MiNLO-Merging

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Work done in collaboration with Keith Hamilton, Paolo Nason, Carlo Oleari
1206.3572 and 1212.4504

The first three years of the LHC Mainz, 18th-22nd March 2013

NLO+PS

- Next-to-leading order parton showers (NLO+PS) have been realized as practical tools (POWHEG,MC@NLO, Sherpa) and are being today routinely used for LHC analyses
- First only processes with no associated jets in the final state, e.g. Drell-Yan, diboson, tt, VBF Higgs, ...
- Now associated jet production also within reach, e.g. for Higgs production in POWHEG there is
 - inclusive Higgs production (H)
 - Higgs plus one jet (HJ)
 - Higgs plus two jets (HJJ)

[same for W and Z]

Campbell, Ellis, Frederix, Nason, Oleari, Williams 1202.5475

To generate these processes need a generation cut on the jets or a socalled Born-suppression factor (see later)

Why merging

- These generators overlap in phase-space, e.g.
 - H describes the first jet only at LO accuracy, and additional jets only at PS accuracy
 - HJ describes quantities inclusive in Higgs plus one jet at NLO accuracy, the second jet at LO accuracy, additional jets at PS accuracy
 - HJ can not be used inclusive jet cross-sections, since the NLO calculation diverges
 - HJJ describes two jet at NLO accuracy, but can not be used without the requirement of two jets

• ...

It is then natural to want to merge these calculations, so that NLO accuracy is guaranteed for all classes of observables in end results

Standard merging

Standard strategy to the merging problem

- 1. separate the output of each simulation according to the jetmultiplicity (⇒ jet-measure merging scale), discarding events for which each generator does not possess NLO accuracy
- 2. finally join events in the inclusive sample
- ⇒ In essence, each generator contributes a single exclusive jet-bin to the final inclusive sample

Lavesson, Lonnblad 0811.2912; Alioli, Hamilton, Re 1108.0909; Hoeche, Krauss, Schonherr, Siegert 1207.5030; Gehrmann, Hoeche, Krauss, Schonherr, Siegert 1207.5031; Frederix, Frixione 1209.6215; Alioli, Bauer et al. 1211.7049; Platzer 1211.5467; Lonnblad, Prestel 1211.7278

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Relative contributions in the inclusive sample determined by the (unphysical) merging scale, but an optimal choice is difficult

- a too large scale forces one to describe relatively hard jets using the H generator, benefits of the HJ and HJJ calculations are lost
- @ with a too small merging scale NLO calculations unreliable

Merging scale issues

Example:

- take NLO+PS accurate calculation for inclusive Higgs (with NLL Sudakov form factors)
- consider now the cross-section integrated up to some (small) pt,cut
- missing NNLL terms in the Sudakov are $\alpha_s^2 L$ with L=log(p_{t,cut}/M_H)
- in the peak region $\alpha_s L^2 \sim 1$, therefore $\alpha_s^2 L \sim \alpha_s^{3/2}$
- NLO quality is lost, since this requires missing terms to be $O(\alpha_s^2)$

Merging scale issues

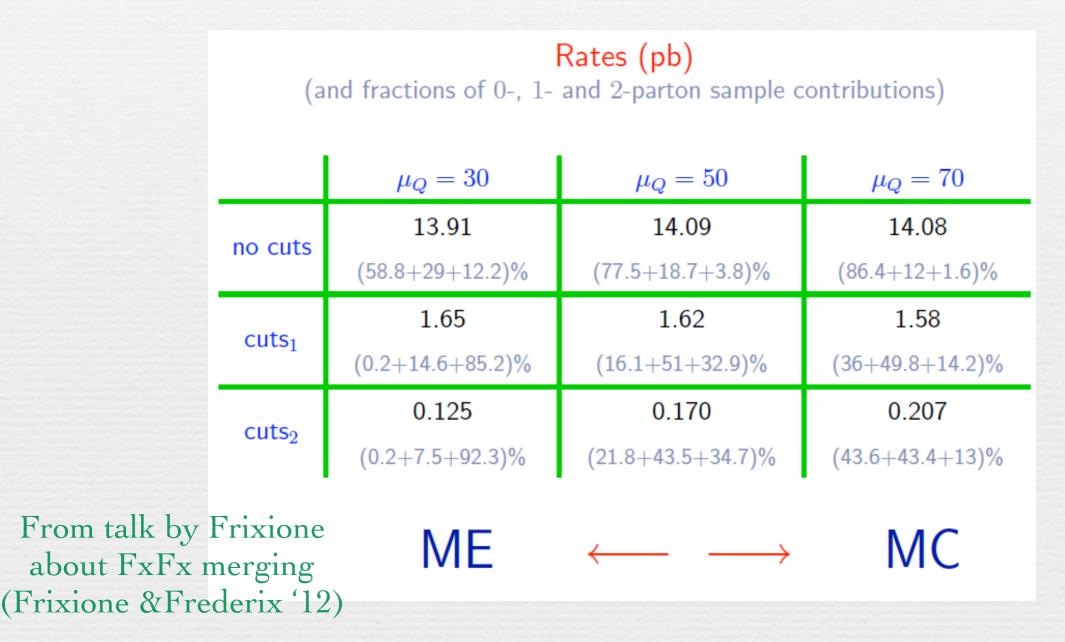
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Moral: even if the inclusive cross-section is NLO accurate, cut cross-section can have a reduced accuracy if the cut is too low

As a consequence, some approaches conservatively avoid low merging scales, others avoid claiming that NLO accuracy is preserved after merging, others include NNLL terms in the Sudakovs to preserve NLO accuracy, yet others add subtraction terms to restore NLO accuracy ...

