$b\to s$ transitions and Lattice QCD

Federico Mescia
ECM & ICC,
Universitat de Barcelona

✓ ***BIG News:*** *November 2012, first evidence of* $B_s\to\mu^+\mu^-$ *from* LHCb

➲ *PANIC:* the measured $Br^{exp}(B_s\to\mu\mu) = (3.2 \pm 1.5)10^{-9}$ is close to the SM, $Br^{SM}(B_s\to\mu\mu) = (3.3 \pm 0.3)10^{-9}$

✓ ***NEWS:*** *information from* $B\to K\mu\mu$ *and* $B\to K^*\mu\mu$ *available* BaBar & LHCb - 2012:

➲ *NO PANIC:* $Br(B\to K^{(*)}\mu\mu)$ and $Br(B_s\to\mu\mu)$ *sensitive to different "b-> s couplings"*

Portoroz, April 14-18, 2013

$Br(B_d \to X_s \gamma) \propto |C_7|$

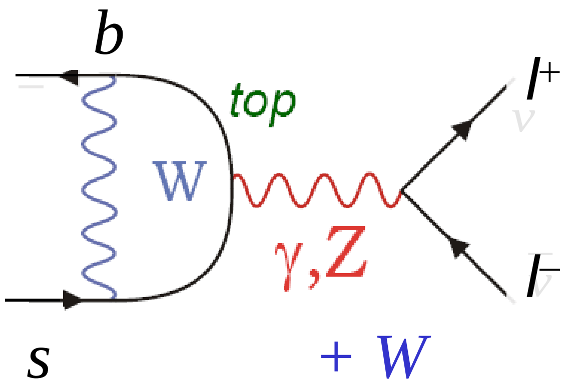
th (7%):	$(3.13 \pm 0.23) \times 10^{-4}$
exp (7%):	$(3.52 \pm 0.24) \times 10^{-4}$

Babar+Belle 1999-2007

$Br(B_d \to X_s l^+ l^-) \propto C_7 C_9 + |C_9| + |C_{10}| + |C_7|$

	exp (30%):	th (15-25%):
$[q^2 \in [0.04, 1.0]\ \mathrm{GeV}^2]$	$(0.6 \pm 0.5) \times 10^{-6}$	$(0.8 \pm 0.2) \times 10^{-6}$
$[q^2 \in [1.0, 6.0]\ \mathrm{GeV}^2]$	$(1.6 \pm 0.5) \times 10^{-6}$	$(1.6 \pm 0.1) \times 10^{-6}$
$[q^2 > 14.4\ \mathrm{GeV}^2]$	$(4.4 \pm 1.3) \times 10^{-7}$	$(2.4 \pm 0.8) \times 10^{-7}$

Babar+Belle 2007



+ W

box

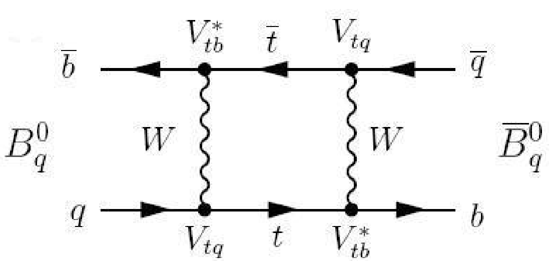
$$Heff = \begin{cases} C_7 \times \bar{b}_R \sigma^{\mu\nu} s_L F_{\mu\nu} + \\ C_9 \times \bar{b}_L \gamma^\mu s_L \bar{l} \gamma^\mu l + C_{10} \times \bar{b}_L \gamma^\mu s_L \bar{l} \gamma^\mu \gamma_5 l \\ C_S \times \bar{b}_L s_R \bar{l} l + C_P \times \bar{b}_L s_R \bar{l} \gamma_5 l + \\ C_T \times \bar{b}_R \sigma^{\mu\nu} s_L \bar{l} \sigma^{\mu\nu} l + C_{T5} \times \bar{b}_R \sigma^{\mu\nu} s_L \bar{l} \sigma^{\mu\nu} \gamma_5 l \\ +L \Leftrightarrow R : (C'_9, C'_{10}, C'_S, C'_P, C_{T5}) \end{cases}$$

SM Ops. (first two lines)

BSM Ops. (last three lines)

ΔM_s

$$C_{box} \times \bar{b}_L \gamma^\mu s_L \bar{b}_L \gamma^\mu s_L$$



Tevatron 2006

$Br(B_s \to \mu^+ \mu^-) \propto C^{(\prime)}_{9,10,S,P}$

$Br(B_d \to K^* \gamma) \propto |C^{(\prime)}_7|$

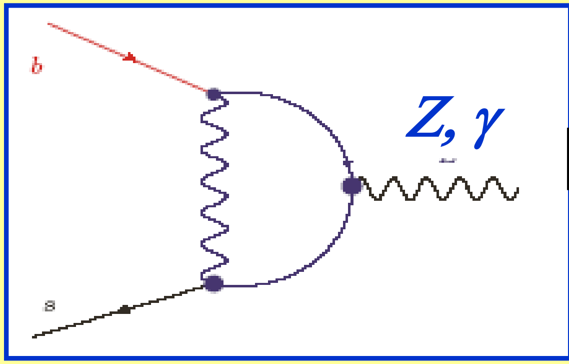
LHCb 2012

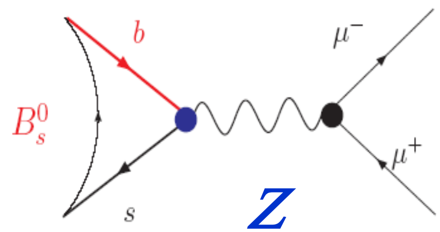
$Br(B_d \to K l^+ l^-) \propto C^{(\prime)}_{9,10,S,P,T}$

$B_d \to K^* l^+ l^- \propto C^{(\prime)}_{9,10,S,P,T}$

here, we expect theory and exp. progress

Theory: Effective lagrangian at $\mu \sim m_b$

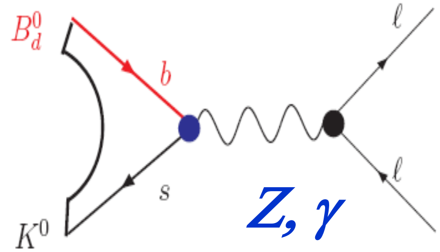




$B_s \to \mu\mu$ **(P=-1)**

$$\langle 0 | \bar{b}\Gamma\gamma_5 s | B_s^0 \rangle \neq 0, \langle 0 | \bar{b}\Gamma s | B_s^0 \rangle = 0$$

Br $\propto$ ($m_l^2(C_{10}$-$C'_{10})$, $C_{P,S}$ -$C'_{P,S}$)



$B \to K\,ll$ **(P=1)**

$$\langle K | \bar{b}\gamma^\mu\gamma_5 s | B_s^0 \rangle = \langle K | \bar{b}\gamma_5 s | B_s^0 \rangle = 0$$

Br $\propto$ (C_7+C'_7, C_9+C'_9, C_{10}+C'_{10}, $C_{P,S}$+$C'_{P,S}$, C_T, C_{T5})

