Can minimal SUSY SU(5) scenario be made realistic?

work in progress with B. Bajc and S. Lavignac

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Introduction

Can *minimal renormalizable supersymmetric SU(5) GUT model* be reconciled with all the phenomenological constraints and remain perturbative?

Excluded according to [*Murayama-Pierce, hep-ph/0108104*]

- gauge coupling unification ($m_T \lesssim 1.4 \cdot 10^{15}\,\mathrm{GeV}$)
- proton decay ($m_T \gtrsim 2.0 \cdot 10^{17}\,\mathrm{GeV}$)

Assumption: all the *sparticles* at the $\mathcal{O}(1\,\mathrm{TeV})$ scale, and *gauginos* around the EW scale ($M_2 \approx 200\,\mathrm{GeV}$, $M_3/M_2 \simeq 3.5$).

Is the model still feasible for some more general superpartner mass spectrum?

Our starting points

1. Why minimal renormalizable SUSY SU(5)?
   - *predictiveness* - probably the only way to ever check the high scale Yukawas
   - smallness of terms $C\,\frac{Q_i Q_j Q_k L_l}{M_P}$; $\quad C \lesssim 10^{-7 \div -8}$
2. assumption of SUSY broken above the GUT scale, as e.g. in supergravity
   $\longrightarrow$ soft terms at the GUT scale SU(5) invariant
3. we concentrated on the mass scales of the theory, not its flavor structure (the only constraint is small FCNCs).

Minimal renormalizable supersymmetric SU(5) GUT (structure and content)

1. Gauge sector:

$$\mathbf{24_g} = (\mathbf{8},\mathbf{1},\mathbf{0}) \oplus (\mathbf{1},\mathbf{3},\mathbf{0}) \oplus (\mathbf{1},\mathbf{1},\mathbf{0}) \oplus (\mathbf{3},\mathbf{2},-\frac{\mathbf{5}}{\mathbf{6}}) \oplus (\overline{\mathbf{3}},\mathbf{2},\frac{\mathbf{5}}{\mathbf{6}})$$

2. Matter (Yukawa) sector:

$$W_Y = \overline{\mathbf{5}}_\mathbf{i}\, Y_5^{i,j} \mathbf{10_j}\, \overline{\mathbf{5}}_\mathbf{H} + \frac{1}{8}\, \mathbf{10_i}\, Y_{10}^{i,j} \mathbf{10_j}\, \mathbf{5_H} \quad , \quad \text{i=1,2,3}$$

$$\mathbf{10_i} = \underbrace{(\mathbf{3},\mathbf{2},\frac{\mathbf{1}}{\mathbf{6}})}_{m_{Qi}} \oplus \underbrace{(\overline{\mathbf{3}},\mathbf{1},-\frac{\mathbf{2}}{\mathbf{3}})}_{m_{ui}} \oplus \underbrace{(\overline{\mathbf{1}},\mathbf{1},\mathbf{1})}_{m_{ei}} \quad , \qquad \overline{\mathbf{5}}_\mathbf{i} = \underbrace{(\overline{\mathbf{3}},\mathbf{1},\frac{\mathbf{1}}{\mathbf{3}})}_{m_{di}} \oplus \underbrace{(\mathbf{1},\mathbf{2},-\frac{\mathbf{1}}{\mathbf{2}})}_{m_{Li}}$$

3. Higgs sector:

$$W_H = \frac{\mu}{2} Tr \mathbf{24_H}^2 + \frac{\lambda}{3} Tr \mathbf{24_H}^3 + \eta \mathbf{\bar{5}_H}(\mathbf{24_H} + 3\sigma)\mathbf{5_H}$$

$$< \mathbf{24_H} > = \sigma \cdot diag(2,2,2,-3,-3) \; : \; \mathrm{SU}(5) \longrightarrow \mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$$

