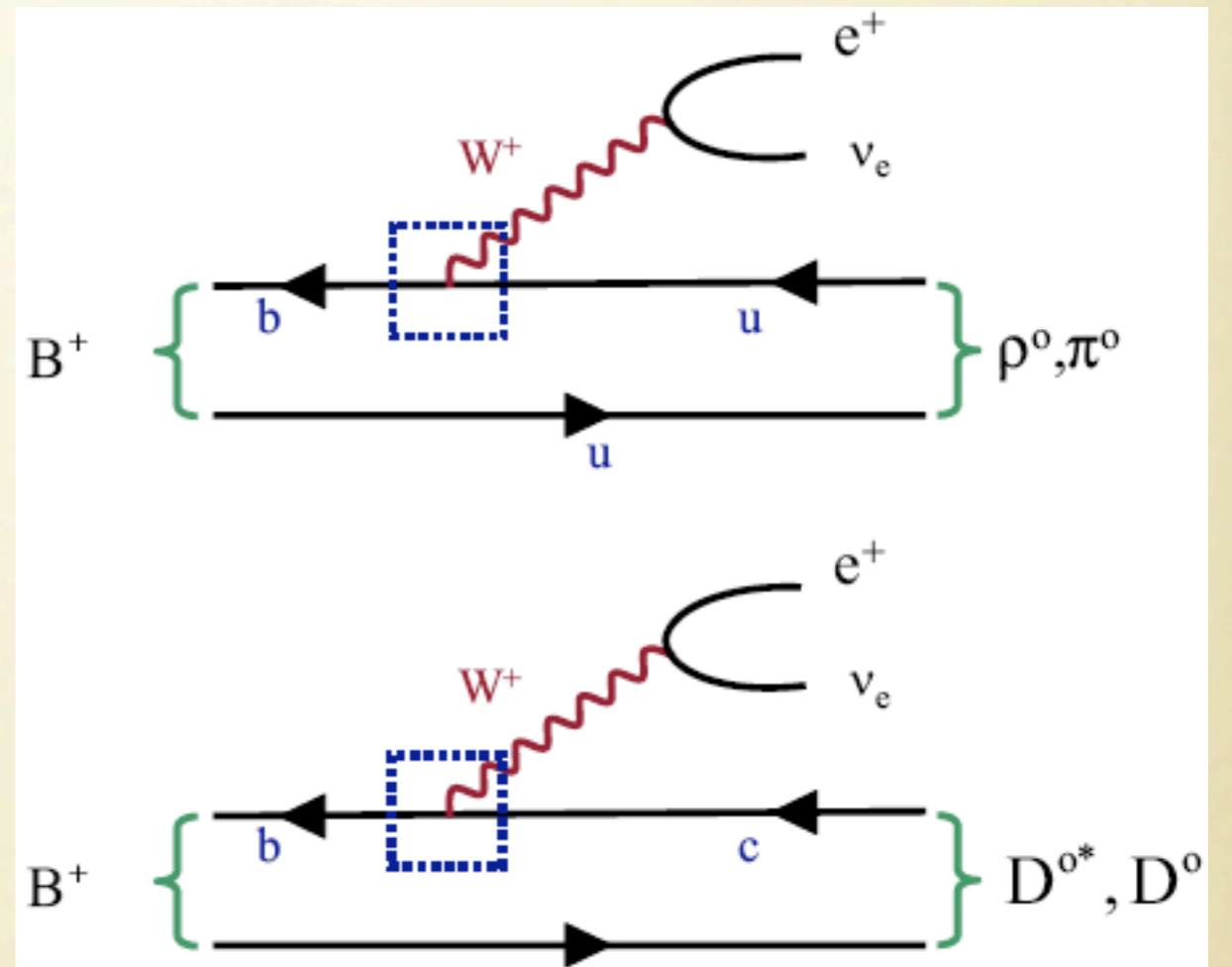
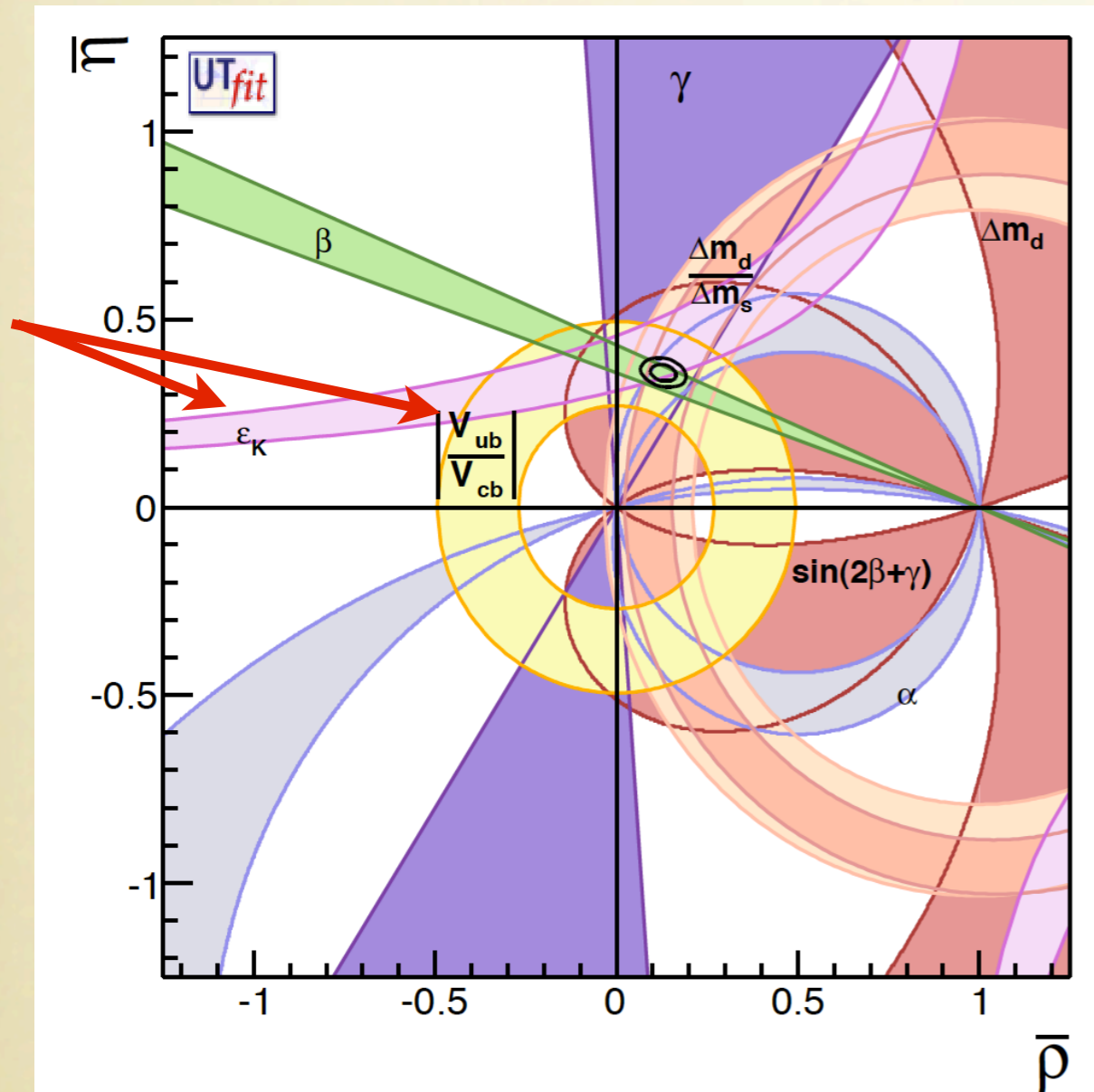