+ $B \to X_s\gamma$ strongly constrains $C_{7(8)}$

$$L_{eff} = C_7 \bar{b}\sigma_L^{\mu\nu} s F_{\mu\nu} + C_7' \bar{b}\sigma_R^{\mu\nu} s F_{\mu\nu} + C_9 \left(\bar{b}\gamma_L^\mu s\right)\bar{\ell}\gamma^\mu \ell + C_9' \left(\bar{b}\gamma_R^\mu s\right)\bar{\ell}\gamma^\mu \ell$$
$$+ C_{10}\left(\bar{b}\gamma_L^\mu s\right)\bar{\ell}\gamma^\mu\gamma_5 \ell + C_{10}'\left(\bar{b}\gamma_R^\mu s\right)\bar{\ell}\gamma^\mu\gamma_5 \ell + C_S\left(\bar{b}Ls\right)\bar{\ell}\ell + C_S'\left(\bar{b}Rs\right)\bar{\ell}\ell$$
$$+ C_P\left(\bar{b}Ls\right)\bar{\ell}\gamma_5 \ell + C_P'\left(\bar{b}Rs\right)\bar{\ell}\gamma_5 \ell + C_T\left(\bar{b}\sigma_L^{\mu\nu} s\right)\bar{\ell}\sigma^{\mu\nu}\ell + C_T'\left(\bar{b}\sigma_R^{\mu\nu} s\right)\bar{\ell}\sigma^{\mu\nu}\ell$$

Theory: Hadronic Uncertainties

$Br(B_s \to \mu^+\mu^-) \propto C_{9,10,S,P}$

$Br(B_d \to K^{(*)} l^+ l^-) \propto C_{9,10,S,P,T}$

GOAL: calculate Matrix elements of 2-quark operators between hadrons (decay constants & Form factors)

SM operators

$$O_7 = \bar{b}_R \sigma^{\mu\nu} s_L F_{\mu\nu}$$

$$O_9 = \left(\bar{b}\gamma_L^\mu s\right)\bar{\ell}\gamma^\mu \ell$$

$$O_{10} = \left(\bar{b}\gamma_L^\mu s\right)\bar{\ell}\gamma^\mu \gamma_5 \ell$$

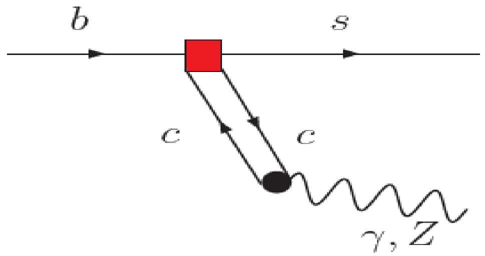
$$O_2 = \left(\bar{b}\gamma_L^\mu c\right)\left(\bar{c}\gamma_L^\mu s\right)$$

BSM operators

$$O_{S(P)} = \left(\bar{b}_R s_L\right)\bar{\ell}\ell_{S(P)}, \quad O_T = \left(\bar{b}_R \sigma^{\mu\nu} s_L\right)\bar{\ell}\sigma^{\mu\nu}\ell$$

$$+L \leftrightarrow R$$

Charm Loops



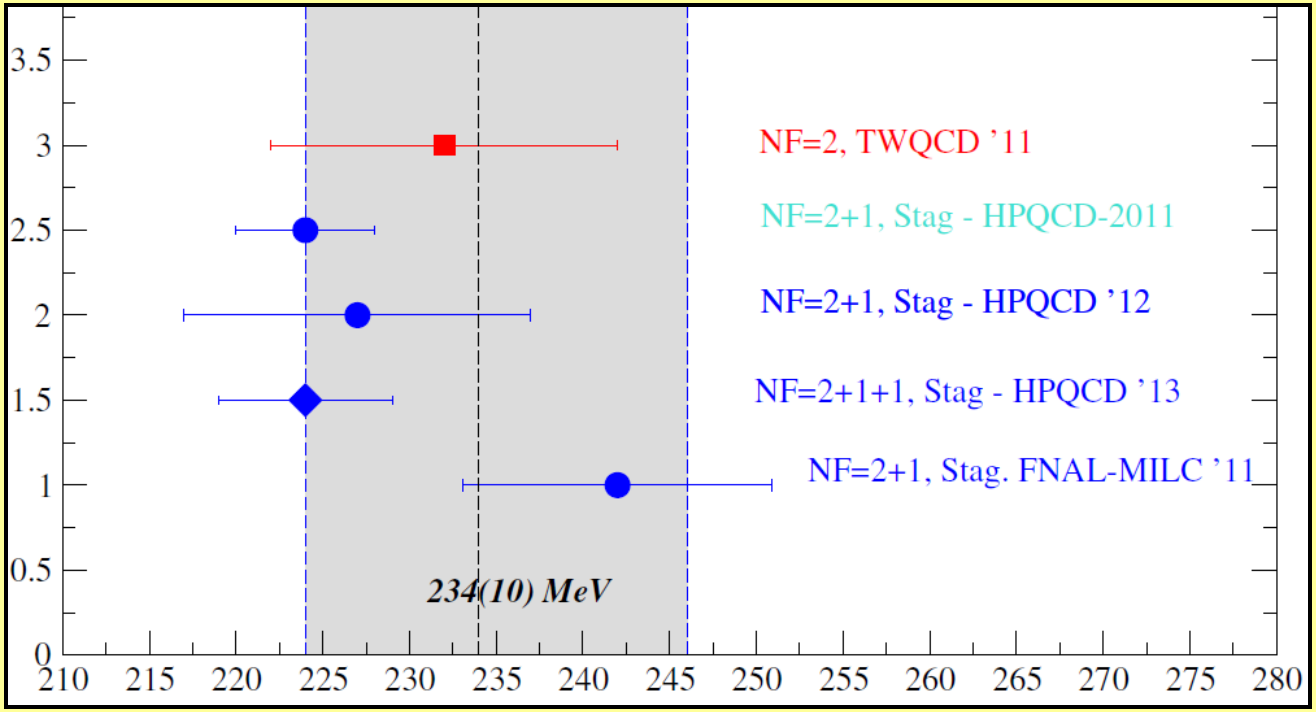
Under control (to some extent)
at low and large q^2, *out of resonance region*

Khodjamirian's talk

Theory: Hadronic Uncertainties

$B_s \to \mu\mu$

f_{Bs}



$B_s \to \mu\mu$

Only one hadronic parameter: f_{Bs}

$$\langle 0 | \bar{b}\gamma^{\mu}\gamma_5 s | B_s^0 \rangle = i p^{\mu} f_{Bs}$$

$$\langle 0 | \bar{b}\gamma_5 s | B_s^0 \rangle = -i f_{Bs} M_{Bs}^2 / m_b$$

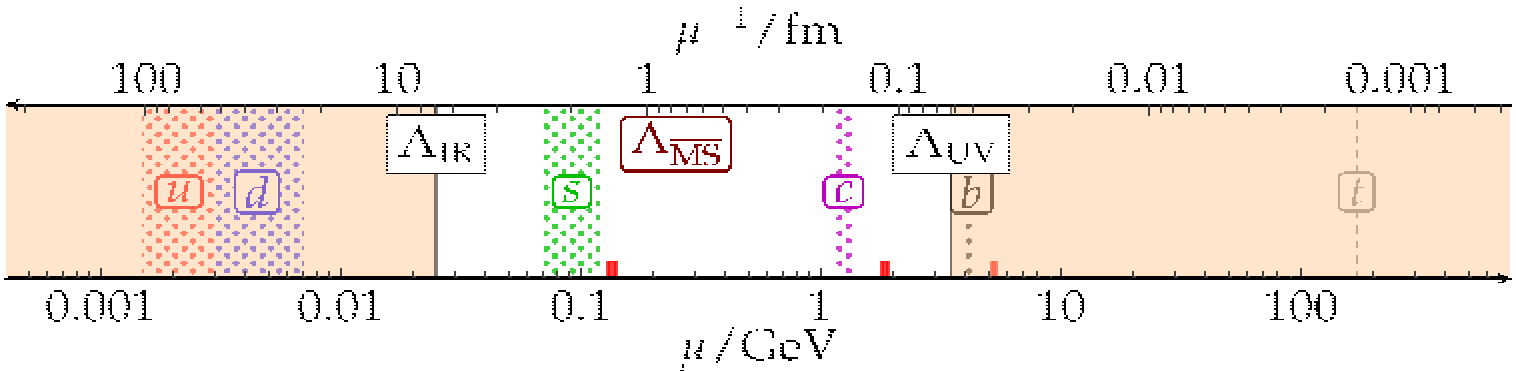
$$f_{Bs} = (234 \pm 10)\,\mathrm{MeV}$$

4% hadronic uncertainty

Lattice: ETMC, MILC, HPQCD

$Br(B_s \to \mu\mu)^{SM} = (3.3 \pm 0.3) \times 10^{-9}$ (6.5%)