Example from FxFx



(In?) dependence of merging scale studied a posteriori. Can lead to observable/cut dependent conclusions

MiNLO-merging

We instead want to reformulate the NLO X+(n+1) jet calculation, such that on integration NLO accurate results for X+n jets are recovered, i.e. we achieve the goal of merging without doing any merging at all (so there is simply NO merging scale)

In essence, the idea behind the MiNLO-merging is to suitable upgrade the high multiplicity calculations (via MiNLO), so that when jets are iteratively integrated out the exact NLO calculation with lower jetmultiplicity is recovered

NB: this is not possible in a standard NLO calculation (without MiNLO) since the integration simply diverges

MiNLO

Multiscale improved NLO

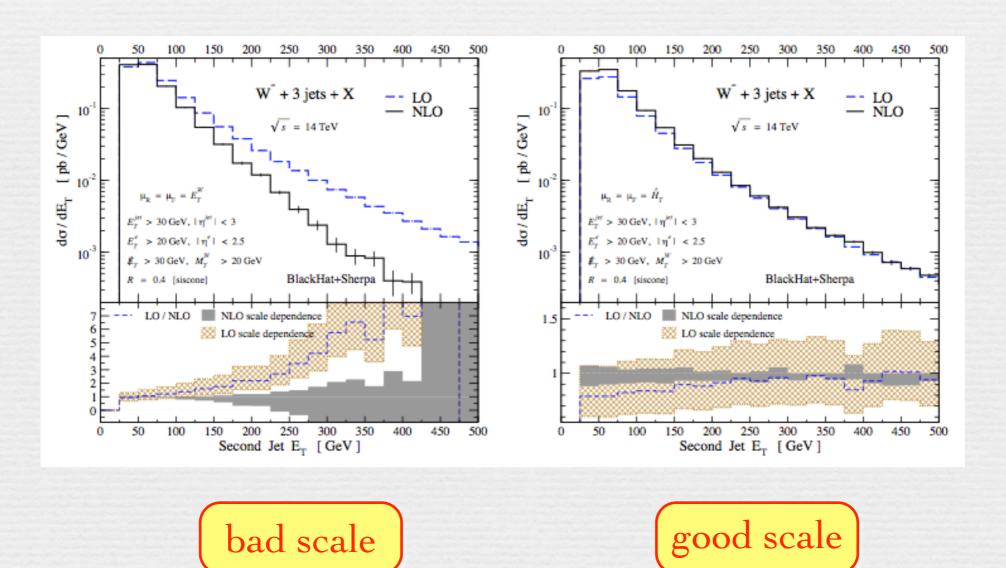
The observation triggering the first idea behind MiNLO was in a paper with K. Melnikov [0910.3671]

• the impact of NLO calculations is often discussed using the same scale choice at LO and NLO, however more advanced LO calculations exist that rely on the CKKW procedure for scale setting (see later) and inclusion of Sudakov effects

Even at NLO the scale choice is an issue and different choices can lead to a different picture/contrasting conclusions, so it seemed natural to look for an extension of the CKKW method to NLO

Scale choice at NLO

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Reason: bad scale is large logs large NLO, large scale dependence

But we also know that large NLO bad scale choice, since NLO corrections can have a "genuine" physical origin (new channels opening up, Sudakov logarithms, color factors, large gluon flux ...)

Furthermore, double logarithmic corrections can never be absorbed by a choice of scale (single log). So a "stability criterion" can be misleading.

Scale choice at LO

LO calculations in matrix elements generators that follow the CKKW procedure are quite sophisticated in the scale choice:

they use optimized/local scales at each vertex and Sudakov form factors at internal/external lines

Catani, Krauss, Kuehn, Webber '01 extension to hh collisions Krauss '02

Reminder:

a Sudakov form factor encodes the probability of evolving from one scale to the next without branching above a resolution scale Q_0

Recap of CKKW

The CKKW prescription in brief:

- $\label{eq:section}$ use the k_t algorithm to reconstruct the most likely branching history
- $\stackrel{>}{\triangleright}$ evaluate each α_s at the local transverse momentum of the splitting
- for each internal line between nodes at scale Q_i and Q_j include a Sudakov form factor $\Delta_{ij}=D(Q_0,Q_i)/D(Q_0,Q_j)$ that encodes the probability of evolving from scale Q_i to scale Q_j without emitting. For external lines include the Sudakov factor $\Delta_{i}=D(Q_0,Q_i)$
- match to a parton shower to include radiation below Q0

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- match to a parton shower to include radiation below Q0

Scale choice intertwined with inclusion of Sudakov form factors

MiNLO

Born as an extension to NLO of the CKKW procedure, such that the procedure to fix the scales is unbiased and decided *a priori*

In particular, the focus is on processes involving many scales (e.g. X+multi-jet production) and on soft/collinear branchings, i.e. on the region where it is more likely that associated jets are produced

[MiNLO has nothing to say about processes like tt etc.]

