Invariants and Flavour Aspects of the Standard Model and Beyond

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based on work done in collaboration with
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The gauge sector of the SM with 3 generations has a large global flavour symmetry :

$$Q_L^j\,(3,\ 2,\ 1/6),\quad u_R^j\,(3,1,2/3);\ \ell_L\,(1,2,-1/2)\quad \ell_R^j\,(1,1,-1)$$

$$\underset{SU(3)}{3},\ \underset{SU(2)}{2},\ \underset{U(1)_Y}{1/6}$$

One can make unitary transformations acting in flavour space, under which :

$$Q_L \to Q'_L\ ,\ \text{with}\quad Q_{L_j} = (W_L)_{jk}\, Q'_{Lk}$$

$$u_R \to u'_R\ ,\ \text{with}\quad (u_R)_j = (W_R^u)_{jk}\, u'_{Rk}\ ,\ \text{etc}$$

where W_L, W_R^u etc are unitary matrices

So there is a global $U(3)^5$ flavour symmetry which is explicitly broken by the Yukawa couplings:

$$-\mathcal{L}^{SM}_{Yukawa} = (Y_d)_{ij}\,\overline{Q}^i \phi\, d^j_R + (Y_u)_{ij}\,\overline{Q}^i \tilde{\phi}\, u^j_R + (Y_e)_{ij}\,\overline{L}^i_L \phi\, e^j_R + h.c.$$

The flavour symmetry leads to a large redundancy in the Yukawa couplings which are not invariant under a weak-basis transformation.

Under a weak-basis transformation, the Yukawa flavour matrices transform as :

$$Y_d \rightarrow Y_d' = W_L^\dagger \, Y_d \, W_R^d$$

$$Y_u \rightarrow Y_u' = W_L \, Y_u \, W_R^u$$

The sets (Y_d, Y_u) and (Y_d', Y_u') contain the same physics

Assume that spontaneous symmetry breaking of $SU(2)\times U(1)\times SU(3)_c$ has already taken place:

$$\mathcal{L}_{CP} = \mathcal{L}_{gauge} = \overline{u}^{\,0}_{iL}\,\gamma_\mu\, d^{\,0}_{iL}\, W^\mu + \ldots$$

$i = 1 \ldots N$; $N \rightarrow$ number of families

One has to consider the most general CP transformation which leaves $\mathcal{L}_{CP}$ invariant. In particular, one has to keep in mind that gauge interactions do not distinguish the various families of fermions. All families enter on equal footing

In the case of the SM, the most general CP transformations which leave $\mathcal{L}_{CP}$ invariant are :

$$(CP)\,(u_L^0)\,(CP)^\dagger = e^{i\xi_W} K_L \, \gamma^0 C \, \bar{u}_L^{0\,T}$$

$$(CP)\,(d_L^0)\,(CP)^\dagger = K_L \, \gamma^0 C \, \bar{d}_L^{0\,T}$$

$$(CP)\,(u_R^0)\,(CP)^\dagger = K_R^u \, \gamma^0 C \, \bar{u}_R^{0\,T}$$

$$(CP)\; d_R^0 \;(CP)^\dagger = K_R^d \, \gamma^0 C \, \bar{d}_R^{0\,T}$$

where K_L , K_R^u , K_R^d are unitary matrices acting in flavour space.

It can be readily shown that in order for $\mathcal{L}_{Yukawa}$ (or equivalently M_u, M_d) to be **CP invariant** the following relations have to be satisfied:

J. Bernabeu, M. Gronau, G.C.B

$$K_L^\dagger M_u K_R^u = M_u^*$$
$$K_L^\dagger M_d K_R^d = M_d^*$$

The existence of the matrices K_L, K_R^u, K_R^d is a necessary and sufficient condition for

CP invariance in the SM

$$\underbrace{K_L^\dagger M_u K_R^u = M_u^* \quad ; \quad K_L^\dagger M_d K_R^d = M_d^*}$$

$$\Downarrow$$

$$\underbrace{K_L^\dagger M_u M_u^\dagger K_L = (M_u M_u^\dagger)^* = H_u^T \; ; \; K_L^\dagger M_d M_d^\dagger K_L = (M_d M_d^\dagger)^* = H_d^T}$$

$$\Downarrow$$

$$\underbrace{K_L^\dagger [H_u, H_d] K_L = [H_u^T, H_d^T] = -[H_u, H_d]^T}$$

$$K_L^\dagger [H_u, H_d]^r K_L = -\left\{[H_u, H_d]^r\right\}^T \quad r \text{ odd}$$