Challenge of B-physics: the multi scale-problem of QCD



hierarchy of disparate physical scales to be covered:

$$\Lambda_{IR} = L^{-1} \ll m_\pi, \ldots, m_D, m_B \ll a^{-1} = \Lambda_{UV}$$

$$\left\{ O(e^{-L m_\pi}) \Rightarrow \boxed{L \gtrsim \frac{4}{m_\pi} \sim 6\,\mathrm{fm}} \right\} \curvearrowright L/a \gtrsim 120 \curvearrowleft \left\{ a m_D \lesssim \frac{1}{2} \Rightarrow \boxed{a \approx 0.05\,\mathrm{fm}} \right\}$$

Currently a^{-1}<4 GeV, *b* quarks cannot be directly simulate at their physical mass due to large discretization errors ($a\, m_b$« 1)

- *effective theories: like NRQCD action*
- *simulate heavy quark in the charm region and extrapolate to the B + HQET.*

😐 Comments:

- ***Discretized NRQCD action***

  - *Quite Sophisticated procedure!*

    - ➲ *larger set of $1/(am_Q)$ corrections on the lattice w.r.t the continuum*

    - $O[\alpha_s^n/(am_Q)]$ divergences to be subtracted to get the continuum limit

- *On the other hand, large experience from MILC/FNAL/HPQCD*

- ***☺ Successful strategy for f_B when comparing with unquenched results from strategy***

Theory: Hadronic Uncertainties

$B \to K^* \gamma$, $B \to K^* ll$

$$\langle V(p',\varepsilon)|\bar{q}\hat{\gamma}^\mu b|B(p)\rangle = \frac{2iV(q^2)}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p'_\rho p_\sigma$$

$$\begin{aligned}\langle V(p',\varepsilon)|\bar{q}\hat{\gamma}^\mu\hat{\gamma}^5 b|B(p)\rangle = & \; 2m_V A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu \\ & +(m_B+m_V)A_1(q^2)\left(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\right) \\ & -A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_V}\left((p+p')^\mu-\frac{m_B^2-m_V^2}{q^2}q^\mu\right)\end{aligned}$$

$$q^\nu\langle V(p',\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}b|B(p)\rangle = 2T_1(q^2)\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^\tau p'^\sigma$$

→ $Br(B \to K^* \gamma)$ one ff. at $q^2=0$

$$\begin{aligned}q^\nu\langle V(p',\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}\hat{\gamma}^5 b|B(p)\rangle = & \; iT_2(q^2)[\varepsilon^*_\mu(m_B^2-m_V^2)-(\varepsilon^*\cdot q)(p+p')_\mu] \\ & +iT_3(q^2)(\varepsilon^*\cdot q)\left[q_\mu-\frac{q^2}{m_B^2-m_V^2}(p+p')_\mu\right]\end{aligned}$$

$Br(B \to K^* ll)$: 7 form factors in QCD

*! LATTICE QCD: only to compute the full ff basis at **large** q^2:*

☺ no $O(\Lambda/m_b)$ uncertainty from Isgur-Wise relation at LO!

Theory: Hadronic Uncertainties

$B \to K ll$

$B \to K ll$

Dominant uncertainties come from the form 3 factors: *$f_+(q^2)$, $f_0(q^2)$, $f_T(q^2)$*

$$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle = \left(p^\mu + k^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu\right) f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2} q^\mu f_0(q^2)$$

$$\langle B(p)|\bar{b}\sigma^{\mu\nu} s|K(k)\rangle = \frac{i f_T}{m_B + m_K}\left[(p^\mu + k^\mu)q^\nu - (p^\nu + k^\nu)q^\mu\right]$$

- $C_{9,10}^{(\prime)} \to f_+(q^2), f_0(q^2), C_{S,P}^{(\prime)} \to f_0/m_b \quad C_7^{(\prime)} \to f_T,$

- Wide range of $q^2=[0,(m_B-m_K)^2]$ -> **Opportunities for different nonperturbative techniques: Lattice QCD-QCSR** – relative th. error 30% -> large room for improvement

*! LATTICE QCD: only to compute the full ff basis at **large** q^2:*
☺ no $O(\Lambda/m_b)$ uncertainty from Isgur-Wise relation at LO!

Studies of form-factor calculations on the Lattice:

$B \to Kll$

N_F=0: Quenched lattice QCD: **relativistic fermions**

- ❖ D. Becirevic, N. Kosnik, F. M., E. Schneider, 2012

N_F=2+1 staggered fermions: NRQCD

- ❖ FNAL/MILC, 2012
- ❖ HPQCD, 2012
- ❖ Cambridge (***prelims***), 2012

→ $f_+(q^2)$, $f_+(q^2)$, $f_T(q^2)$

$B \to K^*ll$

N_F=0: Quenched lattice QCD: **relativistic fermions**

- ❖ D. Becirevic, V. Lubicz & F. M. 2007 → $T_{12}(q^2)$

N_F=2+1 staggered fermions: **NRQCD**

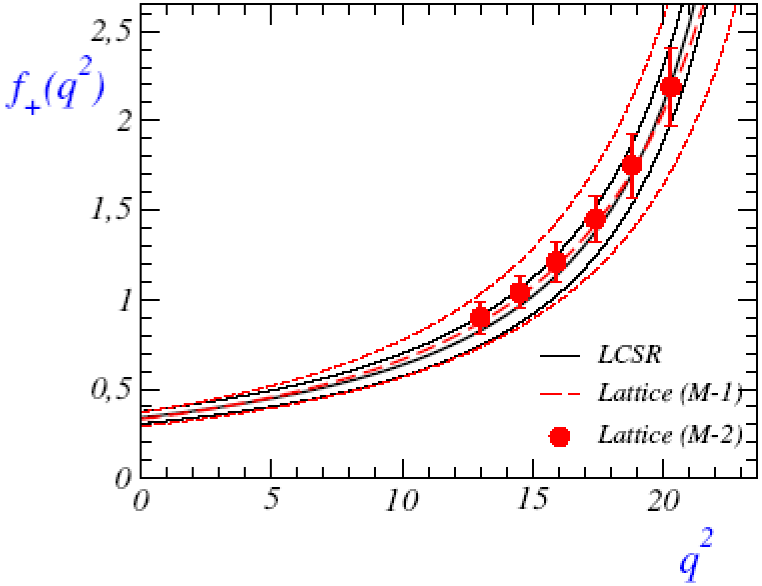
- ❖ Cambridge (***prelims***), 2012 → $T_{12}(q^2)$, $V(q^2)$, $A_{012}(q^2)$