# RECENT DEVELOPMENTS IN SEMILEPTONIC B DECAYS

PAOLO GAMBINO  
UNIVERSITÀ DI TORINO

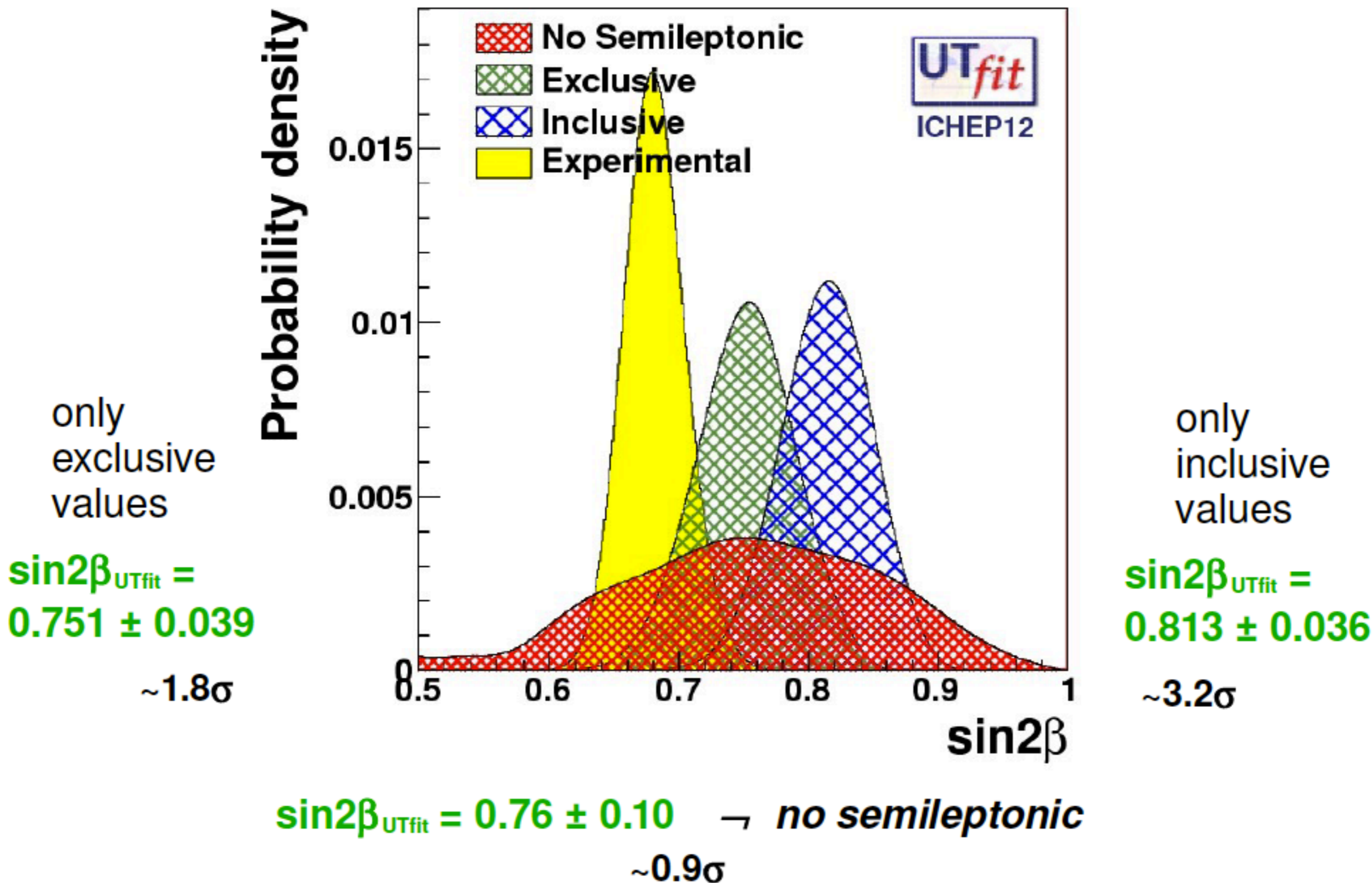
PORTOROZ, 15/4/2013

# S.L. DECAYS DETERMINE $|V_{ub}|$ AND $|V_{cb}|$

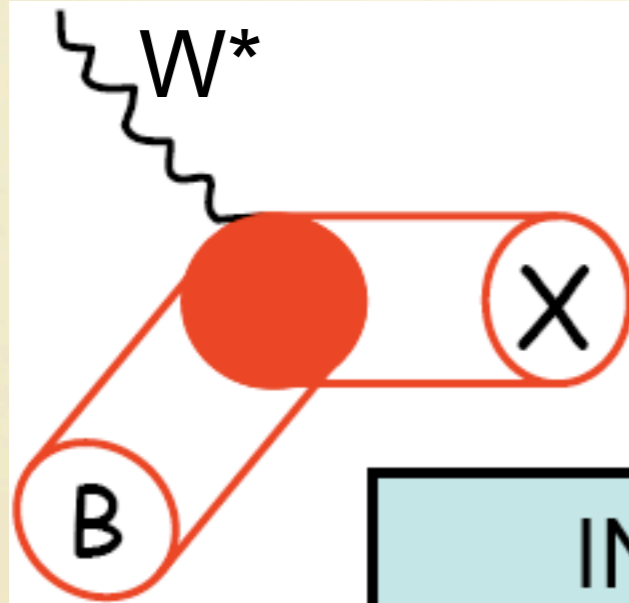


Since several years, exclusive decays prefer smaller  $|V_{ub}|$  and  $|V_{cb}|$

**inclusives vs exclusives**



# INCLUSIVE VS EXCLUSIVE B DECAYS



Simplicity: ew (or em) currents probe the B dynamics

INCLUSIVE	EXCLUSIVE
<b>OPE</b> : non-pert physics described by B matrix elements of local operators can be extracted by exp suppressed by $1/m_b^2$	<b>Form factors</b> : in general computed by non pert methods (lattice, sum rules,...) symmetry can provide normalization

# THE TOTAL WIDTH IN THE OPE

$$\Gamma[B \rightarrow X_c l \bar{\nu}] = \Gamma_0 g(r) \left[ 1 + \frac{\alpha_s}{\pi} c_1(r) + \frac{\alpha_s^2}{\pi^2} c_2(r) - \frac{\mu_\pi^2}{2m_b^2} + c_G(r) \frac{\mu_G^2}{m_b^2} + c_D(r) \frac{\rho_D^3}{m_b^3} + c_{LS}(r) \frac{\rho_{LS}^3}{m_b^3} + O\left(\alpha_s \frac{\mu_{\pi,G}^2}{m_b^2}\right) + O\left(\frac{1}{m_b^4}\right) \right]$$

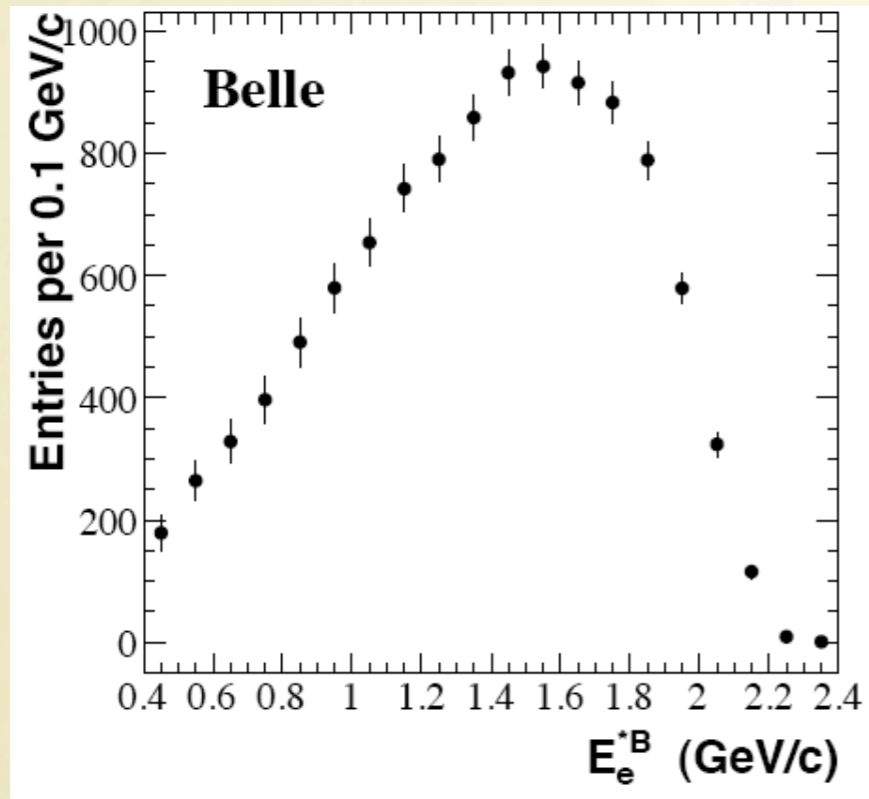
$$r = \frac{m_c^2}{m_b^2} \quad \Gamma_0 = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3}$$

OPE valid for inclusive enough measurements, away from perturbative singularities  $\Rightarrow$  moments

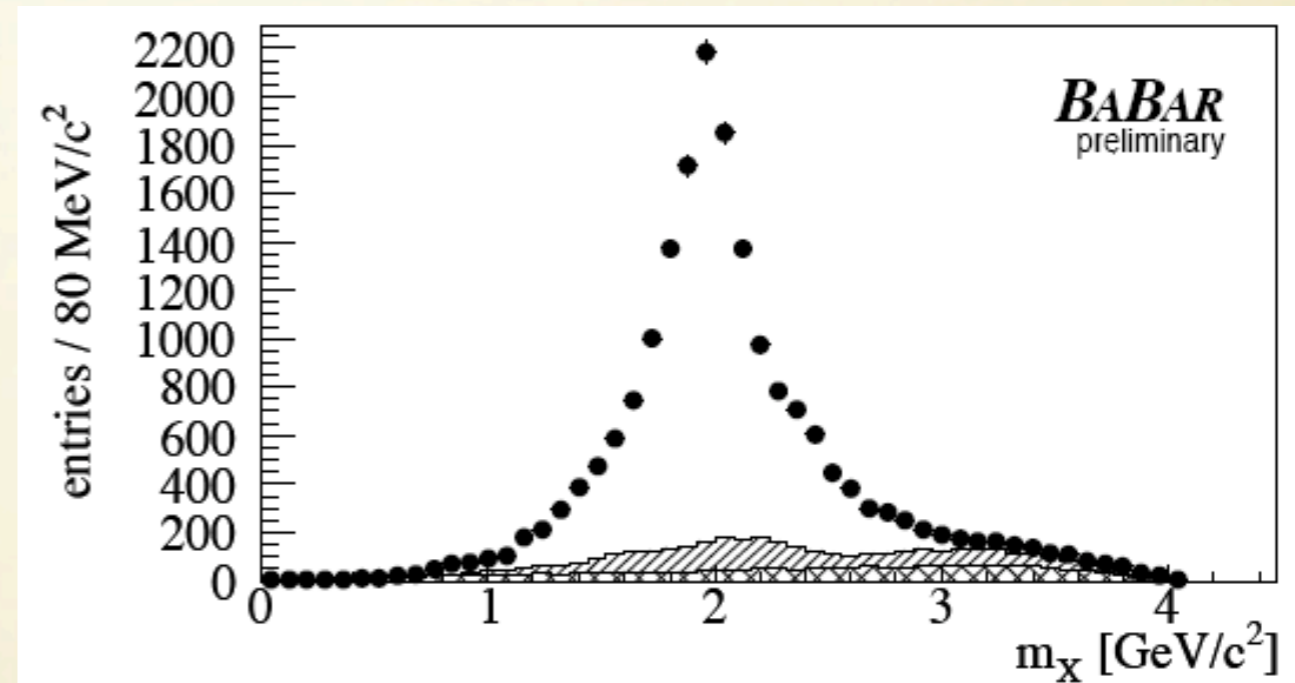
Present implementations include all terms through  $O(\alpha_s^2, 1/m_b^3)$ :  $m_{b,c}, \mu_{\pi,G}^2, \rho_{D,LS}^3$  6 parameters

# FITTING OPE PARAMETERS TO THE MOMENTS

$E_1$  spectrum



$m_x$  spectrum



Total **rate** gives  $|V_{cb}|$ , global **shape** parameters (first moments of the distributions) tell us about  $B$  structure,  $m_b$  and  $m_c$

*OPE parameters describe universal properties of the  $B$  meson and of the quarks  $\rightarrow$  useful in many applications*

# FITS AND MASS CONSTRAINTS

Inputs	$ V_{cb}  \cdot 10^3$	$m_b^{\text{kin}}$	$\chi^2/\text{ndf}$
$b \rightarrow c$ & $b \rightarrow s\gamma$	41.94(43)(58)	4.574(32)	29.7/59
$b \rightarrow c$ & $m_c$	41.88(44)(58)	4.560(23)	33.3/48

