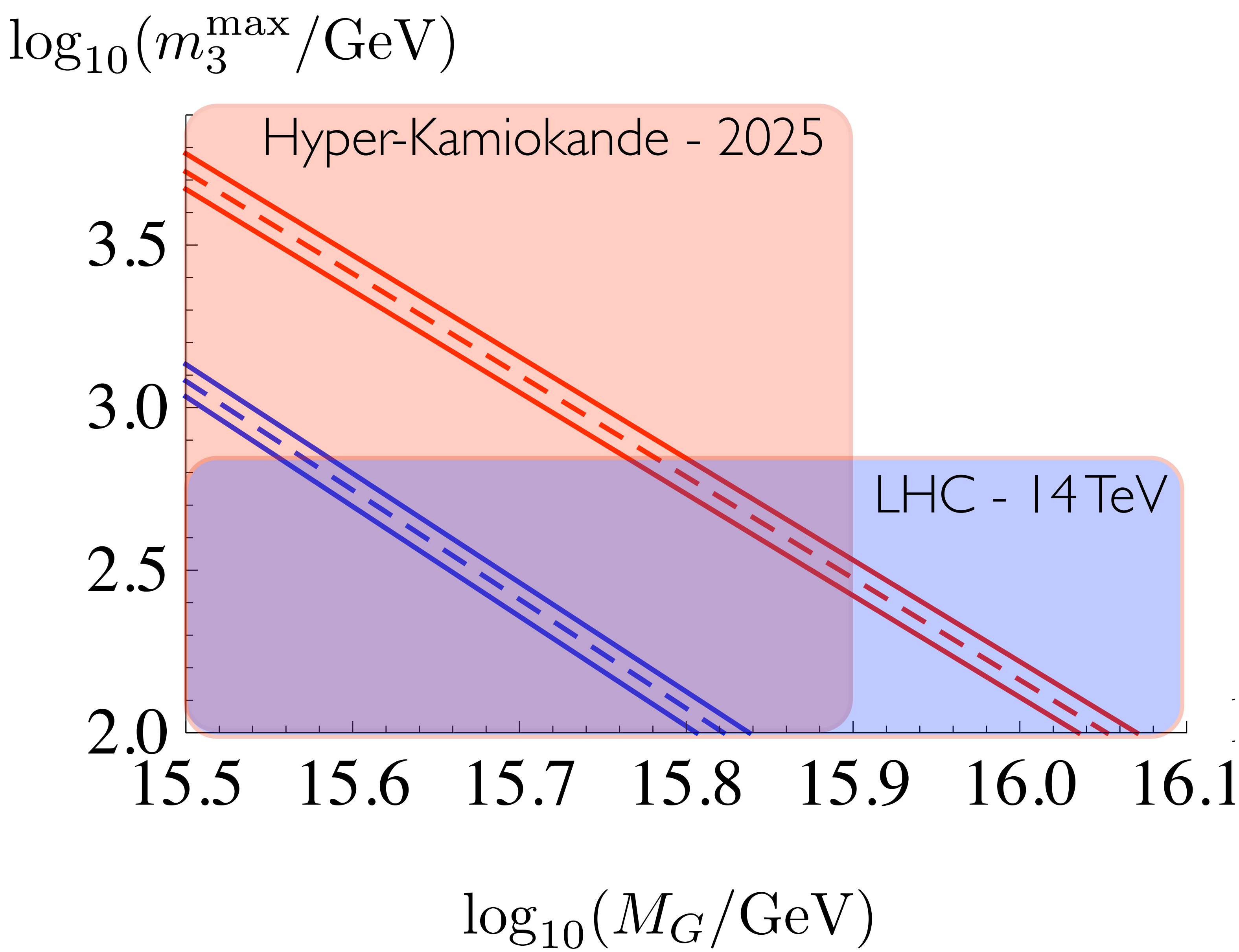
- one-loop ⟶ two-loops $\left(\frac{\Delta m_3}{m_3}\right)^{1\to 2-\text{loop}} = 340\%$ $\left(\frac{\Delta m_3}{m_3}\right)^{\text{exp}} = 25\%$



- requires three-loop accuracy !