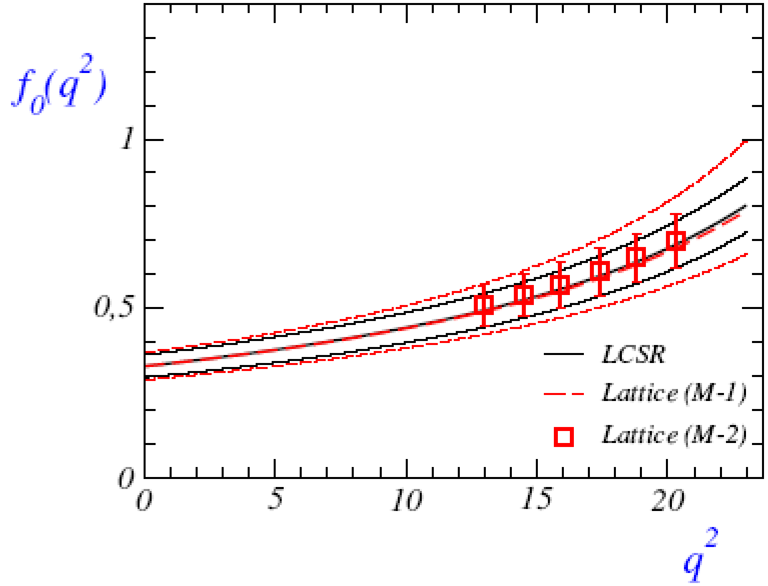
☹ *very preliminary unquenched activities:*

overall agreement between Quenched and LCSR

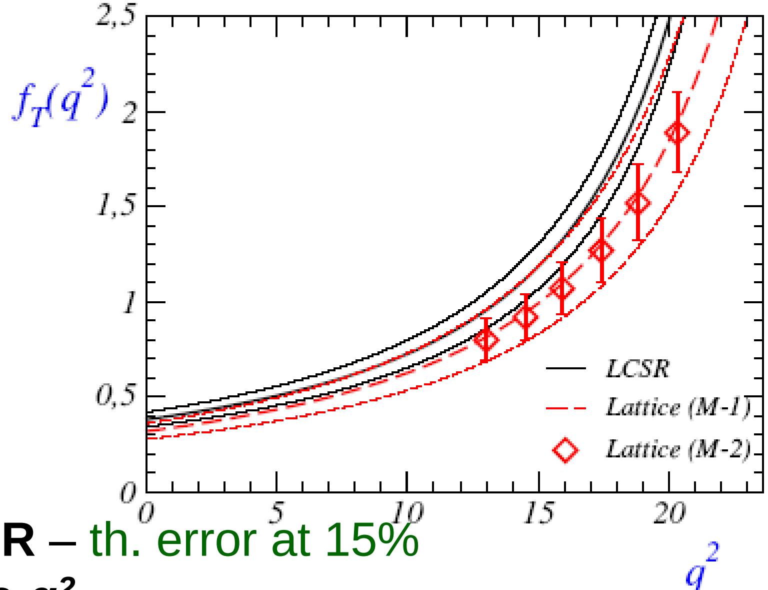
☹ *q^2 dependence: further complication with respect to* f_B *or* B_B

B→Kll* - *STRATEGY 1: QCD + extrapolating from charm region:





Lattice QCD Quenched:
***f_+, f_0, f_T* ->**D. Becirevic et al. 2012



Light cone QCD sum rules [Ball'05, Khodjamirian'07, '10]

1) **Lattice QCD and LCSR** – th. error at 15%
2) **Lattice points at large *q^2***
3) **Agreement with LQSR**

Theory: Hadronic Uncertainties

$B \to K ll$

$B \to K ll$

Dominant uncertainties come from the 3 form factors: $f_+(q^2)$, $f_0(q^2)$, $f_T(q^2)$

$$\langle K | \bar{b}\gamma^{\mu}\gamma_5 s | B \rangle \Leftrightarrow f_{+,0}(q^2) \qquad \langle K | \bar{b}\sigma^{\mu\nu} s | B \rangle \Leftrightarrow f_T(q^2)$$

❖ $C_{9,10}^{(\prime)} \to f_+(q^2), f_0(q^2), C_{S,P}^{(\prime)} \to f_0/m_b \quad C_7^{(\prime)} \to f_T,$

❖ Wide range of $q^2=[0,(m_B-m_K)^2]$ -> Opportunities for different nonperturbative techniques: Lattice QCD and LCSR – relative error 30%

$$\mathrm{Br}\left(B \to K\ell^+\ell^-\right)_{\mathrm{SM}} = \begin{cases} (7.5 \pm 1.4) \times 10^{-7} & \mathrm{LQCD}\,, \\ (6.8 \pm 1.6) \times 10^{-7} & \mathrm{LCSR}\,. \end{cases},$$

Our average

$$\mathrm{Br}\left(B \to K\ell^+\ell^-\right)_{\mathrm{SM}} = (7.0 \pm 1.8) \times 10^{-7}$$

