Perturbative computations for the LHC

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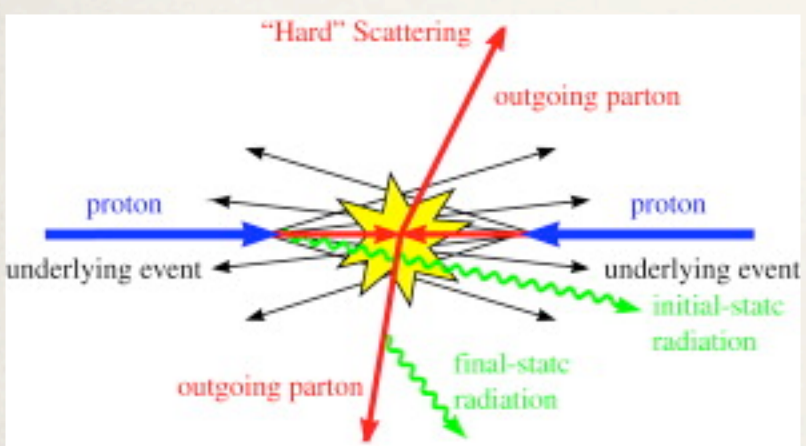
Perturbative calculations

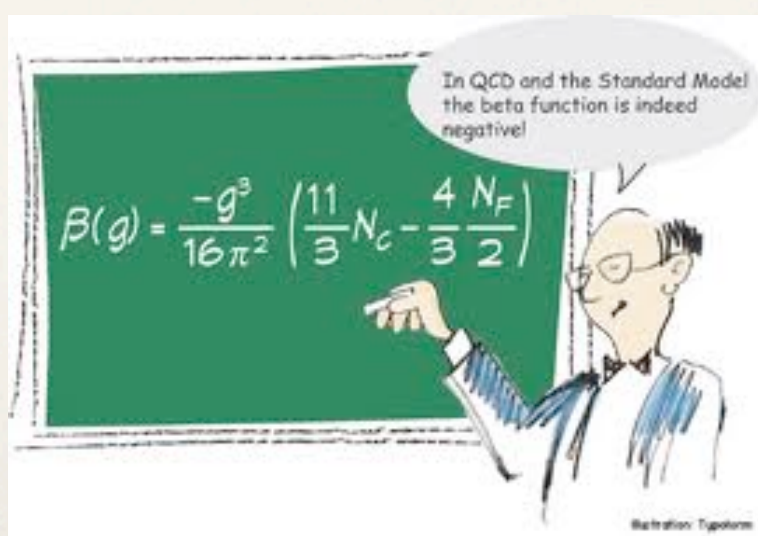
- We will talk about serious issues in hadron collder physics that have some bearing on questions like
  - What is the true SM value of the top forward-backward asymmetry?
  - How well can we predict the shape of dijet invariant mass distribution in W+jj ?
  - What is the true uncertainty in the Higgs boson production rate in gluon fusion?
  - What is the value of the strong coupling constant at the Z-mass and what is the error on it?
- Perturbative computations provide the only known technique to address dynamical questions related to hard processes at colliders in a model-independent and sharp way
- In this talk I will try to give an overview of the field of perturbative computations and provide some perspective on future direction

