On the structure of the charged lepton mass matrix

Andrea Romanino

SISSA/ISAS

work in progress with David Marzocca

Rise and fall of correlations

$$m_\nu = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

3 independent correlations

$m_{12} = m_{13}$ $m_{22} = m_{33}$

$m_{11} + m_{12} = m_{22} + m_{23}$

$$\sin^2\theta_{13} = 0$$

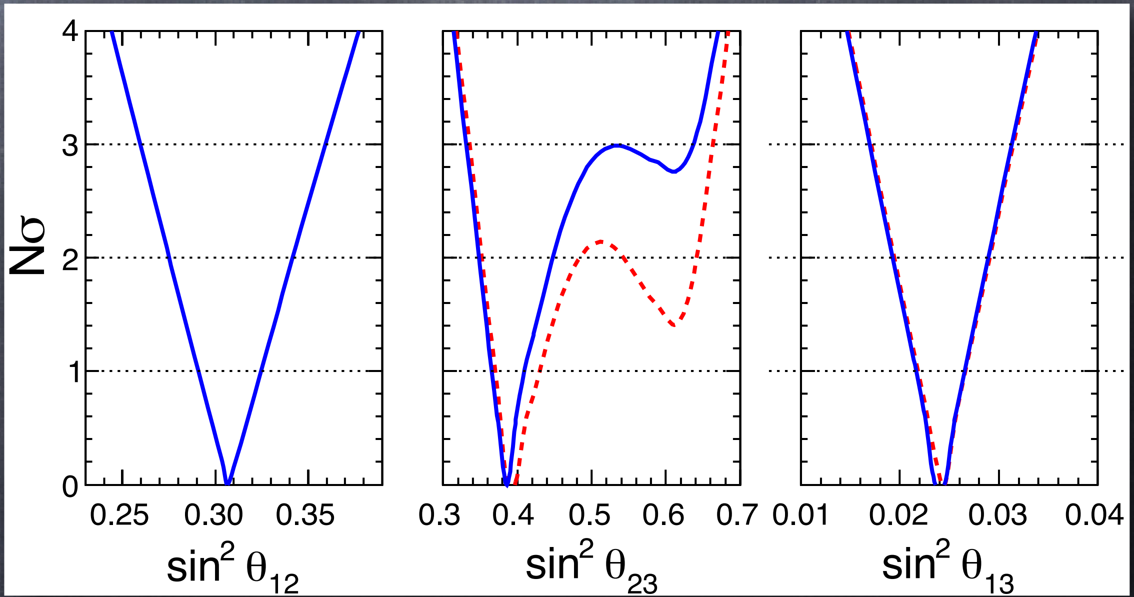
$$\sin^2\theta_{23} = 1/2$$

$$\sin^2\theta_{12} = 1/3$$

symmetry?

non-abelian, discrete

full lagrangian



- Data do not seem to show hints of TBM or of other correlations in m_ν

- Waiting for those to appear...

  - Study possible corrections (from charged leptons)

  - Face the possibility that no correlations are forced by symmetries

- Data do not seem to show any hint of TBM or of other correlations in m_ν

- Waiting for those to appear...

  - Study possible corrections (from charged leptons)

    Xing, Giunti Tanimoto, Frampton Petcov Rodejohann, Altarelli Feruglio Masina, Romanino, King, Antusch King, Dev Gupta Gautam, Antusch Maurer, Meroni Petcov Spinrath, Duarah Das Singh, Chao Zheng, Meloni, Antusch Gross, Maurer Sluka, Altarelli Feruglio Merlo Stamou, Gollu Deepthi Mohanta

  - Face the possibility that no correlations are forced by symmetries

- Data do not seem to show any hint of TBM or of other correlations in m_ν

- Waiting for those to appear...