still th. error large 30%

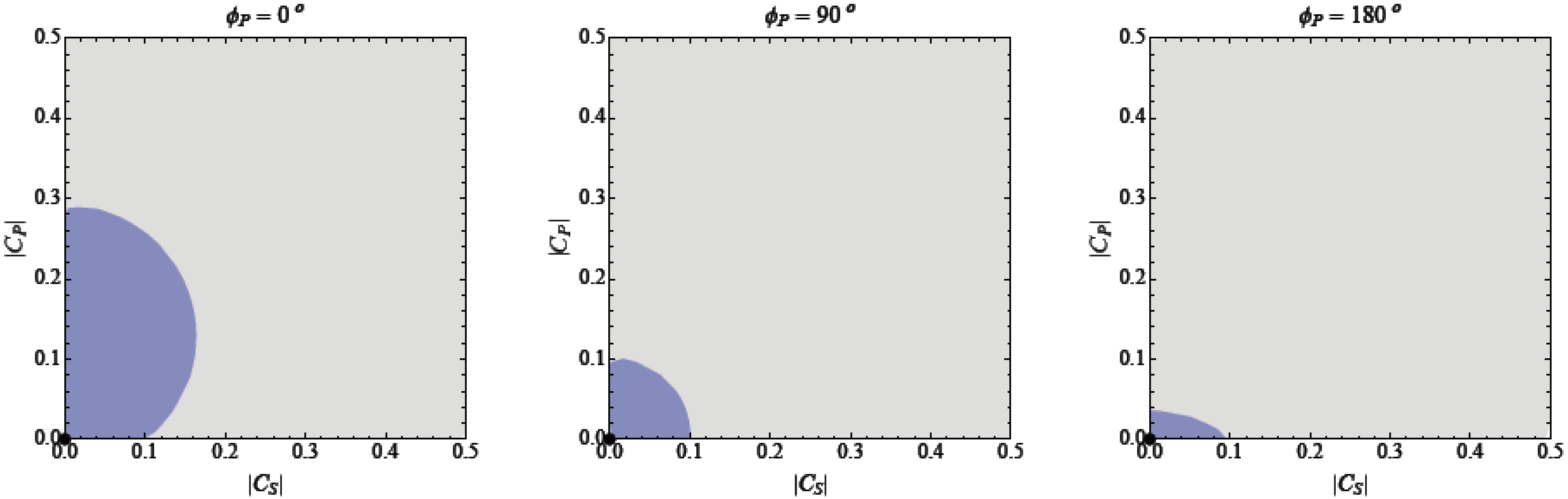
BaBar’12

Br($B \to K\, ll$) = $(4.7 \pm 0.6) \times 10^{-7}$

LHCb’12

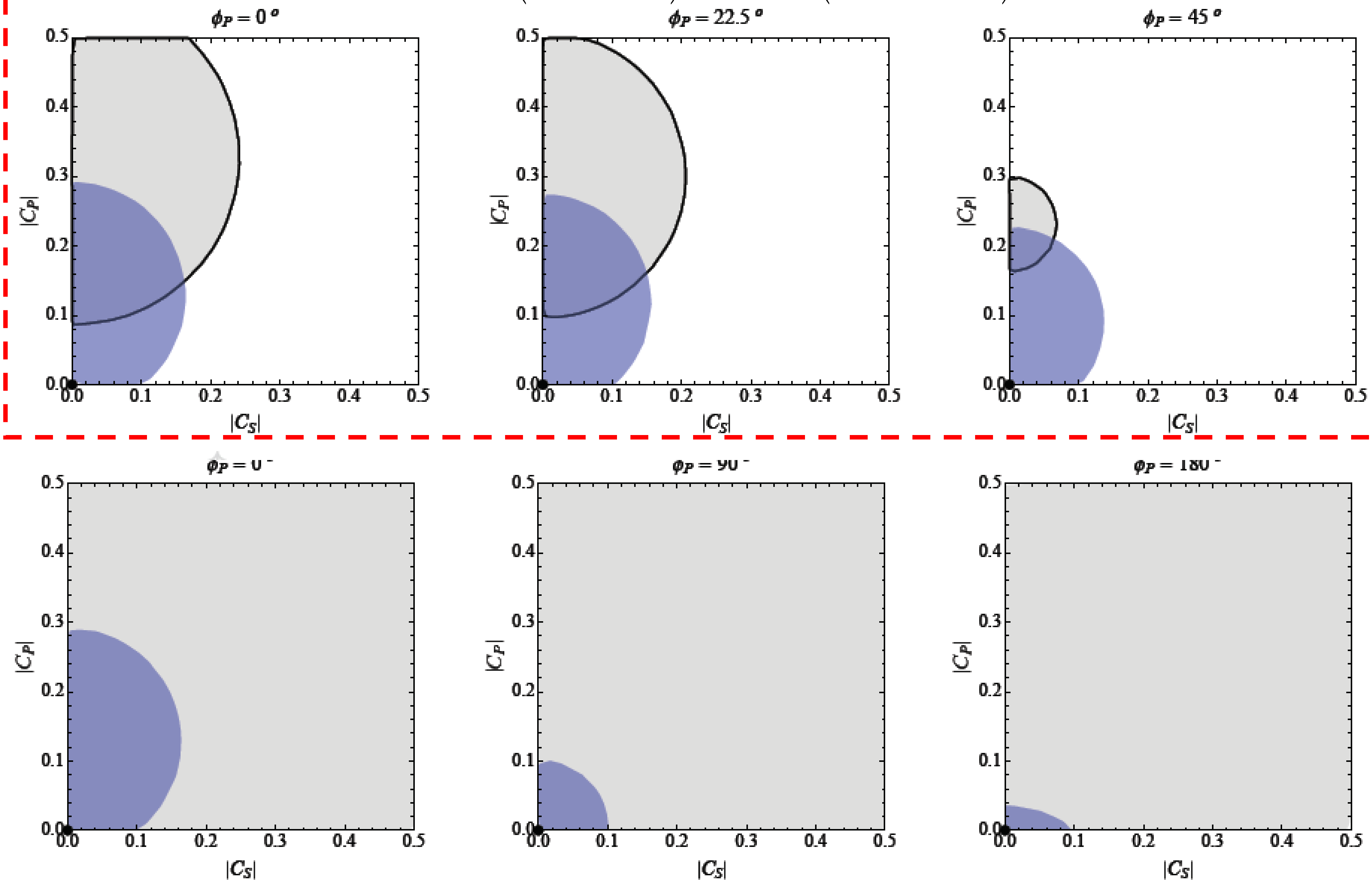
Br($B^+ \to K^+ \mu\mu$) = $(3.1 \pm 0.7) \times 10^{-7}$

New Physics: SM + $C_S\left(\bar{b}(1-\gamma_5)s\right)\bar{\ell}\gamma_5\ell + C_P\left(\bar{b}\gamma_5(1-\gamma_5)s\right)\bar{\ell}\gamma_5\ell$



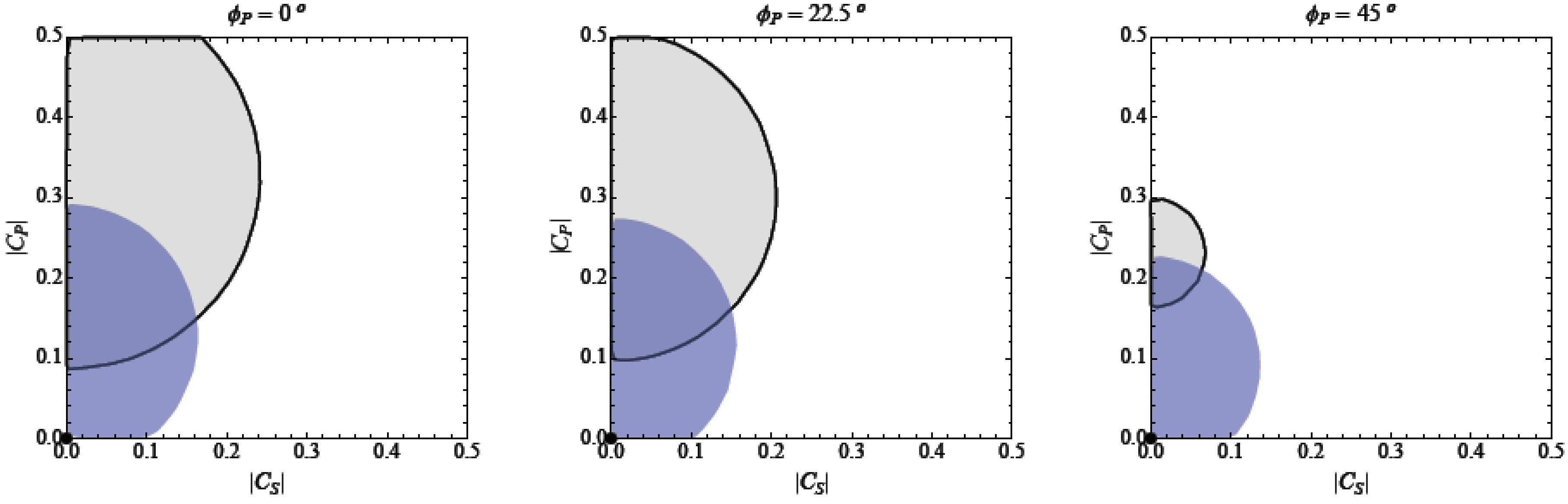
Lowering th. error on *B->Kll: 20% smaller than now*

New Physics: SM + $C_S\left(\bar{b}\left(1-\gamma_5\right)s\right)\bar{\ell}\gamma_5\ell+C_P\left(\bar{b}\gamma_5\left(1-\gamma_5\right)s\right)\bar{\ell}\gamma_5\ell$



Lowering th. error on *B->Kll: 20% smaller than now*

New Physics: SM + $C_S\left(\bar{b}(1-\gamma_5)s\right)\bar{\ell}\gamma_5\ell + C_P\left(\bar{b}\gamma_5(1-\gamma_5)s\right)\bar{\ell}\gamma_5\ell$

