

Top cross-section @ NNLO

Alexander Mitov
Theory Division, CERN



- ✓ Will discuss total inclusive x-section
 - ✓ Useful for normalizations
 - ✓ Aim at precision

- ✓ Differential in future work (2013+)

The story of top pair production

- ✓ Early NLO QCD results (inclusive, semi-inclusive)
Nason, Dawson, Ellis '88
Beenakker et al '89
- ✓ First fully differential NLO
Mangano, Nason, Ridolfi '92
- ✓ 1990's: the rise of the soft gluon resummation at NLL
Catani, Mangano, Nason, Trentadue '96
Kidonakis, Sterman '97
Bonciani, Catani, Mangano, Nason '98
- ✓ NNLL resummation developed (and approximate NNLO approaches)
Beneke, Falgari, Schwinn '09
Czakon, Mitov, Sterman '09
Beneke, Czakon, Falgari, Mitov, Schwinn '09
Ahrens, Ferroglia, Neubert, Pecjak, Yang '10-'11
- ✓ Electroweak effects at NLO known (small $\sim 1.5\%$)
Beenakker, Denner, Hollik, Mertig, Sack, Wackerroth '93
Hollik, Kollar '07
Kuhn, Scharf, Uwer '07
- ✓ NNLO QCD corrections
Bärnreuther, Czakon, Mitov '12

✓ Until 6 months ago σ_{TOT} analyzed exclusively in approximate NNLO QCD

Beneke, Falgari, Klein, Schwinn '09-'11

Ahrens, Ferroglia, Neubert, Pecjak, Yang '10-'11

Kidonakis '03-'11

Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedermann '10

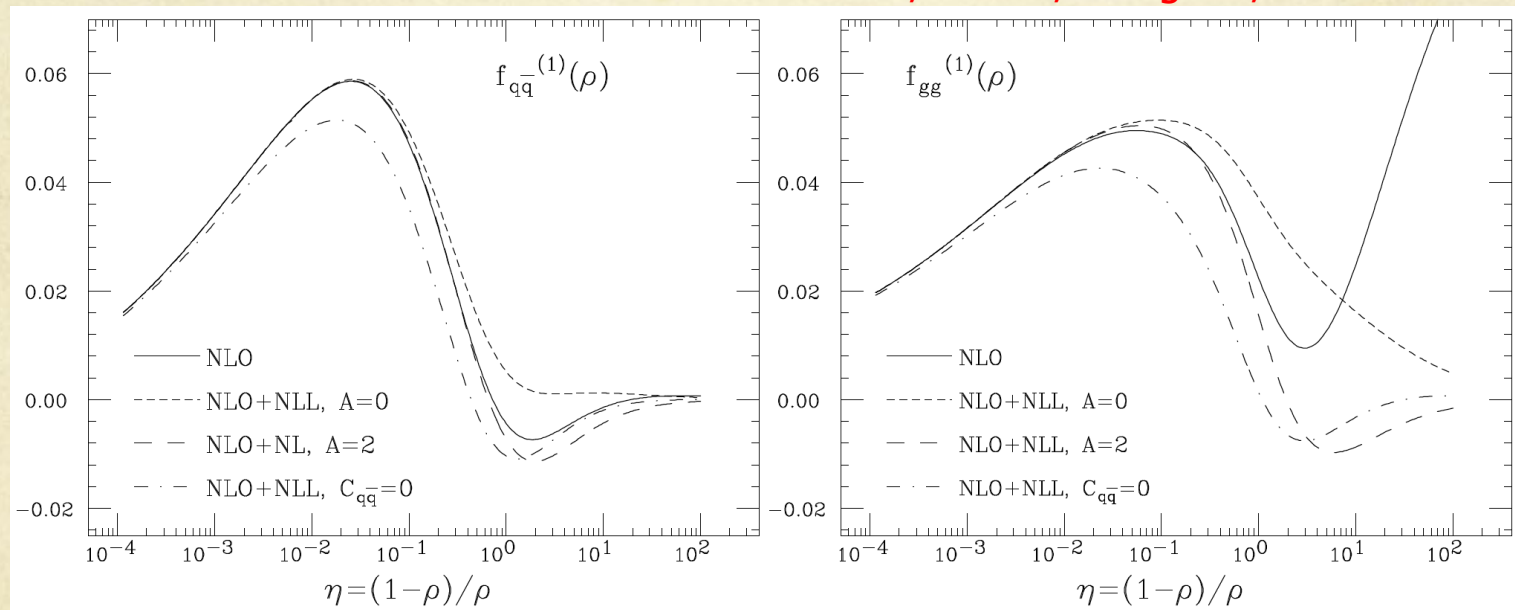
Cacciari, Czakon, Mangano, Mitov, Nason '11

✓ To study approx NNLO is theoretically very interesting but is it pheno game changer?

✓ No.

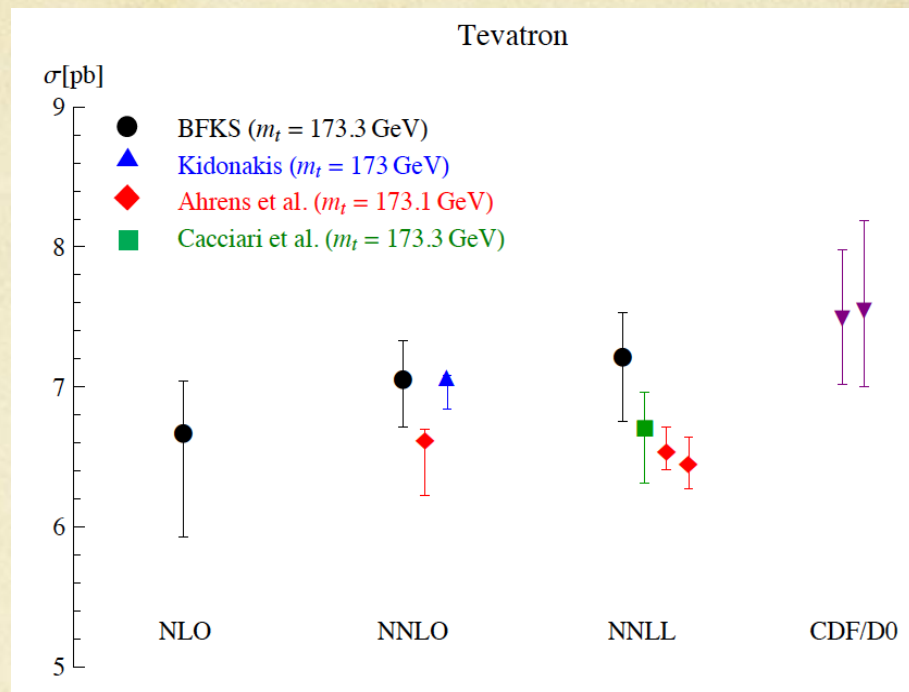
✓ This should not come as surprise. This was first noticed in 1998 at NLO+NLL

Bonciani, Catani, Mangano, Nason '98



Conclusion: resummed result alone does not approximate the exact NLO very well.
additional power suppressed terms are needed.