One concludes :

$$\boxed{\mathrm{tr}\,[H_u, H_d]^r = 0}$$

Therefore, one concludes that in the SM **CP invariance implies** $\mathrm{tr}\,[H_u, H_d]^r = 0$, for r odd

For $r = 1 \rightarrow$ trivially satisfied

For $r = 3$: For 2 generations, automatically satisfied for any Hermitian H_u, H_d

For 3 generations :

$$\mathrm{tr}\,[H_u, H_d]^3 = 6\,i\,(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)\times$$
$$\times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\,\mathrm{Im}\,Q$$

$Q \rightarrow$ invariant quartet of V_{CKM} e.g. $(V_{us} V_{cb} V_{cs}^* V_{ub}^*)$

Flavour in the Higgs Sector

Consider an extension of the SM with n_d Higgs doublets. The most general gauge invariant, renormalizable Higgs potential can be written:

$$\mathcal{L}_\phi = Y_{ab}\, \phi_a^\dagger \phi_b + Z_{abcd} (\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

Hermiticity of $\mathcal{L}_\phi$ implies:

$$\boxed{Y_{ab}^* = Y_{ba} \qquad Z_{abcd}^* = Z_{badc}}$$

The kinetic term of the Higgs Sector is invariant under the following Higgsbasis transformations (HBT):

$$\phi_a \xrightarrow{HBT} \phi'_a = V_{ai}\phi_i \; ; \; \phi^\dagger_a \to (\phi')^\dagger_{ai} = V^*_{ai}(\phi')^\dagger_i$$

Under a HBT, the physics content of $\mathcal{L}_\phi$ does not change, but the couplings Y, Z transform as:

$$Y_{ab} \xrightarrow{HBT} Y'_{ab} = V_{am} Y_{mn} V^\dagger_{nb}$$

$$Z_{abcd} \xrightarrow{HBT} Z'_{abcd} = V_{am} V_{cp} Z_{mnpq} V^\dagger_{nb} V^\dagger_{qd}$$

The most general CP transformation that leaves the kinetic energy term invariant is:

$$\phi_a \xrightarrow{CP} U_{ai}\,\phi_i^*$$
$$\phi_a^\dagger \xrightarrow{CP} U_{ai}^*\,\phi_i^T$$

where U is a $n_d \times n_d$ unitary matrix, acting in Higgs doublets space. The necessary and sufficient condition for $\mathcal{L}_\phi$ to conserve CP is that the following relations be satisfied

$$(Y^*)_{ab} = U^\dagger_{am}\,Y_{mn}\,U_{nb}\;;\quad Z^*_{abcd} = U^\dagger_{am}\,U^\dagger_{cp}\,Z_{mnpq}\,U_{nb}\,U_{qd}$$

CP-odd HBT invariants are useful in the study of the CP properties of the Higgs sector. One has the freedom to make HB transformations which do not change the physical content of the model, but do change the quadratic and quartic couplings.

Couplings that are complex in one weak-basis may become real in another basis

The general Two Higgs doublets Model

$$V_{H_2} = m_1\,\phi_1^\dagger\phi_1 + \mu e^{i\varphi}\phi_1^\dagger\phi_2 + \mu e^{-i\varphi}\phi_2^\dagger\phi_1 + m_2\,\phi_2^\dagger\phi_2 +$$

$$+\, a_1(\phi_1^\dagger\phi_1)^2 + a_2(\phi_2^\dagger\phi_2)^2 + b\,(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + b'(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) +$$

$$+\, c_1\, e^{i\theta_1}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) + c_1\, e^{-i\theta_1}(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + c_2\, e^{i\theta_2}(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1) +$$

$$+\, c_2\, e^{-i\theta_2}(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + d\, e^{i\delta}(\phi_1^\dagger\phi_2)^2 + d\, e^{-i\delta}(\phi_2^\dagger\phi_1)^2$$

It is clear that this potential contains an excess of parameters. Without loss of generality, one can choose a Higgs basis where the quadratic terms are diagonal. Of the three remaining phases, one can still eliminate one of the phases by rephasing one of the Higgs fields. $\Rightarrow$ Only 2 phases