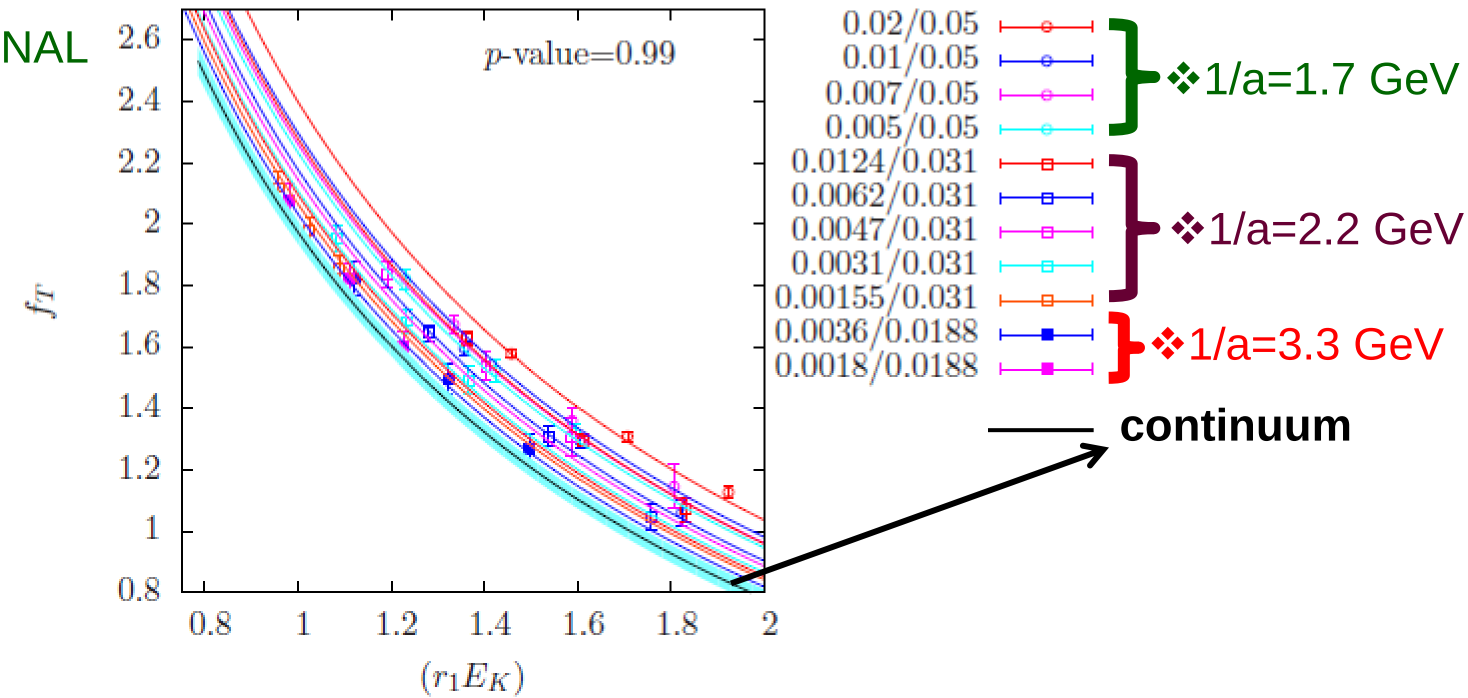


This "toy-scenario" would prefer nonzero C_P.

Lowering th. error on *B->Kll: 20% smaller than now*

B→Kll - STRATEGY 2: (NRQCD) effective action for the b quark, v<<c

❖ MILC-FNAL



m_π (MeV)
~300
~460
~320

❖ **fit in q^2, m_{sea}, and lattice spacing (a)**

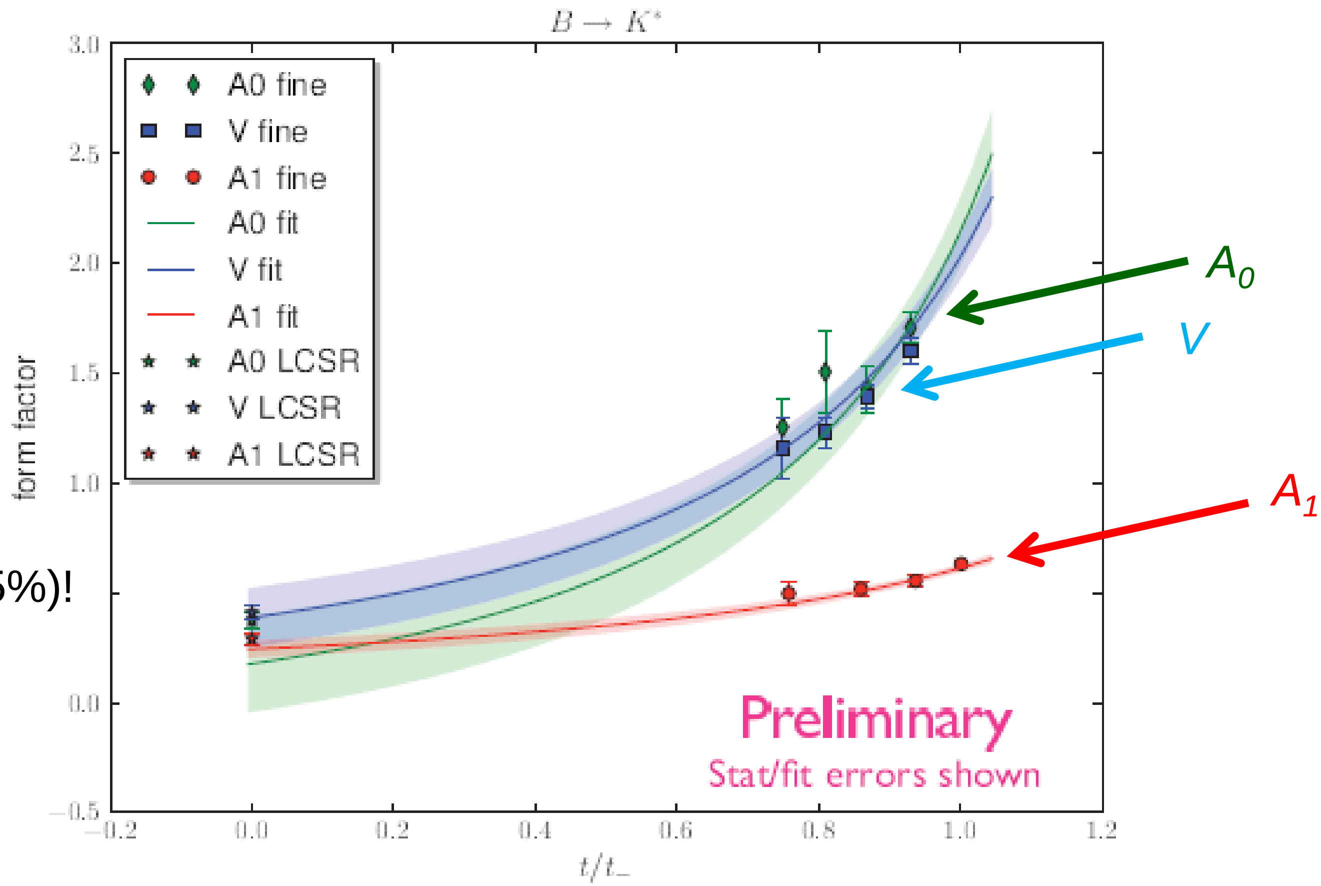
$$f_{\|} = \frac{C_{\|}^{(0)}}{f_\pi}\left[1 + \text{logs} + C_{\|}^{(1)} m_l + C_{\|}^{(2)}(2m_l + m_s) + C_{\|}^{(3)} E_K + C_{\|}^{(4)} E_K^2 + C_{\|}^{(5)} a^2\right],$$

