Top cross-section @ NNLO

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- ✓ Will discuss total inclusive x-section
    - ✓ Useful for normalizations
    - ✓ Aim at precision
- ✓ Differential in future work (2013+)

The story of top pair production

- ✓ Early NLO QCD results (inclusive, semi-inclusive)

  Nason, Dawson, Ellis ‘88
  Beenakker et al ‘89

- ✓ First fully differential NLO

  Mangano, Nason, Ridolfi’ 92

- ✓ 1990’s: the rise of the soft gluon resummation at NLL

  Catani, Mangano, Nason, Trentadue ’96
  Kidonakis, Sterman ’97
  Bonciani, Catani, Mangano, Nason `98

- ✓ NNLL resummation developed (and approximate NNLO approaches)

  Beneke, Falgari, Schwinn ‘09
  Czakon, Mitov, Sterman `09
  Beneke, Czakon, Falgari, Mitov, Schwinn `09
  Ahrens, Ferroglia, Neubert, Pecjak, Yang `10-`11

- ✓ Electroweak effects at NLO known (small ~ 1.5%)

  Beenakker, Denner, Hollik, Mertig, Sack, Wackeroth `93
  Hollik, Kollar `07
  Kuhn, Scharf, Uwer ‘07

- ✓ NNLO QCD corrections

  Bärnreuther, Czakon, Mitov `12

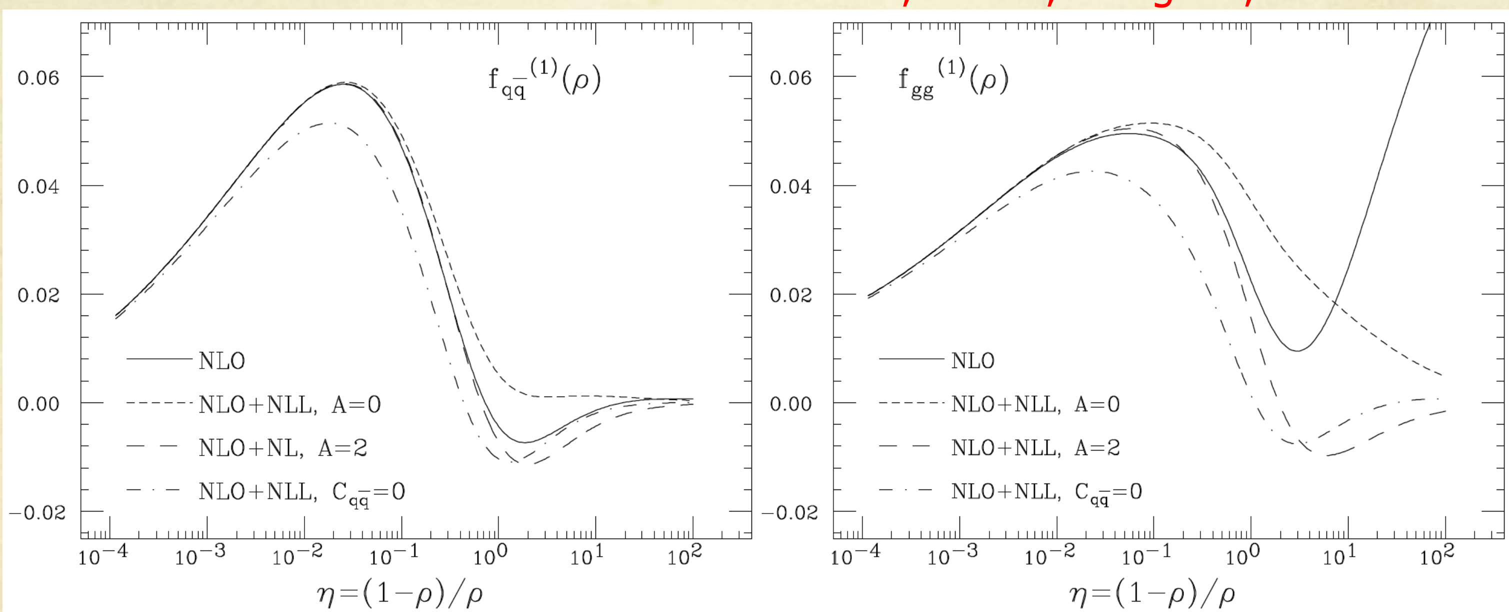
✓ Until 6 moths ago σ_{TOT} analyzed exclusively in approximate NNLO QCD

Beneke, Falgari, Klein,Schwinn `09-`11
Ahrens, Ferroglia, Neubert, Pecjak, Yang `10-`11
Kidonakis `03-`11
Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedermann ’10
Cacciari, Czakon, Mangano, Mitov, Nason ‘11

✓ To study approx NNLO is theoretically very interesting but is it pheno game changer?
- ✓ No.

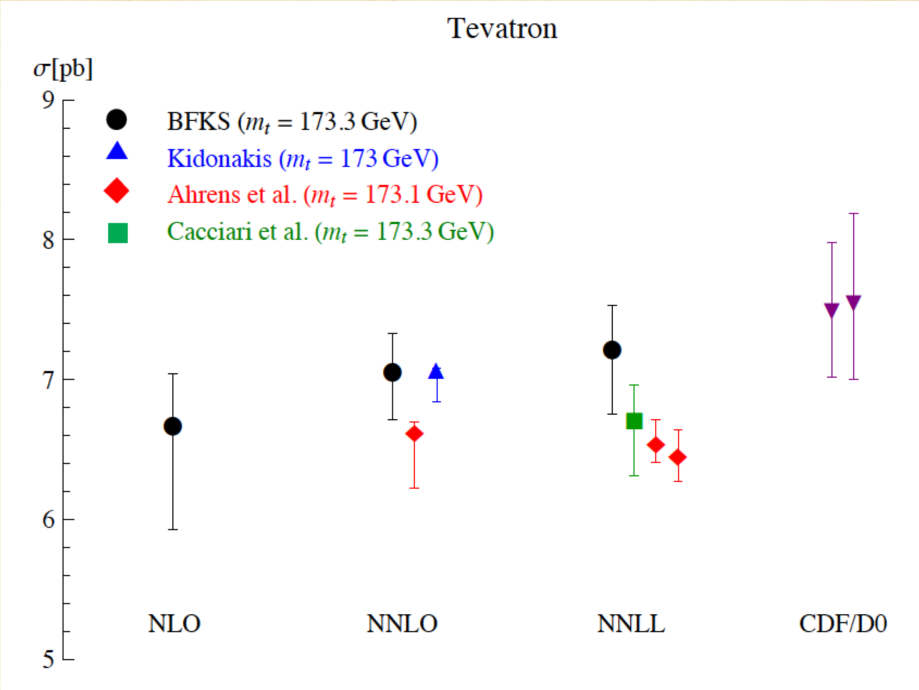
✓ This should not come as surprise. This was first noticed in 1998 at NLO+NLL

Bonciani, Catani, Mangano, Nason ‘98



Conclusion: resummed result alone does not approximate the exact NLO very well.
additional power suppressed terms are needed.

