CP Phases from Non-Abelian Discrete Symmetries

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Outline

- Lepton mixing: parametrization and experimental results
- Dirac phase from non-trivial breaking of flavor symmetry G_f
  - Idea *(Blum et al. ('07), Lam ('07,'08), de Adelhart Toorop et al. ('11))*
  - Some example
- Dirac/Majorana phases from non-trivial breaking of G_f & CP
  - Idea *(Harrison/Scott ('02,'04), Grimus/Lavoura ('03), Feruglio et al. ('12))*
  - Study of S_4 and CP *(Feruglio et al. ('12,'13))*
- Conclusions

Parametrization of lepton mixing

Parametrization ***(PDG)***

$$U_{PMNS} = \tilde{U}\ \mathrm{diag}(1, e^{i\alpha/2}, e^{i(\beta/2+\delta)})$$

with

$$\tilde{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

and $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Jarlskog invariant J_{CP}

$$\begin{aligned} J_{CP} &= \mathrm{Im}\left[U_{PMNS,11}U^*_{PMNS,13}U^*_{PMNS,31}U_{PMNS,33}\right] \\ &= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta \end{aligned}$$

Experimental results on lepton mixing

Latest global fits ***(Gonzalez-Garcia et al. ('12))***

best fit and $1\,\sigma$ error	$3\,\sigma$ range
$\sin^2\theta_{13} = 0.0227^{+0.0023}_{-0.0024}$	$0.0156 \leq \sin^2\theta_{13} \leq 0.0299$
$\sin^2\theta_{12} = 0.302^{+0.013}_{-0.012}$	$0.267 \leq \sin^2\theta_{12} \leq 0.344$
$\sin^2\theta_{23} = \begin{cases} 0.413^{+0.037}_{-0.025} \\ 0.594^{+0.021}_{-0.022} \end{cases}$	$0.342 \leq \sin^2\theta_{23} \leq 0.667$
$\delta = 300^\circ{}^{+\;66^\circ}_{-138^\circ}$	$0^\circ \leq \delta \leq 360^\circ$
$\alpha\;,\;\;\beta$	unconstrained

Non-trivial breaking of G_f

<u>Idea</u>:

Derivation of the lepton mixing from how G_f is broken
Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f

(Blum et al. ('07), Lam ('07,'08), de Adelhart Toorop et al. ('11))

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos — charged leptons

$$G_\nu \qquad\qquad G_e$$

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$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos
assume 3 generations
of Majorana neutrinos

charged leptons
distinguish 3 generations

Non-trivial breaking of G_f

Idea:

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Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos | charged leptons

$G_\nu = Z_2 \times Z_2$ | $G_e = Z_N$ with $N \geq 3$

Non-trivial breaking of G_f

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

neutrinos

$G_\nu = Z_2 \times Z_2$

charged leptons

$G_e = Z_N$ with $N \geq 3$

Further requirements

- two/three non-trivial angles $\Rightarrow$ irred 3-dim rep of G_f
- fix angles through G_ν, G_e $\Rightarrow$ 3 families transform diff. under G_ν, G_e

Non-trivial breaking of G_f

- neutrino sector: $Z_2 \times Z_2$ preserved

neutrino mass term $\nu_a \, m_{\nu,ab} \, \nu_b$
is invariant under $\nu_\alpha \to Z_{i,\alpha\beta} \, \nu_\beta$ with $i = 1, 2$

- charged lepton sector: Z_N, $N \geq 3$, preserved

charged lepton mass term $e^c_a \, m_{e,ab} \, l_b$
is invariant under $l_\alpha \to Q_{e,\alpha\beta} \, l_\beta$

Non-trivial breaking of G_f

- neutrino sector: $Z_2 \times Z_2$ preserved

  $\rightarrow$ neutrino mass matrix m_ν fulfills

$$Z_i^T m_\nu Z_i = m_\nu \quad \text{with} \quad i = 1, 2$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

$$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

Non-trivial breaking of G_f

- neutrino sector: $Z_2 \times Z_2$ preserved and generated by

$$Z_i = \Omega_\nu Z_i^{diag} \Omega_\nu^\dagger \quad \text{with} \quad i = 1, 2$$

$$Z_i^{diag} = \text{diag}\,(\pm 1, \pm 1, \pm 1) \quad \text{and} \quad \Omega_\nu \quad \text{unitary}$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

$\rightarrow$ charged lepton mass matrix m_e fulfills

$$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

Non-trivial breaking of G_f

- neutrino sector: $Z_2 \times Z_2$ preserved

  $\rightarrow$ neutrino mass matrix m_ν fulfills

$$Z_i^{diag} \left[\Omega_\nu^T m_\nu \Omega_\nu\right] Z_i^{diag} = \left[\Omega_\nu^T m_\nu \Omega_\nu\right] \quad \text{with} \quad i = 1, 2$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

$$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

Non-trivial breaking of G_f

- neutrino sector: $Z_2 \times Z_2$ preserved

$\rightarrow$ neutrino mass matrix m_ν fulfills

$$\Omega_\nu^T m_\nu \Omega_\nu \quad \text{is diagonal}$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

$\rightarrow$ charged lepton mass matrix m_e fulfills

$$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

Non-trivial breaking of G_f

- neutrino sector: $Z_2 \times Z_2$ preserved

$\rightarrow$ neutrino mass matrix m_ν fulfills

$$\Omega_\nu^T m_\nu \Omega_\nu \quad \text{is diagonal}$$

- charged lepton sector: Z_N, $N \geq 3$, preserved and generated by

$$Q_e = \Omega_e Q_e^{diag} \Omega_e^\dagger \quad \text{with} \quad \Omega_e \quad \text{unitary}$$
$$Q_e^{diag} = \text{diag}\left(\omega_N^{n_e}, \omega_N^{n_\mu}, \omega_N^{n_\tau}\right)$$
$$\text{and} \quad n_e \neq n_\mu \neq n_\tau \quad \text{and} \quad \omega_N = e^{2\pi i/N}$$

Non-trivial breaking of G_f

- neutrino sector: $Z_2 \times Z_2$ preserved

  $\rightarrow$ neutrino mass matrix m_ν fulfills

  $$\Omega_\nu^T m_\nu \Omega_\nu \quad \text{is diagonal}$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

  $$\Omega_e^\dagger m_e^\dagger m_e \Omega_e \quad \text{is diagonal}$$

Non-trivial breaking of G_f

- neutrino sector: $Z_2 \times Z_2$ preserved

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- charged lepton sector: Z_N, $N \geq 3$, preserved