Two observations

1. A generic NLO cross-section has the form

$$\alpha_{\rm S}^{n}(\mu_{R}) B + \alpha_{\rm S}^{n+1}(\mu_{R}) \left(V(Q) + nb_{0} \log \frac{\mu_{R}^{2}}{Q^{2}} B(Q) \right) + \alpha_{\rm S}^{n+1}(\mu_{R}) R$$

Adopting CKKW scales at LO, this becomes naturally

$$\alpha_s(\mu_1) \dots \alpha_s(\mu_n) B + \alpha_s^{n+1}(\mu_R') \left(V(Q) + b_0 \log \frac{\mu_1^2 \dots \mu_n^2}{Q^{2n}} B \right) + \alpha_s^{n+1}(\mu_R'') R$$

and the scale choices μ_R and μ_R are irrelevant for the scale cancelation

2. Sudakov corrections included at LO via the CKKW procedure lead to NLO corrections that need to be subtracted to preserve NLO accuracy

The original MiNLO

- 1. Find the CKKW n clustering scales $Q_1 < ... < Q_n$. Fix the hard scale of the process Q to the system invariant mass after clustering. Set Q_0 to Q_1 (inclusive on radiation below Q_1)
- 2. Evaluate the n coupling constants at the scales Q_i (times a factor to probe scale variation)
- 3. Set μ_R in the virtual to the geometric average of these scales and μ_F to the softest scale Q_1
- 4. Include Sudakov form factors for Born and virtual terms, and for the real term after the first branching
- 5. Subtract the NLO bit present in the CKKW Sudakov of the Born
- 6. The (n+1)th power of α_s in the real and virtual is evaluated at the arithmetic average of the n α_s in the Born term (since corrections can be thought of as additive at each vertex, but other choices possible)

MiNLO in one equation

Example: take e.g. HJ

In POWHEG it is customary to discuss the \overline{B} function, which for HJ is defined as

$$\bar{B} = \alpha_s^2 (\mu_R) \left[B + V(\mu_R) + \int d\Phi_{\rm rad} R \right]$$

With MiNLO this function becomes

$$\Delta(Q_{0},Q) \qquad \Delta(Q_{0},Q) \qquad \Delta(Q_{0},Q_{0}) = 1$$

$$Q = M_{H} \qquad Q_{0} = q_{T}$$

$$\Delta(Q_{0},Q_{0}) \neq 1$$

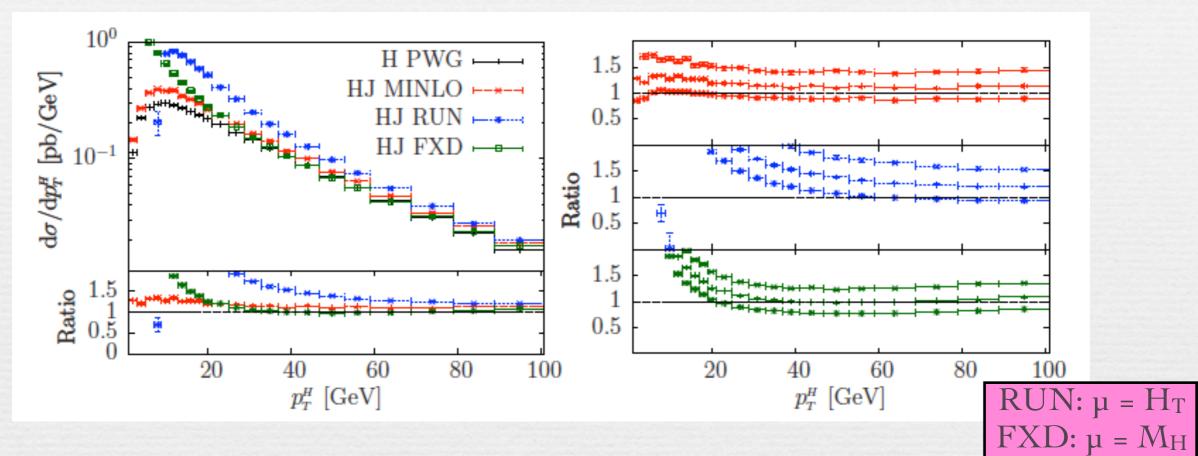
$$\bar{B} = \alpha_s^2 \left(M_H^2 \right) \alpha_s \left(q_T^2 \right) \Delta_g^2 \left(M_H, q_T \right) \left[B \left(1 - 2\Delta_g^{(1)} \left(M_H, q_T \right) \right) + V(\mu') + \int d\Phi_{\text{rad}} R \right]$$