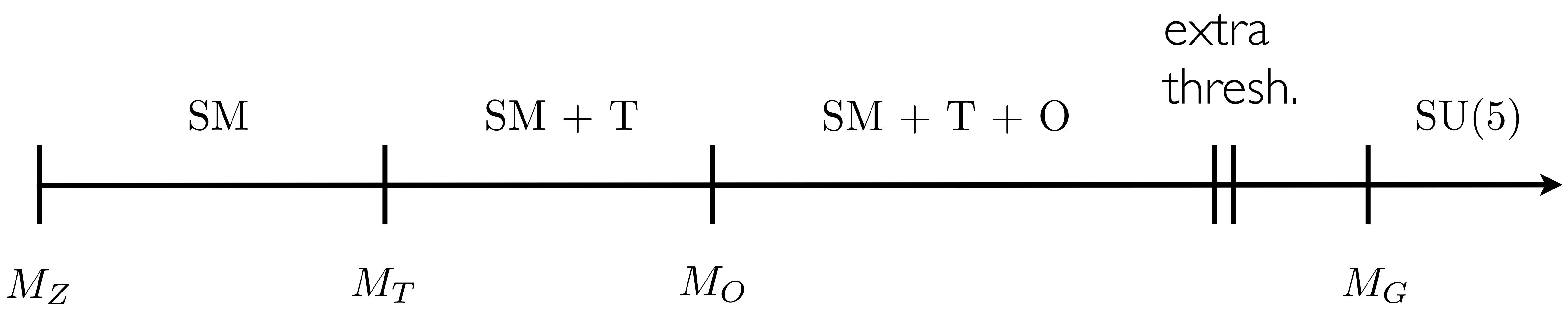
m_3^{max} - M_G correlation

- Interplay btw LHC and HK will cover most of the parameter space



Ingredients for a 3-loop analysis

- Effective field theories: n-loop running + (n-1)-loop matching



- 3-loop beta functions in the SM [Mihaila, Salomon, Steinhauser (2012)]
- 2-loop matching for SM $\longrightarrow$ SM + T $\longrightarrow$ SM + T + O (here)
- 3-loop beta functions in SM + T and SM + T + O (here)
- 2-loop matching at M_G (missing)

3-loop beta functions

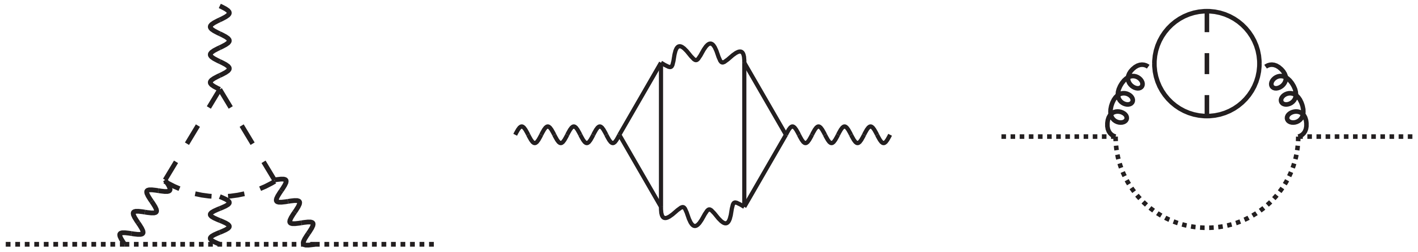
- Dimensional regularization and MSbar scheme

$$\mu^2 \frac{d}{d\mu^2}\frac{\alpha_i}{\pi} \equiv \beta_i(\{\alpha_j\},\epsilon) = -\epsilon\frac{\alpha_i}{\pi} - \left(\frac{\alpha_i}{\pi}\right)^2 \left[a_i + \sum_j \frac{\alpha_i}{\pi} b_{ij} + \sum_{jk} \frac{\alpha_j}{\pi}\frac{\alpha_k}{\pi} c_{ijk} + \ldots\right]$$

$$\alpha_i^{\text{bare}} = \mu^{2\epsilon} Z_{\alpha_i}(\{\alpha_j\},\epsilon)\,\alpha_i \quad \xrightarrow{\frac{d}{d\mu}\alpha_i^{\text{bare}} = 0} \quad \beta_i = f(Z_{\alpha_i})$$

- Calculation of Z_{α_i} at 3 loops: O(10^5) diagrams

$$Z_{\alpha_i} = \frac{(Z_{\text{vrtx}})^2}{\Pi_k Z_{k,\text{wf}}}$$



- One non-zero external momentum & all masses set to zero

3-loop beta functions

- An example: triplets contribution to β_2

$$\mu^2 \frac{d}{d\mu^2}\frac{\alpha_i}{\pi} = \beta_i(\{\alpha_j\},\epsilon)$$

$$\begin{aligned}
\Delta\beta_2 = \frac{\alpha_2^2}{\pi^2}\Bigg\{ & \frac{1}{6}C(G_L)\,N_{T_F} + \frac{1}{24}C(G_L)\,N_{T_H} + \frac{\alpha_2}{\pi}\left(\frac{1}{3}C(G_L)^2\,N_{T_F} + \frac{7}{48}C(G_L)^2\,N_{T_H}\right) \\
& + \frac{\alpha_2^2}{\pi^2}\Bigg[\Bigg(\frac{247}{432}C(G_L)^3 - \frac{7}{108}C(G_L)^2 T(R_L)\left(N(R_C)N_q + N_\ell\right) - \frac{11}{576}C(G_L)C(R_L)T(R_L)\left(N(R_C)N_q + N_\ell\right) \\
& - \frac{127}{3456}C(G_L)^2 T(R_L)N_h - \frac{25}{576}C(G_L)C(R_L)T(R_L)N_h - \frac{145}{3456}C(G_L)^3\,N_{T_F} - \frac{277}{6912}C(G_L)^3\,N_{T_H}\Bigg)N_{T_F} \\
& + \Bigg(\frac{2749}{6912}C(G_L)^3 - \frac{13}{432}C(G_L)^2 T(R_L)\left(N(R_C)N_q + N_\ell\right) - \frac{23}{2304}C(G_L)C(R_L)T(R_L)\left(N(R_C)N_q + N_\ell\right) \\
& - \frac{143}{6912}C(G_L)^2 T(R_L)N_h - \frac{49}{2304}C(G_L)C(R_L)T(R_L)N_h - \frac{145}{13824}C(G_L)^3\,N_{T_H}\Bigg)N_{T_H}\Bigg]\Bigg\}
\end{aligned}$$