✓ Indeed, comparison between various $NNLO_{approx}$ groups shows:



Beneke, Falgari, Klein, Schwinn `11

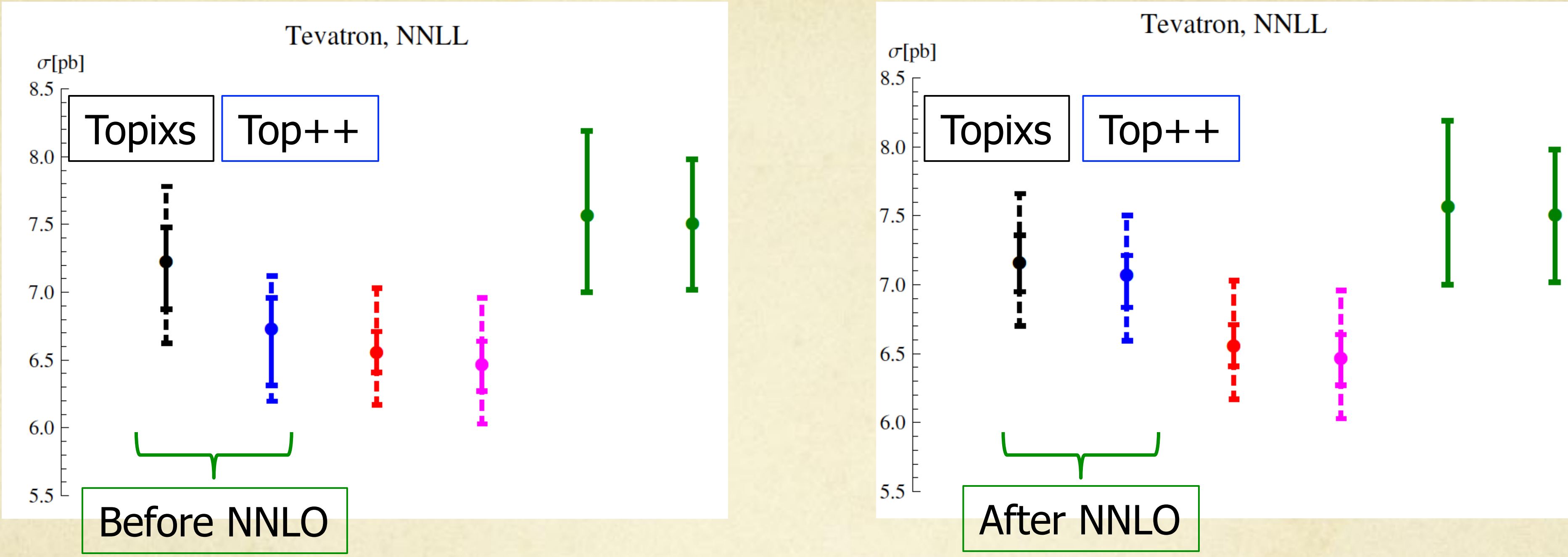
✓ Significant differences between various predictions

✓ Suggests the true uncertainty of approximate NNLO (originates beyond the approximation)

Cacciari, Czakon, Mangano, Mitov, Nason '11

Here is the proof we understand the physics well

Plots: M. Beneke, CKM 2012



✓ It was established that approx NNLO is dominated by unknown NNLO effects, not resummation

Cacciari, Czakon, Mangano, Mitov, Nason '11

✓ The inclusion of the full NNLO proves that (see above): perfect agreement now between different resummations

➢ Mellin space resummation with ***Top++(1.3)***
Current version 1.4 (includes all available NNLO results + resummation)

Czakon, Mitov arXiv:1112.5675

➢ x-space resummation with ***Topixs***

Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan '12

Towards complete NNLO result

✓ First ever hadron collider calculation at NNLO with more than 2 colored partons.

✓ First ever NNLO hadron collider calculation with massive fermions.

- Published qQ → tt +X

  Bärnreuther, Czakon, Mitov `12

- Published all fermionic reactions (qq,qq',qQ')

  Czakon, Mitov `12

- Published gq

  Czakon, Mitov `12

- Work on the only remaining reaction gg progressing well:
  - Barring unexpected computing slowdown, sound estimate for gg (if not the full result) should be available within 1 month.

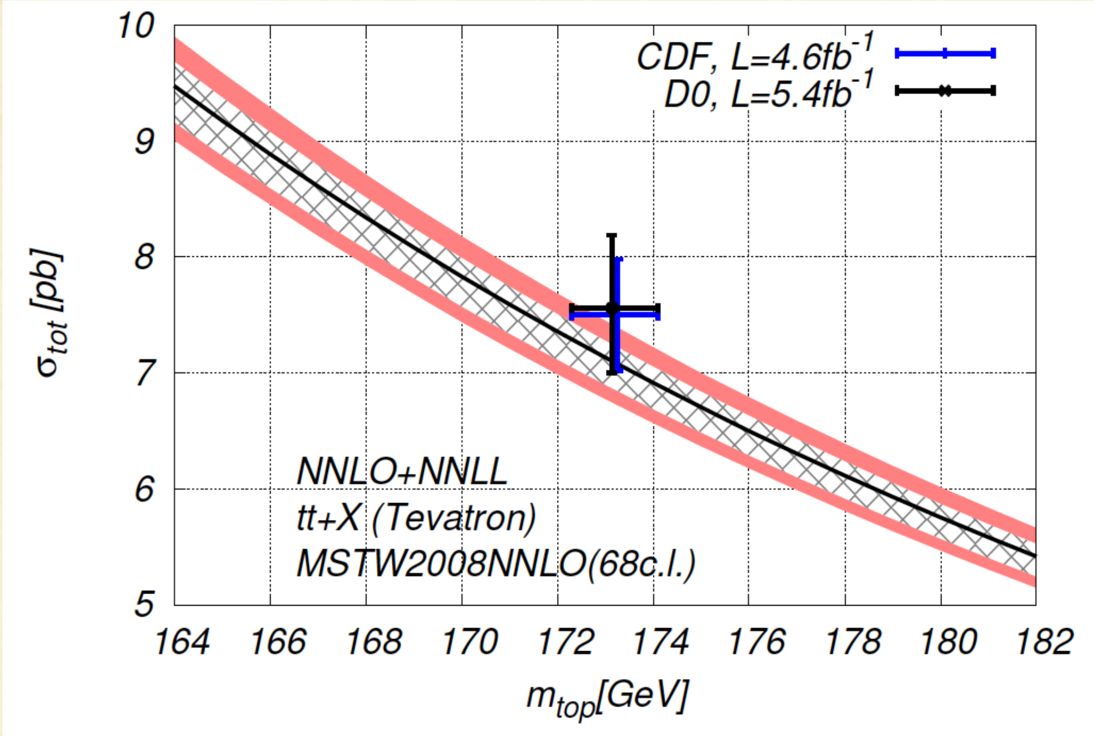
NNLO phenomenology at the Tevatron:

P. Bärnreuther et al arXiv:1204.5201

✓ Independent F/R scales
✓ MSTW2008NNLO
✓ mt=173.3

NNLO

$$\sigma_{\rm tot}^{\rm NNLO} = 7.005 \, {}^{+0.202\,(2.9\%)}_{-0.310\,(4.4\%)} \,[\text{scales}]\, {}^{+0.170\,(2.4\%)}_{-0.122\,(1.7\%)} \,[\text{pdf}]$$



$$\sigma_{\rm tot}^{\rm res} = 7.067 \, {}^{+0.143\,(2.0\%)}_{-0.232\,(3.3\%)} \,[\text{scales}]\, {}^{+0.186\,(2.6\%)}_{-0.122\,(1.7\%)} \,[\text{pdf}]$$

Best prediction at NNLO+NNLL

✓ Two loop hard matching coefficient extracted and included

✓ Very week dependence on unknown parameters (sub 1%): gg NNLO, A, etc.

✓ ~ 50% scales reduction compared to the NLO+NNLL analysis of

Cacciari, Czakon, Mangano, Mitov, Nason '11

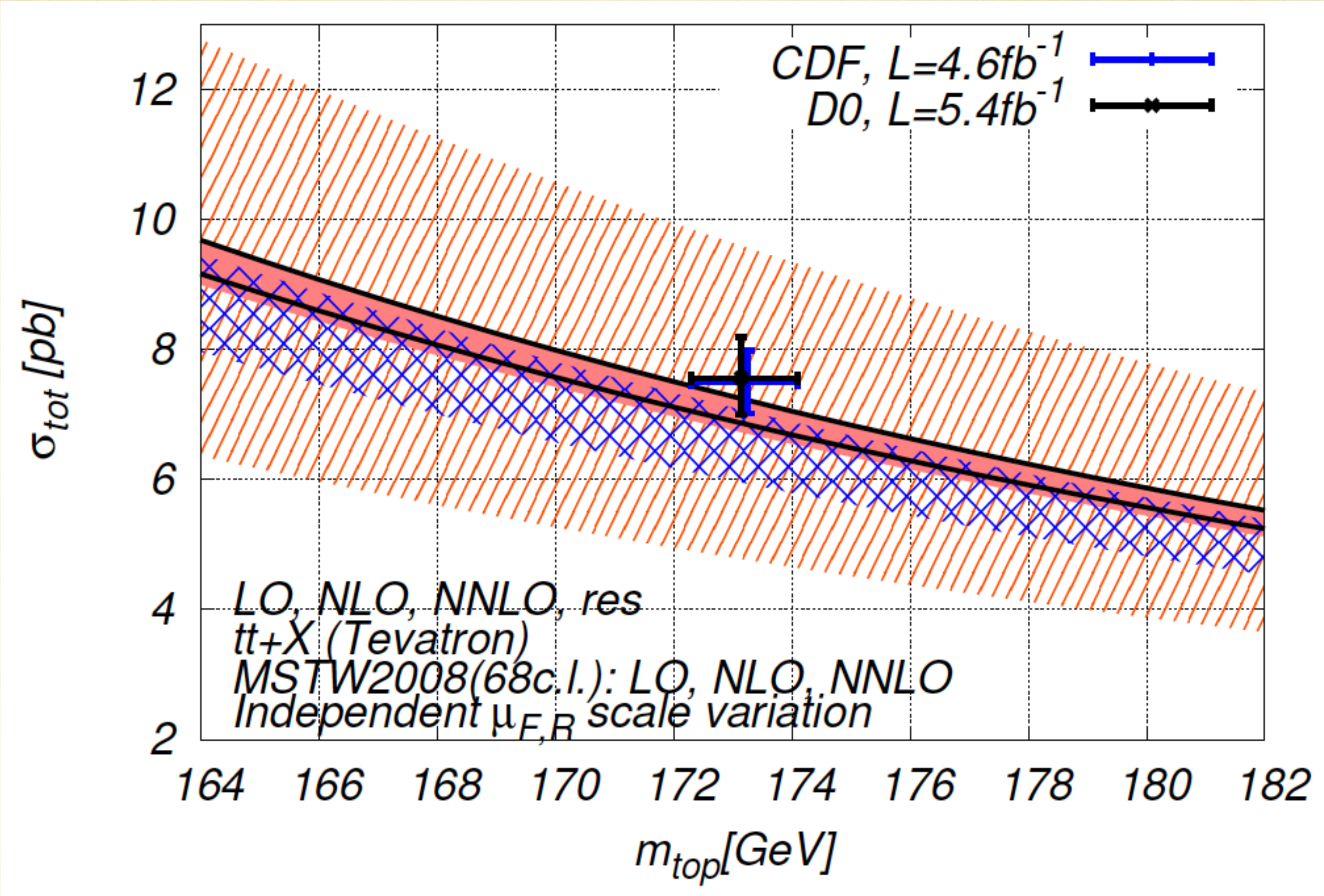
$$6.722 \, {}^{+0.238\,(3.5\%)}_{-0.410\,(6.1\%)} \,[\text{scales}]\, {}^{+0.160\,(2.4\%)}_{-0.115\,(1.7\%)} \,[\text{PDF}]$$

Resumed (approximate NNLO)

Good perturbative convergence:

✓ Independent F/R scales
✓ mt=173.3

P. Bärnreuther et al arXiv:1204.5201



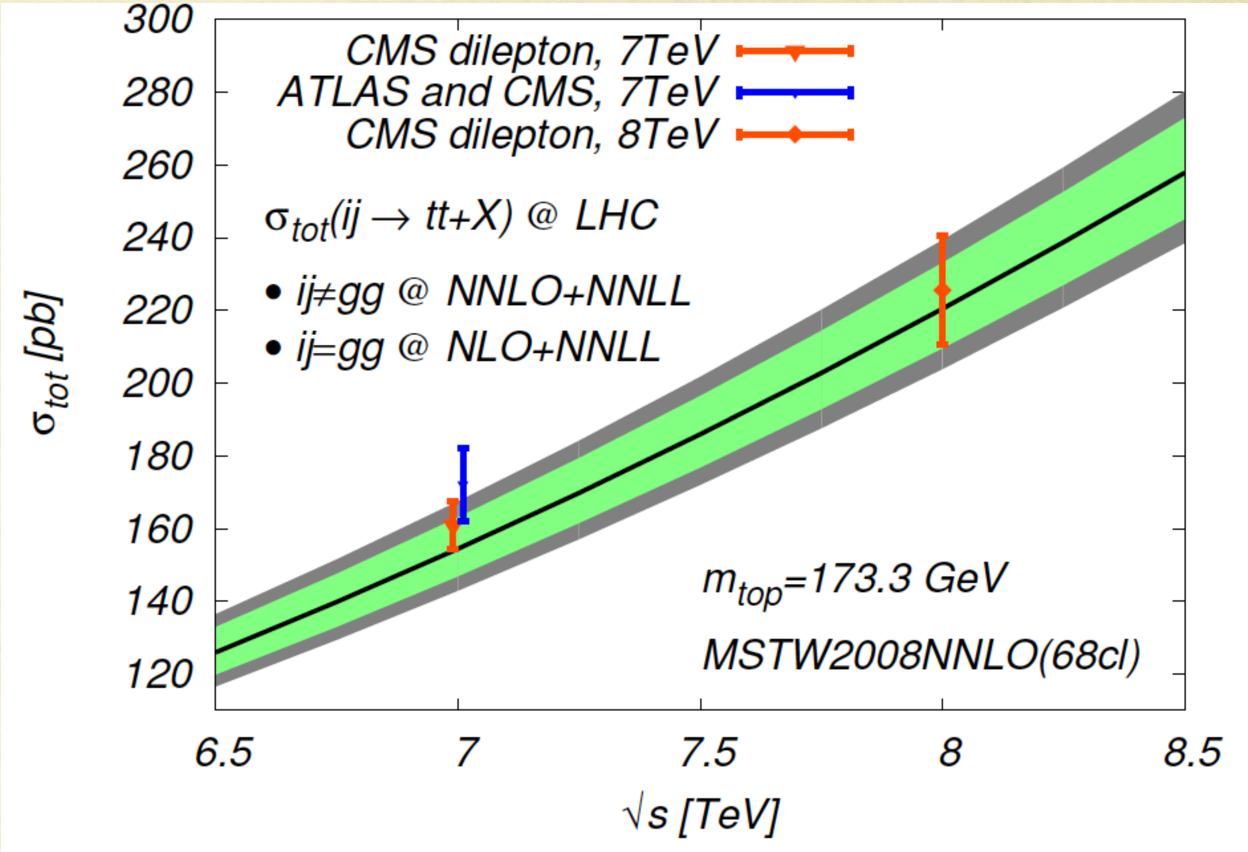
✓ Good overlap of various orders (LO, NLO, NNLO).
✓ Suggests our (restricted) independent scale variation is good

NNLO phenomenology at the LHC:

Czakon, Mitov arXiv:1210.6832

✓ Independent F/R scales
✓ MSTW2008NNLO
✓ mt=173.3

Best prediction at (N)NLO+NNLL



✓ 5% scale uncertainty
✓ Good agreement with LHC measurements
✓ Clearly, main uncertainty from unknown NNLO gg terms

Decrease of scale variation due to currently known NNLO corrections @ LHC:
by ±1% (from qQ)
by ±2% (from gq)

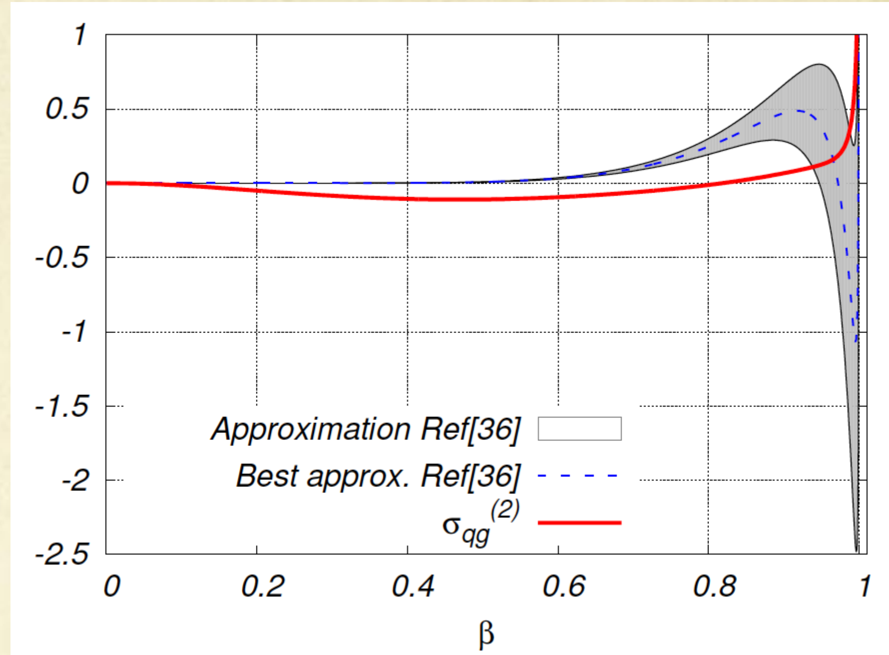
❖ An often asked question (recall CMS α_S measurement):

which one of the many theory predictions should we use?

✓ It was suggested to use the high-energy limit of the X-section to predict it everywhere:

Moch, Uwer, Vogt '12

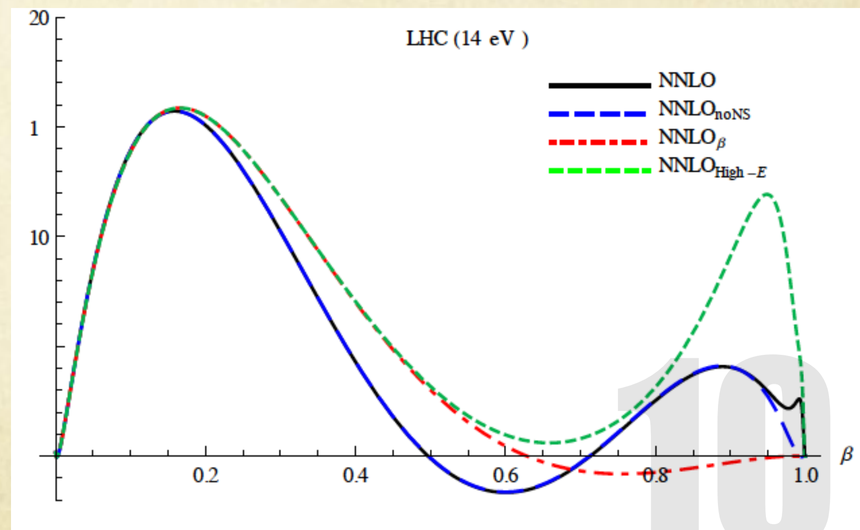
(included in Hathor 1.3 by Aliev et al '10)



✓ MUV approximation dramatically deviates from the exact gq NNLO result

✓ Leads to large difference for the x-section O(5%) from gq alone !