$$\mathbf{24_H} = \underbrace{(\mathbf{8},\mathbf{1},\mathbf{0})}_{m_\Sigma(m_8)} \oplus \underbrace{(\mathbf{1},\mathbf{3},\mathbf{0})}_{m_\Sigma(m_3)} \oplus (\mathbf{1},\mathbf{1},\mathbf{0}) \oplus \underbrace{(\mathbf{3},\mathbf{2},-\frac{\mathbf{5}}{\mathbf{6}}) \oplus (\mathbf{\bar{3}},\mathbf{2},\frac{\mathbf{5}}{\mathbf{6}})}_{m_V}$$

$$\mathbf{5_H} = \underbrace{(\mathbf{3},\mathbf{1},-\frac{\mathbf{1}}{\mathbf{3}})}_{m_T} \oplus \underbrace{(\mathbf{1},\mathbf{2},\frac{\mathbf{1}}{\mathbf{2}})}_{m_H} \quad , \quad \mathbf{\bar{5}_H} = \underbrace{(\mathbf{\bar{3}},\mathbf{1},\frac{\mathbf{1}}{\mathbf{3}})}_{m_T} \oplus \underbrace{(\mathbf{1},\mathbf{2},-\frac{\mathbf{1}}{\mathbf{2}})}_{m_H}$$

$$\left.\begin{array}{rcl} m_T &=& 5\eta\sigma \\ m_\Sigma &=& m_8 = m_3 = \frac{5}{2}\mu \\ m_V &=& 5\sqrt{2}\, g_{GUT}\, \sigma \end{array}\right\} \Longrightarrow perturbativity\,(m_T \lesssim m_V)$$

Theoretical and experimental constraints

- $\tau_p^{exp}(p \to K^+\overline{\nu}) > 4 \cdot 10^{33}\,\mathrm{yrs}$
- $m_{h^0} \simeq 125.5\,\mathrm{GeV}$
- correct down-sector fermion mass relations (δm_d, δm_s ; $\delta m_b \approx 0$)
- small FCNCs
- LHC bounds on sfermion and gluino masses
- gauge coupling unification
- $m_V \gtrsim m_T$ (perturbativity)
- (absolute) vacuum stability

μ-term

From tree-level EWSB condition at the $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ scale (where the 1-loop corrections are small)

$$m_{H_u}^2 + |\mu|^2 - \frac{b}{\tan\beta} + \frac{m_Z^2}{2}\left(\frac{\tan^2\beta - 1}{\tan^2\beta + 1}\right) = 0$$

$$m_{H_d}^2 + |\mu|^2 - b\tan\beta - \frac{m_Z^2}{2}\left(\frac{\tan^2\beta - 1}{\tan^2\beta + 1}\right) = 0$$

and assuming *small* $\tan\beta \approx 2 - 3$ because of the proton decay and $m_{H_u}(m_{GUT}) \approx m_{H_d}(m_{GUT})$ not to produce tachyons from soft masses RGEs, we typically get *large* $|\mu| \sim \mathcal{O}(m_{H_d}/\tan\beta)$

$$|\mu|^2 = \frac{m_{H_d}^2 - m_{H_u}^2\tan^2\beta}{\tan^2\beta - 1} - \frac{m_Z^2}{2}$$

$$b = \frac{(m_{H_d}^2 - m_{H_u}^2)\tan\beta}{\tan^2\beta - 1} - m_Z^2\frac{\tan\beta}{\tan^2\beta + 1}$$

Corrections to light fermion masses

1. in minimal renormalizable SU(5)

$$y^D(M_{GUT}) = y^E(M_{GUT})$$

2. mismatch after the running with the measured masses at the weak scale

$$m^E_{exp}(M_Z) = y^E(M_Z) v \cos\beta \approx diag(0.5\,\mathrm{MeV}, 100\,\mathrm{MeV}, 1.7\,\mathrm{GeV})$$
$$m^D_{exp}(M_Z) \approx diag(3\,\mathrm{MeV}, 55\,\mathrm{MeV}, 2.9\,\mathrm{GeV})$$

$$\delta m^D = m^D_{exp} - y^D(M_Z) v \cos\beta$$

$$\frac{\delta m_d}{y_d(M_Z) v \cos\beta} \approx 2$$
$$\frac{\delta m_s}{y_s(M_Z) v \cos\beta} \approx -0.75$$
$$\frac{\delta m_b}{y_b(M_Z) v \cos\beta} \approx +0.05$$