3-loop beta functions

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$$+\frac{\alpha_2^2}{\pi^2}\Bigg[\Bigg(\frac{247}{432}C(G_L)^3 - \frac{7}{108}C(G_L)^2 T(R_L)\left(N(R_C)N_q + N_\ell\right) - \frac{11}{576}C(G_L)C(R_L)T(R_L)\left(N(R_C)N_q + N_\ell\right)$$

$$-\frac{127}{3456}C(G_L)^2 T(R_L)N_h - \frac{25}{576}C(G_L)C(R_L)T(R_L)N_h - \frac{145}{3456}C(G_L)^3\,N_{T_F} - \frac{277}{6912}C(G_L)^3\,N_{T_H}\Bigg)N_{T_F}$$

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2-loop matching coefficients

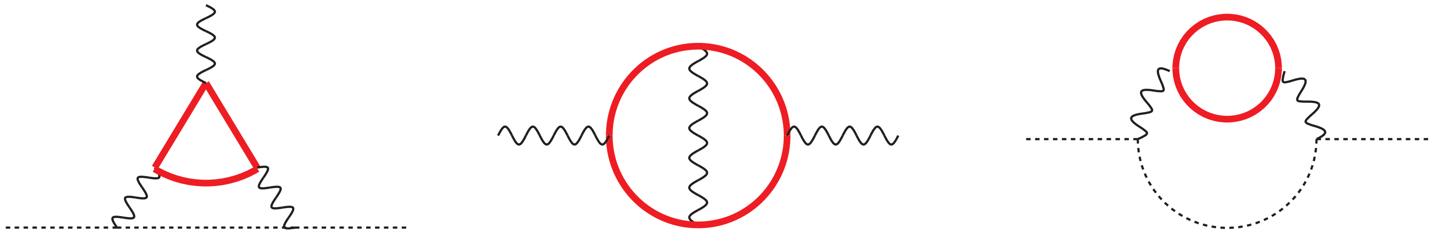
- Matching of effective fields theories [Weinberg (1980), Hall (1981)]

$$\mathcal{L}_{\rm full}(\alpha_i^{(\rm full)}, M_{\rm heavy}, \ldots)$$

$$\downarrow \quad \text{@ energy } \mu \approx M_{\rm heavy} \qquad \alpha_i^{(\rm eff)} = \zeta_i(\alpha_i, M_{\rm heavy}, \mu)\, \alpha_i^{(\rm full)}$$

$$\mathcal{L}_{\rm eff}(\alpha_i^{(\rm eff)}, \ldots)$$

- Match Green's functions of the light particles in the full and eff theory



- Zero external momenta & and all masses set to zero but $M_{\rm heavy}$

[Chetyrkin, Kniehl, Steinhauser (1998)]

2-loop matching coefficients

- An example: triplets contribution to ζ_{α_2} $\qquad \alpha_i^{(\text{eff})} = \zeta_i \, \alpha_i^{(\text{full})}$

$$\zeta_{\alpha_2} = 1 + \frac{\alpha_2}{\pi}\left(-\frac{1}{6}C\left(G_L\right)\ln\frac{\mu^2}{m_{T_F}^2}N_{T_F} - \frac{1}{24}C\left(G_L\right)\ln\frac{\mu^2}{m_{T_H}^2}N_{T_H}\right)$$
$$+\frac{\alpha_2^2}{\pi^2}\left[\left(-\frac{7}{288}C\left(G_L\right)^2 - \frac{1}{12}C\left(G_L\right)^2\ln\frac{\mu^2}{m_{T_F}^2} + \frac{1}{36}C\left(G_L\right)^2\ln^2\frac{\mu^2}{m_{T_F}^2}N_{T_F}\right)N_{T_F}\right.$$
$$+\left(\frac{37}{576}C\left(G_L\right)^2 - \frac{11}{96}C\left(G_L\right)^2\ln\frac{\mu^2}{m_{T_H}^2} + \frac{1}{576}C\left(G_L\right)^2\ln^2\frac{\mu^2}{m_{T_H}^2}N_{T_H}\right)N_{T_H}$$
$$\left.+\frac{1}{72}C\left(G_L\right)^2\ln\frac{\mu^2}{m_{T_F}^2}\ln\frac{\mu^2}{m_{T_H}^2}N_{T_F}N_{T_H}\right]$$