Schwanda, PG 2012, using  $m_c$  from Hoang et al

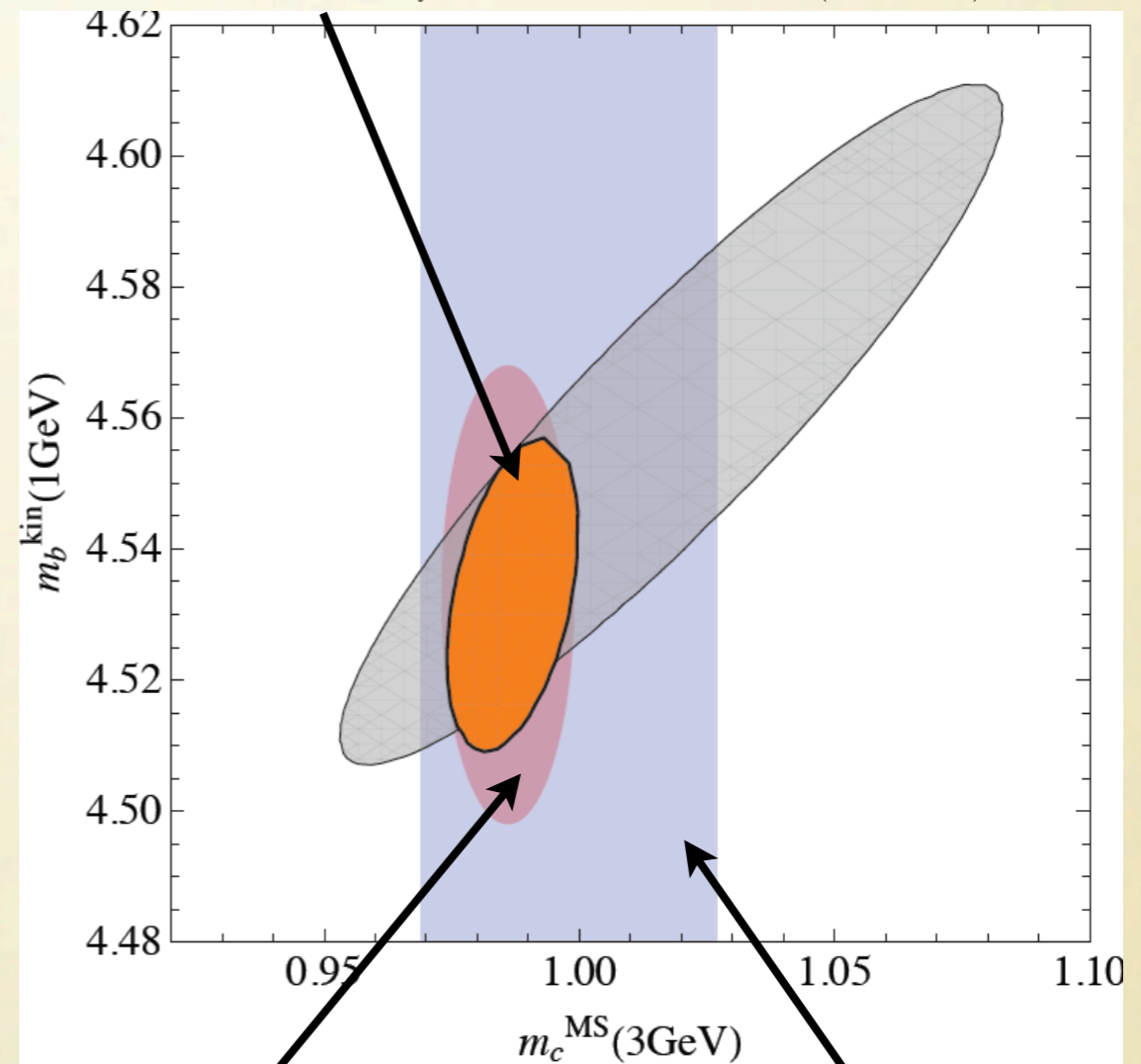
Similar results in 1S scheme (Bauer et al)

Recent sum rules studies of  $\sigma(e^+e^- \rightarrow \text{hadrons})$

and lattice calculations give very precise NNNLO determinations of  $m_c$  (and  $m_b$ ).

They are consistent with our NNLO fit and can be used to improve precision

SL fit with only  $m_c$  constraint (Kuhn)



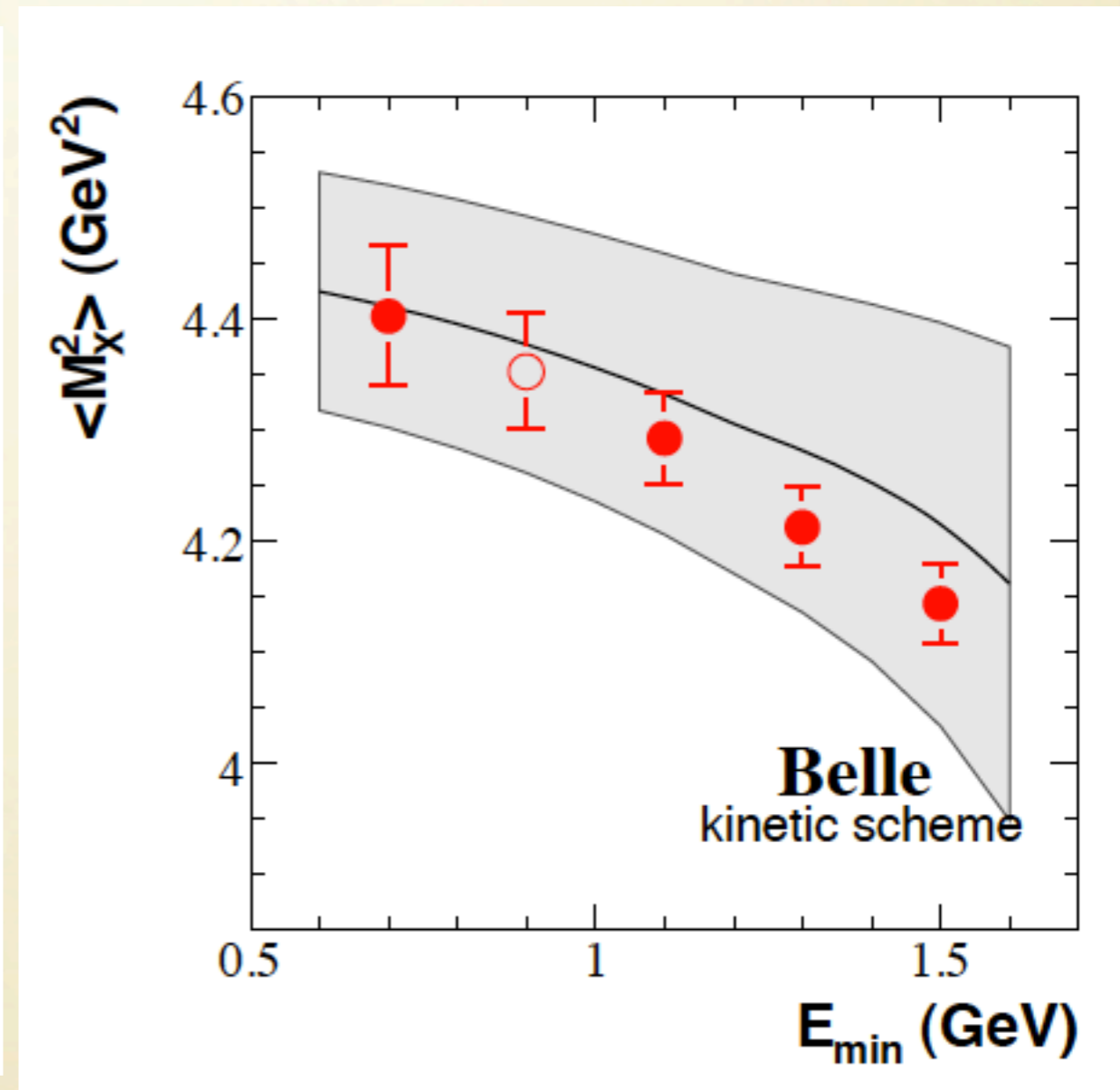
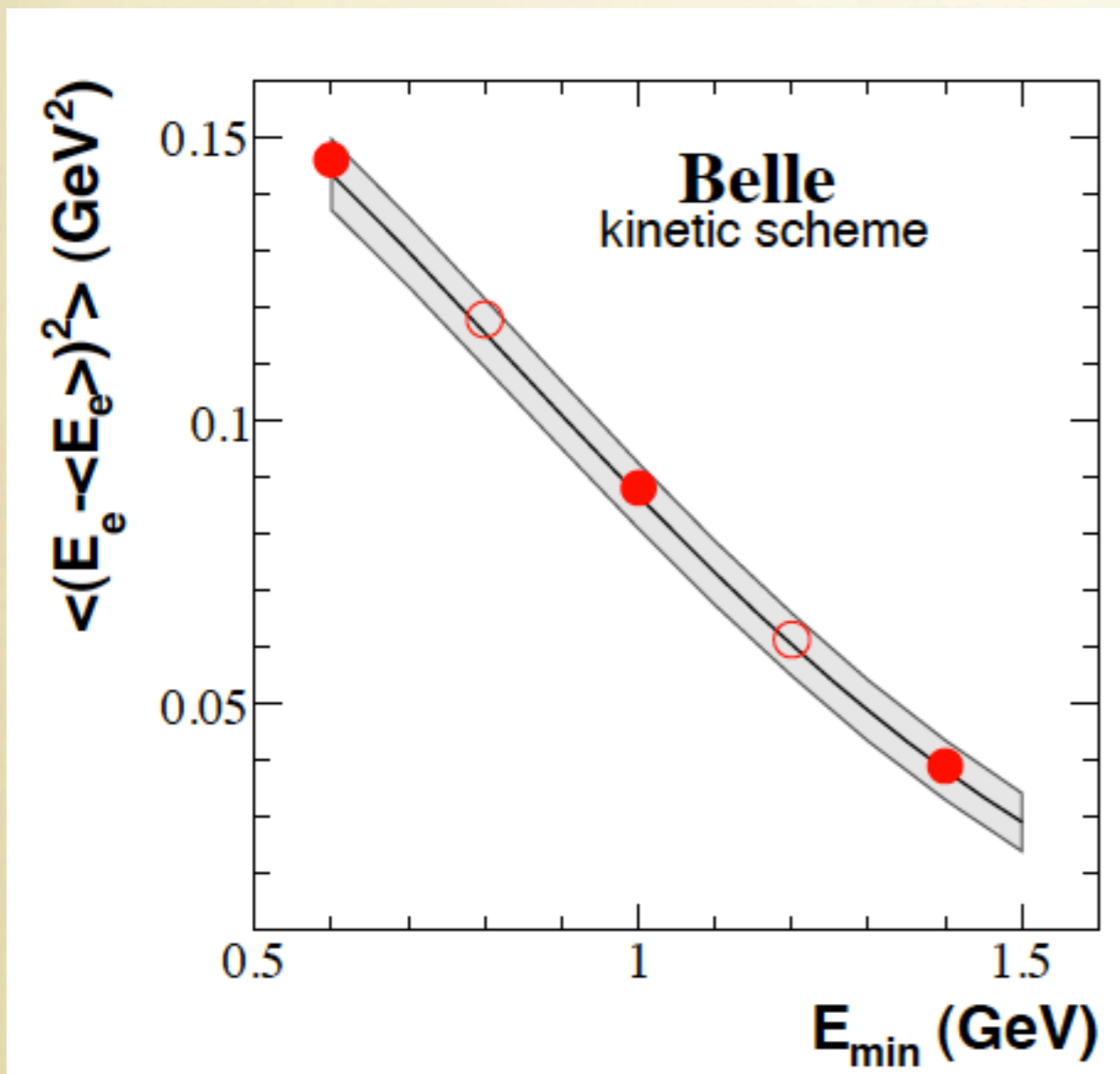
Kuhn et al 2009