  $\rightarrow$ charged lepton mass matrix m_e fulfills

  $$\Omega_e^\dagger m_e^\dagger m_e \Omega_e \quad \text{is diagonal}$$

- conclusion: $\Omega_{\nu,e}$ diagonalize m_ν and $m_e^\dagger m_e$

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu \quad \text{in} \quad \bar{l} \not{W}^- U_{PMNS} \, \nu$$

Non-trivial breaking of G_f

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu$$

- 3 unphysical phases are removed by $\Omega_e \to \Omega_e K_e$
- neutrino masses are made real and positive through $\Omega_\nu \to \Omega_\nu K_\nu$
- permutations of columns of Ω_e, Ω_ν are possible: $\Omega_{e,\nu} \to \Omega_{e,\nu} P_{e,\nu}$

$$\Downarrow$$

Predictions:

Mixing angles up to exchange of rows/columns

Dirac CP phase δ up to π

Majorana phases undetermined

Some example

Mixing with $\theta_{13} \neq 0$ from $G_f = \Delta(384)$, $G_e = Z_3$

(de Adelhart Toorop et al. ('11))

$$||U_{PMNS}|| = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{2}\sqrt{4+\sqrt{2}+\sqrt{6}} & 1 & \frac{1}{2}\sqrt{4-\sqrt{2}-\sqrt{6}} \\ \frac{1}{2}\sqrt{4+\sqrt{2}-\sqrt{6}} & 1 & \frac{1}{2}\sqrt{4-\sqrt{2}+\sqrt{6}} \\ \sqrt{1-\frac{1}{\sqrt{2}}} & 1 & \sqrt{1+\frac{1}{\sqrt{2}}} \end{pmatrix}$$

$$\sin^2\theta_{13} \approx 0.011 \ , \quad \sin^2\theta_{12} \approx 0.337 \ , \quad \sin^2\theta_{23} \approx \begin{cases} 0.424 \\ 0.576 \end{cases}$$

and

$$\sin\delta = 0$$

Some example

Mixing with $\delta \neq 0$ from $G_f = A_4$, $G_e = Z_3$

(Cabibbo ('78), Wolfenstein ('78))

$$||U_{PMNS}|| = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\sin^2\theta_{13} = \frac{1}{3}\ , \quad \sin^2\theta_{12} = \frac{1}{2}\ , \quad \sin^2\theta_{23} = \frac{1}{2}$$

and

$$|\sin\delta| = 1\ , \quad |J_{CP}| = \frac{1}{6\sqrt{3}}$$

Non-trivial breaking of G_f and CP

Idea:

Relate lepton mixing to how G_f and CP are broken

(Feruglio et al. ('12,'13), Holthausen et al. ('12), Grimus/Rebelo ('95))

G_f & CP

$\swarrow \qquad \searrow$

neutrinos	charged leptons
$G_\nu = Z_2 \times$ CP	$G_e = Z_N$ with $N \geq 3$

An example: $\mu\tau$ reflection symmetry *(Harrison/Scott ('02,'04), Grimus/Lavoura ('03))*

Non-trivial breaking of G_f and CP

In principle, procedure like in case above, but some consistency conditions have to be fulfilled:

- definition of generalized CP transformation (X being unitary and symmetric) ***(see e.g. Branco et al. ('11))***

$$\phi \xrightarrow{\text{CP}} X\phi^\star$$

- assume ϕ transforms as 3-dim rep of G_f, then

$$\phi \xrightarrow{\text{CP}} X\phi^\star \xrightarrow{G_f} XA^\star\phi^\star \xrightarrow{\text{CP}} XA^\star X^\star \phi = (X^\star A X)^\star \, \phi$$

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- definition of generalized CP transformation (X being unitary and symmetric) ***(see e.g. Branco et al. ('11))***

$$\phi \xrightarrow{\text{CP}} X\phi^\star$$

- assume ϕ transforms as 3-dim rep of G_f, then

$$(X^\star A X)^\star = A' \quad \text{with in general} \quad A \neq A'$$

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- definition of generalized CP transformation (X being unitary and symmetric) ***(see e.g. Branco et al. ('11))***

$$\phi \stackrel{\text{CP}}{\longrightarrow} X\phi^\star$$

- assume $Z_2 \subset G_\nu$ is given by Z and is "combined" with CP

$$\phi \stackrel{\text{CP}}{\longrightarrow} X\phi^\star \stackrel{Z_2}{\longrightarrow} XZ^\star\phi^\star \quad \text{and} \quad \phi \stackrel{Z_2}{\longrightarrow} Z\phi \stackrel{\text{CP}}{\longrightarrow} ZX\phi^\star$$

Non-trivial breaking of G_f and CP

In principle, procedure like in case above, but some consistency conditions have to be fulfilled:

- definition of generalized CP transformation (X being unitary and symmetric) ***(see e.g. Branco et al. ('11))***

$$\phi \xrightarrow{\text{CP}} X\phi^\star$$

- assume $Z_2 \subset G_\nu$ is given by Z and is "combined" with CP

$$XZ^\star - ZX = 0$$

Non-trivial breaking of G_f and CP

Now we just need to re-consider the neutrino sector: $G_\nu = Z_2 \times$ CP

- invariance conditions for m_ν

$$Z^T m_\nu Z = m_\nu \quad \text{and} \quad X m_\nu X = m_\nu^\star$$

- notice we can choose a basis such that ($\nu = \Omega_\nu \, \nu'$)

$$X = \Omega_\nu \Omega_\nu^T \quad \text{and} \quad Z = \Omega_\nu Z^{diag} \Omega_\nu^\dagger \quad , \quad \Omega_\nu \text{ unitary}$$

- in this basis the conditions read

$$Z^{diag} [\Omega_\nu^T m_\nu \Omega_\nu] Z^{diag} = [\Omega_\nu^T m_\nu \Omega_\nu] \quad \text{and} \quad [\Omega_\nu^T m_\nu \Omega_\nu] = [\Omega_\nu^T m_\nu \Omega_\nu]^\star$$