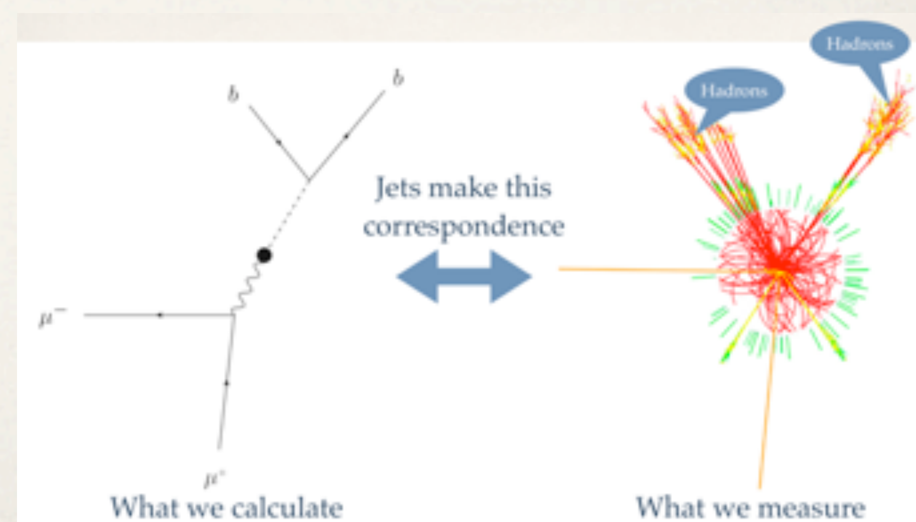
Physics of parton collider

- We must take QCD Lagrangian very seriously because we deal with a quark-gluon collider !
    - We are interested in rare processes with large momentum transfer where asymptotic freedom is at work
    - Since confinement is soft, jets provide reliable probe of short-distance physics accessible at large distances
    - Factorization theorems imply that protons are beams of quarks a gluon

$$\mathcal{L}_{\text{QCD}} = \bar{q}(i\gamma^\mu \partial_\mu - g_s \gamma^\mu A_\mu) q - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$

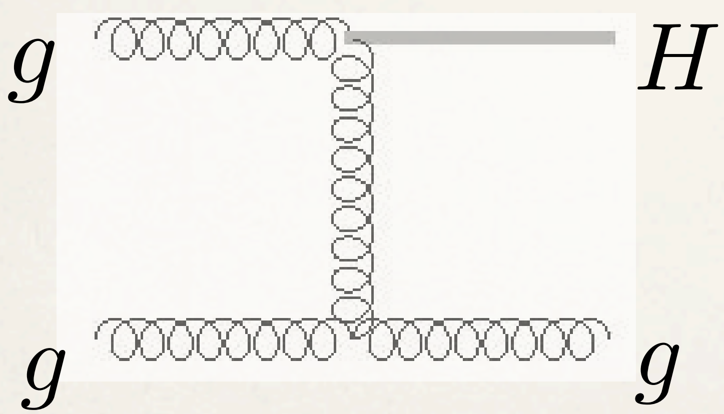






Perturbation theory for quark-gluon S-matrix

- Collisions of quarks and gluons are described by ordinary perturbation theory where the small parameter is the strong coupling constant
- Perturbative expansion consists of elastic (loops) and inelastic (real emission) processes; both must be combined for a consistent calculation
- For multi-jet processes, we can currently only deal with two-loop diagrams for 2 -> 2 proceses; this provides a limit on what a fixed-order computation can accomplish. Note that this also limits available number of real emissions, which makes our experimental colleagues somewhat unhappy.



Leading order contributions

Next-to-leading order contributions: virtual(left) and real(right)