3. $\delta m_d > 0$, $\delta m_s < 0$
$\longrightarrow$ the down-sector quark masses (m_d, m_s) corrected through the *generation dependent* supersymmetric threshold corrections (a-terms).

$$\delta m_i^D = -\frac{2\alpha_s}{3\pi} v m_{\tilde{g}} (a_i^D \cos\beta - \mu y_i^D \sin\beta) I_3(m^2_{\tilde{d}_{Ri}}, m^2_{\tilde{d}_{Li}}, m^2_{\tilde{g}})$$
$$\approx -\frac{2\alpha_s}{3\pi} \frac{(a_i^D \cos\beta - \mu y_i^D \sin\beta) v}{2\widetilde{m}} \quad ; \qquad m_{\tilde{g}} \approx m_{\tilde{d}_{Li}} \approx m_{\tilde{d}_{Ri}} \equiv \widetilde{m}$$

$\longrightarrow$ $|a_{d,s}| \gtrsim y_{d,s} \tan\beta\, \mu$

$\longrightarrow$ *not too large* $m_{\tilde{g}}, m_{\tilde{d}_{Li}}, m_{\tilde{d}_{Ri}}$ (split supersymmetry fails)

Vacuum stability

To avoid various problems we demand *global vacuum*, which requires for the first two generations

$$\left|a_i^D\right| \leq y_i^D \sqrt{3} \left(m_{H_d}^2 + m_{\tilde{d}_{Li}}^2 + m_{\tilde{d}_{Ri}}^2 \right)^{1/2}, \qquad i = 1, 2$$

but to have large enough corrections to light fermion masses, the only choice is to have m_{H_d} large

$$\frac{\delta m_{d_i}}{y_i^D v \cos\beta} \approx \frac{\alpha_s}{3\pi} \frac{(\mu \tan\beta - \gamma_i^D \sqrt{3} m_{H_d})}{\widetilde{m}}, \quad |\gamma_i^D| \lesssim 1$$

$\longrightarrow m_{H_d} \gg m_{\tilde{g}} \approx m_{\tilde{d}_{Li}} \approx m_{\tilde{d}_{Ri}} \equiv \widetilde{m}; \;\; (i = 1, 2)$

Gauge coupling unification

1-loop RGEs unification condition

$$\ln m_T = \left[\frac{1}{6}\ln m_Z - \frac{5\pi}{6}\left(2\alpha_3^{-1} - 3\alpha_2^{-1} + \alpha_1^{-1}\right)(m_Z)\right] + \frac{5}{6}\ln m_{susy}$$

$$m_{susy} = m_Z\left(\frac{m_{\tilde{w}}}{m_{\tilde{g}}}\right)^2 \prod_{i=1}^{3}\left(\frac{m^4_{\tilde{Q}_i}}{m^3_{\tilde{u}^c_i} m_{\tilde{e}^c_i}} \frac{m^2_{\tilde{L}_i}}{m^2_{\tilde{d}^c_i}}\right)^{1/10}\left(\frac{m_{\tilde{h}}}{m_Z}\right)^{4/5}\left(\frac{m_H}{m_Z}\right)^{1/5} \lesssim \mu$$

also for 2-loop running with 1-loop threshold corrections m_T depends mainly on the *higgsino* mass $m_{\tilde{h}} \approx \mu$

$$m_T \approx 10^{15}\ \mathrm{GeV}\left(\frac{\mu}{1\ \mathrm{TeV}}\right)^{5/6}$$

Color Higgs triplet m_T mediates $d = 5$ proton decay!