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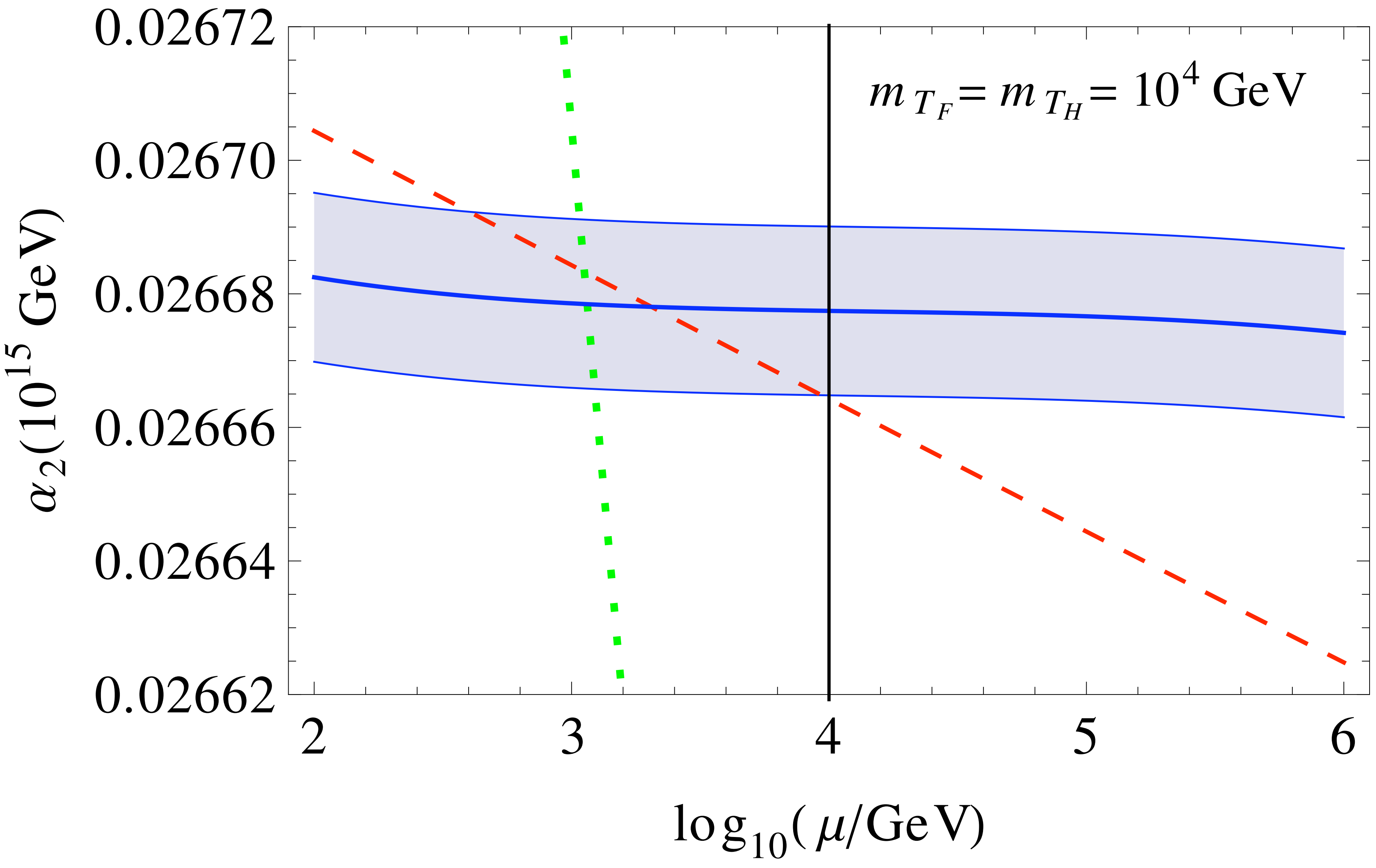