- ✓ Indeed, comparison between various NNLO_{approx} groups shows:



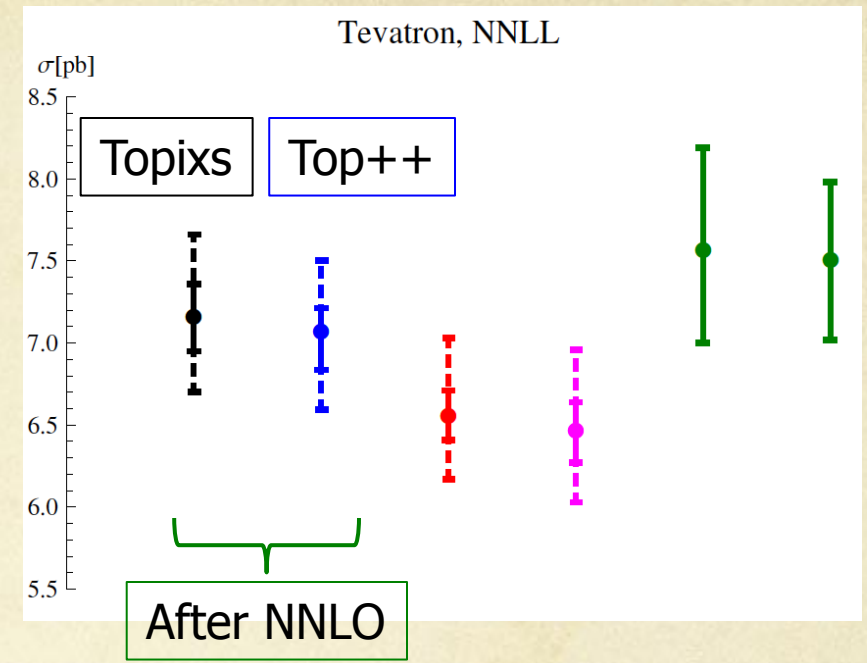
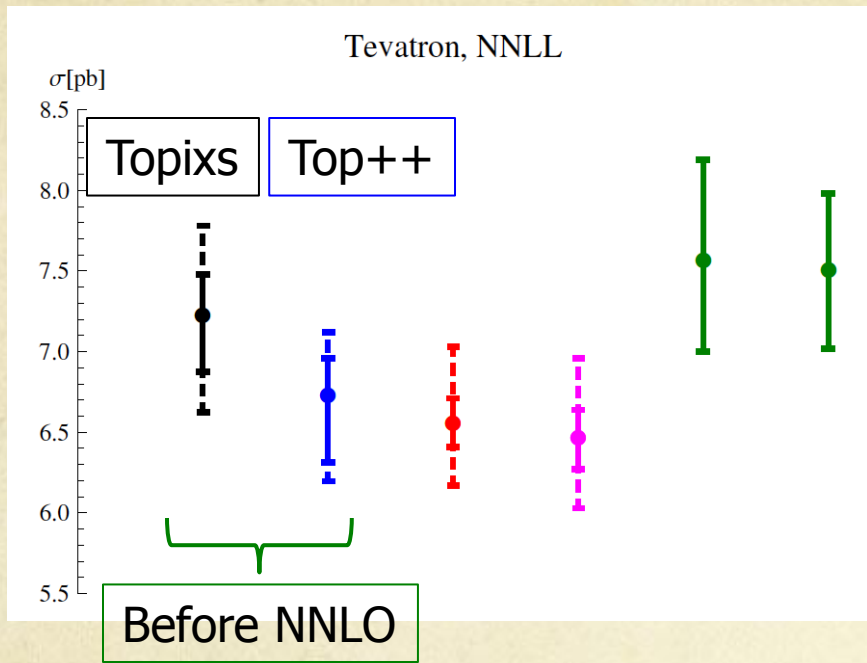
Beneke, Falgari, Klein, Schwinn '11

- ✓ Significant differences between various predictions
- ✓ Suggests the true uncertainty of approximate NNLO (originates beyond the approximation)

Cacciari, Czakon, Mangano, Mitov, Nason '11

Here is the proof we understand the physics well

Plots: M. Beneke, CKM 2012



✓ It was established that approx NNLO is dominated by unknown NNLO effects, not resummation
Cacciari, Czakon, Mangano, Mitov, Nason '11

✓ The inclusion of the full NNLO proves that (see above): perfect agreement now between different resummations

- Mellin space resummation with **Top++(1.3)**
Current version 1.4 (includes all available NNLO results + resummation)

Czakon, Mitov arXiv:1112.5675

- x-space resummation with **Topixs**

Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan '12



Towards complete NNLO result

- ✓ First ever hadron collider calculation at NNLO with more than 2 colored partons.
- ✓ First ever NNLO hadron collider calculation with massive fermions.

➤ Published $qQ \rightarrow tt + X$

Bärnreuther, Czakon, Mitov `12

➤ Published all fermionic reactions (qq, qq', qQ')

Czakon, Mitov `12

➤ Published gq

Czakon, Mitov `12

➤ Work on the only remaining reaction gg progressing well:

- Barring unexpected computing slowdown, sound estimate for gg (if not the full result) should be available within 1 month.

NNLO phenomenology at the Tevatron:

P. Bärnreuther et al arXiv:1204.5201

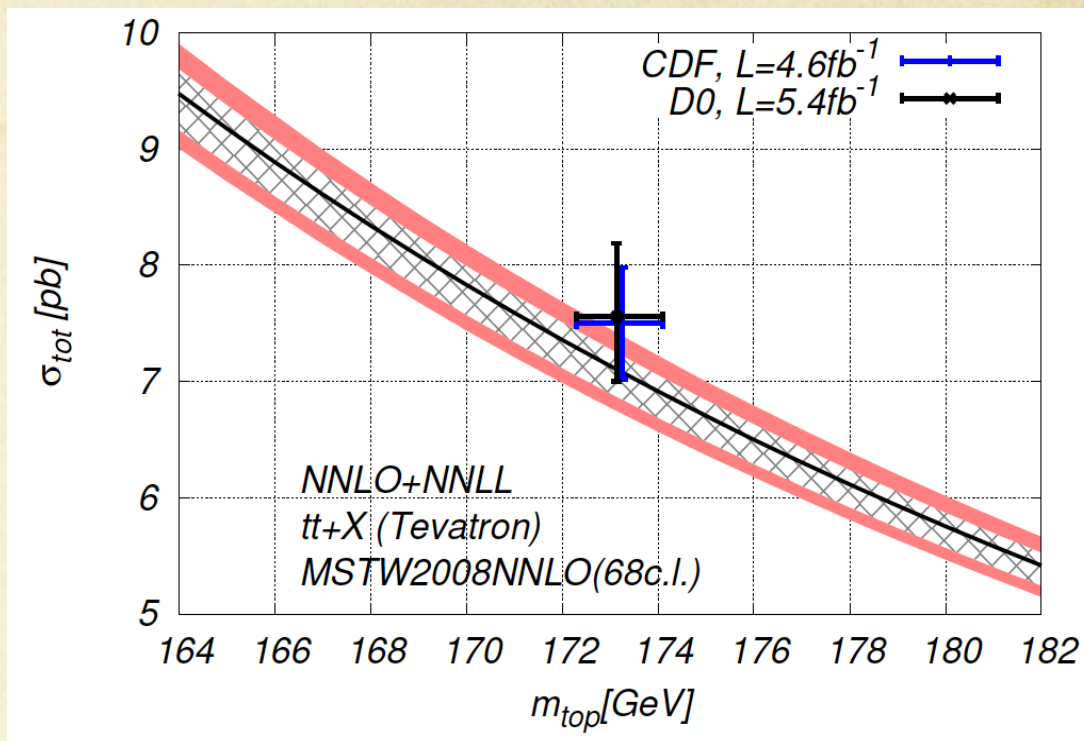
- ✓ Independent F/R scales
- ✓ MSTW2008NNLO
- ✓ $m_t=173.3$