Czakon, Mitov arXiv:1210.6832



plots by C. Schwinn

✓ Similar deviation for qq->tT+X (flux included)

Summary and Conclusions

- ✓ Moving from approximate NNLO to complete NNLO

- ✓ Resummation does improve theory, but alone is not a game changer:

    - ➢ *Approximate NNLO* (in name)

    - ➢ *Almost NLO+NLL* (in predictive power)

- ✓ Besides gg, all partonic reactions known to NNLO.

    - ✓ At Tevatron this is sufficient for full NNLO phenomenology
    - ✓ At LHC 5% non-pdf theory uncertainty.

- ✓ gg will be available very soon O(weeks) + time needed to make it public.

- ✓ Any suggestions about how to make the results more useful will be highly appreciated

Backup slides

Structure of the cross-section

$$\sigma = \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_0^{\beta_{\max}} \mathcal{L}_{ij}(\beta)\hat{\sigma}(\beta)$$

Relative velocity of tT

$$\rho = \frac{4m_t^2}{s} \qquad \beta = \sqrt{1-\rho}$$

- ✓ The partonic cross-section computed numerically in 80 points. Then fitted.
- ✓ Many contributing partonic channels:

Computed. Dominant at Tevatron (~85%)

$$q\bar{q} \to t\bar{t} \qquad gg \to t\bar{t} \qquad qg \to t\bar{t}q$$
$$q\bar{q} \to t\bar{t}g \qquad gg \to t\bar{t}g \qquad qg \to t\bar{t}qg$$
$$q\bar{q} \to t\bar{t}gg \qquad gg \to t\bar{t}gg \qquad qq' \to t\bar{t}qq' \,, \quad q \neq \bar{q}'$$
$$q\bar{q} \to t\bar{t}q'\bar{q}' \,, \quad q \neq q' \qquad gg \to t\bar{t}q\bar{q} \qquad q\bar{q} \to t\bar{t}q\bar{q}$$

All of the same complexity. No more conceptual challenges expected (just lots of CPU)

Results @ parton level: qqbar -> ttbar +X

Notable features:

- ✓ Small numerical errors
- ✓ Agrees with limits

Partonic cross-section through NNLO:

$$\sigma_{ij}\left(\beta,\frac{\mu^2}{m^2}\right)=\frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)}+\alpha_S\left[\sigma_{ij}^{(1)}+L\,\sigma_{ij}^{(1,1)}\right]+\alpha_S^2\left[\sigma_{ij}^{(2)}+L\,\sigma_{ij}^{(2,1)}+L^2\sigma_{ij}^{(2,2)}\right]+\mathcal{O}(\alpha_S^3)\right\},$$

The NNLO term:

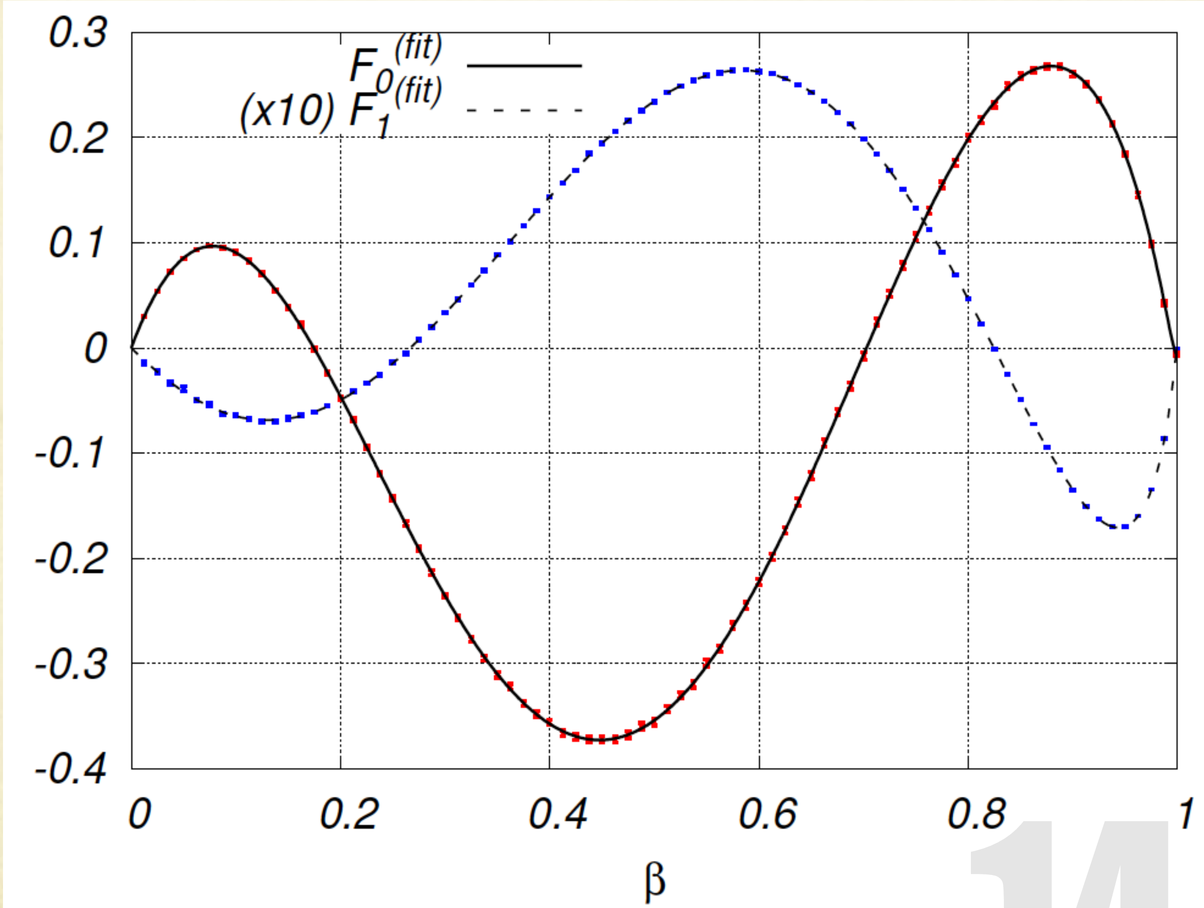
$$\sigma_{q\bar q}^{(2)}(\beta)=F_0(\beta)+F_1(\beta)N_L+F_2(\beta)N_L^2$$

Numeric (→ F_0, F_1) Analytic (→ F_2)

$$F_i\equiv F_i^{(\beta)}+F_i^{(\mathrm{fit})},\, i=0,1$$

The known threshold approximation (→ $F_i^{(\beta)}$)