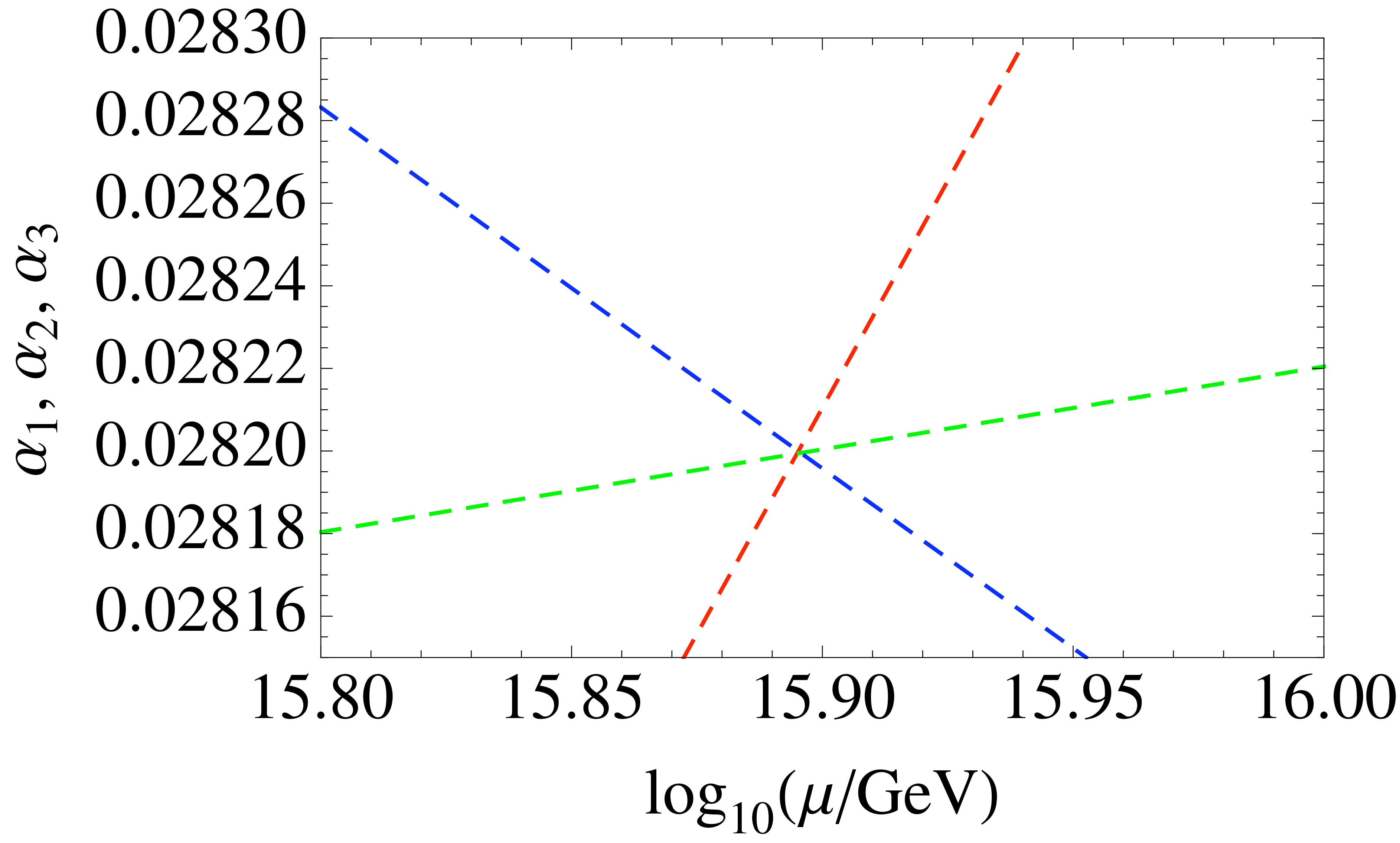
Scale dependence