NNLO

$$\sigma_{\text{tot}}^{\text{NNLO}} = 7.005^{+0.202 (2.9\%)}_{-0.310 (4.4\%)} [\text{scales}]^{+0.170 (2.4\%)}_{-0.122 (1.7\%)} [\text{pdf}]$$

$$\sigma_{\text{tot}}^{\text{res}} = 7.067^{+0.143 (2.0\%)}_{-0.232 (3.3\%)} [\text{scales}]^{+0.186 (2.6\%)}_{-0.122 (1.7\%)} [\text{pdf}]$$

Best prediction at NNLO+NNLL



- ✓ Two loop hard matching coefficient extracted and included
- ✓ Very weak dependence on unknown parameters (sub 1%): gg NNLO, A, etc.
- ✓ ~ 50% scales reduction compared to the NLO+NNLL analysis of

Cacciari, Czakon, Mangano, Mitov, Nason '11

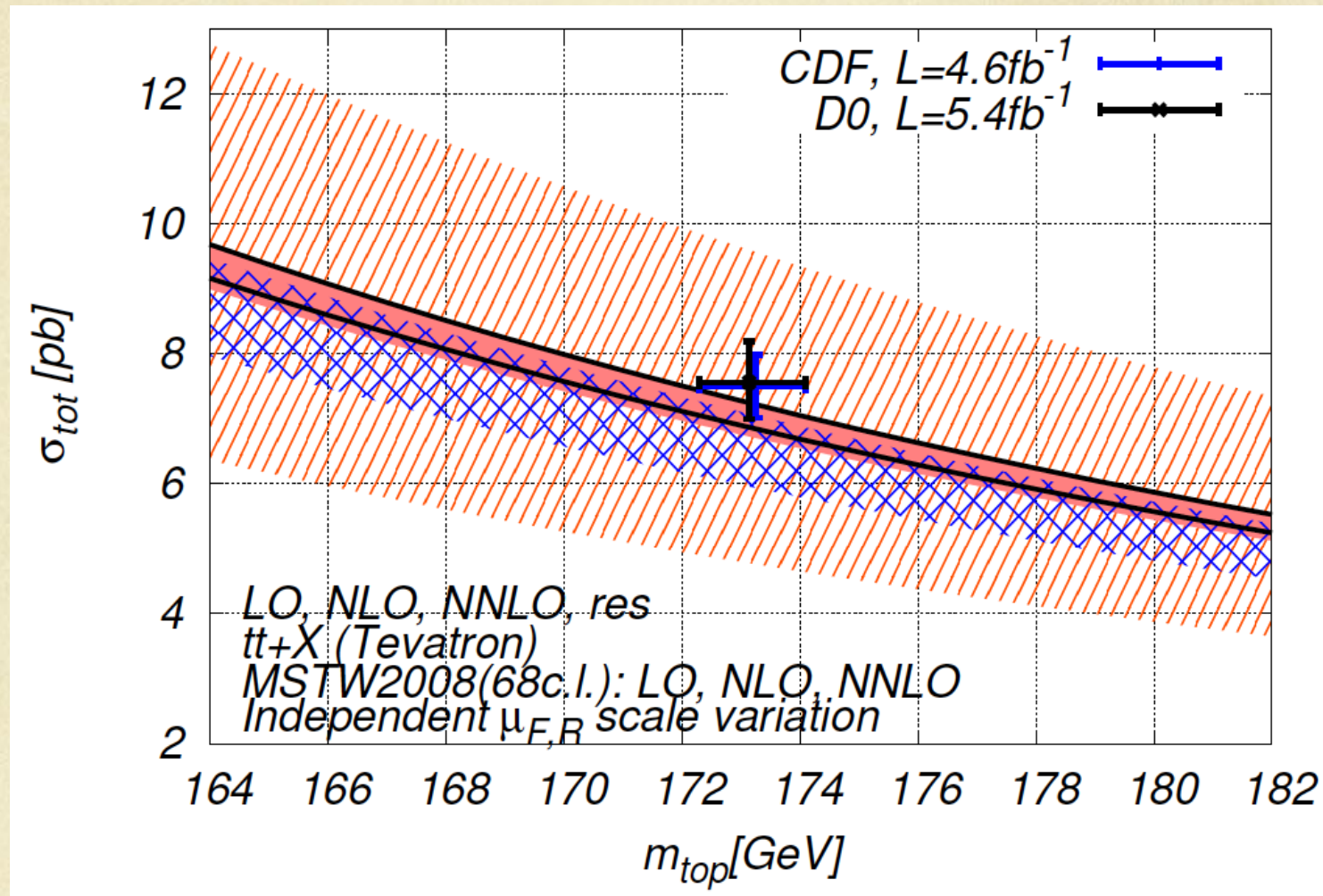
$$6.722^{+0.238 (3.5\%)}_{-0.410 (6.1\%)} [\text{scales}]^{+0.160 (2.4\%)}_{-0.115 (1.7\%)} [\text{PDF}]$$

Resummed (approximate NNLO)

Good perturbative convergence:

- ✓ Independent F/R scales
- ✓ $m_t=173.3$

P. Bärnreuther et al arXiv:1204.5201



- ✓ Good overlap of various orders (LO, NLO, NNLO).
- ✓ Suggests our (restricted) independent scale variation is good