  - Study possible corrections (from charged leptons)

    Marzocca Petcov R Spinrath
    Marzocca Petcov R Sevilla

  - Face the possibility that no correlations are forced by symmetries

Corrections to θ_{13} from charged leptons

- $U = U_e U_\nu^\dagger$: assume both U_e, U_ν would give $\theta_{13} = 0$:

$$U_e = \Psi_e R_{12}(\theta^e_{12}) R_{23}(\theta^e_{23}) \Phi_e$$
$$U_\nu = \Psi_\nu R_{12}(\theta^\nu_{12}) R_{23}(\theta^\nu_{23}) \Phi_\nu$$ (TBM, BM specific cases)

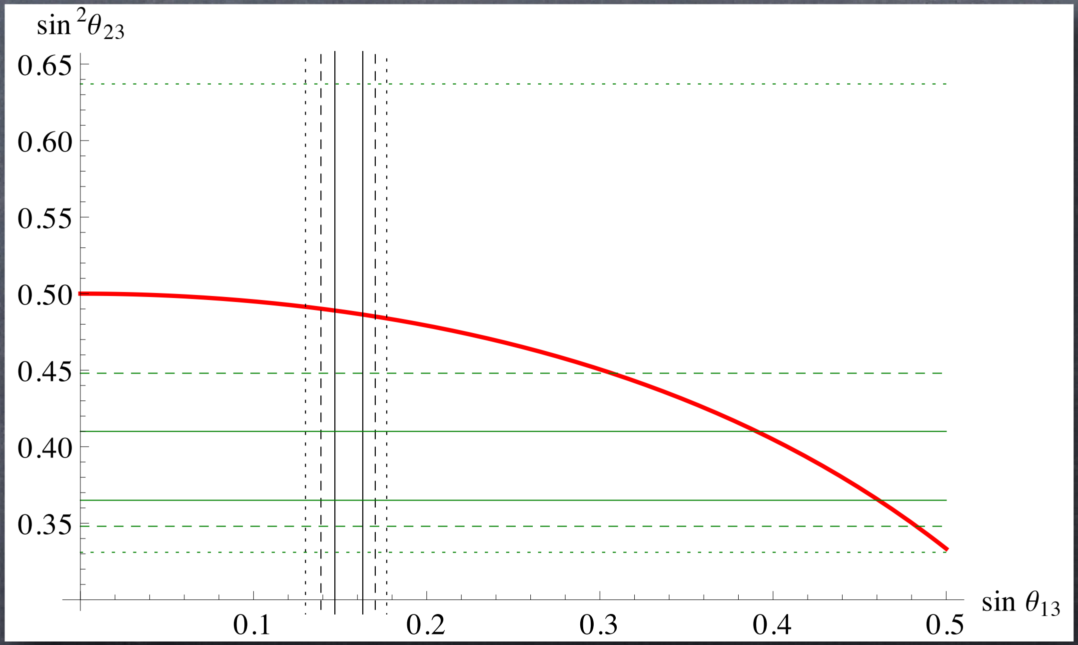
- Physical parameters: $U = R_{12}(\theta^e_{12}) \Phi R_{23}(\hat{\theta}_{23}) R_{12}(\theta^\nu_{12}) Q$

- $\sin\theta_{13} = \sin\theta^e_{12} \sin\hat{\theta}_{23},$ ← generated from the interplay of 12/23