Properties of MiNLO

MiNLO satisfies the following requirements

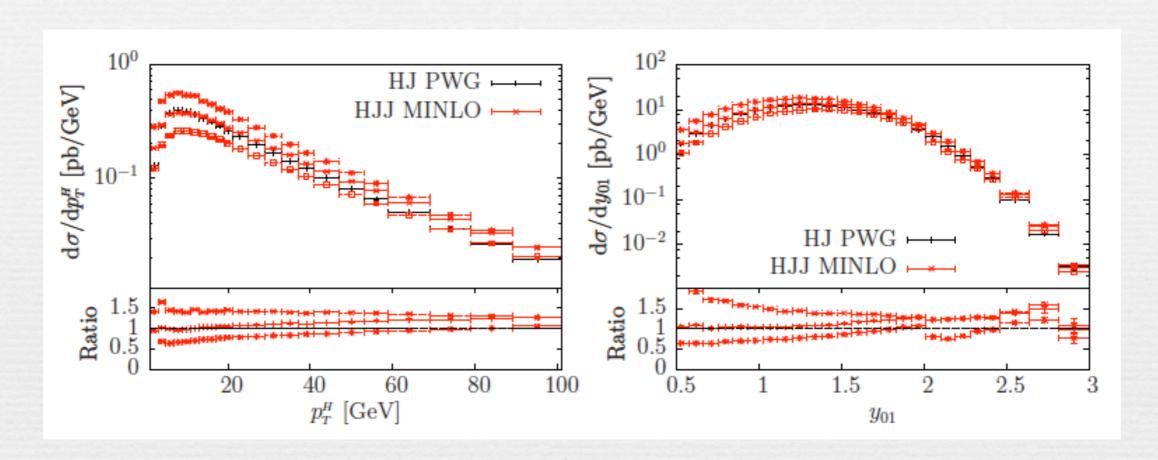
- the result is accurate at NLO, i.e. the scale dependence is NNLO
- the accuracy in the Sudakov region depends on the observable and the form of the Sudakov used
- the smooth behaviour of the CKKW scheme in the singular regions is preserved
 - X+n-jet cross-sections are finite even without jet cuts (do not need generation cuts or Born suppression factors)
 - X+n-jet cross-sections reproduce the inclusive cross-section accurate to LO (and better, see later)
- the procedure is simple to implement in any NLO calculation, i.e. the improvement requires only a very modest amount of work

First MiNLO results



- MiNLO mimics POWHEG all the way down to very small pt,H where standard H+1jet NLO calculations diverge
- MiNLO uncertainty band compatible with POWHEG all the way down to low transverse momenta
- MiNLO more compatible with fixed rather than running scales (surprising? No, running scale misses Sudakov)

H+2jets



- without cuts impossible to compare to standard NLO
- again, MiNLO uncertainty band compatible with POWHEG all the way down to low transverse momenta

If NLO+PS calculations upgraded with MiNLO (without any merging) describe inclusive distributions so well the natural questions become ...

MiNLO & merging

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 - χ in the original MiNLO formulation terms neglected are $O(\alpha_s^{3/2})$, so almost NLO, but not quite ...

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 - **X** in the original MiNLO formulation terms neglected are $O(\alpha_s^{3/2})$, so almost NLO, but not quite ...
- Can one modify the MiNLO procedure to guarantee NLO accuracy for also inclusive quantities?
 - ✓ yes, our explicit study of the case of H/V+jet shows that this is possible there. This requires some changes that were part of the freedom in the formulation of MiNLO

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- Can one modify the MiNLO procedure to guarantee NLO accuracy for also inclusive quantities?
 - ✓ yes, our explicit study of the case of H/V+jet shows that this is possible there. This requires some changes that were part of the freedom in the formulation of MiNLO
- Can one also solve the general case? The facts that
 - the simplest MiNLO already works well (see also later ...)
 - the HJ/VJ case could be solved in a relatively simple way make us confident that this is possible

Here I'll only sketch the idea (two versions of full proof in 1212.4504)
Consider for simplicity the explicit case of H and H+1jet

The HJ-MiNLO formula reads

$$\bar{B} = \alpha_s^2 \left(M_H^2 \right) \alpha_s \left(q_T^2 \right) \Delta_g^2 \left(M_H, q_T \right) \left[B \left(1 - 2\Delta_g^{(1)} \left(M_H, q_T \right) \right) + V + \int d\Phi_{\text{rad}} R \right]$$

with

$$\Delta_g\left(Q, q_T\right) = \exp\left\{-\int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \left[A\left(\alpha_s\left(q^2\right)\right) \log \frac{Q^2}{q^2} + B\left(\alpha_s\left(q^2\right)\right)\right]\right\}$$

$$\Delta_g(Q, q_T) = 1 + \Delta_g^{(1)}(Q, q_T) + \mathcal{O}(\alpha_s^2) \qquad \Delta_g^{(1)}(Q, q_T) = \alpha_s \left[-\frac{1}{2} A_1 \log^2 \frac{q_T^2}{Q^2} + B_1 \log \frac{q_T^2}{Q^2} \right]$$