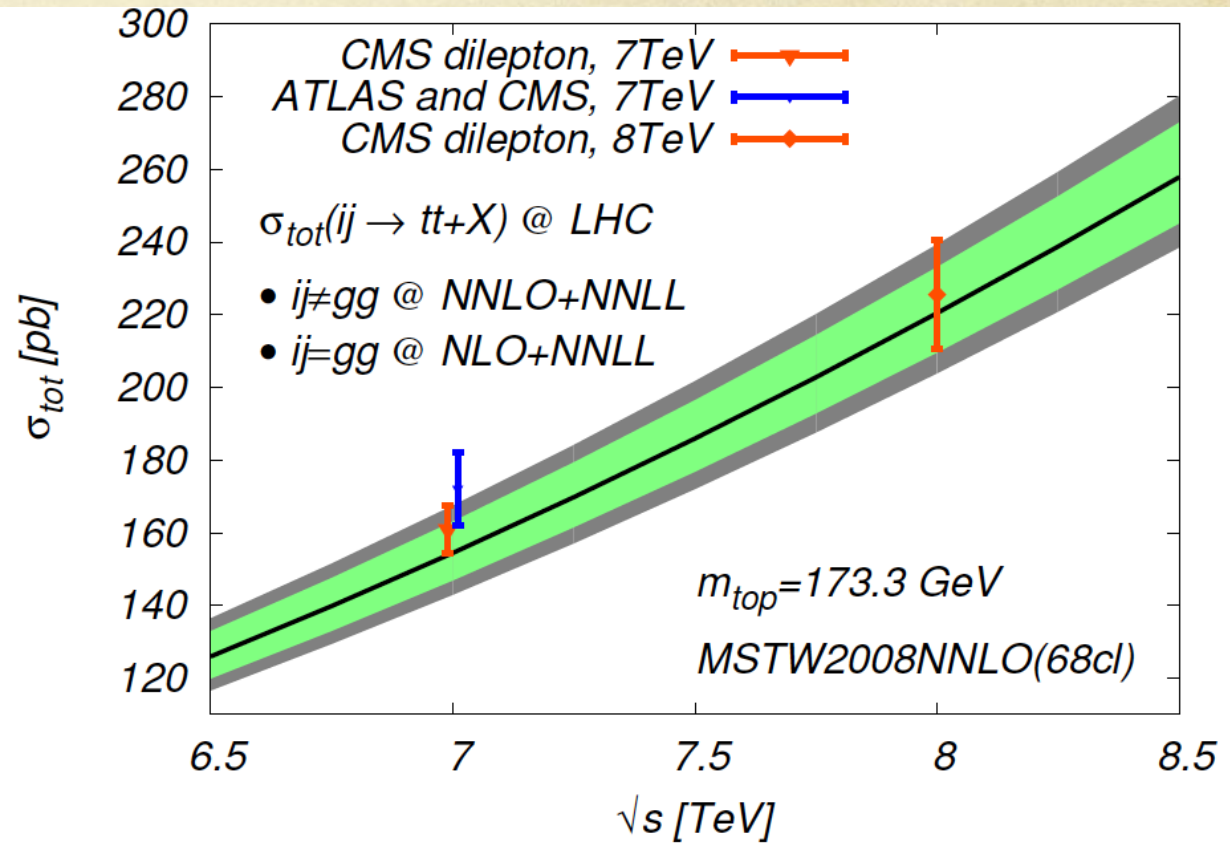
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NNLO phenomenology at the LHC:

Czakon, Mitov arXiv:1210.6832

- ✓ Independent F/R scales
- ✓ MSTW2008NNLO
- ✓ $m_t=173.3$

Best prediction at (N)NLO+NNLL



- ✓ 5% scale uncertainty
- ✓ Good agreement with LHC measurements
- ✓ Clearly, main uncertainty from unknown NNLO gg terms

Decrease of scale variation due to currently known NNLO corrections @ LHC:
by $\pm 1\%$ (from qQ)
by $\pm 2\%$ (from gg)

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❖ An often asked question (recall CMS α_s measurement):

which one of the many theory predictions should we use?

✓ It was suggested to use the high-energy limit of the X-section to predict it everywhere:

Moch, Uwer, Vogt '12

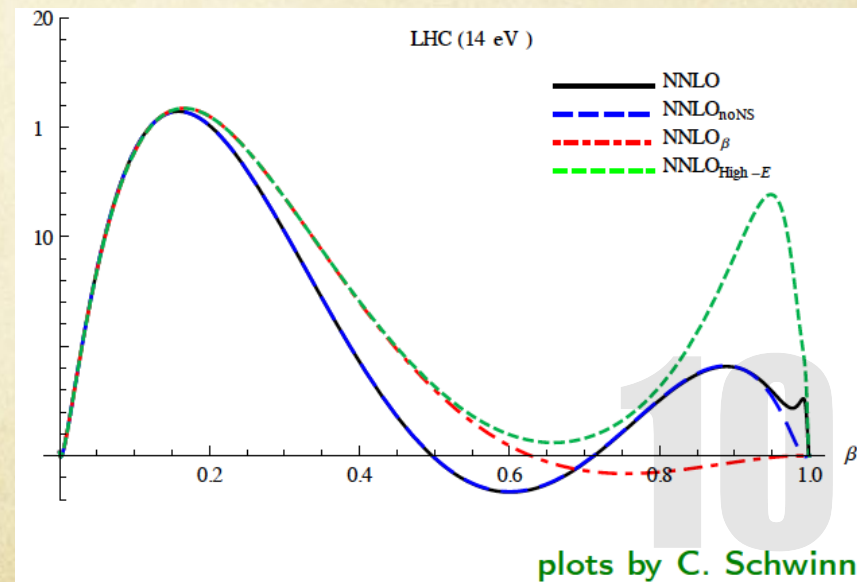
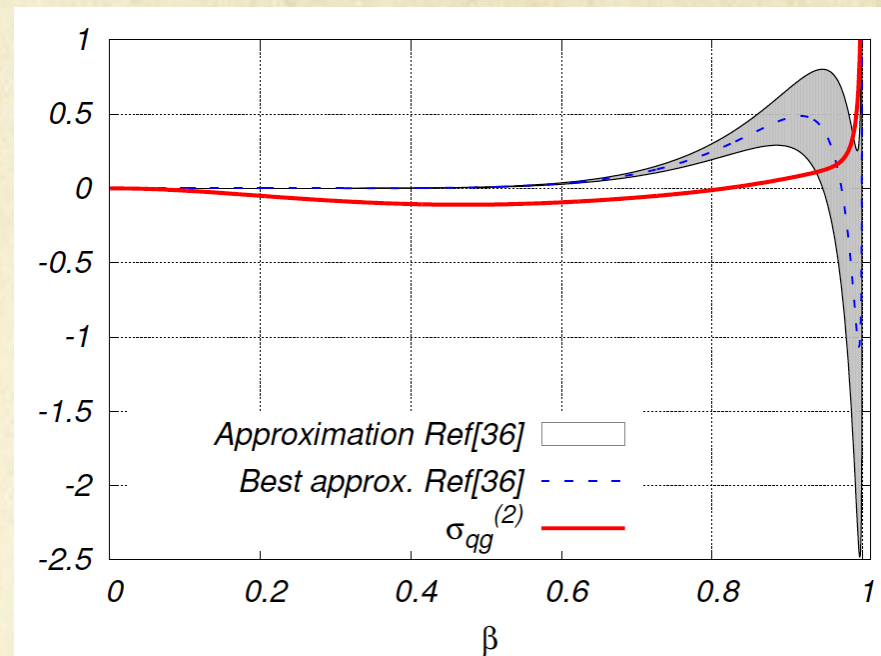
(included in Hathor 1.3 by Aliev et al '10)

✓ MUV approximation dramatically deviates from the exact NNLO result

✓ Leads to large difference for the x-section O(5%) from gq alone !