Hoang et al 2011

$$m_c(3\text{GeV}) = 0.986(13)\text{GeV}$$

$$m_b(m_b) = 4.163(16)\text{GeV}$$

# THEORETICAL ERRORS DOMINATE





# HIGHER ORDER EFFECTS

- The reliability of the inclusive method depends on our ability to control higher order effect and quark-hadron duality violations.
- **Purely perturbative corrections** complete  $O(\alpha_s^2)$  known.  
Melnikov, Czarnecki, Pak, PG
- **Power corrections** included up to  $O(1/m_b^3)$ ,  $O(1/m_b^{4,5})$  known but involve many new parameters, numerical relevance under study. In vacuum saturation approx small effect on  $V_{cb}$   
Mannel, Turczyk, Uraltsev
- **Mixed** *perturbative corrections to power suppressed coefficients* at  $O(\alpha_s/m_b^2)$  almost finished, already known for  $b \rightarrow s\gamma$   
Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, PG

# $O(\alpha_s/m_b^2)$ EFFECTS

Boos,Becher,Lunghi 2007  
Alberti,Ewerth,Nandi,PG 2012

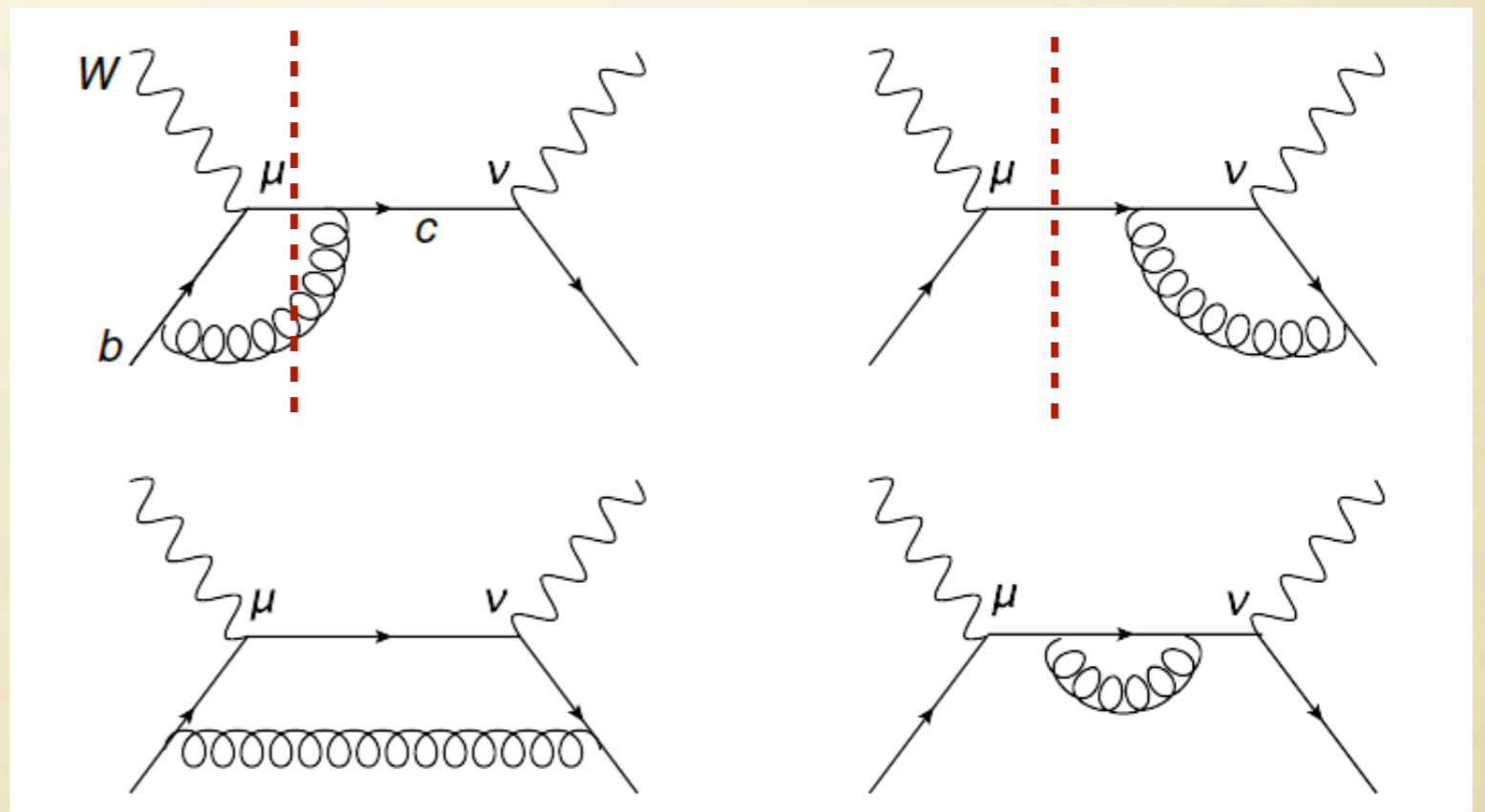
They can be computed using reparameterization invariance which relates different orders in the HQET

$$W_i = W_i^{(0)} + \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,0)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,0)} + \frac{C_F \alpha_s}{\pi} \left[ W_i^{(1)} + \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,1)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,1)} \right]$$

For  $i=3$  at all orders  $W_3^{(\pi,n)} = \frac{5}{3} \hat{q}_0 \frac{dW_3^{(n)}}{d\hat{q}_0} - \frac{\hat{q}^2 - \hat{q}_0^2}{3} \frac{d^2 W_3^{(n)}}{d\hat{q}_0^2}$  Manohar 2010

good testing ground for the calculation.

Proliferation of power divergences, up to  $1/u^3$ , and complex kinematics ( $q^2, q_0, m_c, m_b$ )



# RESULT FOR $W_1$

$$\begin{aligned}
 W_1^{(\pi,1)} &= 2E_0 B_{(1,\pi)} + \frac{8}{3}(1 - E_0 I_{1,0})(1 - E_0) \left( E_0 \left[ \frac{1}{\hat{u}^2} \right]_+ - 2 \left[ \frac{1}{\hat{u}} \right]_+ \right) + R_1^{(\pi)} \theta(\hat{u}) \\
 &+ \frac{8E_0}{3} \left[ \frac{1 - E_0}{2} S_1 - \lambda_0(1 - E_0 I_{1,0}) - \frac{E_0^2}{\rho} + (3E_0^2 - \rho) I_{1,0} - 2E_0(1 + I_{1,0}) + 3 \right] \delta'(\hat{u}) \\
 &+ \frac{8}{3} \left[ S_2 - E_0 S_1 + E_0(1 - E_0 I_{1,0}) \left( \frac{\lambda_0}{2} - \frac{(1 - E_0)^2}{\lambda_0} + E_0 \right) + \left( E_0^2 - \frac{3}{4} E_0 - \frac{\rho}{4} \right) I_{1,0} \right. \\
 &\left. + \frac{\frac{3}{4} E_0^3 - \frac{\rho}{4} (E_0^2 + 2E_0 - 2\rho(1 - E_0))}{\rho^2} \right] \delta(\hat{u}), \tag{4.2}
 \end{aligned}$$

$$B_{(i,\pi)} = \frac{\lambda_0}{3} \left\{ \left[ S_i + 3(1 - E_0 I_{1,0}) \right] \delta''(\hat{u}) - 4(1 - E_0 I_{1,0}) \left[ \frac{1}{\hat{u}^3} \right]_+ \right\}$$

$$\begin{aligned}
 R_1^{(\pi)} &= \frac{8}{3} E_0 \left( \frac{(1 - E_0)(E_0 - 2\hat{u})}{\hat{u}^2} - \frac{E_0 \lambda_0}{\hat{u}^3} \right) I_{1,0} - \frac{\frac{4}{3} \hat{u}^2 + 2(1 - 5E)\hat{u} + \frac{28}{3}(E^2 - E)}{\lambda} \left( I_1 - \frac{1}{E} \right) \\
 &+ \frac{8}{3} \left( \frac{7}{8} E - \frac{3}{8} - \frac{\hat{u}}{4} + \frac{E - z}{\hat{u}} + \frac{E^2 \lambda}{\hat{u}^3} + \frac{E(E - E^2 - \lambda)}{\hat{u}^2} \right) I_1 + \frac{16E^2(E - z)}{3\hat{u}^2 z} \\
 &- \frac{\hat{u}^2 (4E^2 - Ez - 2z^2)}{6Ez^3} + \frac{-8E^3 - 3E^2 z + 5Ez(z + 1) + (5 - 2z)z^2}{3z^3} \tag{4.7} \\
 &+ \frac{4E(2E^2 - 5Ez + 4z^2)}{3\hat{u} z^2} + \frac{\hat{u} (24E^4 - 12E^3 z + 4E^2(z - 3)z + E(z^2 - 6z^3) + 3z^3)}{6Ez^4},
 \end{aligned}$$

# EXCLUSIVE DECAY $B \rightarrow D^* \ell \nu$

At zero recoil, where rate vanishes, the  $ff$  is

$$\mathcal{F}(1) = \eta_A \left[ 1 + O\left(\frac{1}{m_c^2}\right) + \dots \right]$$