Non-trivial breaking of G_f and CP

Now we just need to re-consider the neutrino sector: $G_\nu = Z_2 \times$ CP

- in this basis the conditions read

$$Z^{diag}[\Omega_\nu^T m_\nu \Omega_\nu] Z^{diag} = [\Omega_\nu^T m_\nu \Omega_\nu] \quad \text{and} \quad [\Omega_\nu^T m_\nu \Omega_\nu] = [\Omega_\nu^T m_\nu \Omega_\nu]^\star$$

- choose $Z^{diag} = \text{diag}\,(-1, 1, -1)$
- the form of m_ν is constrained by

$$\Omega_\nu^T m_\nu \Omega_\nu = \begin{pmatrix} m_{11} & 0 & m_{13} \\ 0 & m_{22} & 0 \\ m_{13} & 0 & m_{33} \end{pmatrix} \quad \text{with} \quad m_{ij} \text{ real}$$

Non-trivial breaking of G_f and CP

Now we just need to re-consider the neutrino sector: $G_\nu = Z_2 \times$ CP

- $\Omega_\nu^T m_\nu \Omega_\nu$ is diagonalized by

$$R(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \quad \text{with} \quad \tan 2\theta = \frac{2\, m_{13}}{m_{33} - m_{11}}$$

- diagonal matrix K_ν $(\pm 1, \pm i)$ renders neutrino masses positive

$$\Downarrow$$

m_ν is diagonalized by $\Omega_\nu R(\theta) K_\nu$

Non-trivial breaking of G_f and CP

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu R(\theta) K_\nu$$

- 3 unphysical phases are removed by $\Omega_e \to \Omega_e K_e$
- U_{PMNS} contains one parameter θ
- permutations of rows and columns of U_{PMNS} possible

$$\Downarrow$$

Predictions:

Mixing angles and CP phases are predicted
in terms of one parameter θ only,
up to permutations of rows/columns

Study of S_4 and CP

Generators in rep. $\mathbf{3}'$:

$$S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \;,\quad T = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ -1 & \sqrt{2} & -1 \end{pmatrix} \;,\quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

which fulfill

$$S^2 = \mathbb{1}\;,\;\; T^3 = \mathbb{1}\;,\;\; U^2 = \mathbb{1}\;,$$
$$(ST)^3 = \mathbb{1}\;,\;\; (SU)^2 = \mathbb{1}\;,\;\; (TU)^2 = \mathbb{1}\;,\;\; (STU)^4 = \mathbb{1}$$

Study of $\boldsymbol{S_4}$ and $\boldsymbol{CP}$

Transformations X in rep. $\mathbf{3}'$ for $Z=S$, $Z=U$, $Z=SU$:

$$X_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \; , \; X_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \; ,$$

$$X_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \; , \; X_4 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

which fulfill

$$X X^\dagger = X X^\star = \mathbb{1}$$

$$(X^\star A X)^\star = A' \;\; , \quad X Z^\star - Z X = 0$$

Study of S_4 and CP

Additional transformations X in rep. $\mathbf{3}'$ for $Z = S$

$$X_5 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \; , \; X_6 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

which fulfill

$$X X^\dagger = X X^\star = \mathbb{1}$$

$$(X^\star A X)^\star = A' \; , \quad X Z^\star - Z X = 0$$

Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and $X = X_1$

(Harrison/Scott ('02,'04), Grimus/Lavoura ('03), Feruglio et al. ('12,'13))

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - i\sqrt{3}\cos\theta \\ -\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

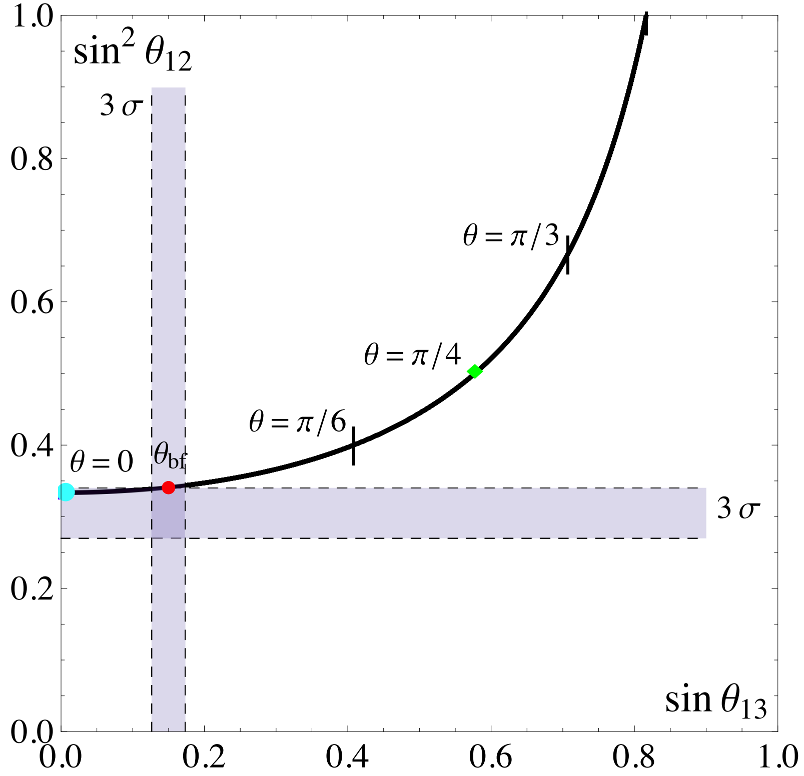
$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta \ , \quad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta} \ , \quad \sin^2\theta_{23} = \frac{1}{2}$$

and

$$|\sin\delta| = 1 \ , \quad |J_{CP}| = \frac{|\sin 2\theta|}{6\sqrt{3}} \ , \quad \sin\alpha = 0 \ , \quad \sin\beta = 0$$

Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and $X = X_1$



Study of S_4 and CP

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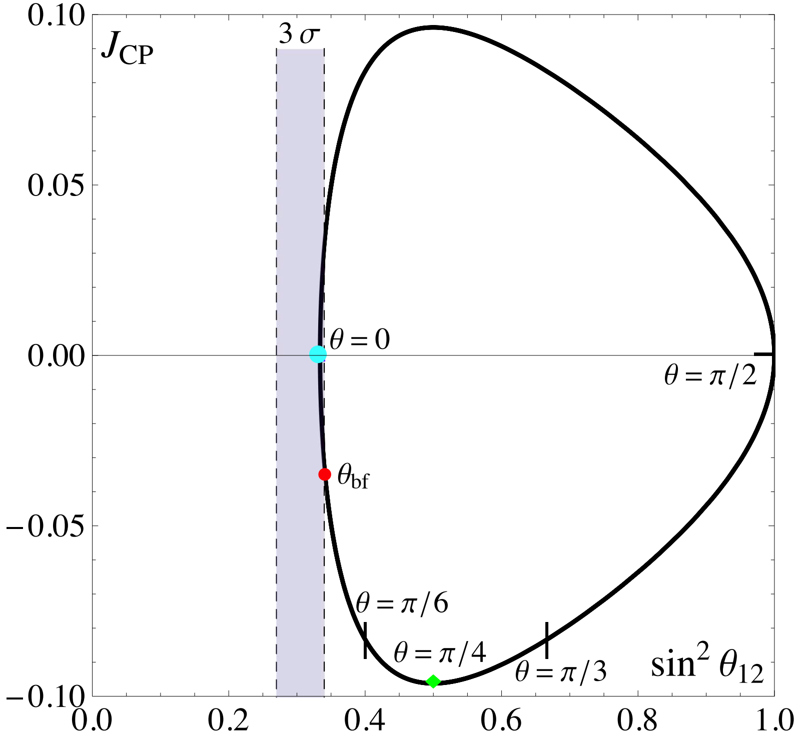
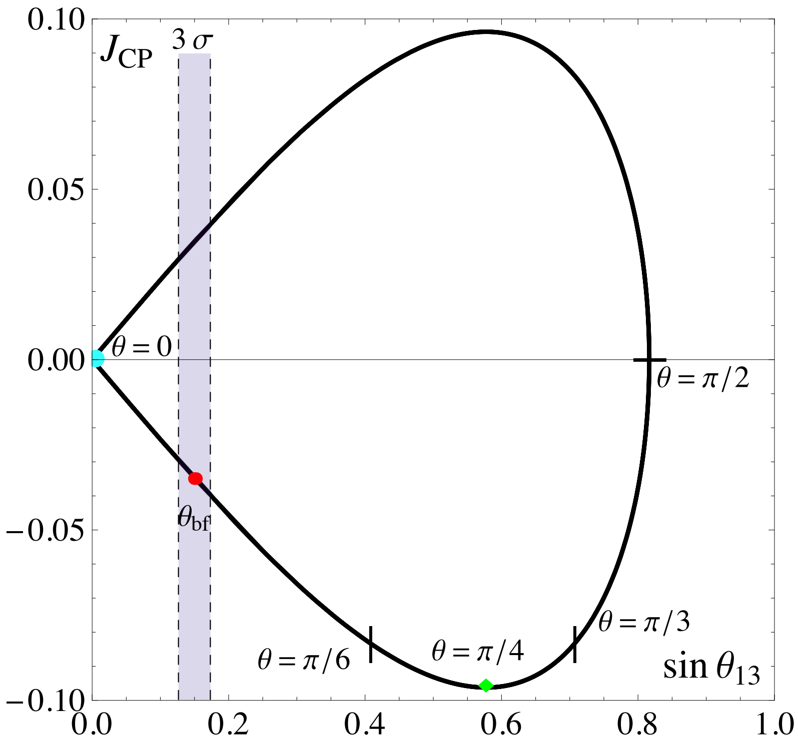


Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and $X = X_1$



Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and $X = X_1$

$$\theta_{\text{bf}} \approx 0.185 \quad , \quad \chi^2_{\text{min}} \approx 18.4 \quad \text{for} \quad \theta_{23} < \pi/4$$

$$\sin^2\theta_{13}(\theta_{\text{bf}}) = 0.023 \quad , \quad \sin^2\theta_{12}(\theta_{\text{bf}}) = 0.341 \quad ,$$
$$|J_{CP}(\theta_{\text{bf}})| = 0.0348$$

Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = SU$ and $X = X_1$

(Feruglio et al. ('12))

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2}\cos\theta & \sqrt{2}\sin\theta \\ -1 & \sqrt{2}\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2}\sin\theta - i\sqrt{3}\cos\theta \\ -1 & \sqrt{2}\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2}\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

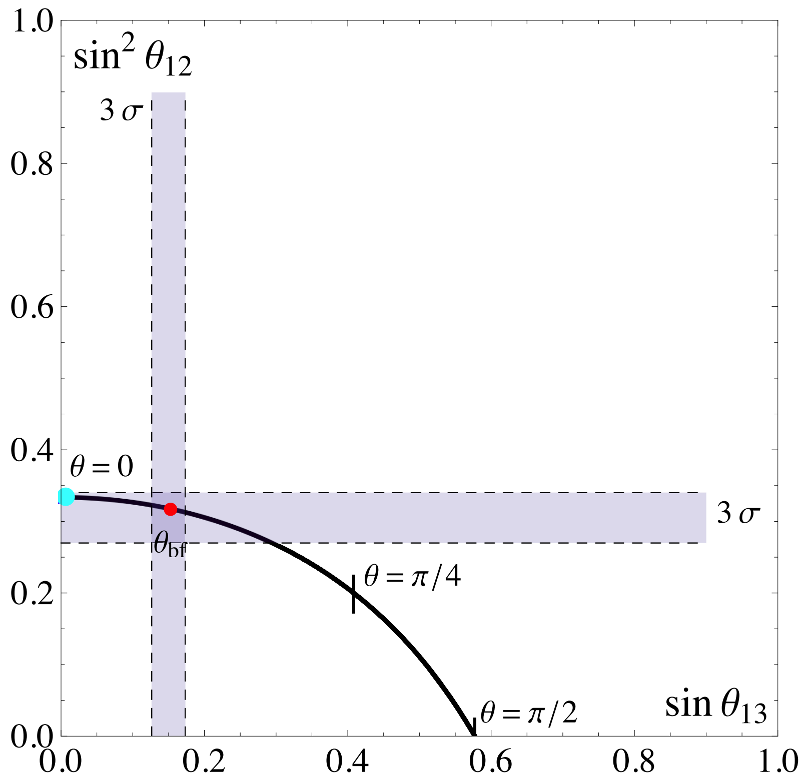
$$\sin^2\theta_{13} = \frac{1}{3}\sin^2\theta\ , \quad \sin^2\theta_{12} = \frac{\cos^2\theta}{2+\cos^2\theta}\ , \quad \sin^2\theta_{23} = \frac{1}{2}$$

and

$$|\sin\delta| = 1\ \ , \quad |J_{CP}| = \frac{|\sin 2\theta|}{6\sqrt{6}}\ , \quad \sin\alpha = 0\ \ , \quad \sin\beta = 0$$