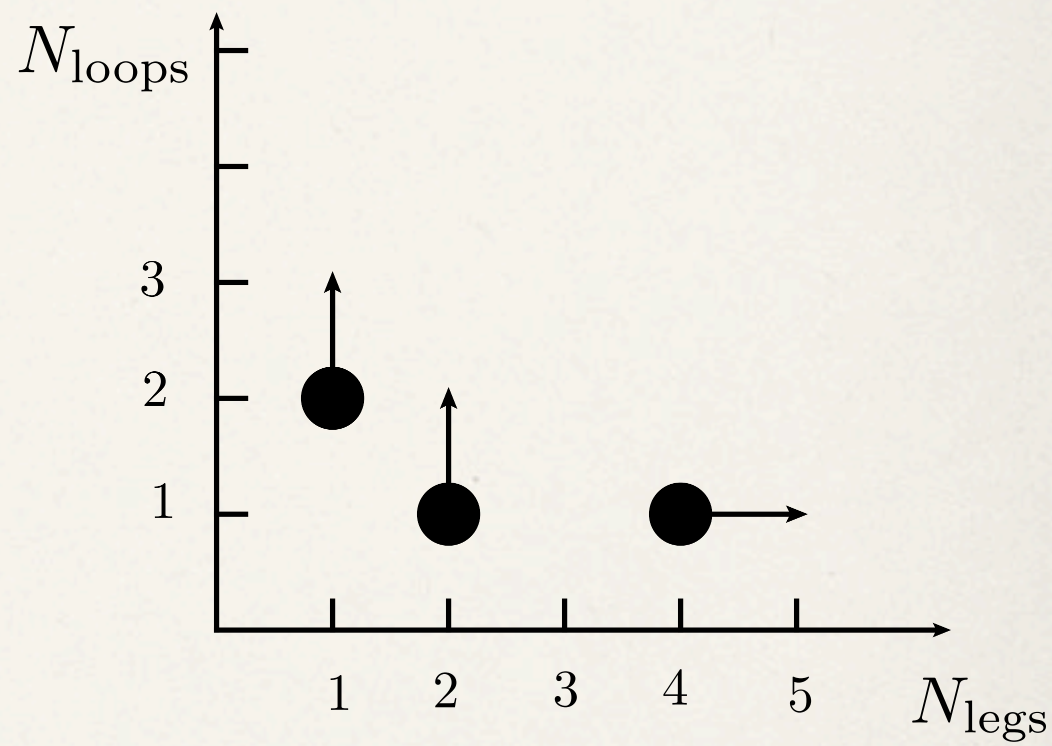
Kinematic approximations

- Additional kinematic approximations within perturbation theory combined with the understanding that soft and collinear singularities must cancel in suitably defined observables make an all-order treatment possible and give us some idea about the importance of multiple emissions, both real and virtual

- This is the main idea behind parton showers and resummations

- Unfortunately, the domain of applicability of these approximations is rather narrow and in recent years these techniques were combined with fixed order computations in various ways (CKKW, MLM, MC@NLO, POWHEG), to get the hard physics correctly.

- It turned out that we can push fixed order computations sufficiently far, to describe a large number of processes required for the LHC phenomenology. In a certain sense, this is a game changer.



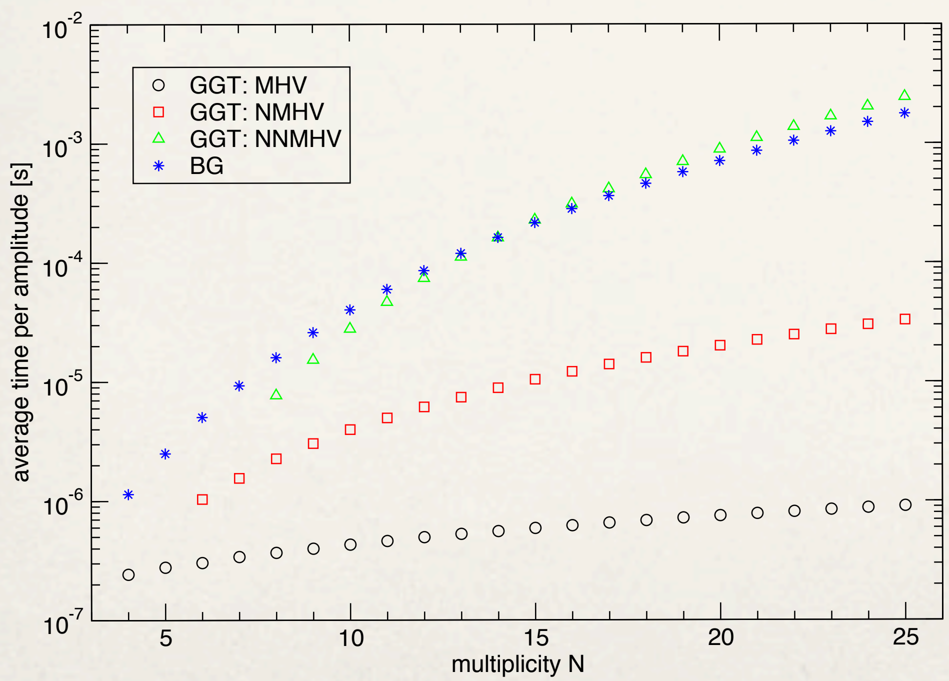
Progress with leading order

- For leading order computations, the significant progress occurred in the 1990s with the appearance of spinor helicity methods, color-stripped amplitudes and Berends-Giele (BG) recursion. With the significant increase in computing power, calculations of amplitudes and cross-sections for high multiplicity processes became possible