$$\text{SM} \longrightarrow \text{SM} + \text{triplets}$$



Sample Unification



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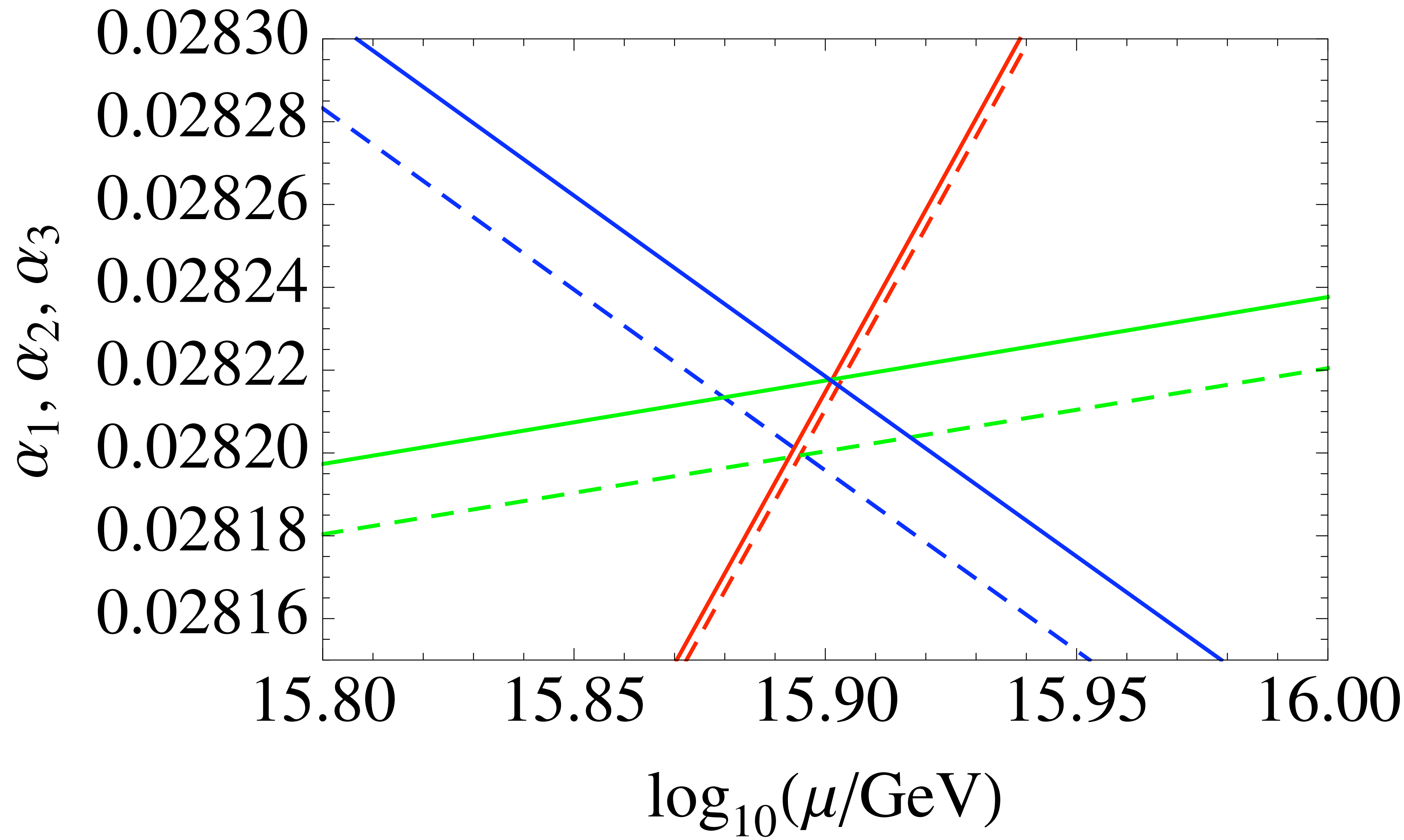
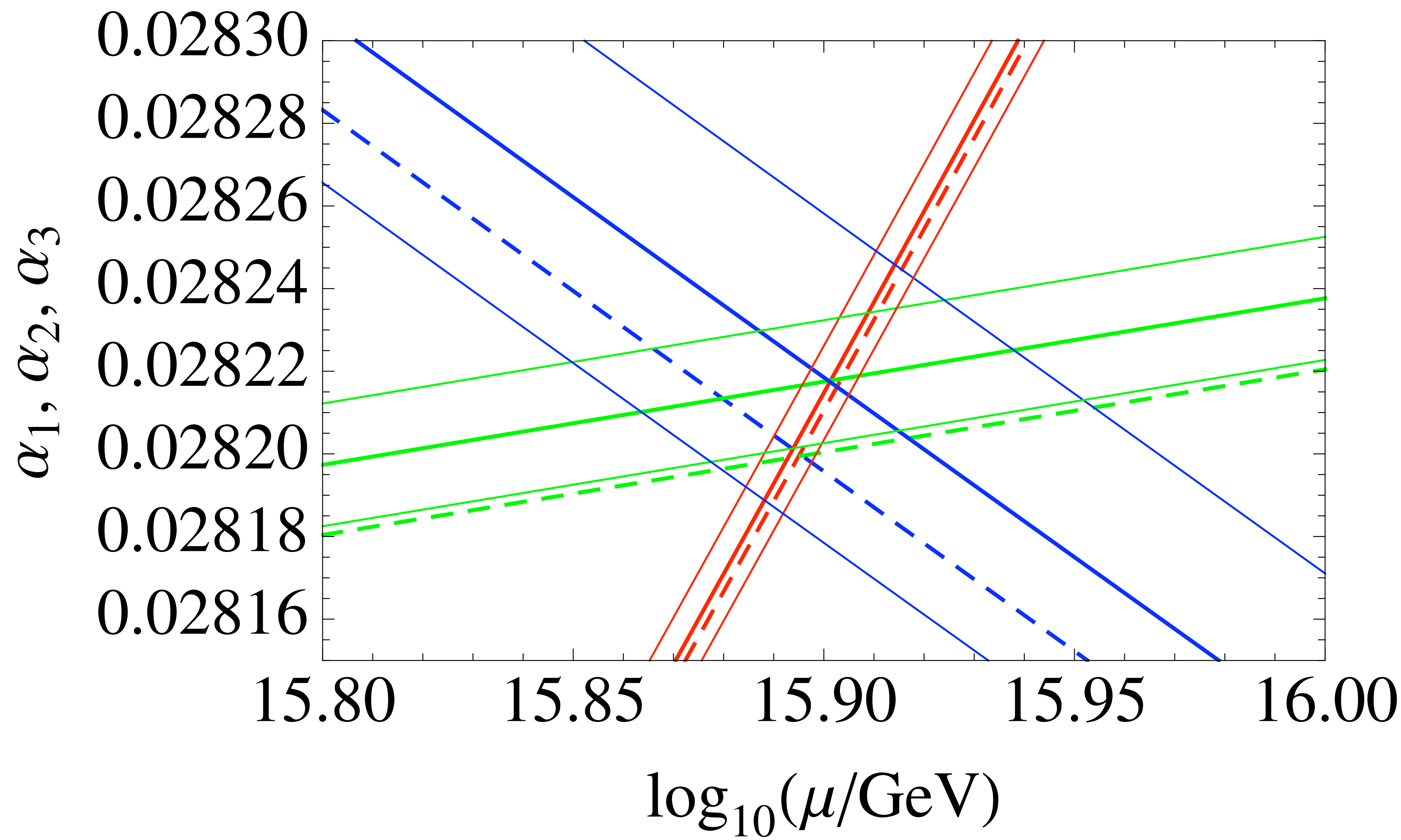