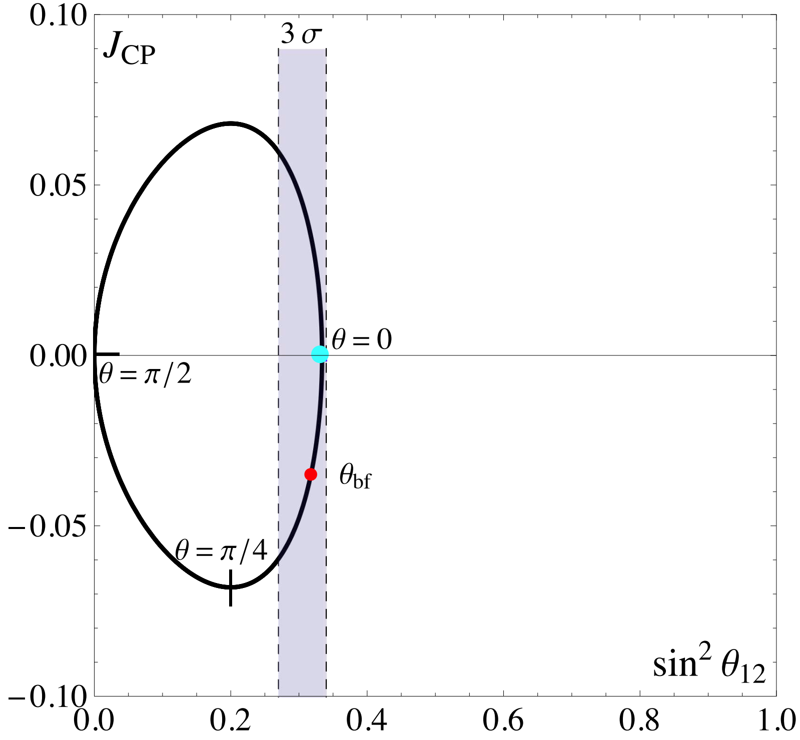
Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = SU$ and $X = X_1$



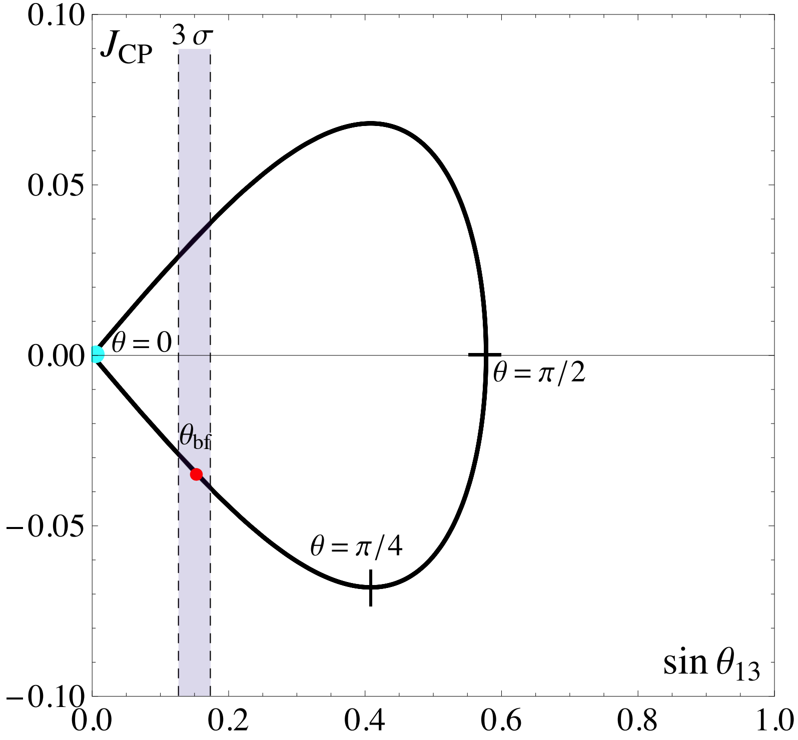
Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = SU$ and $X = X_1$



Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = SU$ and $X = X_1$



Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = SU$ and $X = X_1$

$$\theta_{\text{bf}} \approx 0.268 \quad , \quad \chi^2_{\text{min}} \approx 10.2 \quad \text{for} \quad \theta_{23} < \pi/4$$

$$\sin^2\theta_{13}(\theta_{\text{bf}}) = 0.023 \quad , \quad \sin^2\theta_{12}(\theta_{\text{bf}}) = 0.317 \quad ,$$
$$|J_{CP}(\theta_{\text{bf}})| = 0.0348$$

Study of S_4 and CP

Non-trivial Majorana phases from $G_e = Z_3$, $Z = S$ and $X = X_5$

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{2}\, e^{-i\theta}\, (i\sqrt{3} + e^{2i\theta}) & 1 & \frac{1}{2}\, e^{-i\theta}\, (-i\sqrt{3} + e^{2i\theta}) \\ -e^{i\theta} & 1 & -e^{i\theta} \\ \frac{1}{2}\, e^{-i\theta}\, (-i\sqrt{3} + e^{2i\theta}) & 1 & \frac{1}{2}\, e^{-i\theta}\, (i\sqrt{3} + e^{2i\theta}) \end{pmatrix} K_\nu$$

$$|\sin\delta| = \left| \frac{(4+\sqrt{3}\sin 2\theta)\cos 2\theta \sqrt{4 - 2\sqrt{3}\sin 2\theta}}{5 + 3\cos 4\theta} \right| \;\; , \quad |J_{CP}| = \frac{|\cos 2\theta|}{6\sqrt{3}} \;\; ,$$

$$|\sin\alpha| = \left| \frac{\sqrt{3} + 2\sin 2\theta}{2 + \sqrt{3}\sin 2\theta} \right| \;\; , \quad |\sin\beta| = \left| \frac{4\sqrt{3}\cos 2\theta}{5 + 3\cos 4\theta} \right|$$