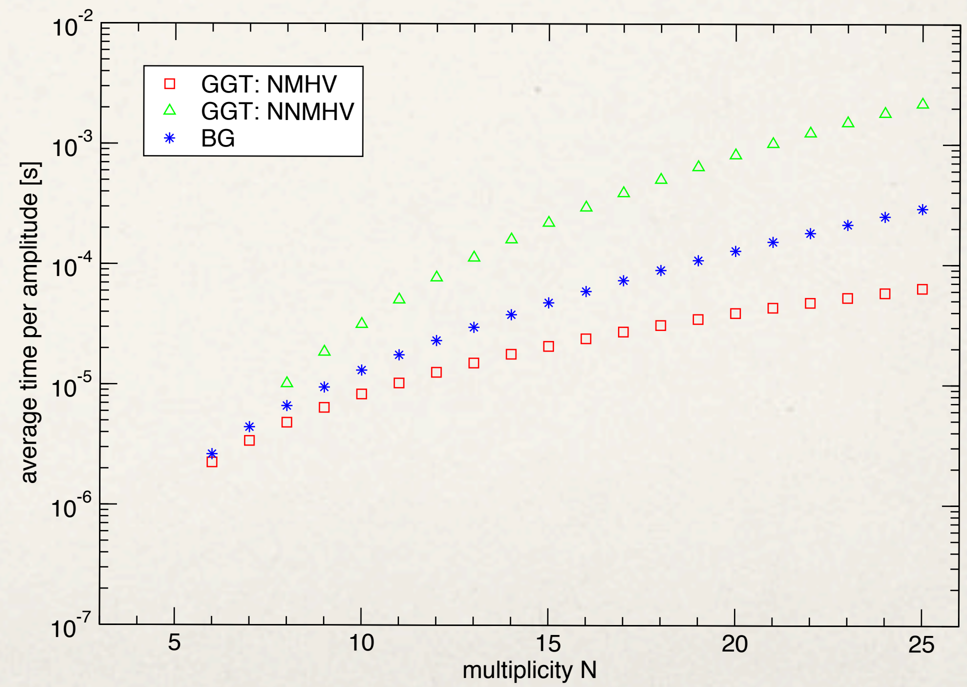
- Recently, explicit analytic results for tree QCD amplitudes became available, as a byproduct of N=4 SYM computations. The explicit comparison of analytic and BG results reveals that MHV and NMHV amplitudes are computed more efficiently with analytic methods; after that, BG recursion becomes more efficient

$$A_{jk}^{\text{MHV}} = i\frac{\langle jk\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