Czakon, Mitov arXiv:1210.6832

✓ Similar deviation for $qq \rightarrow tT + X$ (flux included)



Summary and Conclusions

- ✓ Moving from approximate NNLO to complete NNLO
- ✓ **Resummation does improve theory**, but alone is not a game changer:
 - *Approximate NNLO* (in name)
 - *Almost NLO+NLL* (in predictive power)
- ✓ Besides gg, all partonic reactions known to NNLO.
 - ✓ At Tevatron this is sufficient for full NNLO phenomenology
 - ✓ At LHC 5% non-pdf theory uncertainty.
- ✓ gg will be available very soon O(weeks) + time needed to make it public.
- ✓ Any suggestions about how to make the results more useful will be highly appreciated

Backup slides

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Structure of the cross-section

$$\sigma = \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_0^{\beta_{\max}} \mathcal{L}_{ij}(\beta) \hat{\sigma}(\beta)$$

$$\rho = \frac{4m_t^2}{s}$$

$$\beta = \sqrt{1 - \rho}$$

Relative velocity
of tT

- ✓ The partonic cross-section computed numerically in 80 points. Then fitted.
- ✓ Many contributing partonic channels:

Computed. Dominant at Tevatron (~85%)

$$q\bar{q} \rightarrow t\bar{t}$$

$$q\bar{q} \rightarrow t\bar{t}g$$

$$q\bar{q} \rightarrow t\bar{t}gg$$

$$q\bar{q} \rightarrow t\bar{t}q'q', \quad q \neq q'$$

$$gg \rightarrow t\bar{t}$$

$$gg \rightarrow t\bar{t}g$$

$$gg \rightarrow t\bar{t}gg$$

$$gg \rightarrow t\bar{t}q\bar{q}$$

$$qg \rightarrow t\bar{t}q$$

$$qg \rightarrow t\bar{t}qg$$

$$qq' \rightarrow t\bar{t}qq', \quad q \neq q'$$

$$q\bar{q} \rightarrow t\bar{t}q\bar{q}$$

All of the same complexity. No more conceptual challenges expected (just lots of CPU)

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Results @ parton level: $q\bar{q} \rightarrow t\bar{t} + X$

Notable features:

Partonic cross-section through NNLO:

$$\sigma_{ij} \left(\beta, \frac{\mu^2}{m^2} \right) = \frac{\alpha_S^2}{m^2} \left\{ \sigma_{ij}^{(0)} + \alpha_S \left[\sigma_{ij}^{(1)} + L \sigma_{ij}^{(1,1)} \right] + \alpha_S^2 \left[\sigma_{ij}^{(2)} + L \sigma_{ij}^{(2,1)} + L^2 \sigma_{ij}^{(2,2)} \right] + \mathcal{O}(\alpha_S^3) \right\},$$

The NNLO term:

$$\sigma_{q\bar{q}}^{(2)}(\beta) = F_0(\beta) + F_1(\beta)N_L + F_2(\beta)N_L^2$$