Beneke, Czakon, Falgari, Mitov, Schwinn `09



P. Bärnreuther et al arXiv:1204.5201

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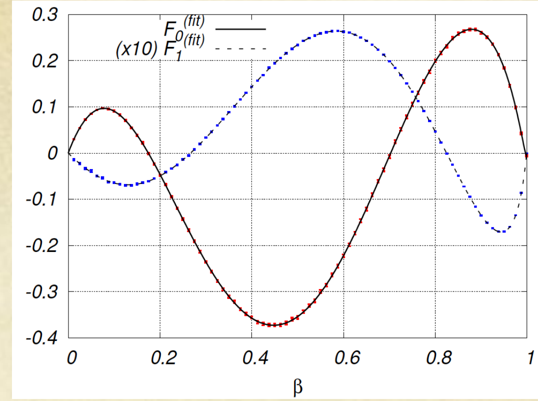
P. Bärnreuther et al arXiv:1204.5201

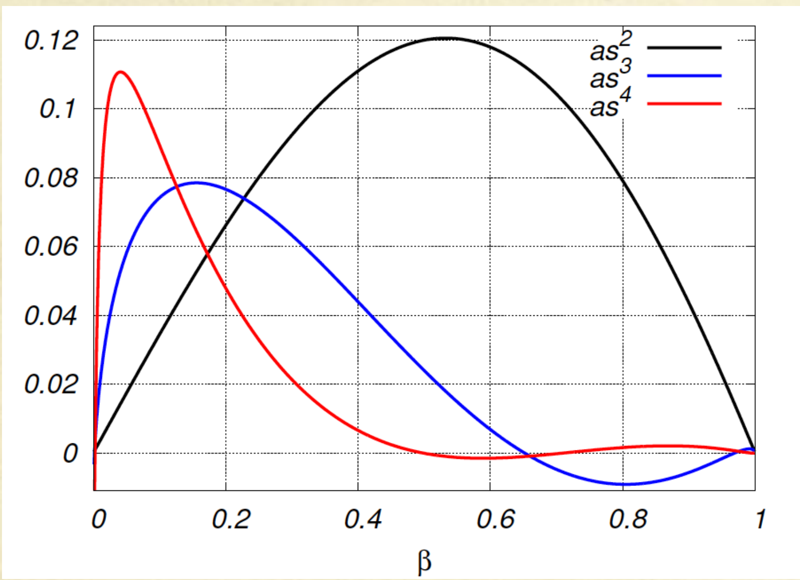
Results @ parton level: qqbar -> ttbar +X

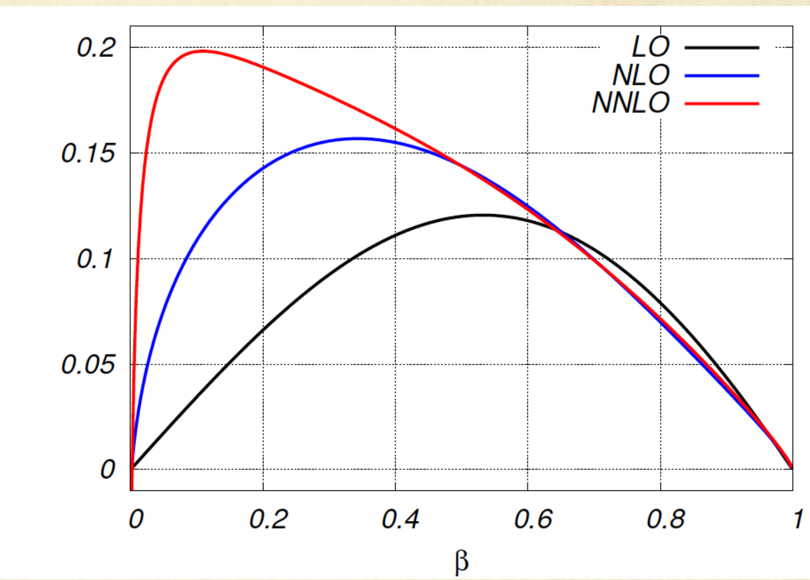
$$\sigma_{ij}\left(\beta, \frac{\mu^2}{m^2}\right) = \frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)} + \alpha_S\left[\sigma_{ij}^{(1)} + L\,\sigma_{ij}^{(1,1)}\right] + \alpha_S^2\left[\sigma_{ij}^{(2)} + L\,\sigma_{ij}^{(2,1)} + L^2\sigma_{ij}^{(2,2)}\right] + \mathcal{O}(\alpha_S^3)\right\},$$



An alternative view of the partonic cross-sections:

Bärnreuther, Czakon, Mitov `12

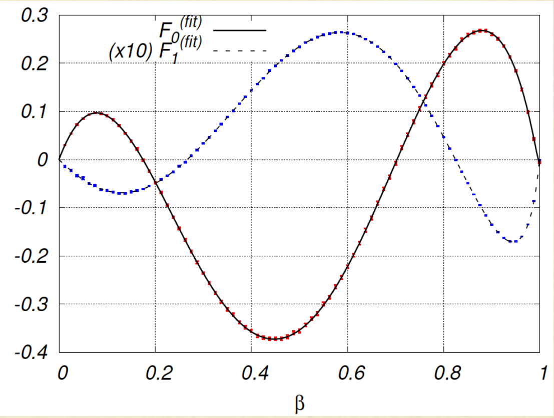




$$\hat{\sigma}(\beta) = \frac{\alpha_S^2}{m^2}\left(\sigma^{(0)} + \alpha_S\sigma^{(1)} + \alpha_S^2\sigma^{(2)} + \ldots\right) \equiv \frac{\alpha_S^2}{m^2}\left(f_{\alpha_S^2} + f_{\alpha_S^3} + f_{\alpha_S^4} + \ldots\right)$$

Results @ parton level:
The all-fermionic reactions

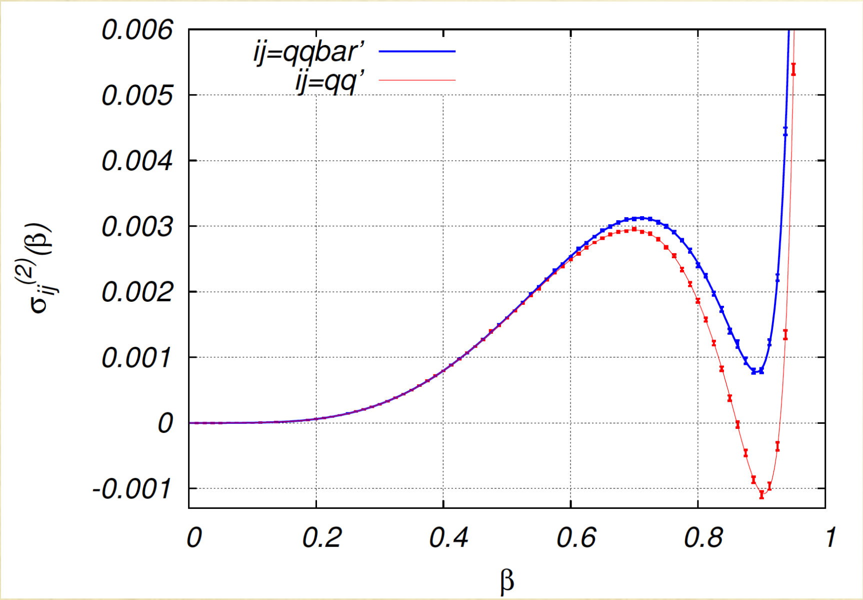
$$
\begin{aligned}
q\bar{q} &\rightarrow t\bar{t} + q\bar{q}\big|_{\rm NS}\,, \\
q\bar{q}' &\rightarrow t\bar{t} + q\bar{q}'\,, \\
qq' &\rightarrow t\bar{t} + qq'\,, \\
qq &\rightarrow t\bar{t} + qq\,.
\end{aligned}
$$