Proton decay

Proton decay dominated by the *wino* exchange $d = 5$ operators, where the lifetime of $p \to K^+\bar{\nu}$ channel scales approximately as

$$\tau_p^{d=5} \approx 4.10^{33}\ \mathrm{yrs} \left(\frac{2}{\tan\beta}\right)^2 \left(\frac{1}{m_{\widetilde{w}} I_3(m_5^2, m_{10}^2, m_{\widetilde{w}}^2)\ 10\ \mathrm{TeV}}\right)^2 \left(\frac{m_T}{10^{18}\ \mathrm{GeV}}\right)^2$$

$$\approx 4.10^{33}\ \mathrm{yrs} \left(\frac{2}{\tan\beta}\right)^2 \left(\frac{1}{m_{\widetilde{w}} I_3(m_5^2, m_{10}^2, m_{\widetilde{w}}^2)\ 10\ \mathrm{TeV}}\right)^2 \left(\frac{\mu}{100\ \mathrm{TeV}}\right)^{5/3}$$

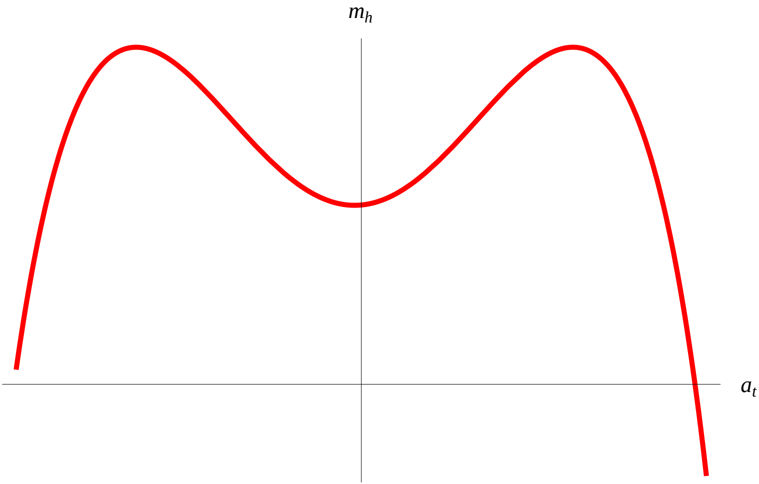
Higgs mass

$$
\begin{aligned}
m_{h^0}^2 &= \overbrace{m_Z^2 \cos^2 2\beta}^{tree-level} + \frac{3}{4\pi^2} \sin^2\beta\, y_t^2 m_t^2 \,[\ln \frac{\frac{1}{2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)}{m_t^2} + \\
&+ \underbrace{\frac{(\frac{a_t}{y_t} - \mu \cot\beta)^2}{\frac{1}{2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)} \left(1 - \frac{1}{12} \frac{(\frac{a_t}{y_t} - \mu \cot\beta)^2}{\frac{1}{2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)}\right)}_{STOP-MIXING\ contribution}] \\
\sin\theta_{\tilde{t}} \cos\theta_{\tilde{t}} &= \frac{(a_t \sin\beta - \mu y_t \cos\beta) v}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}
\end{aligned}
$$

$\longrightarrow a_t(m_{\tilde{t}_1}, m_{\tilde{t}_2})$; $\quad m_h = 125.5\,\mathrm{GeV}$

$\longrightarrow m_{\tilde{t}_1}, m_{\tilde{t}_2}$ chosen so, that $a_t(m_{GUT}) \sim 0$

Higgs mass as a function of a trilinear term a_t



Conclusions

Can minimal SUSY SU(5) scenario be made realistic?

We are proposing a sort of *"split supersymmetry"* model with the SU(5) invariant soft terms and the following INPUT and the preliminary results show it might work, but it is border case at best:

- $\tan\beta \approx 2 - 3$
- $m_{H_u}(m_{GUT}) \approx m_{H_d}(m_{GUT}) \approx m_{10_3}(m_{GUT}) \approx \mathcal{O}(10^2 - 10^3\,\mathrm{TeV})$
- $m_{5_{1,2}}(m_{GUT}) \approx m_{5_3}(m_{GUT}) \approx m_{10_{1,2}}(m_{GUT}) \approx M_\lambda(m_{GUT}) \approx \mathcal{O}(1 - 10\,\mathrm{TeV})$
- not too large *a-terms* at m_{GUT}

The good thing about such model is (if it survives), that is quite predictive for the superpartner spectrum.

Thank you