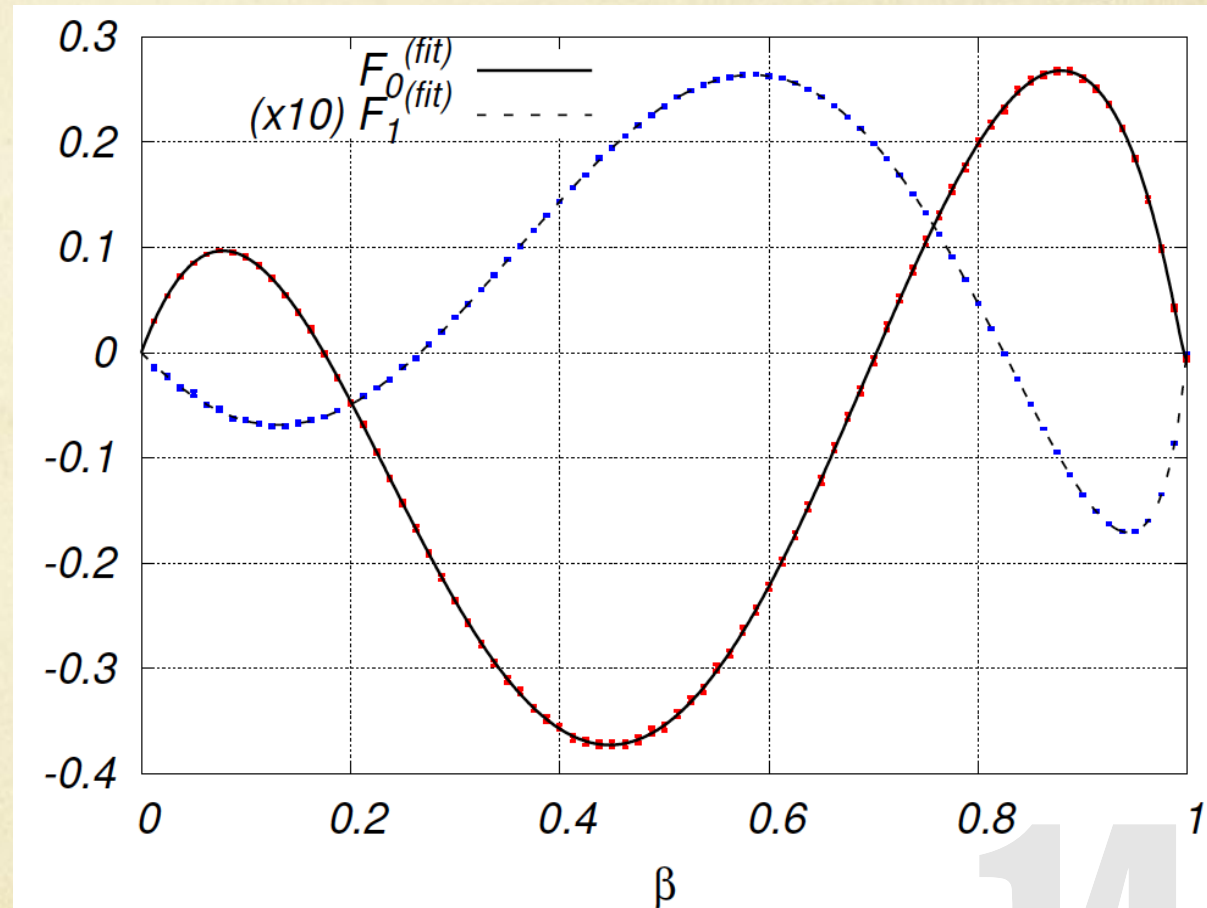
Numeric

Analytic

$$F_i \equiv F_i^{(\beta)} + F_i^{(\text{fit})}, i = 0, 1$$

The known threshold approximation

- ✓ Small numerical errors
- ✓ Agrees with limits

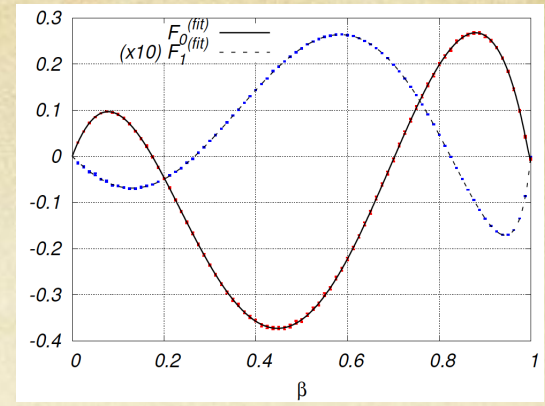


P. Bärnreuther et al arXiv:1204.5201

Beneke, Czakon, Falgari, Mitov, Schwinn `09

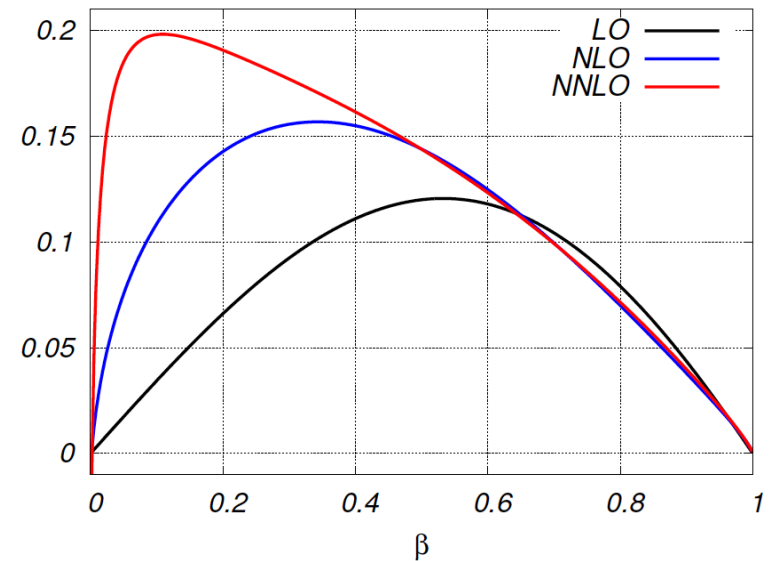
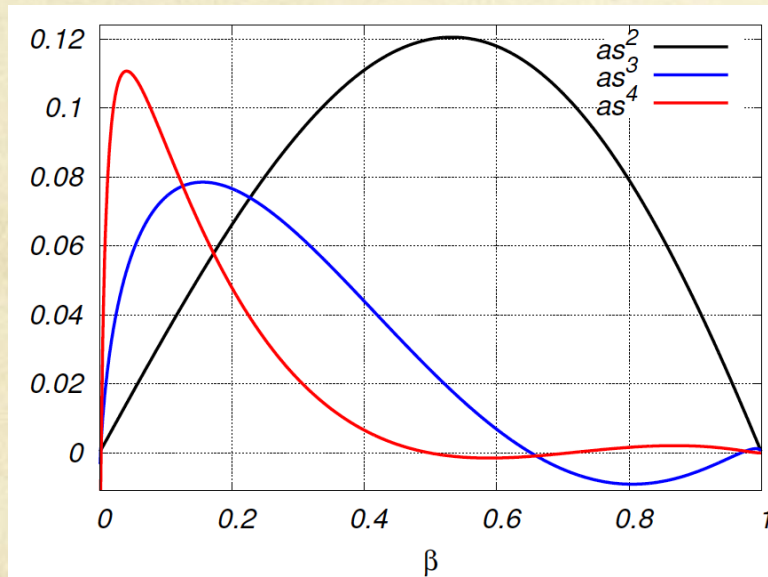
Results @ parton level: $q\bar{q} \rightarrow t\bar{t} + X$

$$\sigma_{ij} \left(\beta, \frac{\mu^2}{m^2} \right) = \frac{\alpha_S^2}{m^2} \left\{ \sigma_{ij}^{(0)} + \alpha_S \left[\sigma_{ij}^{(1)} + L \sigma_{ij}^{(1,1)} \right] + \alpha_S^2 \left[\sigma_{ij}^{(2)} + L \sigma_{ij}^{(2,1)} + L^2 \sigma_{ij}^{(2,2)} \right] + \mathcal{O}(\alpha_S^3) \right\},$$



An alternative view of the partonic cross-sections:

Bärrreuther, Czakon, Mitov '12



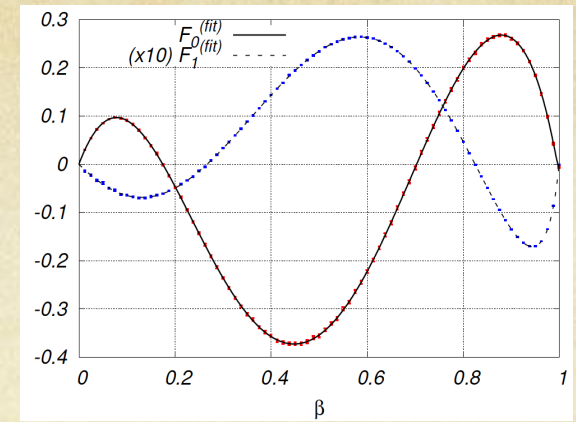
$$\hat{\sigma}(\beta) = \frac{\alpha_S^2}{m^2} \left(\sigma^{(0)} + \alpha_S \sigma^{(1)} + \alpha_S^2 \sigma^{(2)} + \dots \right) \equiv \frac{\alpha_S^2}{m^2} \left(f_{\alpha_S^2} + f_{\alpha_S^3} + f_{\alpha_S^4} + \dots \right)$$

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Results @ parton level:
The all-fermionic reactions