The idea is to compare this with the NNLL resummation (including finite parts to achieve NLO accuracy for Higgs production, i.e. NLO⁽⁰⁾) and just see what is missing in the MiNLO formula

NNLL_Σ Higgs q_T resummation at fixed rapidity can be written as

$$\frac{d\sigma}{dydq_T^2} = \sigma_0 \frac{d}{dq_T^2} \left\{ \left[C_{ga} \otimes f_{a/A} \right] (x_A, q_T) \times \left[C_{gb} \otimes f_{b/B} \right] (x_B, q_T) \times \exp \mathcal{S} \left(Q, q_T \right) \mathcal{F} \right\} + R_f$$

Integrating in qT one gets

$$\frac{d\sigma}{dy} = \sigma_0 \left[C_{ga} \otimes f_{a/A} \right] (x_A, Q) \times \left[C_{gb} \otimes f_{b/B} \right] (x_B, Q) + \int dq_T^2 R_f + \dots$$

i.e. the formula is $NLO^{(0)}$ accurate if $O(\alpha_s)$ corrections to the coefficient functions are included and R_f is $LO^{(1)}$ accurate

Now, need to show that if the derivative is taken explicitly, and some higher orders are neglected, NLO⁽⁰⁾ accuracy is maintained.

$$\sigma_0 \frac{1}{q_T^2} \left[\alpha_s, \, \alpha_s^2, \, \alpha_s^3, \, \alpha_s^4, \, \alpha_s L, \, \alpha_s^2 L, \, \alpha_s^3 L, \, \alpha_s^4 L \right] \exp \mathcal{S} \left(Q, q_T \right)$$

$$\sigma_0 \frac{1}{q_T^2} \left[\alpha_s \right] \alpha_s^2, \ \alpha_s^3, \ \alpha_s^4, \ \alpha_s L, \ \alpha_s^2 L, \ \alpha_s^3 L, \ \alpha_s^4 L \right] \exp \mathcal{S} \left(Q, q_T \right)$$
 B_1

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$$B_1 B_2$$

$$\sigma_0 \frac{1}{q_T^2} \left[\alpha_s \right] \alpha_s^2 \alpha_s^3, \ \alpha_s^4, \alpha_s L \alpha_s^2 L, \ \alpha_s^3 L, \ \alpha_s^4 L \exp \mathcal{S} (Q, q_T)$$

$$B_1 \ B_2 \ \cdots \ A_1$$

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$$B_1 \ B_2 \ \cdots \ A_1 \cdots \ C_1 \otimes C_1 \otimes A_1$$

$$\sigma_0 \frac{1}{q_T^2} \left[\alpha_s \right] \alpha_s^2, \alpha_s^3, \alpha_s^4, \alpha_s L \alpha_s^2 L, \alpha_s^3 L, \alpha_s^4 L \exp \mathcal{S} (Q, q_T)$$

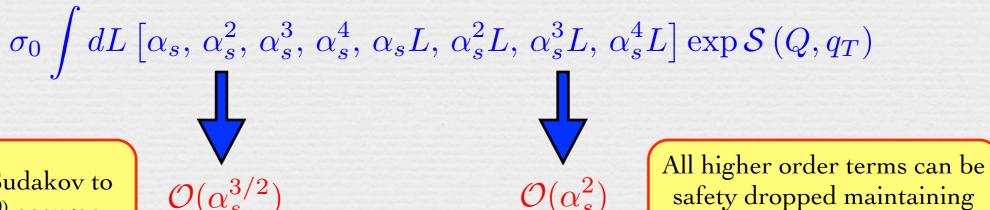
$$B_1 B_2 \cdots A_1 \cdots C_1 \otimes C_1 \otimes A_1 \cdots$$

Taking the derivative one gets

$$\sigma_0 \frac{1}{q_T^2} \left(\alpha_s \right) \left(\alpha_s^2 \right) \alpha_s^3, \ \alpha_s^4, \left(\alpha_s L \right) \alpha_s^2 L, \left(\alpha_s^3 L \right) \alpha_s^4 L \right] \exp \mathcal{S} \left(Q, q_T \right)$$

$$B_1 B_2 \cdots A_1 \cdots C_1 \otimes C_1 \otimes A_1 \cdots$$

After integration with the Sudakov weight, the counting is set by $L \sim dL \sim 1/\sqrt{\alpha_s}$. So these terms contribute, e.g.