$$\left(\frac{\Delta\alpha_1}{\alpha_1}\right)^{2\to 3-\text{loop}} = 0.015\% \qquad \left(\frac{\Delta\alpha_2}{\alpha_2}\right)^{2\to 3-\text{loop}} = 0.061\% \qquad \left(\frac{\Delta\alpha_3}{\alpha_3}\right)^{2\to 3-\text{loop}} = 0.08\%$$

Sample Unification



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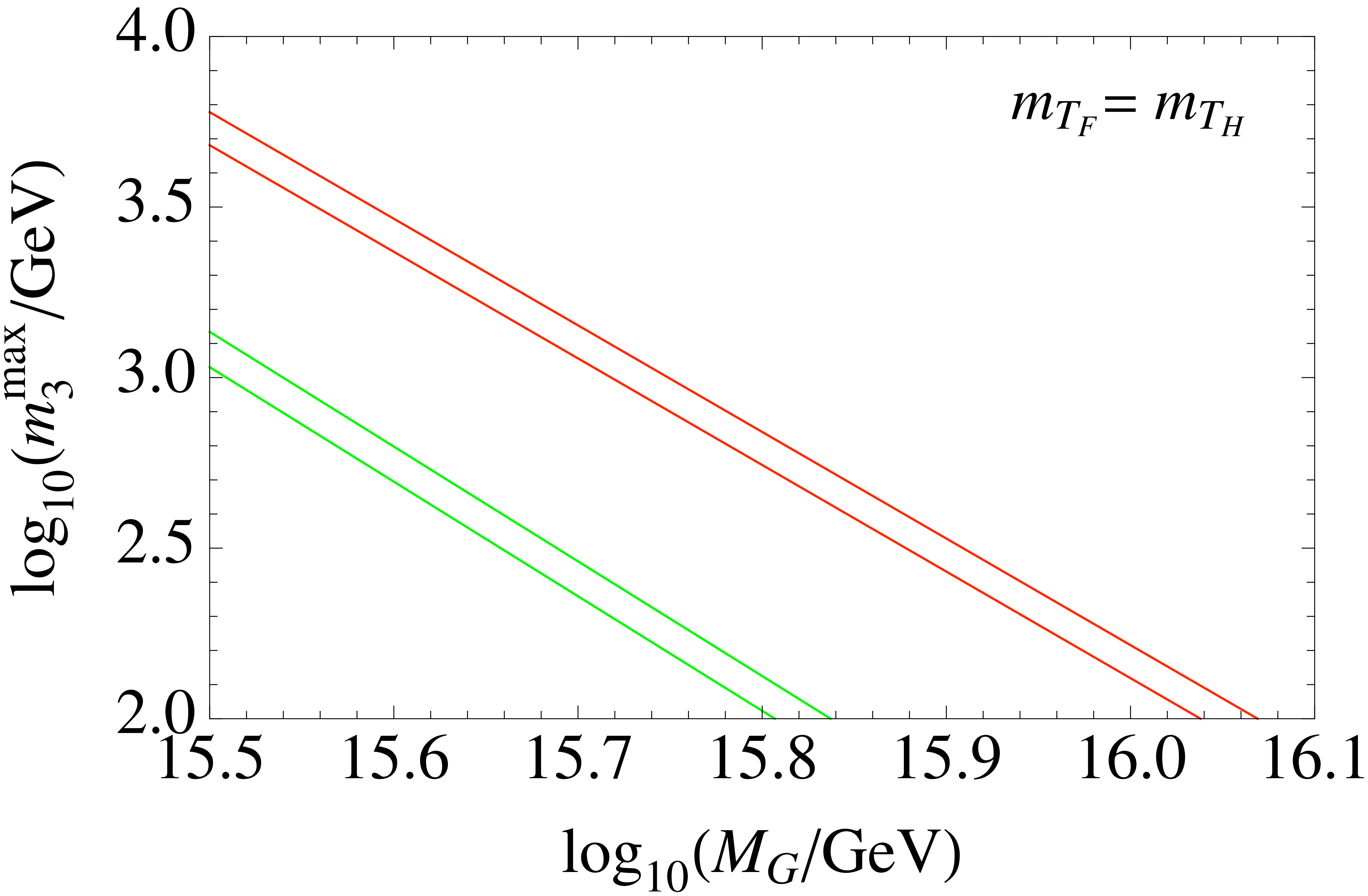
$$\left(\frac{\Delta\alpha_1}{\alpha_1}\right)^{\text{exp}} = 0.023\% \qquad \left(\frac{\Delta\alpha_2}{\alpha_2}\right)^{\text{exp}} = 0.059\% \qquad \left(\frac{\Delta\alpha_3}{\alpha_3}\right)^{\text{exp}} = 0.59\%$$