Recent progress in measurement of slopes and shape parameters, *exp error only*  $\sim 2\%$

The  $ff$   $F(l)$  cannot be experimentally determined. Lattice QCD is the best hope to compute it. Only one unquenched Lattice calculation:

$F(1) = 0.902(17)$  Laiho et al 2010

$|V_{cb}| = 39.6(0.7)(0.6) \times 10^{-3}$

**2.1% error (adding in quadrature)**

**$\sim 2\sigma$  or  $\sim 5\%$  from inclusive determination**

$B \rightarrow D \ell \nu$  has larger errors  $|V_{cb}| = 39.1(1.4)(1.3) \times 10^{-3}$

# ZERO RECOIL SUM RULE

Heavy quark sum rules put bounds on the zero recoil form factor  $\mathcal{F}(1)$  for  $B \rightarrow D^*$

Shifman, Vainshtein, Uraltsev 1996

$$\mathcal{F}(1) = \sqrt{I_0(\varepsilon_M) - I_{inel}(\varepsilon_M)} \quad \mathcal{F}(1) \leq \sqrt{I_0(\varepsilon_M)}$$

Unitarity bound

$$\mathcal{F}(1) < 0.935$$

- Starting point OPE for axial vector current at zero recoil: expansion of  $I_0$  in  $1/m_c$  and  $1/m_b$  and  $\alpha_s$
- Recent calculation incorporates higher order effects and estimates inelastic contributions
- Estimate of inelastic (non-resonant) contribution is hard

Mannel, Uraltsev, PG 2012

# THE INELASTIC CONTRIBUTION

$$I_1(\varepsilon_M) = -\frac{1}{2\pi i} \oint_{|\varepsilon|=\varepsilon_M} T(\varepsilon) \varepsilon d\varepsilon \quad I_{inel}(\varepsilon_M) = \frac{I_1(\varepsilon_M)}{\bar{\varepsilon}}$$

$\bar{\varepsilon}$  represents the average excitation energy mainly controlled by the lowest radial ( $1/2^+$ ) and D-wave ( $3/2^+$ ) excitations, therefore about 700MeV

OPE: 
$$I_1 = \frac{-(\rho_{\pi G}^3 + \rho_A^3)}{3m_c^2} + \frac{-2\rho_{\pi\pi}^3 - \rho_{\pi G}^3}{3m_c m_b} + \frac{\rho_{\pi\pi}^3 + \rho_{\pi G}^3 + \rho_S^3 + \rho_A^3}{4} \left( \frac{1}{m_c^2} + \frac{2}{3m_c m_b} + \frac{1}{m_b^2} \right) + \mathcal{O}\left(\frac{1}{m_Q^3}\right)$$

in terms of little known non-local correlators of the form

$$\frac{i}{2M_B} \int d^4x \langle B | T \{ O_i(x), O_j(x) \} | B \rangle' \quad O \sim \bar{b} \pi_k \pi_l b$$

$$\rho_{\pi\pi}^3 + \rho_{\pi G}^3 + \rho_S^3 + \rho_A^3 \geq 0$$

each of them is integral of spectral function with specific spin structure e.g.

$$\rho_{\pi\pi}^3 = \int_{\omega>0} d\omega \frac{\rho_p^{(\frac{1}{2}^+)}}{\omega}$$

# ESTIMATING THE NON-LOCAL GUYS

Hyperfine splitting

$$\Delta M_Q^2 = M_{Q^*}^2 - M_Q^2 = \frac{4}{3}c_G(m_Q)\mu_G^2 + \frac{2}{3}\frac{\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 + 2\bar{\Lambda}\mu_G^2}{m_Q} + O\left(\frac{1}{m_Q^2}\right)$$

Experimentally  $\Delta M_B^2 \simeq \Delta M_D^2$

$$\rho_{\pi G}^3 + \rho_A^3 \approx -0.45\text{GeV}^3$$

within a  $\sim 25\%$  uncertainty

From  $\bar{M}_B - \bar{M}_D$  and moments fits

$$\rho_{\pi G}^3 + \rho_A^3 \lesssim -0.33\text{GeV}^3$$

with somewhat larger uncertainty

These are strong indications that non-local guys are larger than expected.

Based on a BPS expansion we get a minimum  $I_{inel}(\varepsilon_M \sim 0.75\text{GeV}) \gtrsim 0.14 \pm 0.03$

using the lowest value of  $I_{inel}$  and interpreting the total uncertainty as gaussian

$$\mathcal{F}(1) = 0.86 \pm 0.02$$

in perfect agreement with inclusive  $V_{cb}$  which would lead to  $0.85 \pm 0.03$

# “RADIAL” CONTRIBUTIONS TO TOTAL WIDTH

- Non-local guys determined by transitions to + parity light d.o.f.:  $\frac{1^+}{2^-}$  ,  $\frac{3^+}{2^-}$  ,  $\frac{5^+}{2^-}$
- Large  $I_{inel}$  implies strong transitions to “radial” excitations (radial &  $D$ -wave states)
- Assuming a single multiplet of “radials” for each  $j_q$  hyperfine splitting constrains the strength of  $B \rightarrow$  “radials”
- At leading order in the heavy quark expansion and neglecting  $v$  dependence of the form factors one typically gets

$$\frac{\Gamma_{rad}}{\Gamma_{sl}} \approx 6 \div 7\%$$

Mannel, Uraltsev, PG

suggesting “radials” contribute significantly to the broad resonances, a possible solution of  $1/2 > 3/2$  puzzle

see also Bernlochner, Ligeti, Turczyk



# $|V_{ub}|$ DETERMINATIONS

**Inclusive: 5-6% total error**

HFAG 2012	Average $ V_{ub}  \times 10^3$
DGE	$4.45(15)_{\text{ex}}^{+15}_{-16}$
BLNP	$4.40(15)_{\text{ex}}^{+19}_{-21}$
GGOU	$4.39(15)_{\text{ex}}^{+12}_{-14}$

**Exclusive: 10-15% total error**

$$|V_{ub}| = (3.25 \pm 0.31) \times 10^{-3} \quad \text{MILC}$$

$$|V_{ub}| = \left( 3.50^{+0.38}_{-0.33} \Big|_{\text{th.}} \pm 0.11 \Big|_{\text{exp.}} \right) \times 10^{-3}$$

LCSR, Khodjamirian et al, see also Bharucha

$B \rightarrow \pi l \nu$  data poorly consistent!

2.7-3 $\sigma$  from  $B \rightarrow \pi l \nu$  (MILC-FNAL)

2 $\sigma$  from  $B \rightarrow \pi l \nu$  (LCSR, Siegen)

2.5-3 $\sigma$  from UTfit 2011

UT fit (without direct  $V_{ub}$ ):

$$V_{ub} = 3.62(14) \times 10^{-3}$$

The discrepancy here is around 30% !!

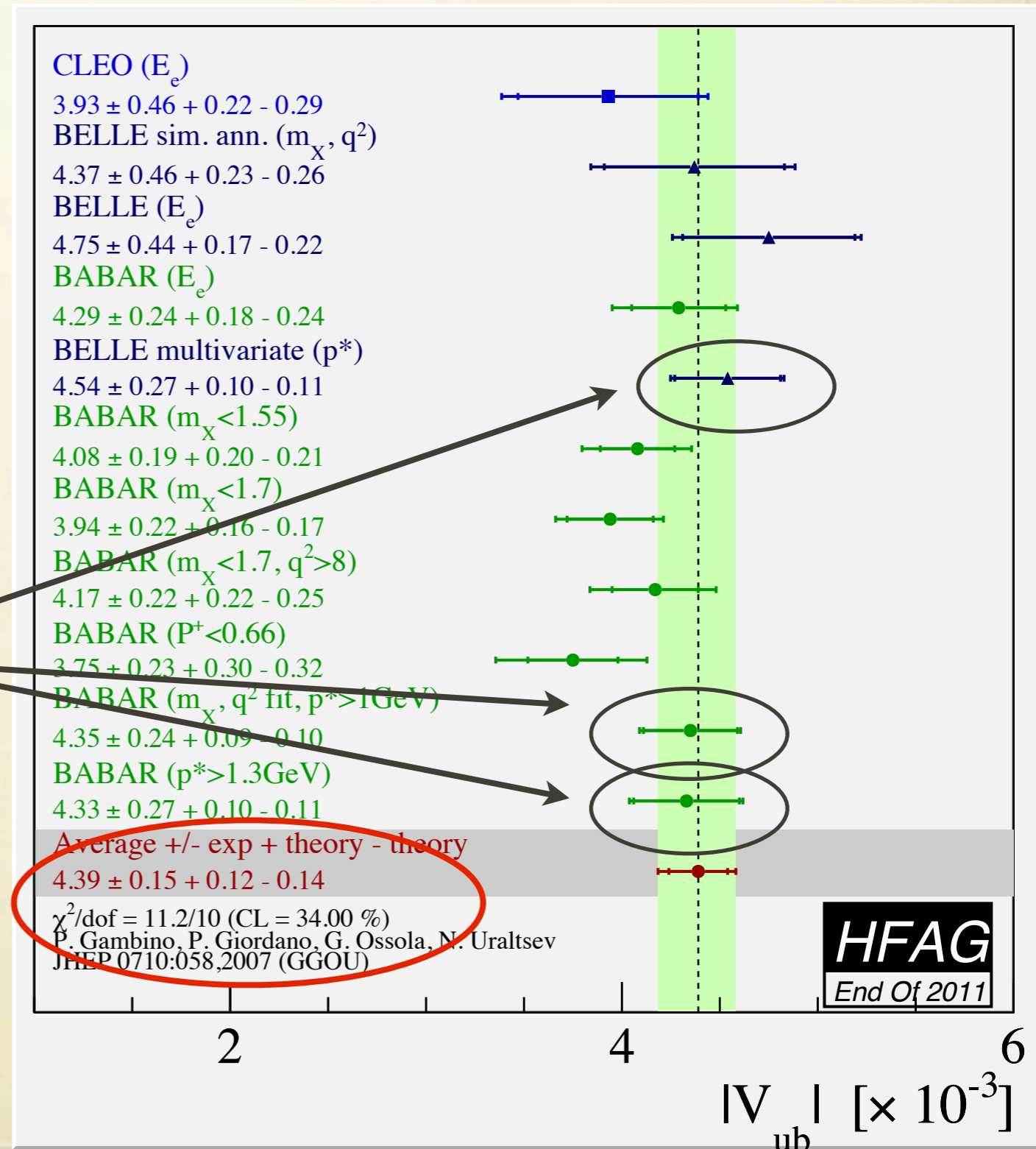
# $V_{ub}$ IN THE GGOU APPROACH

PG, Giordano, Ossola, Uraltsev

Good consistency & small th error.