❖ **<u>preliminary</u> unquenched results (NF=2+1)**

-> stats + (systs) errors at 5%

3 coarse lattice spacings -. Continuum limit

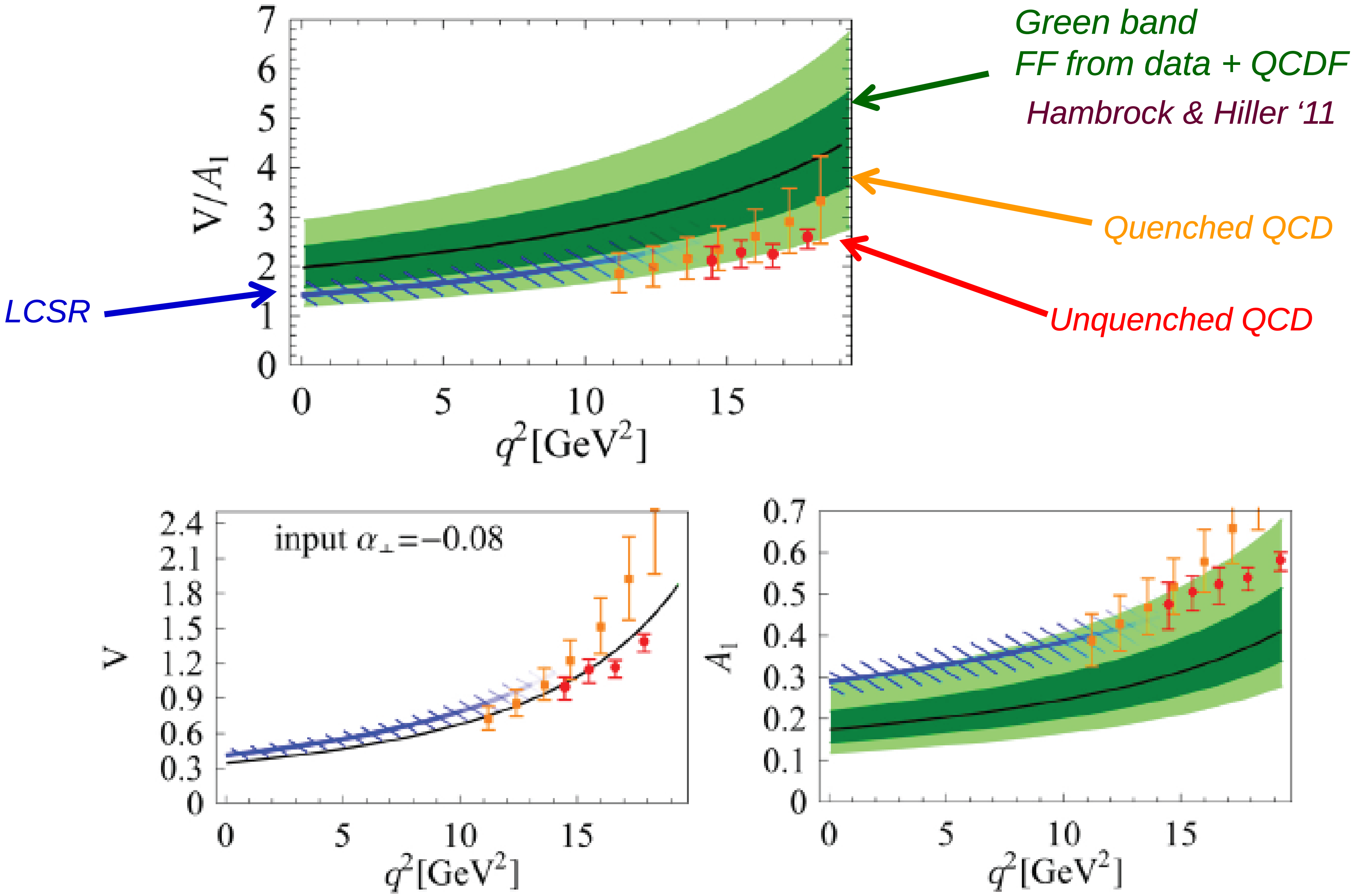
$B \to K^* ll$ form factors from Cambridge/W&M/Edinburgh.



- Only stats errors (at 5%)!
- Promising study

Preliminary results on $B \to K^* ll$ V, A_0, and A_1 vs. q^2/q^2_{max}. (by M. Wingate at lattice 2012)

Comparison of B→K*ll form factor calculations



Conclusions:

LATTICE QCD -> touchable progress in recent years:

- reliable *unquenched simulations with pions close to the physical point* => m_π=156 MeV (PACS-CS), **m_π**=190 MeV (BMW)

- f_K/f_π & f_B *paradigma of present lattice progress!*
- *promising studies at percent level on the way for B Physics ffs*

Still a long work to assess 1%-precision needed for B physics

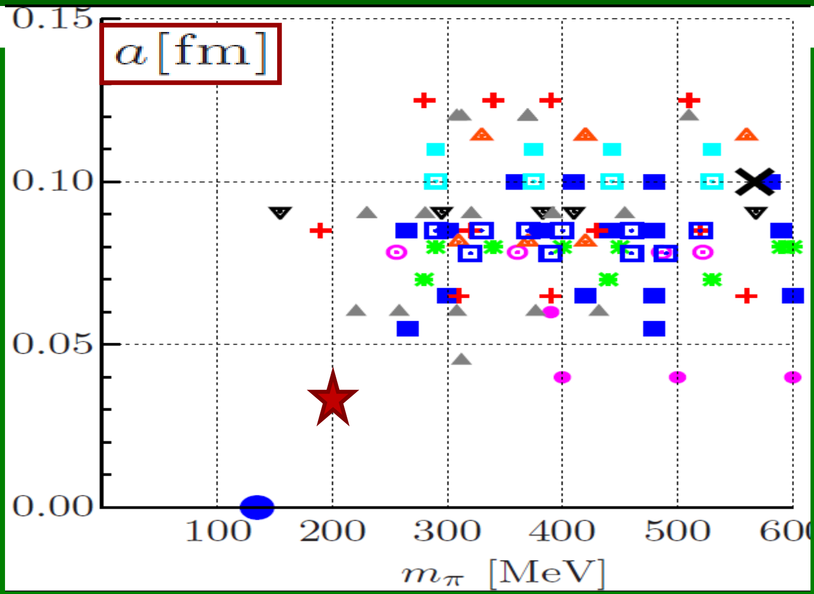
❶ ***discretization errors:*** $a*m_B \ll 1$

=> ***a~ 0.033 fm*** (6 *GeV*): ($a \geq 0.07$ *fm*)

❷ ***finite volume effects:*** $L*m_\pi \gg 1$

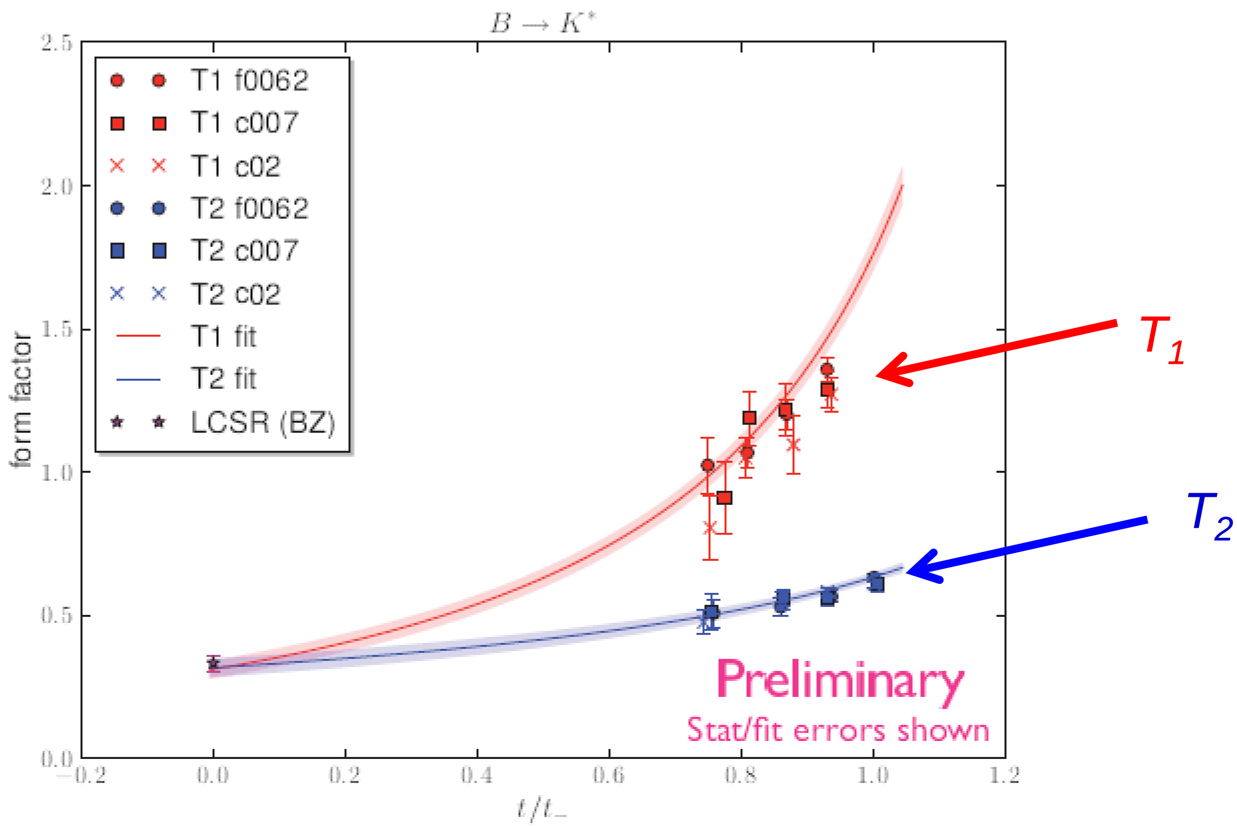
=> ***L ≥ 4.5 fm*** ($L \leq 3$ *fm*)

❸ ***chiral regime***: **$200 \leq m_\pi \leq 300$ MeV**



courtesy of G. Herdoiza

$B \to K^* ll$ form factors from Cambridge/W&M/Edinburgh.