m_3^{max} - M_G correlation @ 3-loops



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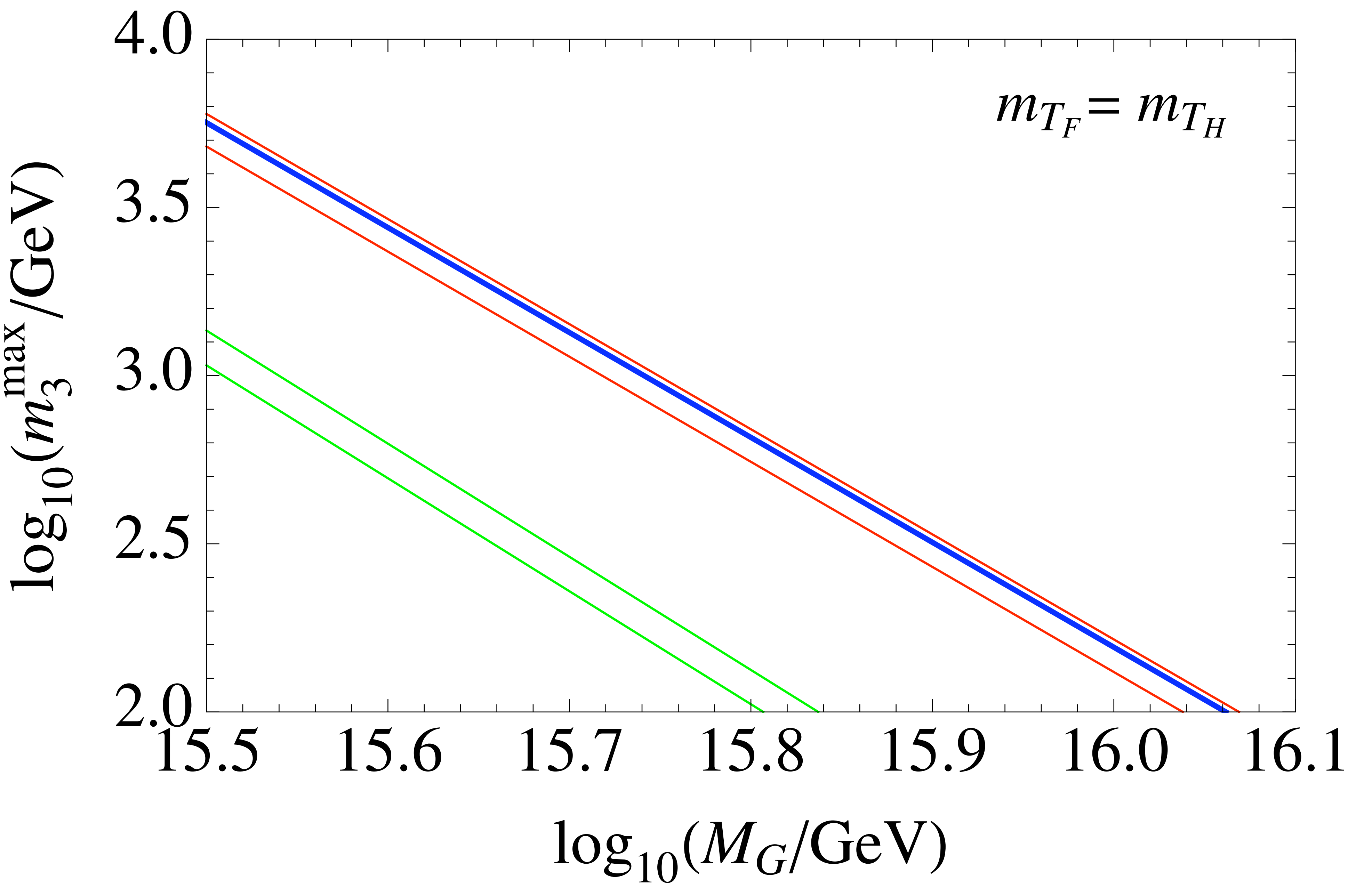

Conclusions

- Minimal extension of GG SU(5) with 24_F :

  - either light O(TeV) electroweak triplets

  - or unification scale < 10^{16} GeV

- Joint effort btw experiments (LHC, HK, ...) and theory

- 3-loops needed to:

  - settle the convergence of the perturbative series

  - match exp precision