**4.7% total error**

strong dependence on  $m_b$   
 recent experimental results  
 are theoretically cleanest  
 but signal simulation relies on  
 theoretical models



# PERTURBATIVE EFFECTS

- $O(\alpha_s)$  implemented by all groups De Fazio,Neubert
- Running coupling  $O(\alpha_s^2\beta_0)$  (PG,Gardi,Ridolfi) in GGOU, DGE lead to -5% & +2%, resp. in  $|V_{ub}|$
- Complete  $O(\alpha_s^2)$  in the SF region Asatrian,Greub,Pecjak-Bonciani,Ferrogli-Beneke,Huber, Li - G. Bell 2008
- In BLNP leads to up 8% increase in  $V_{ub}$  related to resummation, not yet included by HFAG. It is an **artefact** of this approach.

- $P_+ < 0.66$  GeV:

	$\Gamma_u^{(0)}$	$\mu_h$	$\mu_i$
NLO	60.37	+3.52 -3.37	+3.81 -6.67
NNLO	52.92	+1.46 -1.72	+0.09 -2.79

Greub,Neubert,Pecjak arXiv:0909.1609

- $P_+ < 0.66$  GeV:

Fixed-Order	$\Gamma_u^{(0)}$	$\mu$
NLO	49.11	+5.43 -9.41
NNLO	49.53	+0.13 -4.01

**NEW:** full phase space  $O(\alpha_s^2)$  calculation

Brucherseifer,Caola,Melnikov, arXiv:1302.0444

Confirms non-BLM/BLM approx 20% over relevant phase space

# SUMMARY

- Theoretical efforts to develop the OPE approach to semileptonic decays goes on. More results soon. No sign of inconsistency in this approach.
- HQSR determination of zero recoil  $B \rightarrow D^*$  form factor agrees with inclusive determination.
- Exclusive/incl. tension in  $V_{ub}$  remains mysterious ( $2-3\sigma$ ). It could be explained by right-handed current...
- Belle-II will increase significantly the statistics for  $b \rightarrow ul\nu$  decays. Measurement of spectra will enable direct constraints on shape function(s), see e.g. SIMBA.



# NIKOLAI URALTSEV

a first class physicist, an enthusiastic  
colleague, a good friend has died on Feb 13.

# **BACK-UP SLIDES**

# HEAVY QUARK MASSES

**Table 17.1.5.** Recent results for the charm-quark mass. An asterisk indicates that we have obtained this number for  $m_c$  quoted as the main result of the paper using four-loop accuracy (together with  $\alpha_s(M_Z) = 0.1184$  (Nakamura et al. 2010)).

$m_c(3 \text{ GeV})$ (GeV)	$m_c(m_c)$ (GeV)	Method	Reference
$0.986 \pm 0.013$	$1.275 \pm 0.013^*$	LESR	Kühn, Steinhauser, and Sturm (2007)
$0.96 \pm 0.04^*$	$1.25 \pm 0.04$	NRSR	Signer (2009)
$0.986 \pm 0.006$	$1.275 \pm 0.006^*$	LQCD	McNeile, Davies, Follana, Hornbostel, and Lepage (2010)
$0.998 \pm 0.029$	$1.277 \pm 0.026$	LESR	Dehnadi, Hoang, Mateu, and Zebarjad (2011)
$0.987 \pm 0.009$	$1.278 \pm 0.009$	FESR	Bodenstein, Bordes, Dominguez, Penarrocha, and Schilcher (2011)
$0.972 \pm 0.006^*$	$1.262 \pm 0.006$	FESR	Narison (2012)

**Table 17.1.6.** Recent results for the bottom-quark mass.

$m_b(m_b)$	Method	Reference
$4.19 \pm 0.06$	NRSR	Pineda and Signer (2006)
$4.163 \pm 0.016$	LESR	Chetyrkin et al. (2009)
$4.164 \pm 0.023$	LQCD	McNeile, Davies, Follana, Hornbostel, and Lepage (2010)
$4.167 \pm 0.013$	LESR	Narison (2012)

# OPE: POWER CORRECTIONS

$$\Delta_{1/m^2} = \frac{\mu_G^2}{3m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{4} \left( \frac{1}{m_c^2} + \frac{2}{3m_c m_b} + \frac{1}{m_b^2} \right),$$
$$\Delta_{1/m^3} = \frac{\rho_D^3 - \frac{1}{3}\rho_{LS}^3}{4m_c^3} + \frac{1}{12m_b} \left( \frac{1}{m_c^2} + \frac{1}{m_c m_b} + \frac{3}{m_b^2} \right) (\rho_D^3 + \rho_{LS}^3)$$

matrix elements from moments fits & sum rule constraints

$1/m^4$  and  $1/m^5$  also known, matrix elements have been estimated by ground state saturation

Mannel, Turczyk, Uraltsev

$$\Delta_A \simeq 0.090 + 0.029 - 0.023 - 0.013 + \dots$$

heavy quark expansion converges reasonably well

Including all errors for  $\varepsilon_M=0.75\text{GeV}$

$$\mathcal{F}(1) < 0.935$$

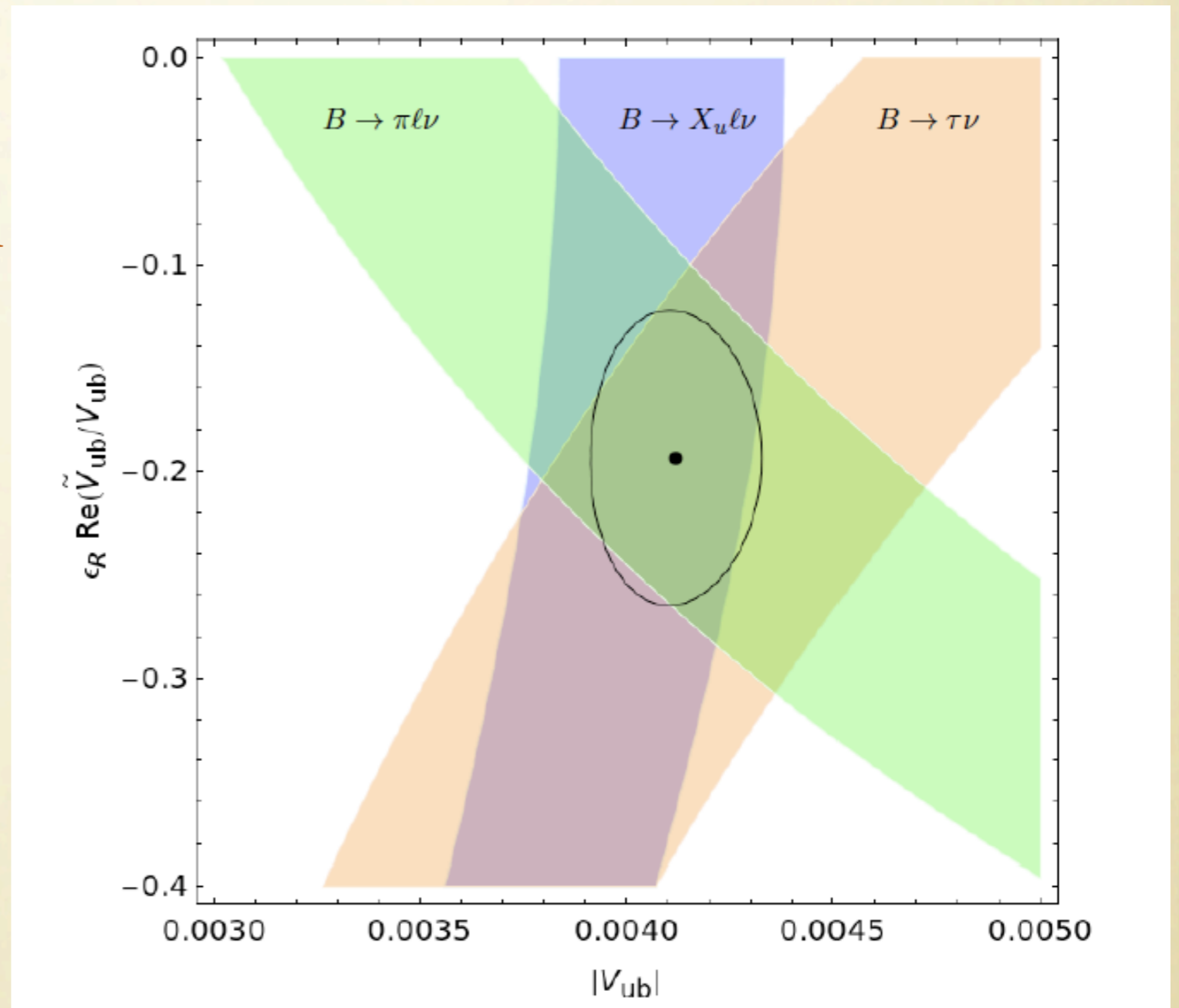


# NEW PHYSICS?

LR models can explain a difference between inclusive and exclusive  $V_{ub}$  determinations (Chen,Nam)

Also in MSSM (Crivellin)

BUT the RH currents affect predominantly the exclusive  $V_{ub}$ , making the conflict between  $V_{ub}$  and  $\sin 2\beta$  ( $\psi K_S$ ) stronger...



Buras, Gemmler, Isidori 1007.1993

# EXCLUSIVE $V_{ub}$ FROM $B \rightarrow \pi l \nu$

Here there is no preferred point in phase space. Lattice and light-cone sum rules estimate form factor.