Preliminary results on $B \to K^* ll$ T_1 and T_2 vs. $q^2/q^2_{\max}$ (by M. Wingate at lattice 2012)

- **Only stats errors at 5%!**
- **Promising study**

Conclusions

- ***Br($B_s \to \mu\mu$) is genuinely sensitive to (pseudo)scalar operators***

$$O'_S = \left(\bar{b} P_{R,L} s\right) \bar{\ell}\ell, \text{ and } O_P = \left(\bar{b} P_{R,L} s\right) \bar{\ell}\gamma_5 \ell$$

  - Only one hadronic parameter enters, ***f_{Bs} -> small th. error***

- ***Br($B \to K ll$) is sensitive to (pseudo)scalar + vector operators (+ tensors)***
  - 3 hadronic parameters, ***$f_{0,+T}$ form factors -> large th. error***
  - **With respect to $B_s \to \mu\mu$ it probes the effective Hamiltonian in the "orthogonal" direction!**
  - **Improvement in form factors calculation** would make the two observables a high resolution probe of scalar operators
    - with tensor operators tested by $A_{FB}(B \to K^{(*)} ll)$
    - with vector ones by $B \to X_s ll$ spectrum and transverse asymmetries in $B \to K^* ll$

New Physics: tensor contributions

- tensor contributions to $B \to K\mu^+\mu^-$

$$\mathcal{O}_T = \frac{e^2}{16\pi^2}(\bar{s}\sigma^{\mu\nu}b)(\bar{\ell}\sigma_{\mu\nu}\ell) \qquad \mathcal{O}_{T5} = \frac{e^2}{16\pi^2}(\bar{s}\sigma^{\mu\nu}b)(\bar{\ell}\sigma_{\mu\nu}\gamma_5\ell)$$

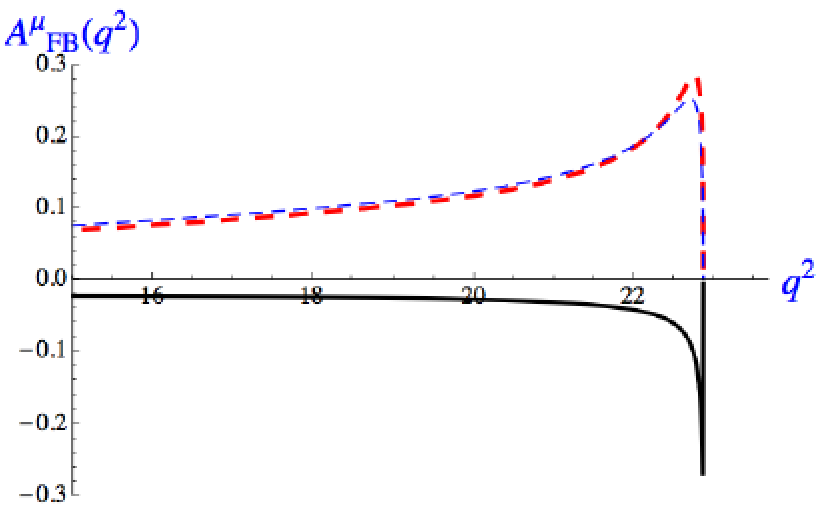
Assumption testable in

- $\mathcal{B}(B \to X_s\mu^+\mu^-)$

$$\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} \frac{d\mathcal{B}(B \to X_s\mu^+\mu^-)}{dq^2}dq^2 = 1.59(17) \times 10^{-6}[1 + 0.59(2)(|C_T|^2 + |C_{T5}|^2)]$$

- Forward-backward asymmetry of $B \to K\mu^+\mu^-$

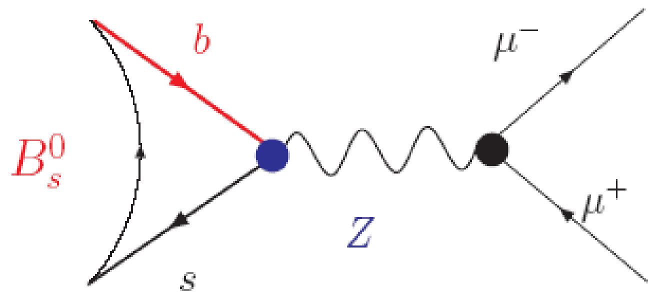
$$A^{\ell}_{FB}(q^2) = \frac{2\,\mathcal{C}(q^2)}{\Gamma_\ell}\,\frac{m_B - m_K}{m_b}\,\sqrt{\lambda(q^2)}\,f_0(q^2)\left\{ (C_S C_T + C_P C_{T5})\ q^2 f_T(q^2) \right.$$
$$\left. + m_\ell\left[C_S\ C_9(m_B + m_K)f_P(q^2) + 2m_b\,(C_S C_7 + 2C_{T5}C_{10})\,f_T(q^2)\right] + \mathcal{O}(m_\ell^2)\right\}$$



- $C_T = C_{T5} = 1.6$
- Thick $\to C_S = C_P = 0$
- Dashed $\to \{C_S, C_P\} = (1, 0)$

2013 Experimental situation

$B_s \to \mu\mu$



LHCb'12

$Br^{exp}(B_s \to \mu\mu) = (3.2 \pm 1.5)10^{-9}$

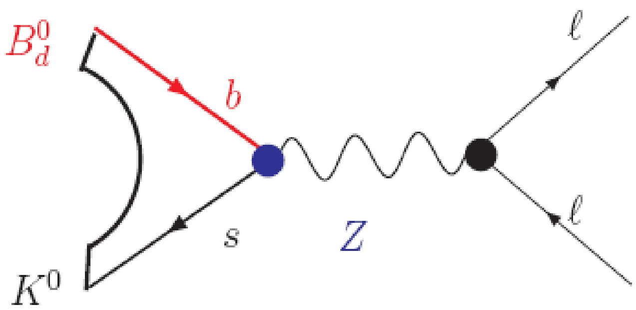
exp. error 50%

$\mathbf{Br(B_s \to \mu\mu)^{SM} = (3.3 \pm 0.3) \times 10^{-9}}$

th. error 6.5%

$B_d^0 \to K^0\, ll$

$B_u^+ \to K^+\, ll$



BaBar'12-04

$Br(B \to Kll) = (4.7 \pm 0.6) \times 10^{-7}$

- averaging B^+/B^0 and e/μ

LHCb'12-05

$Br(B^0 \to K^0 \mu\mu) = (3.1 \pm 0.7) \times 10^{-7}$

- puzzle B^+ and B^0 asymmetries? ☹

(our) $\mathbf{Br(B \to Kll)^{SM} = (7.0 \pm 1.8) \times 10^{-7}}$