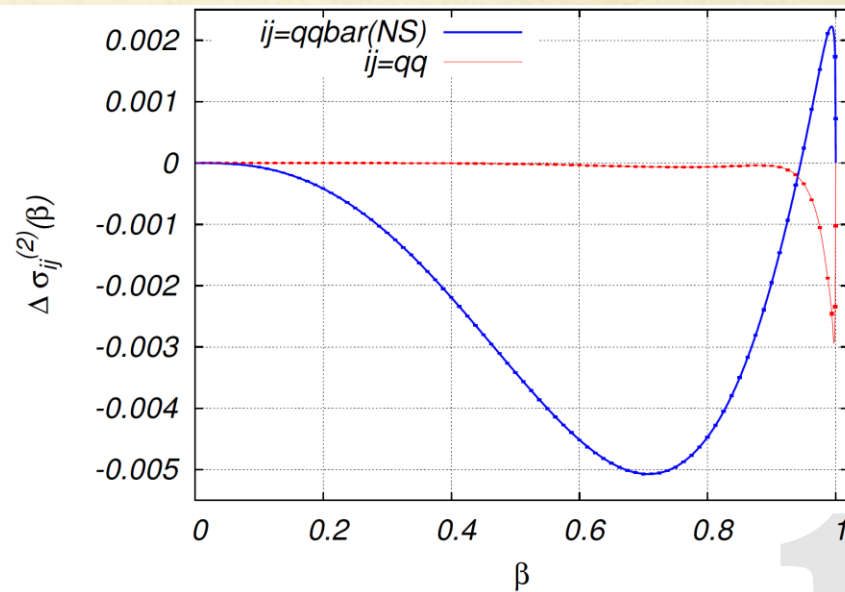
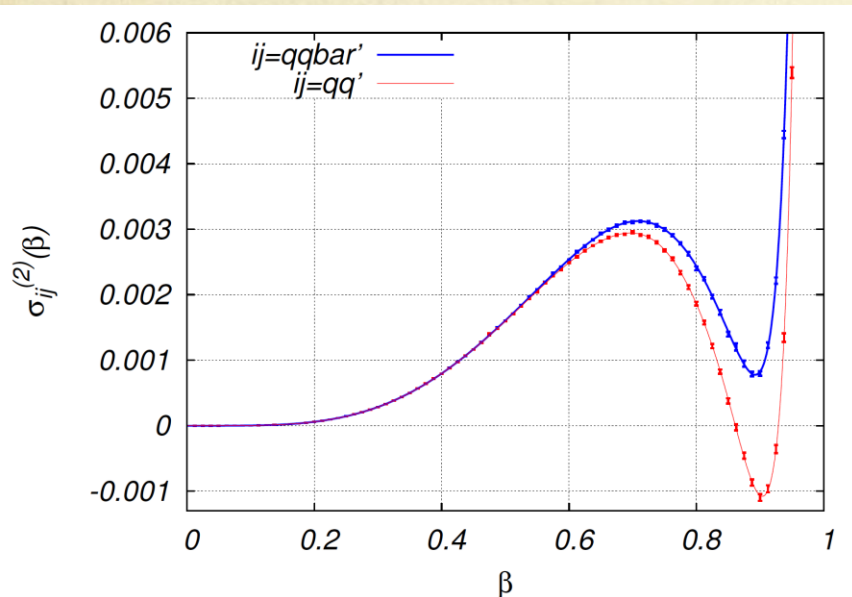
$$\begin{aligned}
 q\bar{q} &\rightarrow t\bar{t} + q\bar{q}|_{\text{NS}}, \\
 q\bar{q}' &\rightarrow t\bar{t} + q\bar{q}', \\
 qq' &\rightarrow t\bar{t} + qq', \\
 qq &\rightarrow t\bar{t} + qq.
 \end{aligned}$$

Czakon, Mitov '12



P. Bärnreuther et al arXiv:1204.5201

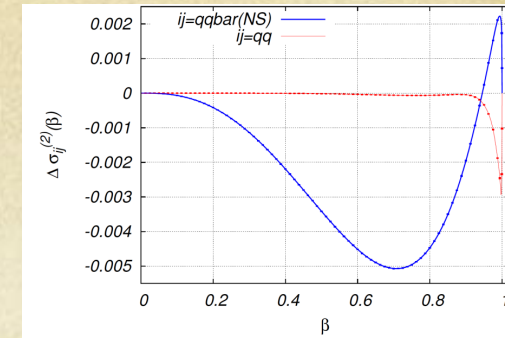
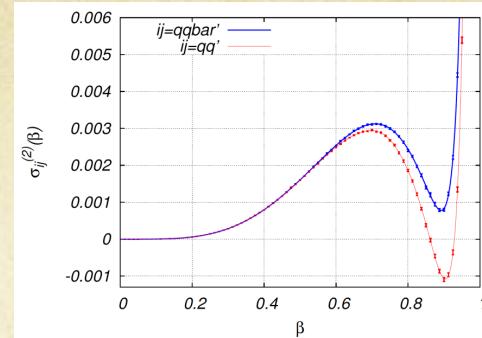
These partonic cross-sections are very small.
Compare to the ones involving gluons!



✧ Had to compute up to beta=0.9999 to get the high-energy behavior right.

Results @ parton level:
The all-fermionic reactions

$$\begin{aligned}
 q\bar{q} &\rightarrow t\bar{t} + q\bar{q}|_{\text{NS}}, \\
 q\bar{q}' &\rightarrow t\bar{t} + q\bar{q}', \\
 qq' &\rightarrow t\bar{t} + qq', \\
 qq &\rightarrow t\bar{t} + qq.
 \end{aligned}$$



The interesting feature: high-energy logarithmic rise:

$$\sigma_{f_1 f_2 \rightarrow t\bar{t} f_1 f_2}^{(2)} \Big|_{\rho \rightarrow 0} \approx c_1 \ln(\rho) + c_0 + \mathcal{O}(\rho)$$

$$\rho = \frac{4m_t^2}{s}$$

$$c_1 = -0.4768323995789214$$

Predicted in analytical form

Ball, Ellis '01

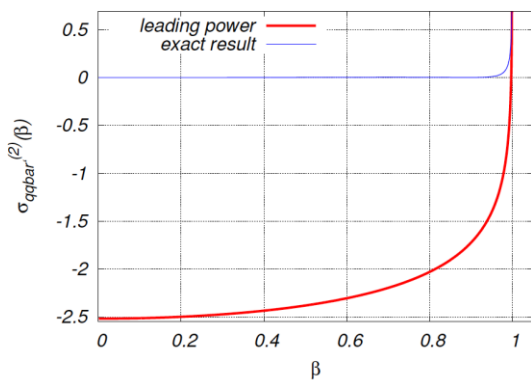
$$c_0 \text{ (from Eqs. (6.3, 6.4))} = \begin{cases} -2.5173 & \text{from } \sigma_{q\bar{q}'}^{(2)} \\ -2.5186 & \text{from } \sigma_{qq'}^{(2)} \end{cases}$$

❖ Direct extraction from the fits.
5% uncertainty.

Czakon, Mitov '12

❖ Agrees with independent prediction.
50% uncertainty.