Study of S_4 and CP

Non-trivial Majorana phases from $G_e = Z_3$, $Z = S$ and $X = X_5$

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{2}\, e^{-i\theta}\, (i\sqrt{3} + e^{2i\theta}) & 1 & \frac{1}{2}\, e^{-i\theta}\, (-i\sqrt{3} + e^{2i\theta}) \\ -e^{i\theta} & 1 & -e^{i\theta} \\ \frac{1}{2}\, e^{-i\theta}\, (-i\sqrt{3} + e^{2i\theta}) & 1 & \frac{1}{2}\, e^{-i\theta}\, (i\sqrt{3} + e^{2i\theta}) \end{pmatrix} K_\nu$$

$$\sin^2\theta_{13} = \frac{1}{3}\left(1 - \frac{\sqrt{3}}{2}\sin 2\theta\right) \;, \quad \sin^2\theta_{12} = \frac{2}{4 + \sqrt{3}\sin 2\theta} \;,$$

$$\sin^2\theta_{23} = \begin{cases} \sin^2\theta_{12} \\ 1 - \sin^2\theta_{12} \end{cases}$$

Study of S_4 and CP

Non-trivial Majorana phases from $G_e = Z_3$, $Z = S$ and $X = X_5$

$$\theta_{\rm bf} \approx \pi/4 \ \ , \quad \chi^2_{\rm min} \gtrsim 100$$

$$\sin^2\theta_{13}(\theta_{\rm bf}) = 0.045 \ \ , \quad \sin^2\theta_{12}(\theta_{\rm bf}) = 0.349 \ \ ,$$

$$\sin^2\theta_{23}(\theta_{\rm bf}) = \begin{cases} 0.349 \\ 0.651 \end{cases}$$

$$\sin\delta(\theta_{\rm bf}) = 0 \ \ , \quad |\sin\alpha|(\theta_{\rm bf}) = 1 \ \ , \quad \sin\beta(\theta_{\rm bf}) = 0$$

Study of S_4 and CP

Trivial CP phases from $G_e = Z_3$, $Z = S$ and $X = X_3$ (Case II)

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta - \sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + \sqrt{3}\cos\theta \\ -\cos\theta + \sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - \sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta \; , \quad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta} \; , \quad \sin^2\theta_{23} = \frac{1}{2}\left(1 - \frac{\sqrt{3}\sin 2\theta}{2+\cos 2\theta}\right)$$

and

$$\sin\delta = 0 \;\; , \quad \sin\alpha = 0 \;\; , \quad \sin\beta = 0$$

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Trivial CP phases from $G_e = Z_3$, $Z = S$ and $X = X_3$ (Case II)

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$$\theta_{\text{bf}} \approx \begin{cases} 0.184, & \theta_{23} < \pi/4 \\ 2.96, & \theta_{23} > \pi/4 \end{cases} \quad , \quad \chi^2_{\text{min}} \approx 10 \quad : \quad \sin^2\theta_{13}(\theta_{\text{bf}}) = 0.022 \ ,$$

$$\sin^2\theta_{12}(\theta_{\text{bf}}) = 0.341 \ , \quad \sin^2\theta_{23}(\theta_{\text{bf}}) = \begin{cases} 0.394 \\ 0.606 \end{cases}$$

Study of S_4 and CP

Trivial CP phases from $G_e = Z_3$, $Z = SU$ and $X = X_2$ (Case V)

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2}\cos\theta & \sqrt{2}\sin\theta \\ -1 & \sqrt{2}\cos\theta + \sqrt{3}\sin\theta & \sqrt{2}\sin\theta - \sqrt{3}\cos\theta \\ -1 & \sqrt{2}\cos\theta - \sqrt{3}\sin\theta & \sqrt{2}\sin\theta + \sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

$$\sin^2\theta_{13} = \frac{1}{3}\sin^2\theta \; , \quad \sin^2\theta_{12} = \frac{\cos^2\theta}{2+\cos^2\theta} \; , \quad \sin^2\theta_{23} = \frac{1}{2}\left(1 - \frac{2\sqrt{6}\sin 2\theta}{5+\cos 2\theta}\right)$$

and

$$\sin\delta = 0 \;\; , \quad \sin\alpha = 0 \;\; , \quad \sin\beta = 0$$

Study of S_4 and CP

Trivial CP phases from $G_e = Z_3$, $Z = SU$ and $X = X_2$ (Case V)

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2}\cos\theta & \sqrt{2}\sin\theta \\ -1 & \sqrt{2}\cos\theta + \sqrt{3}\sin\theta & \sqrt{2}\sin\theta - \sqrt{3}\cos\theta \\ -1 & \sqrt{2}\cos\theta - \sqrt{3}\sin\theta & \sqrt{2}\sin\theta + \sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

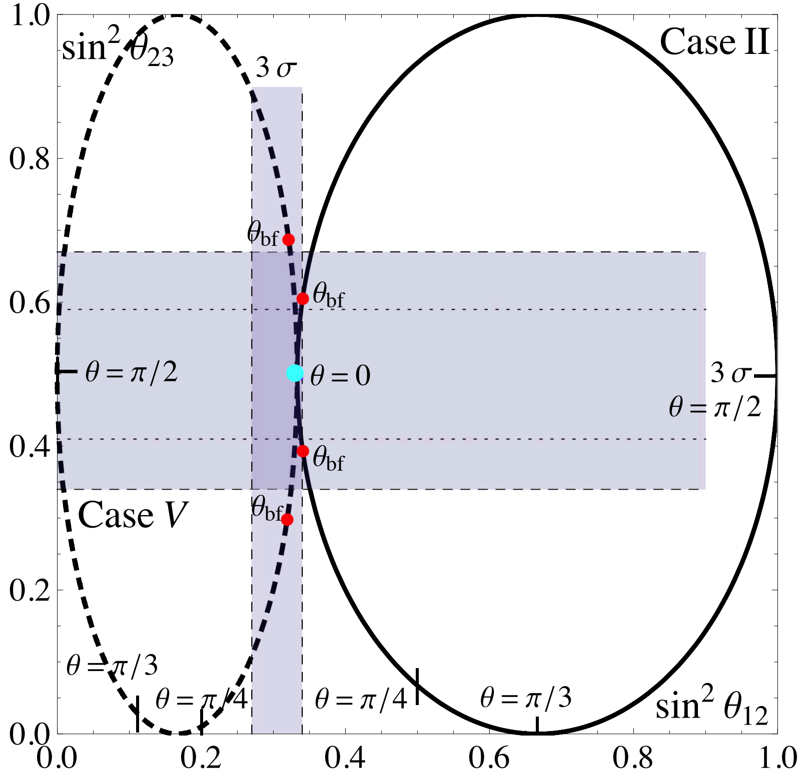
$$\theta_{\text{bf}} \approx 0.251 \quad \text{for} \quad \theta_{23} < \pi/4 \;\;, \quad \chi^2_{\text{min}} \approx 16.1$$