N gluon amplitudes



(N-6) gluon 6 quark amplitudes

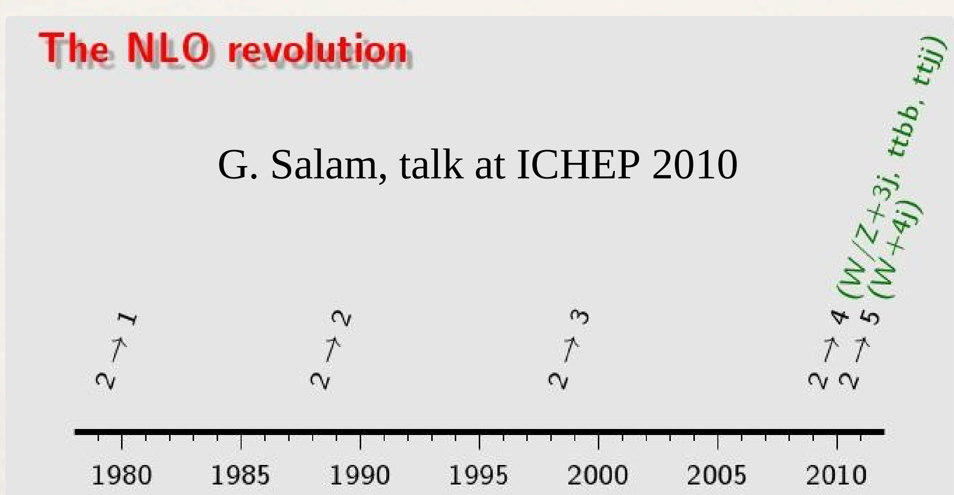


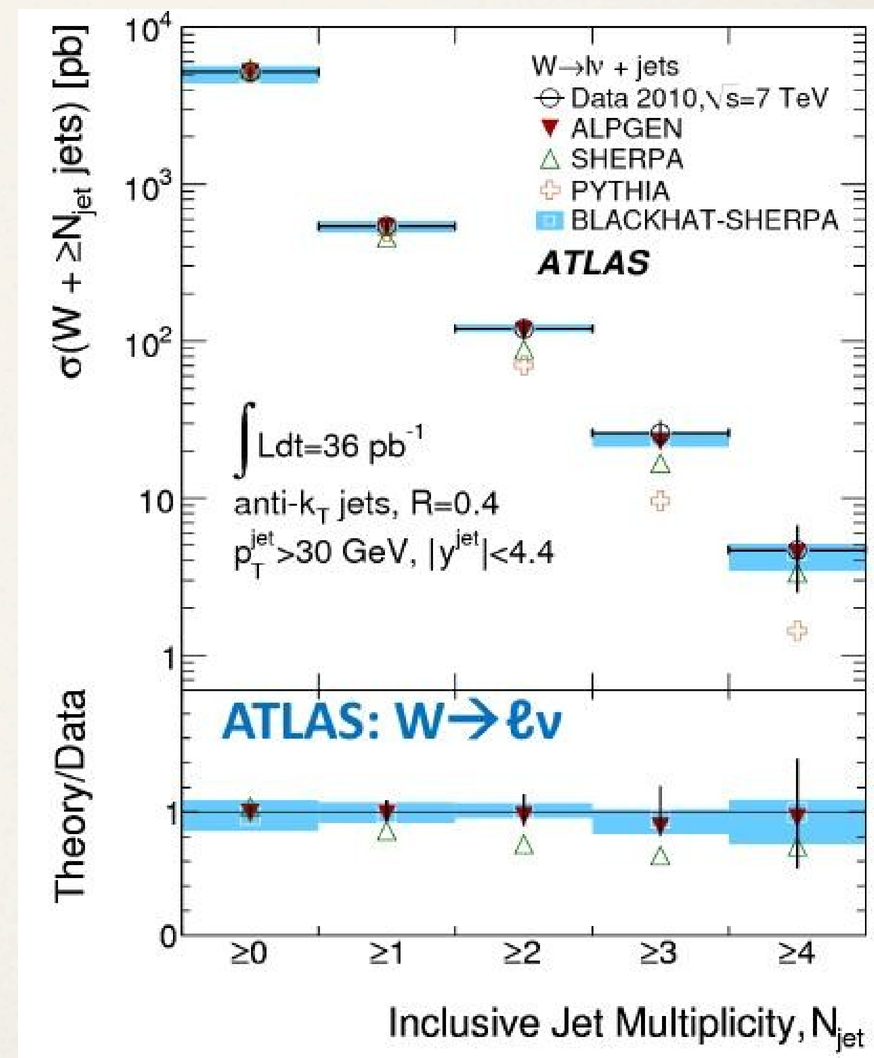
Badger, Biedermann, Hackl, Plefka, Schuster, Uwer

Next-to-leading order computations

- Progress with leading order computations did not translate into progress with next-to-leading order ones right away. Until a few years ago we required a decade to increase the final state multiplicity by one in our NLO computations

- In the past few years - remarkable change in pace which came because of two developments
  - better ways of dealing with Feynman diagrams
  - radically different (on-shell) methods for one-loop computations





A change in the paradigm

- Unitarity (on-shell) techniques allow us to reconstruct one-loop scattering amplitudes directly, by-passing Feynman diagrams

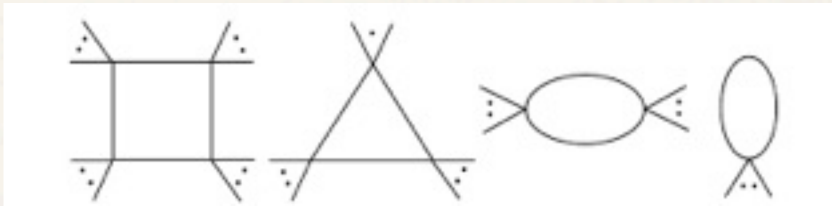
Bern, Dixon, Kosower

- We generalize the old idea of unitarity that proved to be frutiful for low-multiplicity loop computations

$$i\left(T_{ij} - T_{ij}^{+}\right) = \sum T_{in} T_{nj}^{+}$$

- The big boost to this technology came from a new way of tensor reduction for one-loop integrals discovered by Ossola, Pittau and Papadopulous (OPP) and from the observation of Ellis, Giele and Kunszt that generalized unitarity at one-loop can be derived from the OPP

$$\mathcal{A}^{1-\mathrm{loop}} = \sum c_j I_j \qquad I_i =$$



$$\sum c_j \mathrm{Im}(I_j) \propto \sum |\mathcal{A}^{\mathrm{tree}}|^2 \qquad \mathrm{Im}\left(A^{1-\mathrm{loop}}\right) \propto \sum |\mathcal{A}^{\mathrm{tree}}|^2$$