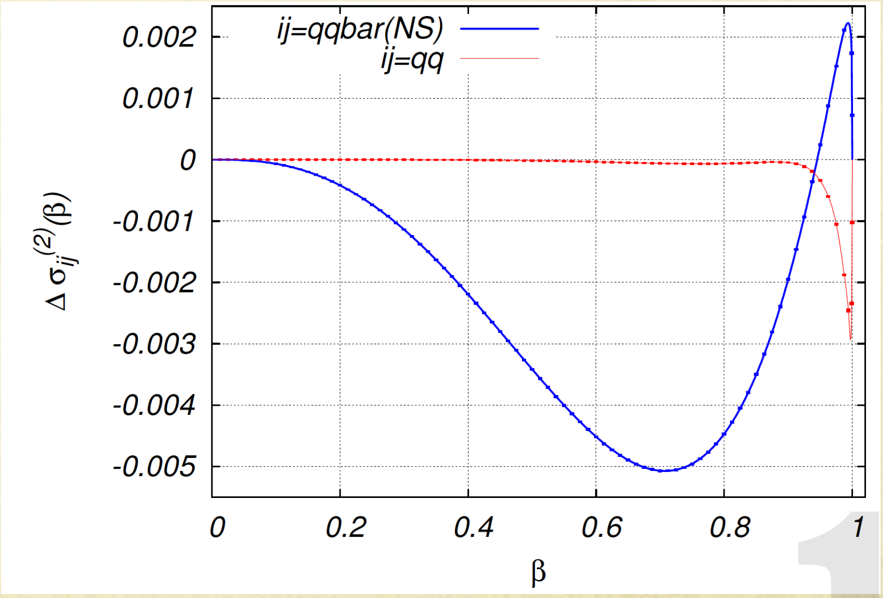


Czakon, Mitov '12

P. Bärnreuther et al arXiv:1204.5201

These partonic cross-sections are very small.
Compare to the ones involving gluons!



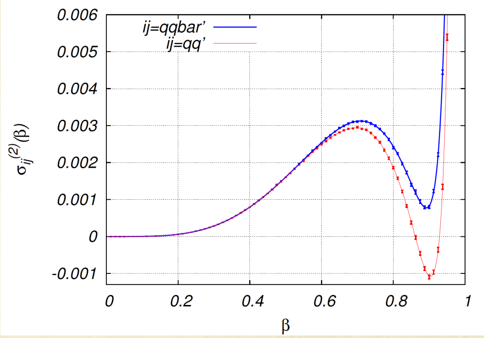


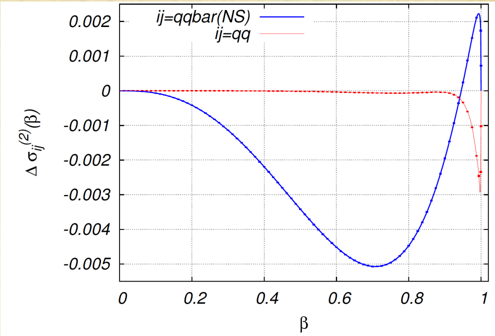
✧ Had to compute up to beta=0.9999 to get the high-energy behavior right.

Results @ parton level:

The all-fermionic reactions

$$q\bar{q} \to t\bar{t} + q\bar{q}|_{\rm NS}\,,\quad q\bar{q}' \to t\bar{t} + q\bar{q}'\,,\quad qq' \to t\bar{t} + qq'\,,\quad qq \to t\bar{t} + qq\,.$$





The interesting feature: high-energy logarithmic rise:

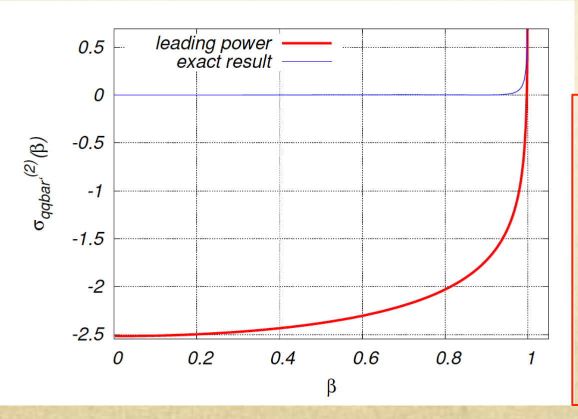
$$\sigma^{(2)}_{f_1 f_2 \to t\bar{t} f_1 f_2}\Big|_{\rho\to 0} \approx c_1 \ln(\rho) + c_0 + \mathcal{O}(\rho) \qquad \rho = \frac{4m_t^2}{s}$$

$$c_1 = -0.4768323995789214$$

Predicted in analytical form Ball, Ellis `01

$$c_0 \text{ (from Eqs. (6.3, 6.4))} = \begin{cases} -2.5173 & \text{from} \quad \sigma^{(2)}_{q\bar{q}'} \\ -2.5186 & \text{from} \quad \sigma^{(2)}_{qq'} \end{cases}$$

- Direct extraction from the fits. 5% uncertainty. Czakon, Mitov '12
- Agrees with independent prediction. 50% uncertainty. Moch, Uwer, Vogt '12



High-energy expansion non-convergent.

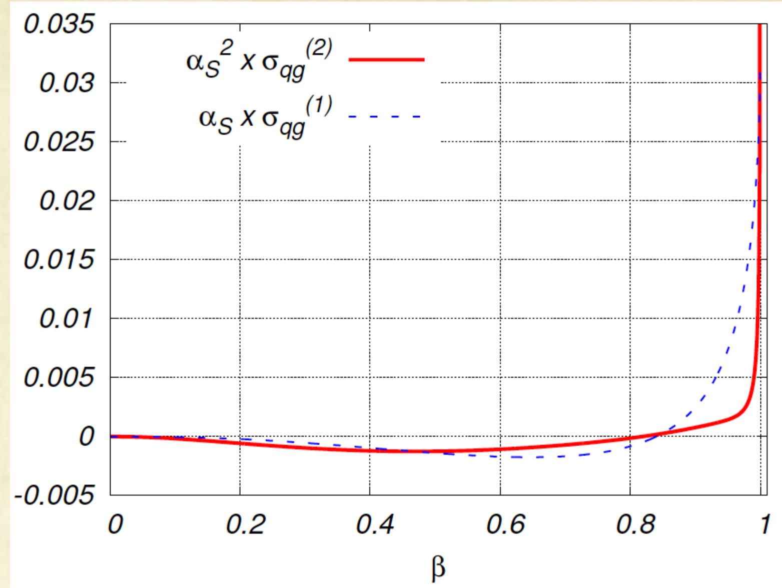
Applies only to the high-energy limit.

	Tevatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
$\Delta\sigma_{q\bar{q},(\rm NS)}$ [pb]	-0.0020	-0.0097	-0.0124	-0.0299
$\sigma_{q\bar{q},(\rm NS)}$ [pb]	-0.0009	-0.0001	0.0021	0.0464
$\sigma_{\rm all}$ [pb]	0.0003	0.0970	0.1504	0.7885
$\sigma_{\rm tot}$ [pb]	7.0056	154.779	220.761	852.177

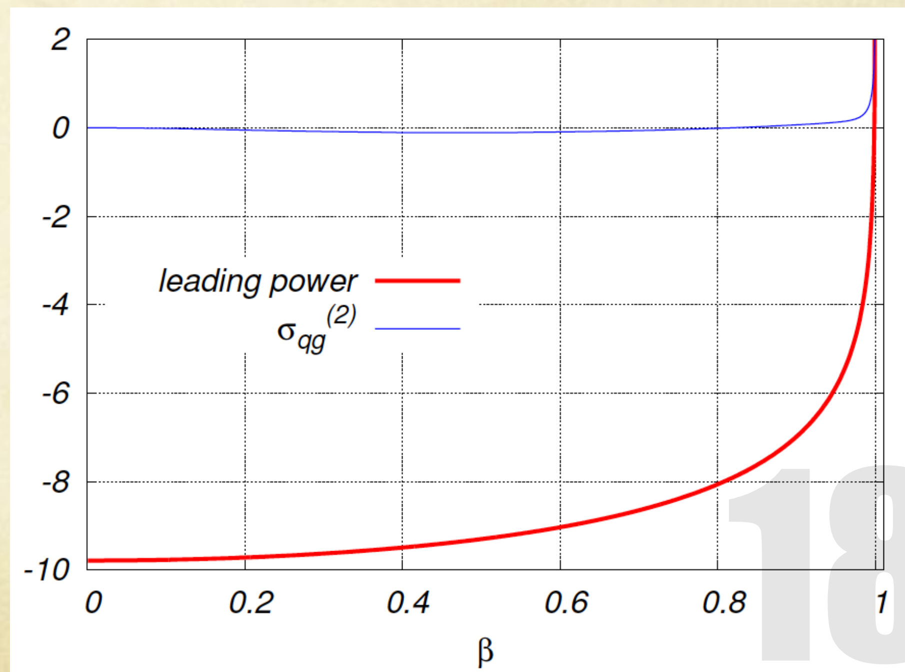
Czakon, Mitov '12

Results @ parton level:qg -> ttbar +X

Czakon, Mitov `12



- ✓ Correction about -1% (Tev and LHC).
- ✓ Notable decrease of scale dependence at LHC.
- ✓ NNLO large compared to NLO.



- ✓ High-energy log-limit correct

Ball, Ellis `01

- ✓ Agree for the constant with

Moch, Uwer, Vogt '12

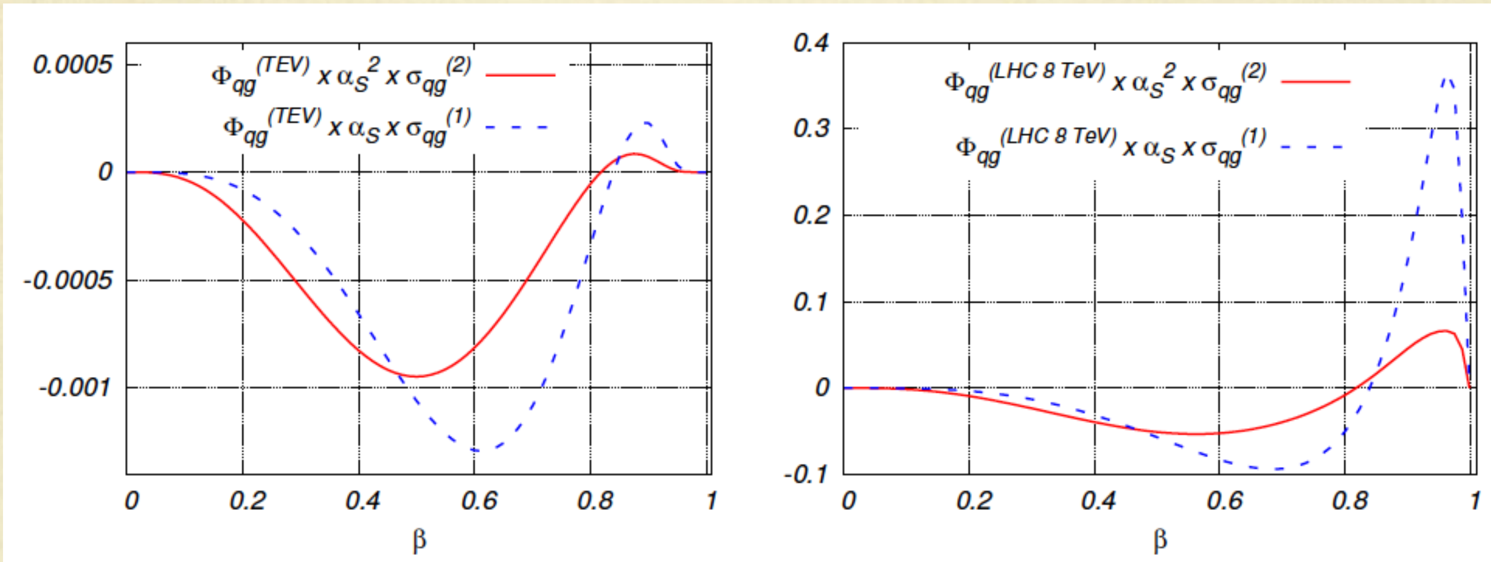
- ✓ The limit itself plays no Pheno role

Results @ parton level: qg -> ttbar +X

Czakon, Mitov `12

X-section times flux

Tevatron | LHC 8 TeV



		Tevatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
I_1	Due to $\sigma_{qg}^{(1)}$ [pb]	-0.068	-0.88	-0.48	9.01
I_2	Due to $\sigma_{qg}^{(2)}$ [pb]	-0.057	-1.82	-2.25	-4.07
I_3	$\sigma_{qg}^{(2)}$(Hathor; $(A+B)/2$) [pb]	0.040	5.78	8.11	27.36
I_4	$(I_3 - I_2)/\sigma_{tot}$ [%]	1.4	4.9	4.7	3.7