$$\sin^2\theta_{13}(\theta_{\text{bf}}) = 0.021 \;\;, \quad \sin^2\theta_{12}(\theta_{\text{bf}}) = 0.319 \;\;,$$

$$\sin^2\theta_{23}(\theta_{\text{bf}}) = 0.299$$

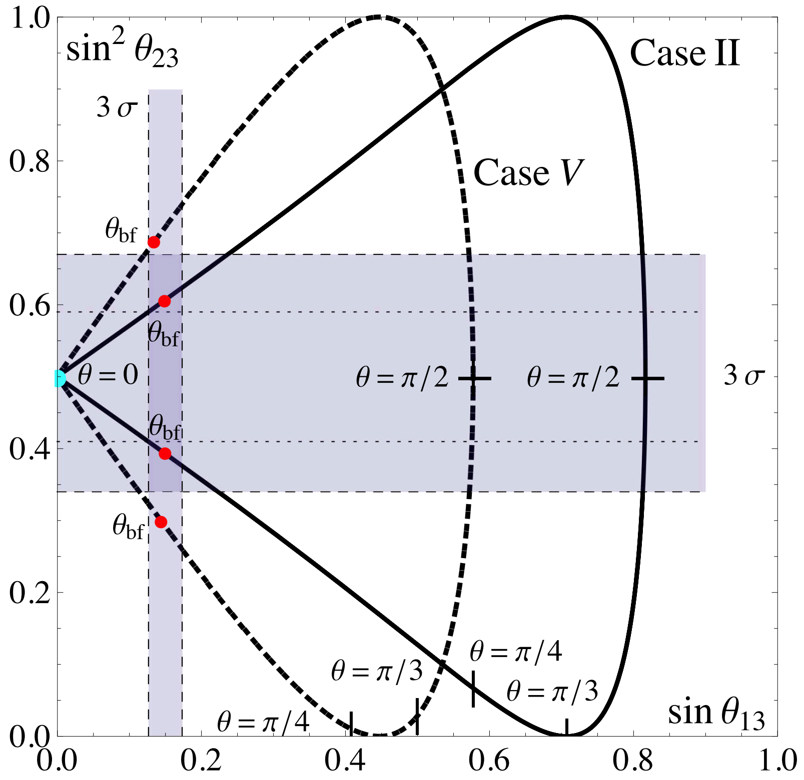
Study of S_4 and CP

Cases with trivial CP phases



Study of S_4 and CP

Cases with trivial CP phases



Study of S_4 and CP

Trivial CP phases from $G_e = Z_4$, $Z = U$ and $X = X_2$
and from $G_e = Z_2 \times Z_2$, $Z = U$ and $X = X_1$

$$U_{PMNS} = \frac{1}{2} \begin{pmatrix} \cos\theta - \sqrt{2}\,\sin\theta & 1 & \sqrt{2}\,\cos\theta + \sin\theta \\ -\sqrt{2}\,\cos\theta & \sqrt{2} & -\sqrt{2}\,\sin\theta \\ \cos\theta + \sqrt{2}\,\sin\theta & 1 & -\sqrt{2}\,\cos\theta + \sin\theta \end{pmatrix} K_\nu$$

$$\sin^2\theta_{13} = \frac{1}{4}\left(\sqrt{2}\cos\theta + \sin\theta\right)^2 , \quad \sin^2\theta_{12} = \frac{2}{5 - \cos 2\theta - 2\sqrt{2}\sin 2\theta} ,$$

$$\sin^2\theta_{23} = \begin{cases} \frac{4\sin^2\theta}{5-\cos 2\theta-2\sqrt{2}\sin 2\theta}, & \text{Case a} \\ 1 - \frac{4\sin^2\theta}{5-\cos 2\theta-2\sqrt{2}\sin 2\theta}, & \text{Case b} \end{cases}$$

Study of S_4 and CP

Trivial CP phases from $G_e = Z_4$, $Z = U$ and $X = X_2$
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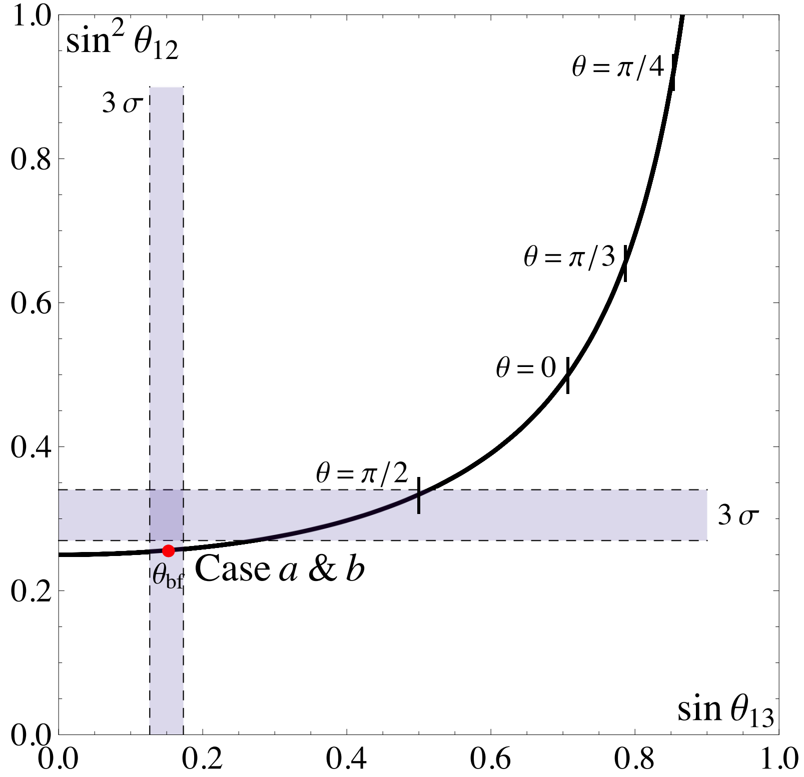
$$\theta_{\text{bf}} \approx 2.01 \ , \quad \chi^2_{\text{min}} \approx 12 \quad : \quad \sin^2\theta_{13}(\theta_{\text{bf}}) = 0.023 \ ,$$

$$\sin^2\theta_{12}(\theta_{\text{bf}}) = 0.256 \ , \quad \sin^2\theta_{23}(\theta_{\text{bf}}) = \begin{cases} 0.420, & \text{Case a} \\ 0.581, & \text{Case b} \end{cases}$$