One-loop calculations

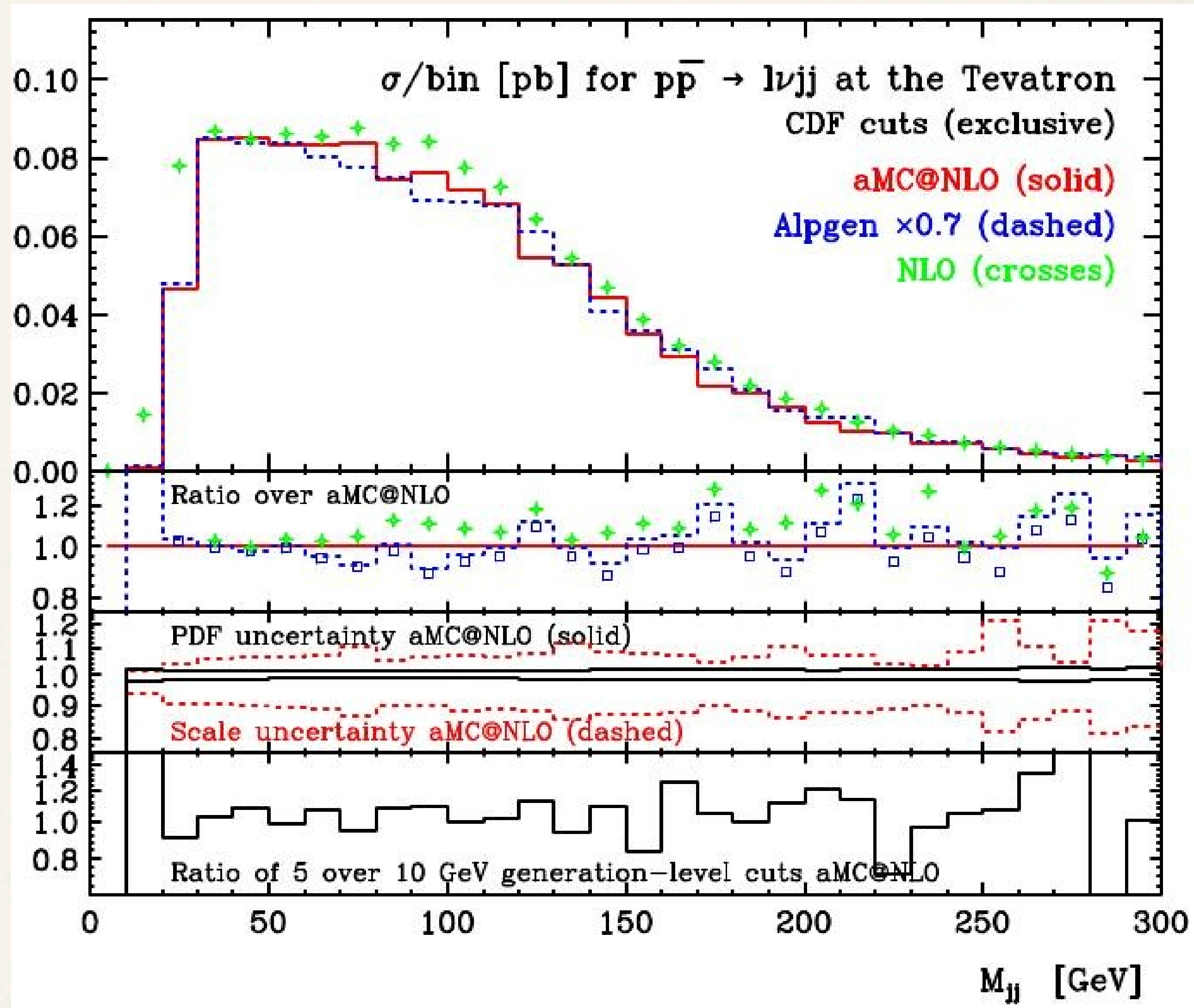
- These developments of one-loop technology lead to a serious accomplishment -- NLO QCD predictions are now available for major collider processes, making rich phenomenology possible
  - multiple jets (up to 4)
  - a gauge boson and up to 5 (!) jets
  - multiple gauge bosons in association with up to 2 jets (up to VV+2jets)
  - top quarks in association with jets (up to two) and gauge photons (W,Z,photon)
  - Higgs and up to two jets