Constructing CP-odd HBT invariants

It is convenient to define :

$$Y_a{}^b \equiv Y_{ab} \quad ; \quad Z_{ac}{}^{bd} \equiv Z_{abcd}$$

Necessary and Sufficient conditions for CP invariance :

$$I_1 \equiv \mathrm{tr}\left[Y Z_Y \hat{Z} - \hat{Z} Z_Y Y \right] = 0$$

$$I_2 \equiv \mathrm{tr}\left[Y Z_2 \tilde{Z} - \tilde{Z} Z_2 Y \right] = 0$$

where we have defined the following 2x2 Hermitian matrices:

$$\hat{Z}_a{}^b \equiv Z_{am}{}^{bm} \quad ; \quad \tilde{Z}_a{}^b \equiv Z_{am}{}^{mb}$$

$$(Z_Y)_a{}^b \equiv Z_{an}{}^{bm}\, Y_m{}^n \quad ; \quad (Z_2)_a{}^b \equiv Z_{an}{}^{bm}\, Z_{mp}{}^{nb}$$

Soft CP-breaking with 2 Higgs Doublets

CP-odd HBT invariants are also useful to find out whether in a given model there is hard or soft CP-breaking.

One may have complex Z-couplings (quartic) in a given Higgs-basis and yet CP be only softly broken.

An example : Consider the following Higgs potential :

$$V = m_1 \phi_1^\dagger \phi_1 + m_2 \phi_2^\dagger \phi_2 + a\left[(\phi_1^\dagger \phi_1)^2 + (\phi_2^\dagger \phi_2)^2\right] +$$

$$b\,(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + c\left[e^{i\theta}(\phi_1^\dagger \phi_1)(\phi_1^\dagger \phi_2) + e^{i\theta}(\phi_2^\dagger \phi_2)(\phi_2^\dagger \phi_1) + h.c.\right] +$$

$$+ d\left[e^{i\delta}(\phi_1^\dagger \phi_2)^2 + h.c.\right]$$

An explicit evaluation gives :

$$I_1 \equiv tr\left[Y Z_Y \hat{Z} - \hat{Z} Z_Y Y\right] = 0$$

$$I_2 \equiv tr\left[Y Z_2 \hat{Z} - \tilde{Z} Z_2 Y\right] \neq 0$$

$$I_3 \equiv tr\left[Z_2 Z_3 \hat{Z} - \hat{Z} Z_3 Z_2\right] = 0 \, ;$$

where

$$(Z_3)_a^b = Z_{ar}^{mp} Z_{mn}^{rs} Z_{ps}^{nb}$$

Keeping in mind that I_3 contains only quartic couplings, the fact that $I_3 = 0$ while $I_2 \neq 0$ provides a hint that CP is only softly broken in this specific model. This can be verified, because the quartic couplings, by themselves conserve CP, since in:

$$\phi_a \xrightarrow{CP} U_{ai} \phi_i^* \quad , \quad \phi_a^+ \longrightarrow U_{ai}^* \phi_i^T$$

one has the freedom to choose:

$$U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Yukawa interactions in the General two Higgs doublet Model

$$-\mathcal{L}_Y = \bar{Q}^0_L \Gamma_1 \phi_1 d^0_R + \bar{Q}^0_L \Gamma_2 \phi_2 d^0_R + \bar{Q}^0 \Delta_1 \tilde{\phi}_1 u^0_R + \bar{Q}_L \Delta_2 \tilde{\phi}_2 u^0_R + h.c.$$

Quark mass matrices:

$$M_d = \frac{1}{\sqrt{2}} \left(v_1 \Gamma_1 + v_2 e^{i\alpha} \Gamma_2 \right) \; ; \; M_u = \frac{1}{\sqrt{2}} \left(v_1 \Delta_1 + v_2 e^{-i\alpha} \Delta_2 \right)$$