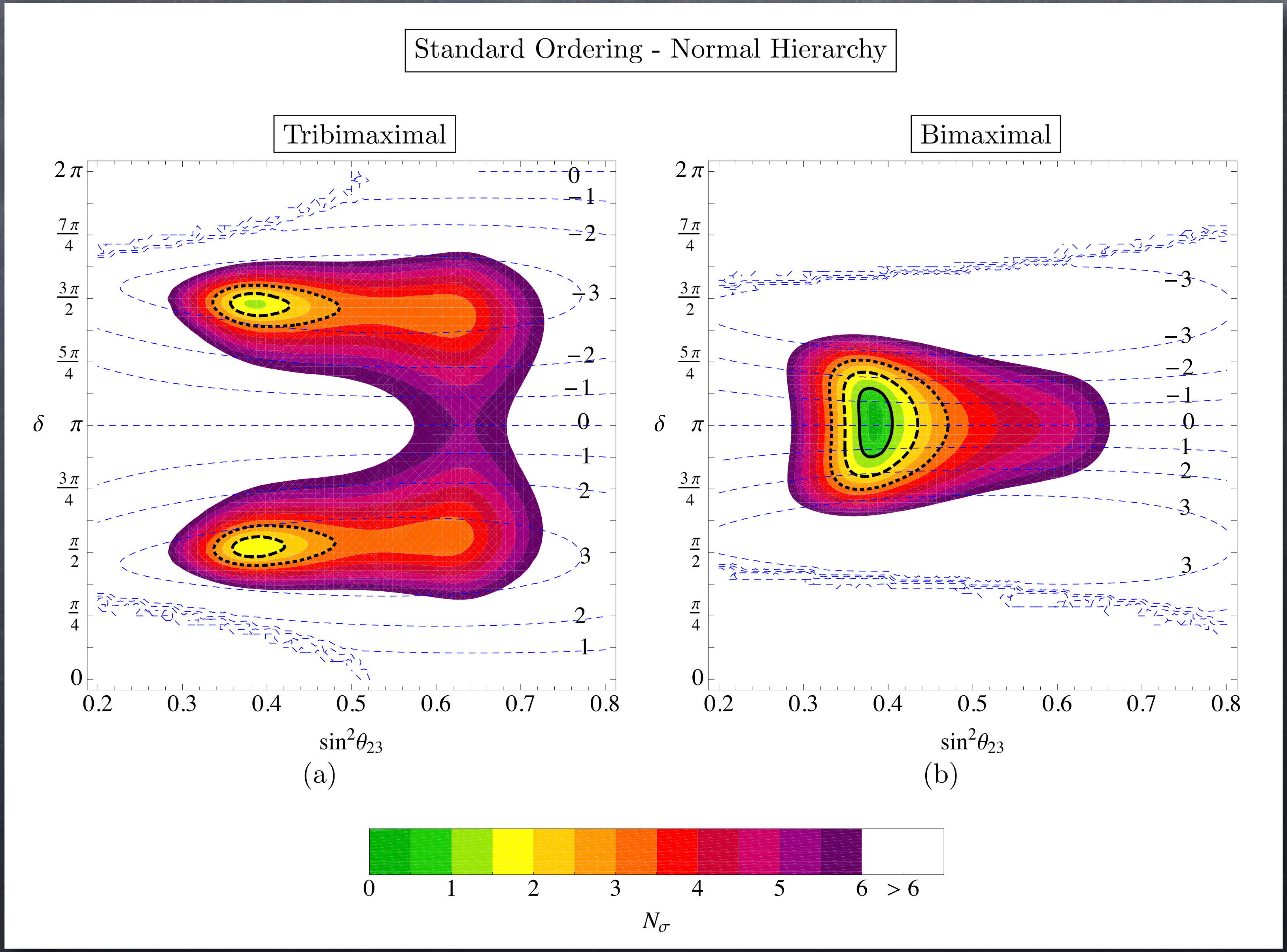
$$\sin^2\theta_{23} = \frac{\sin^2\hat{\theta}_{23} - \sin^2\theta_{13}}{1 - \sin^2\theta_{13}\sin^2\hat{\theta}_{23}}$$

Correction to θ_{23}

Is needed if θ_{23} is indeed significantly < $\pi/4$



- If $\sin^2\theta_{12}^{\nu}$ is given (TBM: 1/3, BM: 1/2): 1 prediction



prefers $\theta_{23} < \pi/4$, $\delta \approx \pi$

- *bimaximal*, $\theta^{\nu}_{12} = \dfrac{\pi}{4}$:

$$\sin^2\theta_{12} \sim \frac{1}{2} + \cot\theta_{23}\sin\theta_{13}\cos\phi$$

- *tri-bimaximal*, $\sin^2\theta^{\nu}_{12} = \frac{1}{3}$:

$$\sin^2\theta_{12} \sim \frac{1}{3}\left[1 + 2\sqrt{2}\,\cot\theta_{23}\sin\theta_{13}\cos\phi\right]$$

- Data do not seem to show any hint of TBM or of other correlations in m_ν

- Waiting for those to appear...