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$ jet	WW jet completed by Dittmaier/Kallweit/Uwer [27, 28]; Campbell/Ellis/Zanderighi [29]. ZZjet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti [30]
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [31]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [32, 33] Interference QCD-EW in VBF channel [34, 35]
3. $pp \to VVV$	ZZZ completed by Lazopoulos/Melnikov/Petriello [36] and WWZ by Hankele/Zeppenfeld [37], see also Binoth/Ossola/Papadopoulos/Pittau [38] VBFNLO [39, 40] meanwhile also contains $WWW, ZZW, WW\gamma, ZZ\gamma, WZ\gamma, W\gamma\gamma, Z\gamma\gamma, \gamma\gamma\gamma$ $WZj, W\gamma j, \gamma jj$ [illegible]
4. $pp \to t\bar{t}\, b\bar{b}$	relevant for $t\bar{t}H$, computed by Bredenstein/Denner/Dittmaier/Pozzorini [41, 42] and Bevilacqua/Czakon/Papadopoulos/Pittau/Worek [43]
5. $pp \to V$+3jets	W+3jets calculated by the Blackhat/Sherpa [44] and Rocket [45] collaborations Z+3jets by Blackhat/Sherpa [46]
Calculations remaining from Les Houches 2005	
6. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$, computed by Bevilacqua/Czakon/Papadopoulos/Worek [47, 48]
7. $pp \to VV\, b\bar{b}$,	Pozzorini et al.[25],Bevilacqua et al.[23]
8. $pp \to VV$+2jets	W^+W^++2jets [49], W^+W^-+2jets [50], VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [51, 52, 53]
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Binoth et al. [54, 55]
NLO calculations added to list in 2009	
10. $pp \to V + 4$ jets	top pair production, various new physics signatures Blackhat/Sherpa: W+4jets [22], Z+4jets [20] see also HEJ [56] for $W + n$jets
11. $pp \to Wb\bar{b}j$	top, new physics signatures, Reina/Schutzmeier [11]
12. $pp \to t\bar{t}t\bar{t}$	various new physics signatures
also: $pp \to 4$ jets	Blackhat/Sherpa [19]

CLOSED

Automation

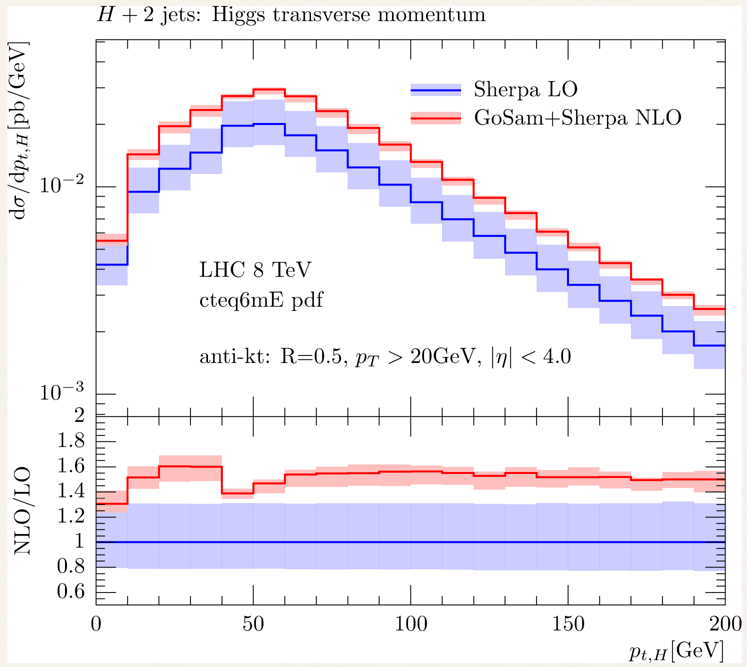
	Process	μ	n_{lf}	Cross section (pb)	
				LO	NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002