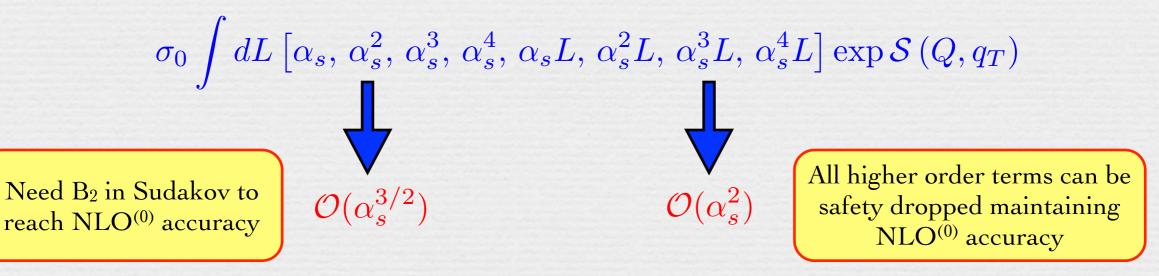
NLO⁽⁰⁾ accuracy

Need B₂ in Sudakov to reach NLO⁽⁰⁾ accuracy

The proof

Taking the derivative one gets

After integration with the Sudakov weight, the counting is set by $L \sim dL \sim 1/\sqrt{\alpha_s}$. So these terms contribute, e.g.



Similarly, the scale in V and R gives the largest difference in the $\alpha_s^2 L$ term, where they give an $\alpha_s^3 L^2$ variation. This contributes $O(\alpha_s^{3/2})$. So an effect of the same size to B_2 .

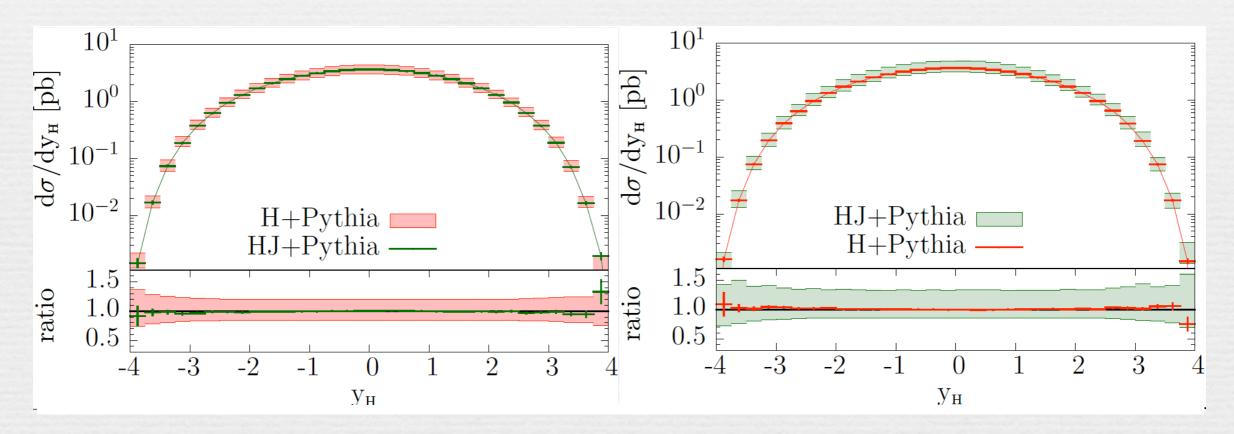
Q. e. d.

- The original MiNLO prescription is less than NLO accurate in the description of inclusive quantities, in that it neglects $O(\alpha_s^{3/2})$ terms
- One way to achieve NLO accuracy from HJ also for inclusive Higgs observables is to
 - ✓ include the B₂ term in the Sudakov form factors
 - ✓ take the scale in the coupling constant in the real, virtual and subtraction terms equal to the Higgs transverse momentum

Provided this is done, merging of e.g. **H** and **HJ** is effectively achieved without doing any merging!

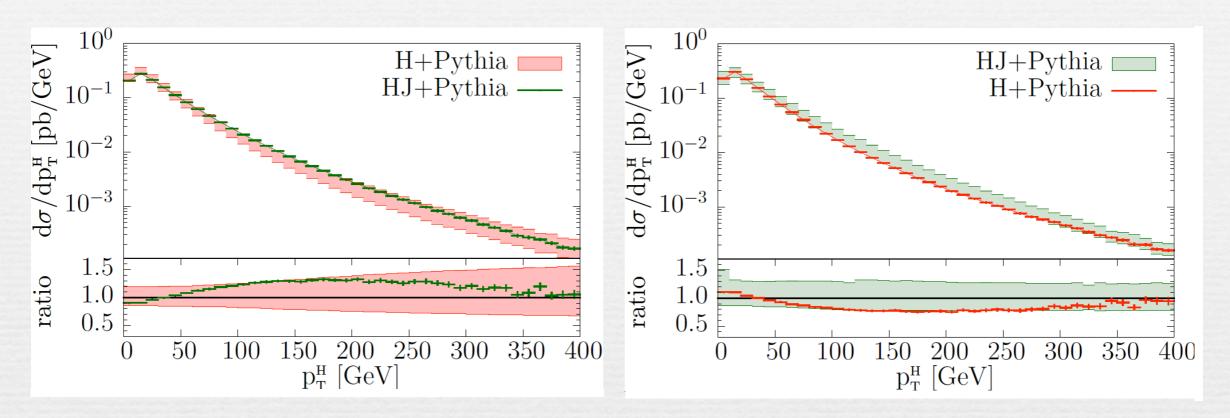
Phenomenology

Higgs (M_H=125 GeV) rapidity of the LHC (8 TeV). Use MSTW8NLO, bands are "7-scale" variation, take hfact = 100 in H