  - Study possible corrections (from charged leptons)

  - Face the possibility that no correlations are forced by symmetries

Marzocca, R

Neutrino mixing without correlations

- (No correlations does not mean anarchy. Symmetries to control small matrix entries. Charged leptons highly disciplined)
- If no correlations are forced by a symmetry (and do not arise accidentally) what is the most general form of a mass matrix?
- What are the charged leptons contributions to neutrino mixing that can be obtained "naturally"?
- New model-building avenues

Example: θ_{23} from charged leptons

$$M^E_{[23]} = \begin{pmatrix} m_{22} & m_{23} \\ m_{32} & m_{33} \end{pmatrix}$$

(RL convention)

$$m_\mu m_\tau = |m_{33} m_{22} - m_{23} m_{32}|$$

no correlations:

$$|m_{33} m_{22}| \lesssim m_\mu m_\tau$$
$$|m_{23} m_{32}| \lesssim m_\mu m_\tau$$

$$\left| \frac{m_{32}}{m_{33}} \right| = \tan\theta_{23} = \mathcal{O}(1)$$

$$|m_{32}| \sim |m_{33}| \sim m_\tau$$

$$M^E_{[23]} \sim \begin{pmatrix} m_\mu & m_\mu \\ m_\tau & m_\tau \end{pmatrix}$$

"Uncorrelated" m_E

- Notation:
  - "≈" : approximately equal
  - " ~ " : equal up to O(1)
  - "≲" : less up to O(1), opposite of "»"
- Notation:
  - $M_{[i_1 \ldots i_n][j_1 \ldots j_m]} : \quad (M_{[i_1 \ldots i_n][j_1 \ldots j_m]})_{kl} = M_{i_k j_l}$
  - $M_{[i_1 \ldots i_n]} \equiv M_{[i_1 \ldots i_n][i_1 \ldots i_n]}$
- Observation:
  - $|m_{33}| = \quad |\det M_{[3]}| \sim m_3$
  - $|\det M_{[23]}| \sim m_2\, m_3$ (up to permutation of rows and columns)
  - $|\det M| = \quad |\det M_{[123]}| = m_1\, m_2\, m_3$

- Theorem
  - $|M_{ij}| \leq m_3$
  - $|\det M_{[ij]\,[hk]}| \leq m_2\, m_3$

- Definition
  - M is "uncorrelated" if the sum of all terms in the expression of $\det M_{[ij]\,[hk]}$ and $\det M$ is not much smaller than the individual terms

What is the most general form of an uncorrelated M?

$$|M_E| = \begin{pmatrix} \sqrt{k_{11}}m_1 & \sqrt{k_{12}m_1m_2}/R_{12} & \sqrt{k_{13}m_1m_3}/R_{13} \\ \sqrt{k_{12}m_1m_2}R_{12} & \sqrt{k_{22}}m_2 & \sqrt{k_{23}m_2m_3}/R_{23} \\ \sqrt{k_{13}m_1m_3}R_{13} & \sqrt{k_{23}m_2m_3}R_{23} & \sqrt{k_{33}}m_3 \end{pmatrix}$$

$$k_{ij} = |M_{ij}M_{ji}|/(m_i m_j) \qquad R_{ij}^2 = |M_{ij}/M_{ji}|$$

$$k_{ij} \lesssim 1 \quad \text{except} \quad k_{13} \lesssim m_2/m_1 \quad k_{13}\sqrt{k_{22}} \lesssim 1 \quad k_{33} \sim 1$$

$$\sqrt{\frac{m_i}{m_j}k_{ij}} \lesssim R_{ij} \lesssim \sqrt{\frac{m_j}{m_i k_{ij}}} \qquad \sqrt{\frac{m_1}{m_2}k_{13}k_{23}} \lesssim \frac{R_{13}}{R_{23}} \lesssim \sqrt{\frac{m_2}{m_1 k_{13} k_{23}}}$$

$$\sqrt{k_{12}k_{23}k_{13}} \lesssim \frac{R_{23}R_{12}}{R_{13}} \lesssim \frac{1}{\sqrt{k_{12}k_{23}k_{13}}}$$

Charged lepton contribution to U

$$U = U_e U_\nu^\dagger$$

$$M_E = U_{e^c}^T M_E^{\text{diag}} U_e = \begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ U_{31}^e & U_{32}^e & U_{33}^e \end{pmatrix} U_{33}^{e^c} m_\tau + \mathcal{O}\left(m_\mu, m_e\right)$$

$$|m_{31}| : |m_{32}| : |m_{33}| = |U_{31}^e| : |U_{32}^e| : |U_{33}^e| \quad \text{up to } \mathcal{O}\left(\frac{m_\mu}{m_\tau}\right) \sim 0.05$$

All from U_e?