Recent lattice based:

MILC collaboration

$$|V_{ub}| = (3.25 \pm 0.31) \times 10^{-3}$$

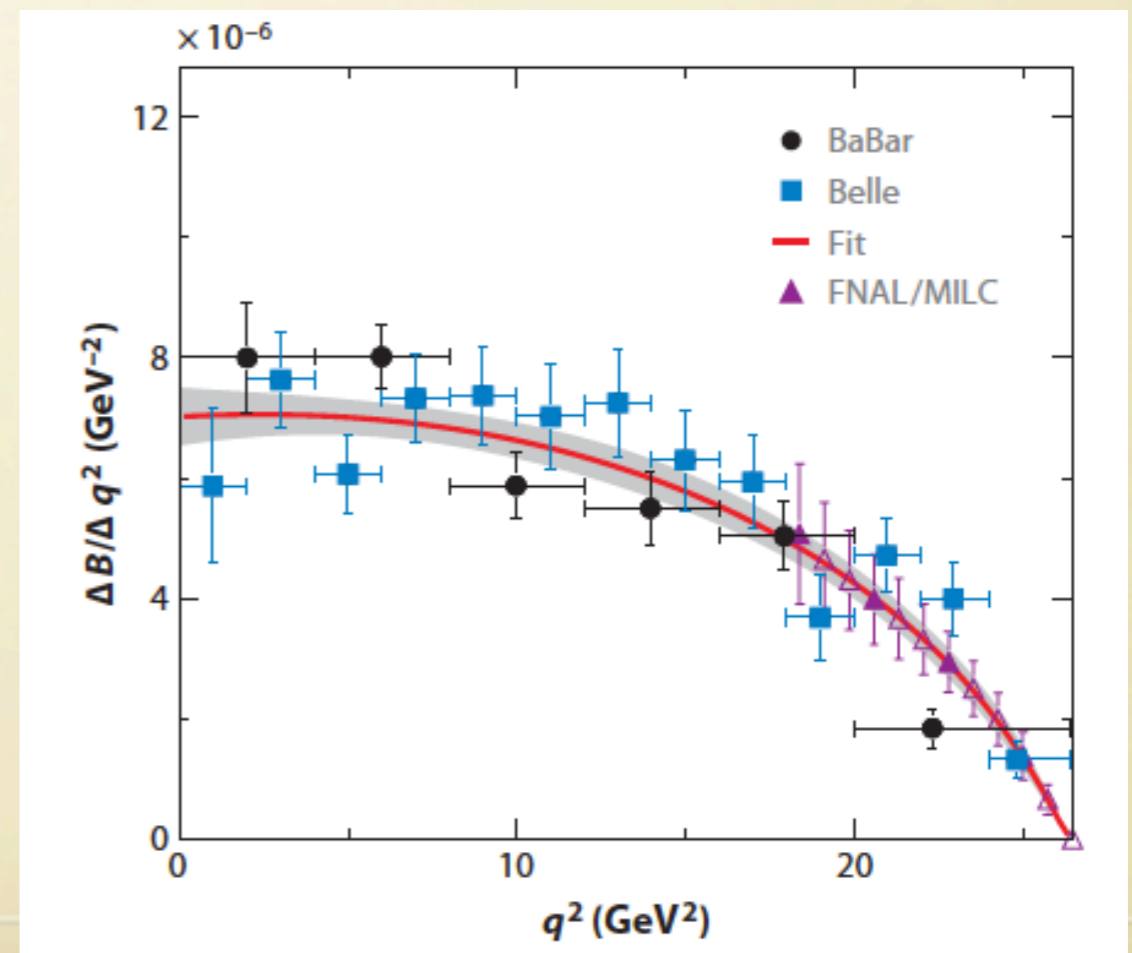
Recent sum-rules based:

Khodjamirian, Mannel, Offen, Wang 2011  
see also Bharucha

$$|V_{ub}| = \left( 3.50^{+0.38}_{-0.33} \Big|_{th.} \pm 0.11 \Big|_{exp.} \right) \times 10^{-3}$$

*Precision is improved by fitting  
lattice/LCSR together with data*

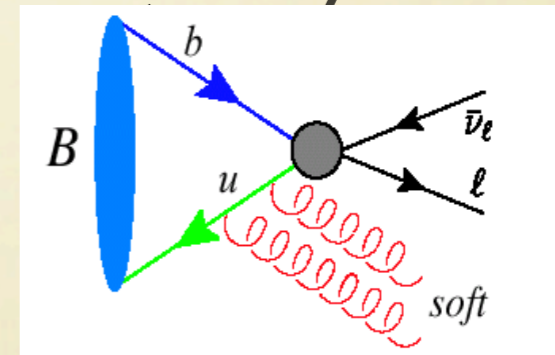
*Experimental data are not well consistent*



# THE TOTAL $B \rightarrow X_U \ell \nu$ WIDTH

$$\Gamma[\bar{B} \rightarrow X_u e \bar{\nu}] = \frac{G_F^2 m_b^5 |V_{ub}|^2}{192\pi^3} \left[ 1 + \frac{\alpha_s}{\pi} p_u^{(1)}(\mu) + \frac{\alpha_s^2}{\pi^2} p_u^{(2)}(r, \mu) - \frac{\mu_\pi^2}{2m_b^2} - \frac{3\mu_G^2}{2m_b^2} \right. \\ \left. + \left( \frac{77}{6} + 8 \ln \frac{\mu_{\text{WA}}^2}{m_b^2} \right) \frac{\rho_D^3}{m_b^3} + \frac{3\rho_{LS}^3}{2m_b^3} + \frac{32\pi^2}{m_b^3} B_{\text{WA}}(\mu_{\text{WA}}) \right] \\ + O\left(\alpha_s \frac{\mu_{\pi, G}^2}{m_b^2}\right) + O\left(\frac{1}{m_b^4}\right)$$

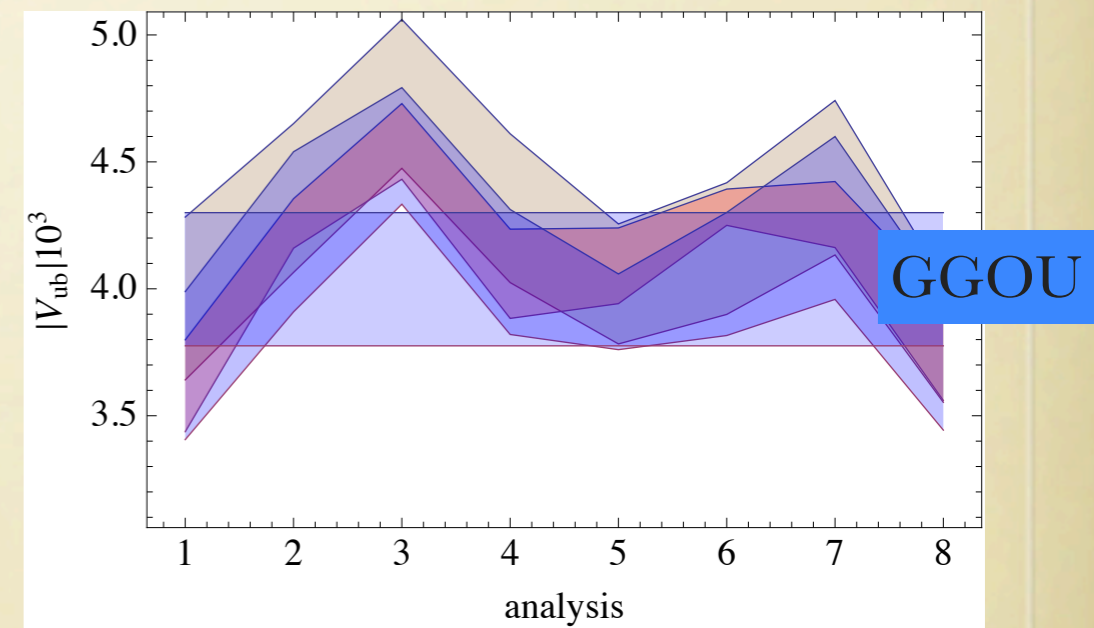
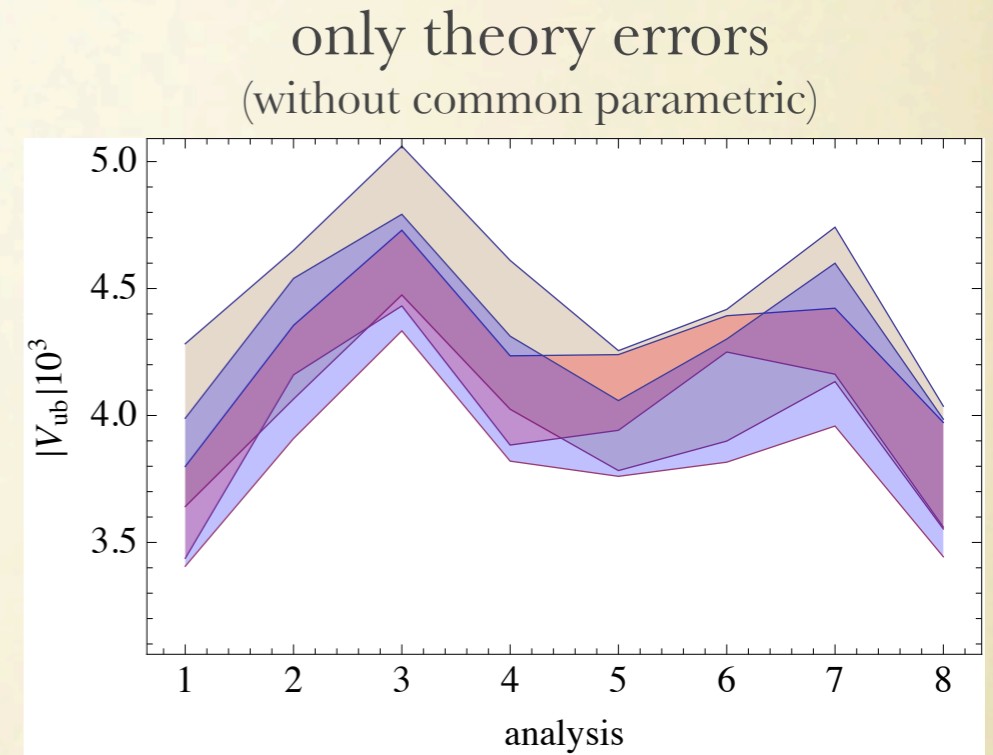
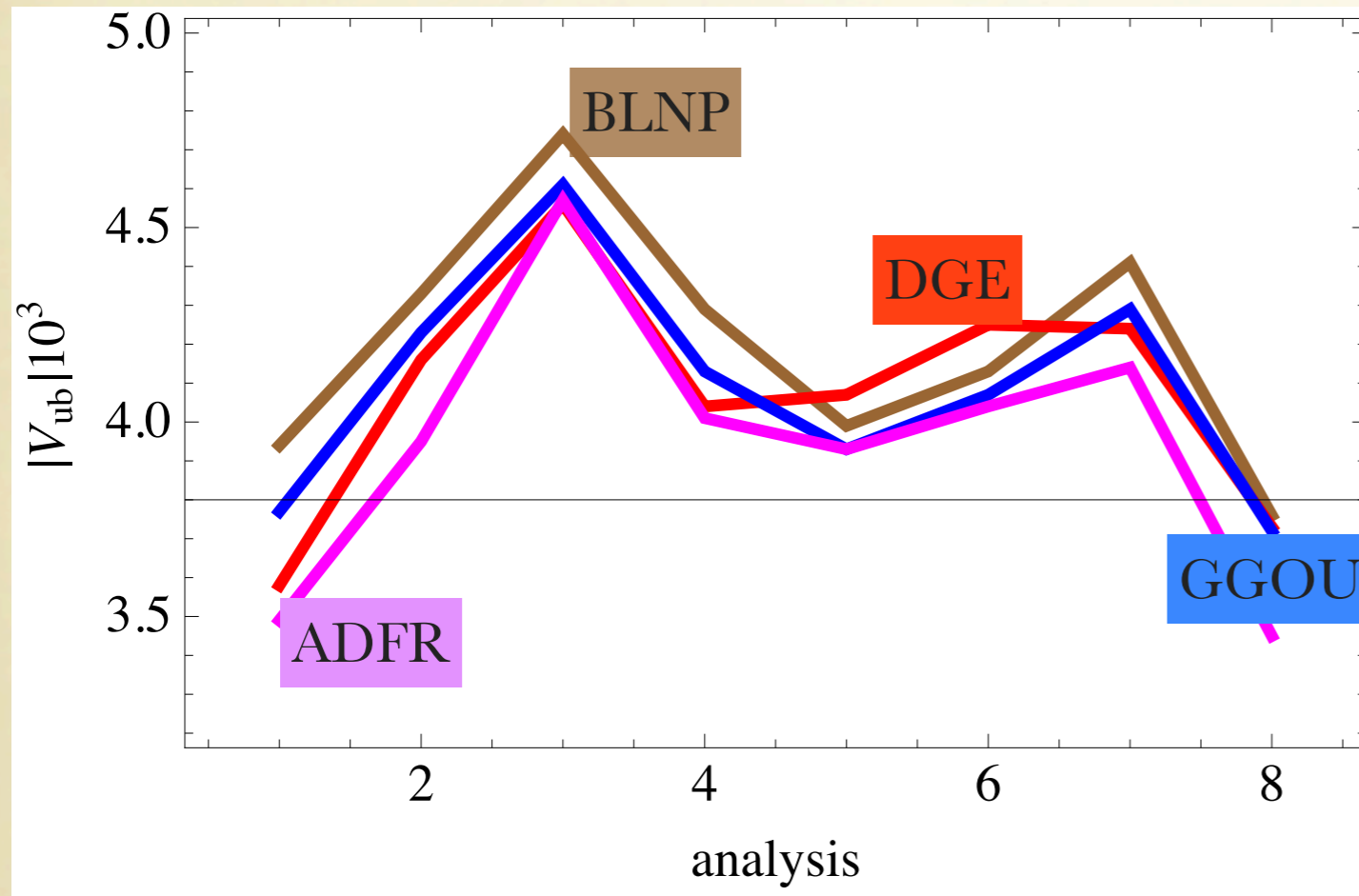
Using the results of the fit,  $V_{ub}$  could be extracted if we had the total width...