Excellent agreement in both in central value and in size of uncertainty bands (less so in W/Z)

Higgs pt



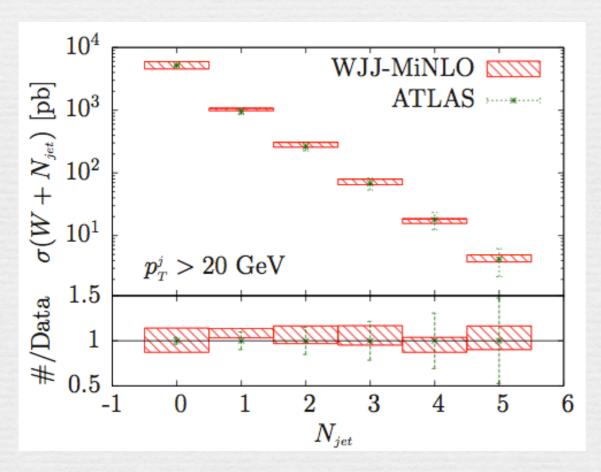
- overall good agreement over the whole region
- p_{t,H} described only at LO accuracy at high p_{t,H} in the H generator,
 (evident from the uncertainty band getting larger)

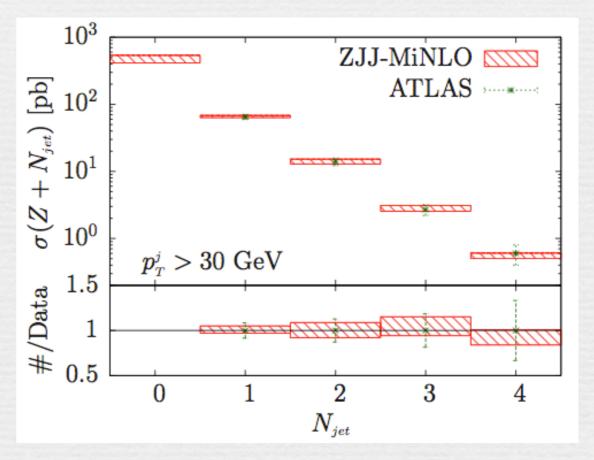
We looked at many more distributions, see 1212.4504 for more. Rather than commenting many distributions, before concluding, I'd like to discuss briefly three more points ...

1 MiNLO-VJJ vs data

Before concluding, more propaganda for MiNLO. We recently implemented Wjj/Zjj in POWHEG, and compared the WJJ/ZJJ-MiNLO generators against ATLAS data from 0 to 5 jets.

Campbell, Ellis, Nason, Zanderighi 1303.xxxx Wjj also in Frederix et al. 1110.5502; Zjj in Re 1204.5433

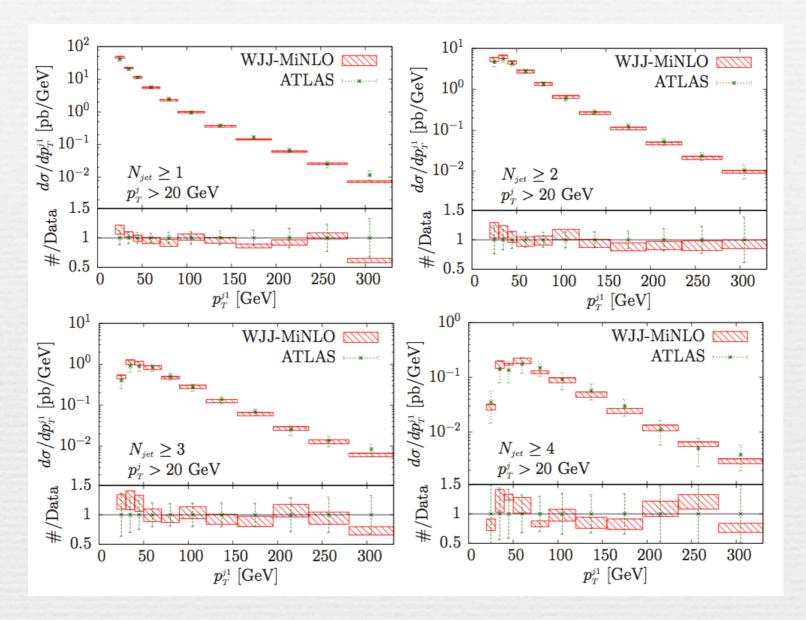




Results out of the box. Nothing has been tuned here.

Agreement is not bad ...

1 MiNLO-VJJ vs data



We looked at all ATLAS distributions in 1201.1276 (Wjj) and 1111.2690 (Zjj) and always found a similar good agreement. These results are very encouraging in terms of extending the merging to more complex processes.