- $U_\nu = 1 \qquad U = U_e$ Altarelli Feruglio Masina: NO

- $|m_{31}| : |m_{32}| : |m_{33}| = |U_{31}| : |U_{32}| : |U_{33}| \approx 0.4 : 0.5 : 0.8$

Same up to O(1)

- $$|M_E| = \begin{pmatrix} \sqrt{k_{11}} m_1 & \sqrt{k_{12} m_1 m_2}/R_{12} & m_1 k_{13} \\ \sqrt{k_{12} m_1 m_2} R_{12} & \sqrt{k_{22}} m_2 & m_2 k_{23} \\ \mathcal{O}(m_3) & \mathcal{O}(m_3) & \mathcal{O}(m_3) \end{pmatrix}$$

$$k_{ij} \lesssim 1 \quad \text{except} \quad k_{13} \lesssim m_2/m_1 \quad k_{13}\sqrt{k_{22}} \lesssim 1$$

$$\sqrt{\frac{m_1}{m_2} k_{12}} \lesssim R_{12} \lesssim \sqrt{\frac{m_2}{m_1 k_{12}}} \min(1, k_{13}^{-1})$$

A simple example

$$|M_E| \sim \begin{pmatrix} m_1 & m_1 & m_1 \\ m_2 & m_2 & m_2 \\ m_3 & m_3 & m_3 \end{pmatrix} \to \begin{cases} U = R_{12}(\omega_{12}) R_{23}(\omega_{23}) R_{12}(\omega'_{12}) \\ \omega_{12}, \omega_{23}, \omega'_{12} = \mathcal{O}(1) \end{cases}$$

$\sin \omega_{23} \approx 0.6$

$\sin \omega_{12} \approx 0.25$ (smallish, by a factor 2)

$\sin \omega'_{12} \sim 0.5$ (depending on δ)

A simple example with $k_{13} > 1$

$$|M_E| \sim \begin{pmatrix} m_1 & m_1 & m_1 k_{13} \\ m_2/k_{13} & m_2/k_{13} & m_2 \\ m_3 & m_3 & m_3 \end{pmatrix} \to \begin{cases} U = R_{12}(\omega_{12}) R_{23}(\omega_{23}) R_{12}(\omega'_{12}) \\ \omega_{12} \sim 1/k_{13}, \quad \omega_{23}, \omega'_{12} = \mathcal{O}(1) \end{cases}$$

$$\sin\omega_{23} \approx 0.6$$

$$\sin\omega_{12} \approx 0.25 \qquad (k_{13} = \mathcal{O}(4))$$

$$\sin\omega'_{12} \sim 0.5 \qquad (\text{depending on } \delta)$$

$k_{13} \gg 1$: counter example to AFM

$k_{13} \sim m_\mu/m_e \sim 200$ (needs $U_\nu \neq 1$)

$$|M_E| \sim \begin{pmatrix} m_1 & m_1 & m_2 \\ m_1 & m_1 & m_2 \\ m_3 & m_3 & m_3 \end{pmatrix} \to \begin{cases} U \approx R_{23}(\omega_{23}) R_{12}(\omega'_{12}) \\ \omega_{23}, \omega'_{12} = \mathcal{O}(1) \end{cases}$$

natural "inverted ordering"
can be used if $U_\nu \neq 1$

U_ν provides a maximal solar angle

$$|M_E| \sim \begin{pmatrix} m_1 & m_1/\theta_C & m_1/\theta_C \\ \theta_C\, m_2 & m_2 & m_2 \\ \theta_C\, m_3 & m_3 & m_3 \end{pmatrix} \to \begin{cases} U = R_{12}(\omega_{12}) R_{23}(\omega_{23}) R_{12}(\omega'_{12}) \\ \omega_{12}, \omega'_{12} = \mathcal{O}\left(\theta_C\right), \quad \omega_{23} = \mathcal{O}\left(1\right) \end{cases}$$

accounts for quark-lepton complementarity
and for $\theta_{13} \sim \theta_C/\sqrt{2}$

In summary

- Despite the large amount of work devoted to charged lepton corrections to neutrino mixing, not all the potential of the charged lepton mass matrix has been explored yet