Weak Annihilation, severely constrained from D decays, see Kamenik, PG, arXiv:1004.0114

# A GLOBAL COMPARISON

0907.5386, Phys Rept



- \* common inputs (except ADFR)
- \* Overall good agreement **SPREAD WITHIN THEORY ERRORS**
- \* NNLO BLNP still missing: will push it up a bit
- \* Systematic offset of central values: normalization? to be investigated

# OPE: PERTURBATIVE EFFECTS

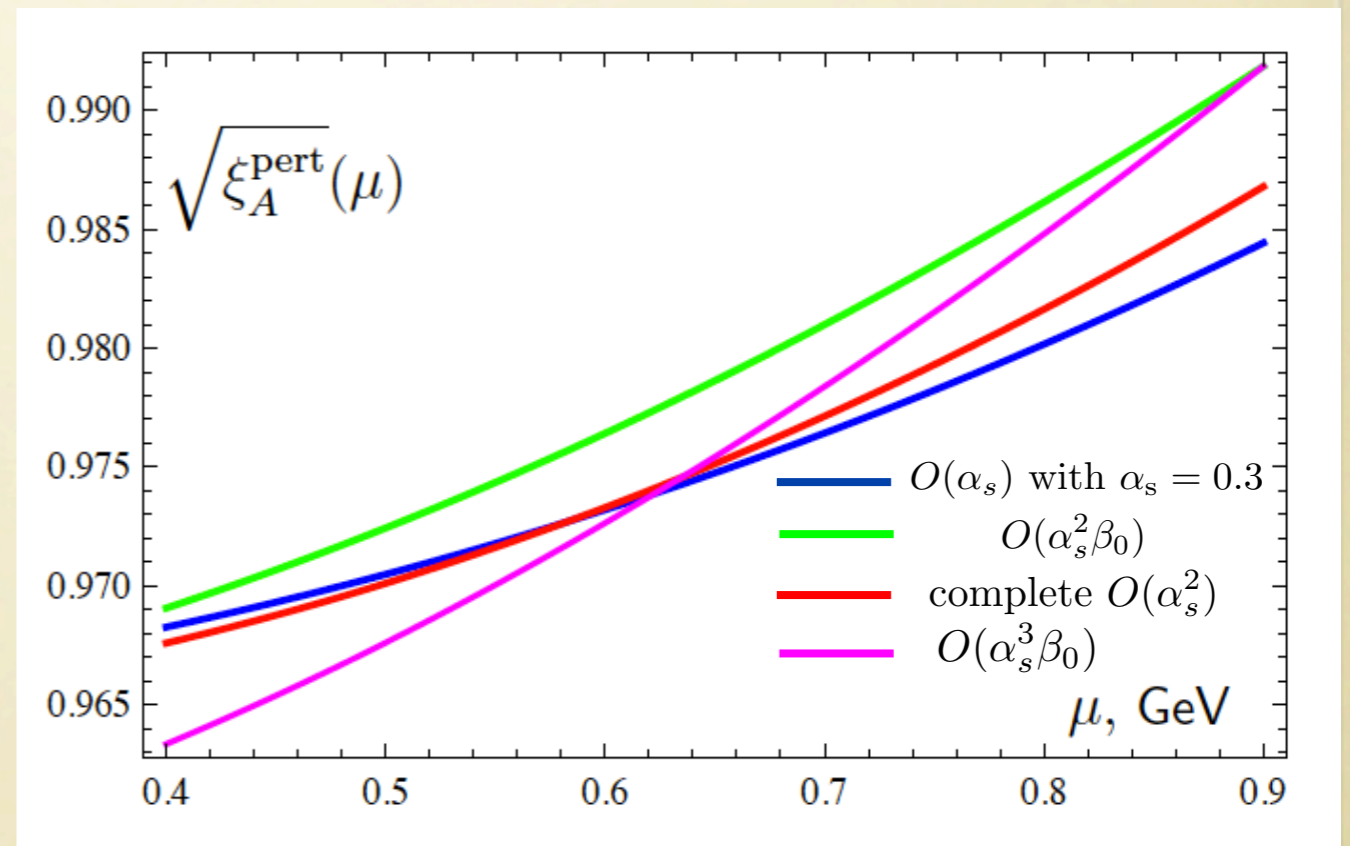
$$I_0(\varepsilon_M) = \xi_A^{\text{pert}}(\varepsilon_M, \mu) + \sum_k C_k(\varepsilon_M, \mu) \frac{\frac{1}{2M_B} \langle B|O_k|B \rangle_\mu}{m_Q^{d_k-3}}$$

the cutoff  $\mu$  separates pert and non-pert physics

Power corrections start with  $1/m_c^2$      $\Lambda_{QCD} \ll \varepsilon_M, \mu \ll 2m_c$

We choose  $\varepsilon_M = \mu = 0.75\text{GeV}$   
and include 1, 2 loop and  
higher BLM corrections  
with no expansion in  $\mu/m_c$

$$\sqrt{\xi_A^{\text{pert}}(0.75\text{GeV})} = 0.98 \pm 0.01$$

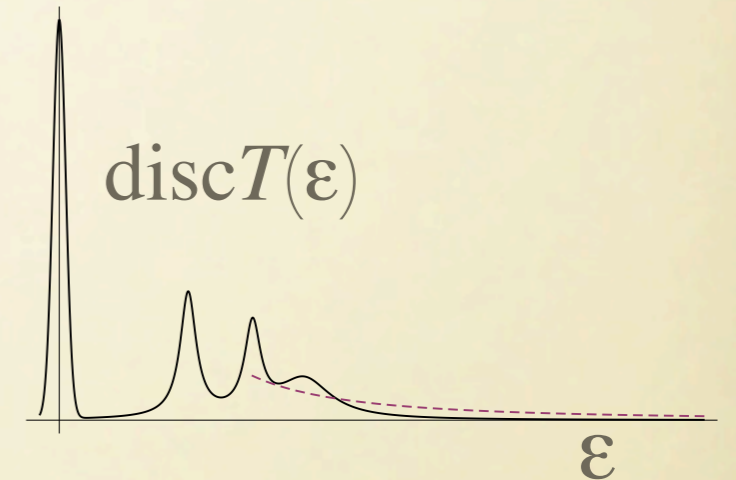


# ZERO RECOIL SUM RULE

$$T(\varepsilon) = \frac{i}{6M_B} \int d^4x e^{-ix_0(M_B - M_{D^*} - \varepsilon)} \langle B | T J_A^k(x) J_{Ak}(0) | B \rangle$$

$$\varepsilon = M_X - M_{D^*}$$

$$I_0(\varepsilon_M) = -\frac{1}{2\pi i} \oint_{|\varepsilon|=\varepsilon_M} T(\varepsilon) d\varepsilon = \mathcal{F}^2(1) + I_{inel}(\varepsilon_M)$$



*Inelastic non-resonant piece*  $I_{inel}(\varepsilon_M) = \frac{1}{2\pi i} \int_{0+}^{\varepsilon_M} \text{disc } T(\varepsilon) d\varepsilon$

$$\mathcal{F}(1) = \sqrt{I_0(\varepsilon_M) - I_{inel}(\varepsilon_M)}$$

$$\mathcal{F}(1) \leq \sqrt{I_0(\varepsilon_M)}$$

Unitarity bound