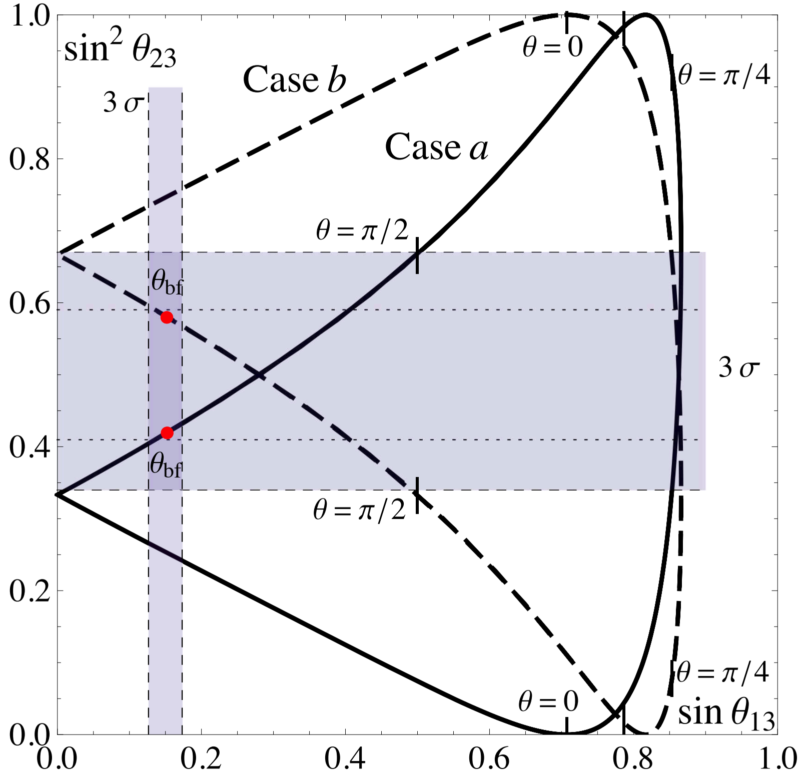
Study of S_4 and CP

Cases with $G_e = Z_4$ or $G_e = Z_2 \times Z_2$



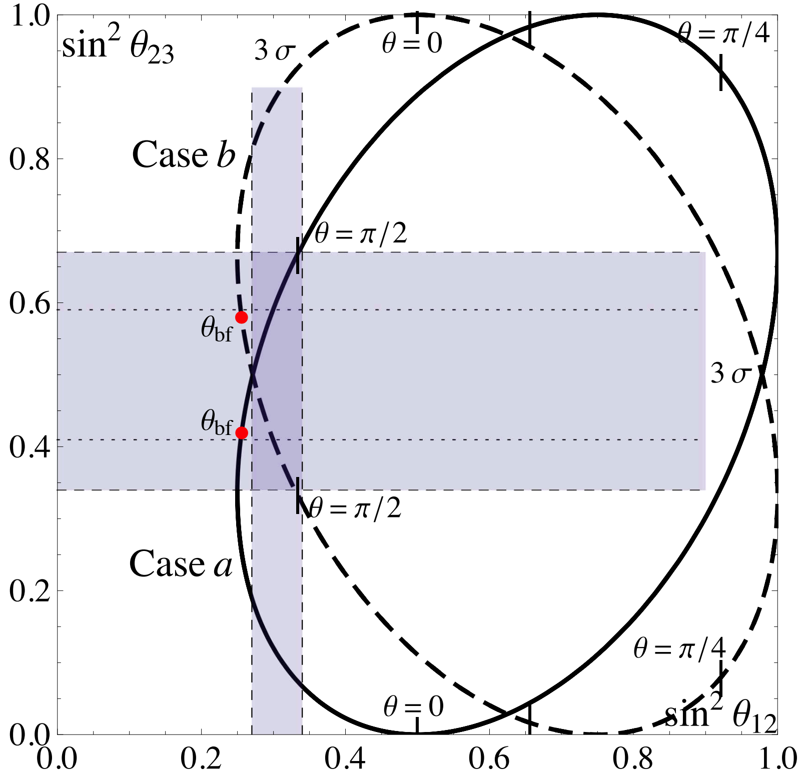
Study of S_4 and CP

Cases with $G_e = Z_4$ or $G_e = Z_2 \times Z_2$



Study of S_4 and CP

Cases with $G_e = Z_4$ or $G_e = Z_2 \times Z_2$



Study of S_4 and CP

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.... because the CP symmetry of neutrinos is also present in the charged lepton sector!

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.... we need to check the following

$$X^\star m_e^\dagger m_e X = (m_e^\dagger m_e)^\star$$

Study of S_4 and CP

Why are the CP phases trivial?
.... because the CP symmetry of neutrinos is also present in the charged lepton sector!
How to see this?
.... we need to check the following

$$X^\star m_e^\dagger m_e X = (m_e^\dagger m_e)^\star$$

.... this is equivalent to checking

$$Q_{e,i} X - X Q_{e,i}^T = 0$$

[compare to condition $X Z^\star - Z X = 0$]

Conclusions

- Large θ_{13} has opened the door for the Dirac phase and $0\nu\beta\beta$ exp. might measure soon a Majorana phase
- Flavor symmetries alone can predict the Dirac phase and, if combined with a CP symmetry, also the Majorana phases
- Flavor & CP approach has one free parameter θ
  $\rightarrow$ explain small θ_{13} in a model via θ, see recent S_4 model
  (Feruglio et al. ('13))

Thank you for your attention.