Moch, Uwer, Vogt '12



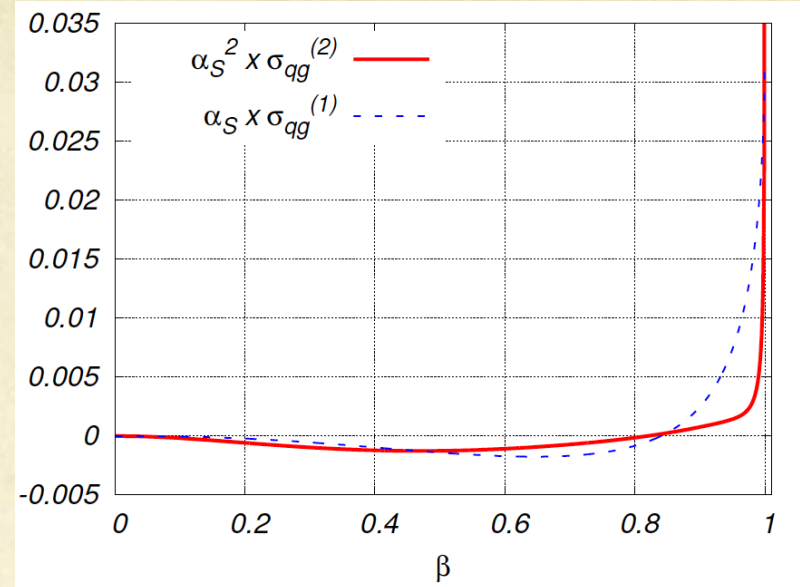
High-energy expansion
non-convergent.

Applies only to the
high-energy limit.

	Tevatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
$\Delta\sigma_{q\bar{q},(\text{NS})}$ [pb]	-0.0020	-0.0097	-0.0124	-0.0299
$\sigma_{q\bar{q},(\text{NS})}$ [pb]	-0.0009	-0.0001	0.0021	0.0464
σ_{all} [pb]	0.0003	0.0970	0.1504	0.7885
σ_{tot} [pb]	7.0056	154.779	220.761	852.177

Czakon, Mitov '12

- ✓ Correction about -1% (Tev and LHC).
- ✓ Notable decrease of scale dependence at LHC.
- ✓ NNLO large compared to NLO.



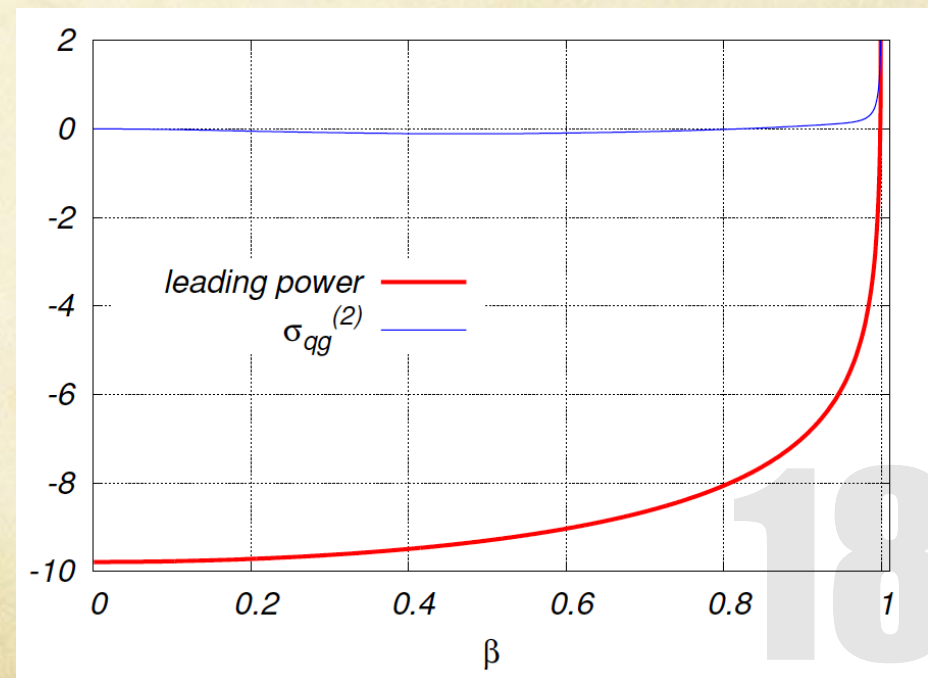
- ✓ High-energy log-limit correct

Ball, Ellis '01

- ✓ Agree for the constant with

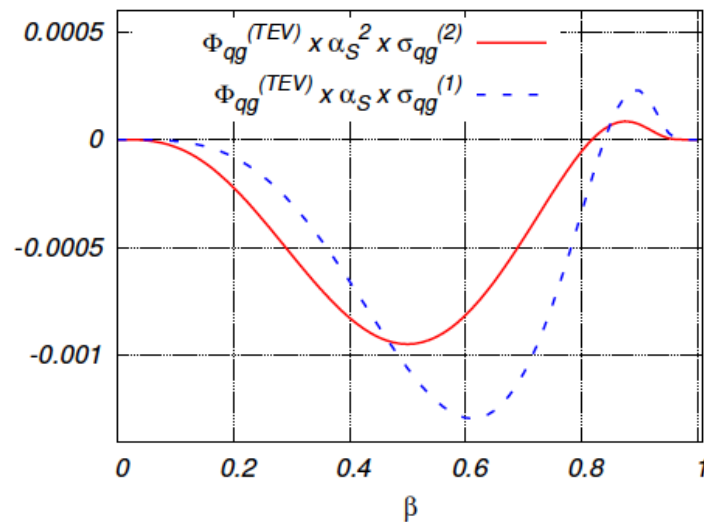
Moch, Uwer, Vogt '12

- ✓ The limit itself plays no Pheno role

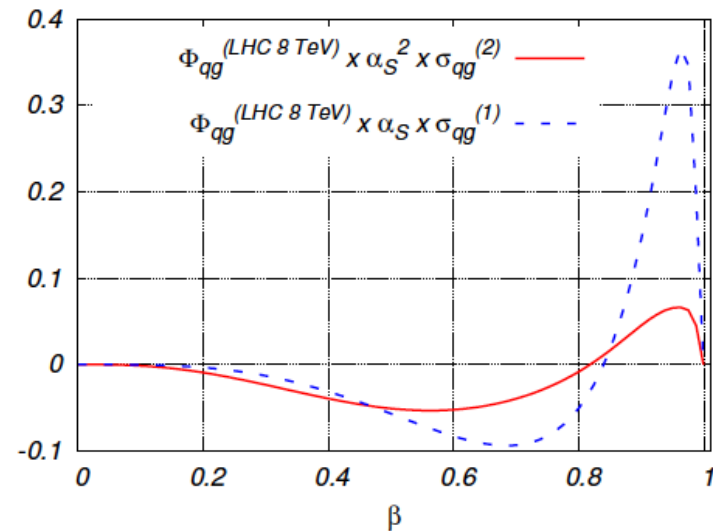


X-section times flux

Tevatron



LHC 8 TeV



		Tevatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
I_1	Due to $\sigma_{qg}^{(1)}$ [pb]	-0.068	-0.88	-0.48	9.01
I_2	Due to $\sigma_{qg}^{(2)}$ [pb]	-0.057	-1.82	-2.25	-4.07
I_3	$\sigma_{qg}^{(2)}$ (Hathor; $(A + B)/2$) [pb]	0.040	5.78	8.11	27.36
I_4	$(I_3 - I_2)/\sigma_{tot}$ [%]	1.4	4.9	4.7	3.7