Reminder: since the Born cross-section of W/Z with associated jets is divergent, one needs a generation cut or a Born suppression factor (i.e. a reweighing factor F that vanishes in divergent regions, so that events have weight 1/F)

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Generation cuts work if low-p_t events that are "cut out" do not contribute to physical distributions, as is usually the case if the cut is low enough.

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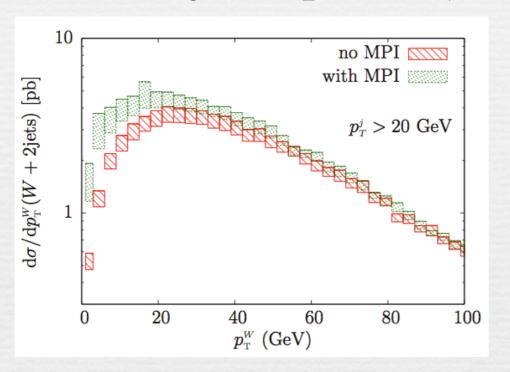
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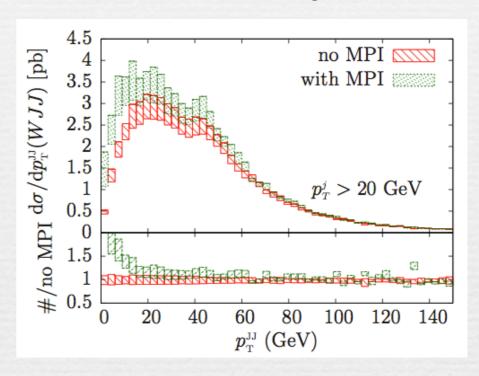
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Mechanism to promote very low pt Born events into high pt events:

- real (ISR or FSR) radiation [⇒ technical improvements in POWHEG]
- parton shower (when the shower emission veto scale is too high)
- hadronization (as expected, does not really happen)
- multi-parton interactions (MPI)

Vjj processes are particularly sensitive to MPI: single boson production at very how pt and dijet event from secondary interaction





- with a generation cut this physical effect is completely missed (checking independence from value of generation cut not enough)
- only when the inclusive cross-section if predicted accurately (e.g. with MiNLO), the estimate of MPI contribution is sensible

NB: MPI made it impossible to us to generate final distributions using H_T-based scale (MiNLO suppresses events with large weights)

3POWHEG@NNLO

How can one use MiNLO to upgrade POWHEG to NNLO?

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For simplicity consider the case of Higgs production

$$\left(\frac{d\sigma}{dy}\right)_{NNLO}$$
 inclusive Higgs rapidity computed at NNLO

$$\left(\frac{d\sigma}{dy}\right)_{\text{HI-MINLO}}$$
 inclusive Higgs rapidity from HJ-MINLO generator

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 inclusive Higgs rapidity computed at NNLO

$$\left(\frac{d\sigma}{dy}\right)_{\text{HJ-MiNLO}}$$
 inclusive Higgs rapidity from **HJ-MINLO** generator

Since HJ-MINLO is NLO accurate, it follows that

$$\frac{\left(\frac{d\sigma}{dy}\right)_{\text{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\text{HJ-MiNLO}}} = \frac{c_2\alpha_s^2 + c_3\alpha_s^3 + c_4\alpha_s^4}{c_2\alpha_s^2 + c_3\alpha_s^3 + d_4\alpha_s^4} \approx 1 + \frac{c_4 - d_4}{c_2}\alpha_s^2 + \mathcal{O}(\alpha_s^3)$$

Thus, reweighing HJ-MINLO results with this factor one obtains NNLO+PS accuracy, exactly in the same way as MC@NLO or POWHEG are NLO+PS accurate

Conclusions

MiNLO born as a simple procedure to assign scales and Sudakov form factors in NLO calculations to account for distinct kinematical scales.

Key features

- results well-behaved in Sudakov region, where standard NLO calculations break down
- away from the Sudakov regions, results are accurate at NLO
- procedure simple to implement in NLO calculations, just try it out ...
- HJ, WJ, ZJ NLO calculations upgraded with (new) MiNLO reproduce NLO results also for inclusive distributions, i.e. merging achieved without doing merging.
- VJJ-MiNLO agree well with data (from 0 to 5jets) without merging
- MiNLO provides a path to upgrade POWHEG to NNLO

Conclusions

MiNLO still very new. Lots of things to learn/do still.

Next:

- extend merging to more complex processes without merging scale
- phenomenological studies with POWHEG at NNLO using MiNLO
- improving logarithmic accuracy of MiNLO in Sudakov regions

A useful integral

$$I(m,n) \equiv \int_{\Lambda^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2} \left(\log \frac{Q^2}{q^2} \right)^m \alpha_s^n \left(q^2 \right) \exp \left\{ - \int_{q^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2} A \, \alpha_s(\mu^2) \log \frac{Q^2}{\mu^2} \right\}$$
$$\approx \left[\alpha_s(Q^2) \right]^{n - \frac{m+1}{2}}$$

i.e. each log "counts" as a square-root of $1/\alpha_s$ after integration over a transverse momentum when a Sudakov weight is present