Diagonalized by:

$$(U_d)^\dagger_L M_d (U_d)_R = D_d = \mathrm{diag}\,(m_d, m_s, m_b)$$

$$(U_u)^\dagger_L M_u U_{uR} = D_d = \mathrm{diag}\,(m_u, m_c, m_t)$$

Expansion around the vev's

$$\phi_j = \begin{pmatrix} \phi_j^+ \\ \frac{e^{i\alpha_j}}{\sqrt{2}}(v_j + \rho_j + i\eta_j) \end{pmatrix}, \quad j = 1,2$$

we perform the following transformation

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = U \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} ; \quad \begin{pmatrix} G^0 \\ I \end{pmatrix} = U \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} ; \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = U \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

$$U = \frac{1}{v}\begin{pmatrix} v_1 e^{-i\alpha_1} & v_2 e^{-i\alpha_2} \\ v_2 e^{-i\alpha_1} & -v_1 e^{-i\alpha_2} \end{pmatrix} ; \quad v = \sqrt{v_1^2 + v_2^2} = \left(\sqrt{2}\, G_F\right)^{-1/2} \simeq 246\,\text{GeV}$$

U singles out

H^0 with couplings to quarks proportional to mass matrices

G^0 neutral pseudo-goldstone boson

G^+ charged pseudo-goldstone boson

Physical neutral Higgs fields are combinations of H^0, R and I

Neutral and charged Higgs interactions in the quark sector

$$-\mathcal{L}_Y = \bar{d}_L^0 \frac{1}{v}\left(M_d H^0 + N_d^0 R + i N_d^0 I\right) d_R^0 +$$

$$+ \bar{u}_L^0 \frac{1}{v}\left(M_u H^0 + N_u^0 R + i N_u^0 I\right) u_R^0 +$$

$$+ \frac{\sqrt{2}}{v} H^+ \left(\bar{u}_L^0 N_d^0 d_R^0 - \bar{u}_R^0 N_u^{0\dagger} d_L^0\right) + h.c.$$

$$N_d^0 = \frac{1}{\sqrt{2}}\left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2\right) ; \quad N_u^0 = \frac{1}{\sqrt{2}}\left(v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2\right)$$

In the quark mass eigenstate basis N_d, N_u are not flavour diagonal.

Physical neutral Higgs are combinations of H^0, R, I

Yukawa Couplings in terms of quark mass eigenstates:

$$\mathcal{L}_Y = \frac{\sqrt{2}\,H^+}{v}\,\bar{u}\left[-V N_d \gamma_R + N_u^{\dagger} V \gamma_L\right] d + \text{h.c.} -$$

$$-\frac{H^0}{v}\left[\bar{u}\, D_u\, u + \bar{d}\, D_d\, d\right] -$$

$$-\frac{R}{v}\left[\bar{u}\left(N_u \gamma_R + N_u^{\dagger}\gamma_L\right) u + \bar{d}\left(N_d \gamma_R + N_d^{\dagger}\gamma_L\right) d\right]$$

$$+\frac{I}{v}\left[\bar{u}\left(N_u \gamma_R - N_u^{\dagger}\gamma_L\right) u - \bar{d}\left(N_d \gamma_R - N_d^{\dagger}\gamma_L\right) d\right]$$

$\gamma_L = \frac{1}{2}(1-\gamma_5)$; $\gamma_R = \frac{1}{2}(1+\gamma_5)$; V is the CKM matrix

Flavour changing neutral currents are controlled by:

$$N_d = \frac{1}{\sqrt{2}} U^\dagger_{d_L} \left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2 \right) U_{d_R}$$

$$N_u = \frac{1}{\sqrt{2}} U^\dagger_{uL} \left(v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2 \right) U_{uR}$$

$$\underbrace{\text{For generic two Higgs doublet models } N_u, N_d \text{ are non-diagonal, arbitrary matrices.}}$$

$\Downarrow$

too large Higgs mediated Flavour changing neutral currents, unless a suppression mechanism is introduced

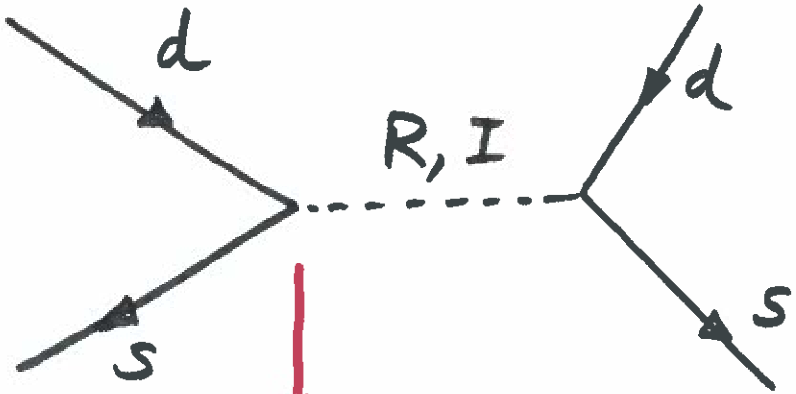
Examples of Suppression mechanisms:

NFC à la Glashow-Weinberg:

↓

no F.C.N.C at tree level

BGL (B., Grimus and Lavoura)



→ λ^{10} suppression

↓

$V_{td} V_{ts}^{*} \sim \lambda^{5}$

We have seen that under a WB transformation:

$$d_L^0 = W_L\, d_L^{0\prime} \quad ; \quad d_R^0 = W_R^d\, d_R^{0\prime}$$

$$u_L^0 = W_L\, u_L^{0\prime} \quad ; \quad u_R^0 = W_R^u\, u_R^{0\prime}$$

Under a WB transformation the flavour matrices transform as:

$$M_d \longrightarrow M_d' = W_L^\dagger M_d W_R^d \quad ; \quad M_u \rightarrow M_u' = W_L^\dagger M_u W_R^u$$

$$N_d^0 \longrightarrow N_d^{0\prime} = W_L^\dagger N_d^0 W_R^d \quad ; \quad N_u^0 \rightarrow N_u^{0\prime} = W_L^\dagger N_u^0 W_R^u$$

In view of the freedom of choice of WB, it is useful to express the physical content of M_d, M_u, N_d^0, N_u^0 in terms of WB invariants.

In the quark mass eigenstate basis flavour mixing can be parametrized by various unitary matrices which arise from the misalignment in flavour space between pairs of various Hermitian flavour matrices. This is entirely analogous to what one encounters in the SM

$V_{CKM} \to$ arises from the misalignment between $H_u \equiv M_u M_u^\dagger$ and $H_d \equiv M_d M_d^\dagger$

Examples of Invariants:

$$I_1 \equiv \mathrm{tr}\left(M_d N_d^{0\dagger}\right) = m_d (N^*)_{11} + m_s (N_d^*)_{22} + m_b (N_b^*)_{33}$$

where N_d is N_d^0 in the basis where the down quark mass matrix is diagonal. This invariant is not sensitive to Higgs mediated FCNC but $\mathrm{Im}\, I_1$ is specially important, since it probes the phases of $(N_d)_{jj}$ which contribute to the electric dipole moment of down quarks.

Analogous comments apply to $\mathrm{tr}\left(M_u N_u^{0\dagger}\right)$

Consider now a WB invariant which is sensitive to the off-diagonal elements of N_d:

$$I_2 \equiv tr\left[M_d N_d^{o\dagger}, M_d M_d^{\dagger}\right]^2 = -2 m_d m_s (m_s^2 - m_d^2)^2 (N_d^*)_{12} N^*_{d\,21}$$

$$- 2 m_d m_b (m_b^2 - m_d^2)(N_d^*)_{13}(N_d^*)_{31} - 2 m_s m_b (m_b^2 - m_s^2)^2 \times (N_d)_{23}(N_d^*)_{32}$$

CP-odd WB invariant:

$(m_u^2 - m_c^2)\ldots$ analogous

$$I_2^{CP} \equiv tr\left[H_u, H_{N_d^o}\right]^3 = 6 i \Delta_u \, \Delta_{N_d} \, \mathrm{Im}\, Q_2$$

$Q_2 \rightarrow$ is a rephasing invariant quartet of V_2

$$V_2 \equiv U_u^{\dagger} (U_{N_d^o})_L$$

These invariants can be very useful to study the CP properties (as well as FCNC) in specific models. For example in BGL models:

$$\mathrm{Im}\,\mathrm{tr}\left(M_d N_d^{0\dagger}\right) = 0$$

In BGL models the lowest order invariant sensitive to CP violation is:

$$I_g^{CP} \equiv \mathrm{tr}\left[M_d N_d^{0\dagger} M_d M_d^\dagger M_u M_u^\dagger M_d M_d^\dagger \ldots\right]$$

This may be relevant for Baryogenesis.

See M.N. Rebelo talk

Conclusions

- WB invariants and Higgs-basis invariants can be a very useful tool to study the flavour structure and the CP properties of the SM and Beyond.
- The Higgs system chosen by nature is likely to be more involved than the SM Higgs structure. In particular one may have two or more Higgs doublets, with a rich flavour structure