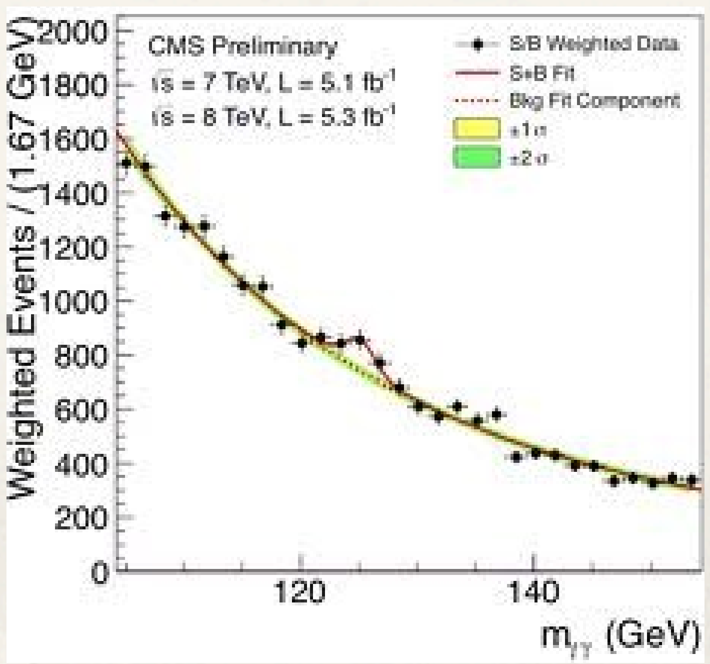
New ideas for NLO computations make LO-style automation possible. MadLoop provides a proof of principle.

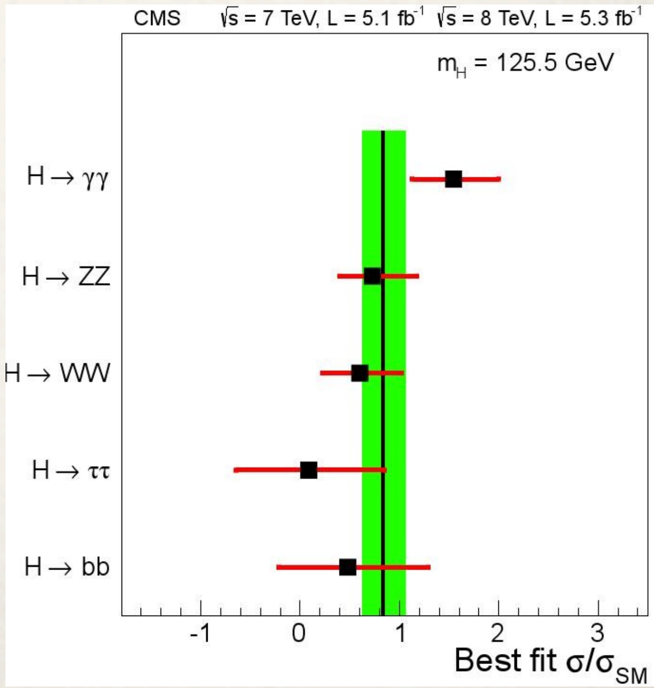
Learning from NLO

- We all know that NLO results reduce the factorization and renormalization uncertainties and provide reliable predictions for the normalization of various cross-sections
- This should be indispensable once the program of extracting coupling constants and making other precise measurements will start in earnest. But that's not all.
- Indeed, since NLO computations take us closer to reality, we should be able to learn quite a bit more from them
- In particular, we should be able to access the validity of various approximations and short-cuts that we may want to use to arrive at reliable results in more complex cases



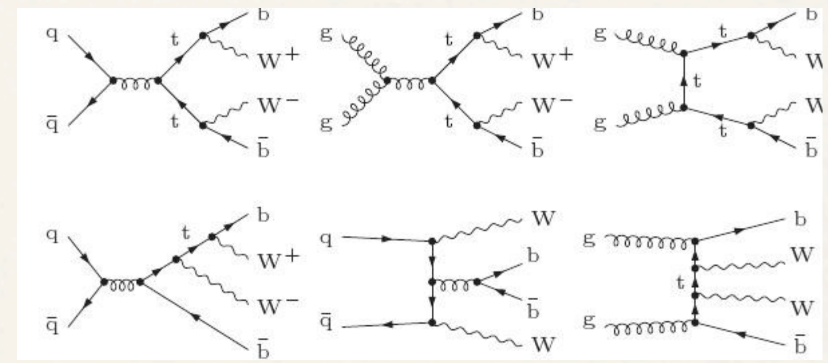
Samurai collaboration



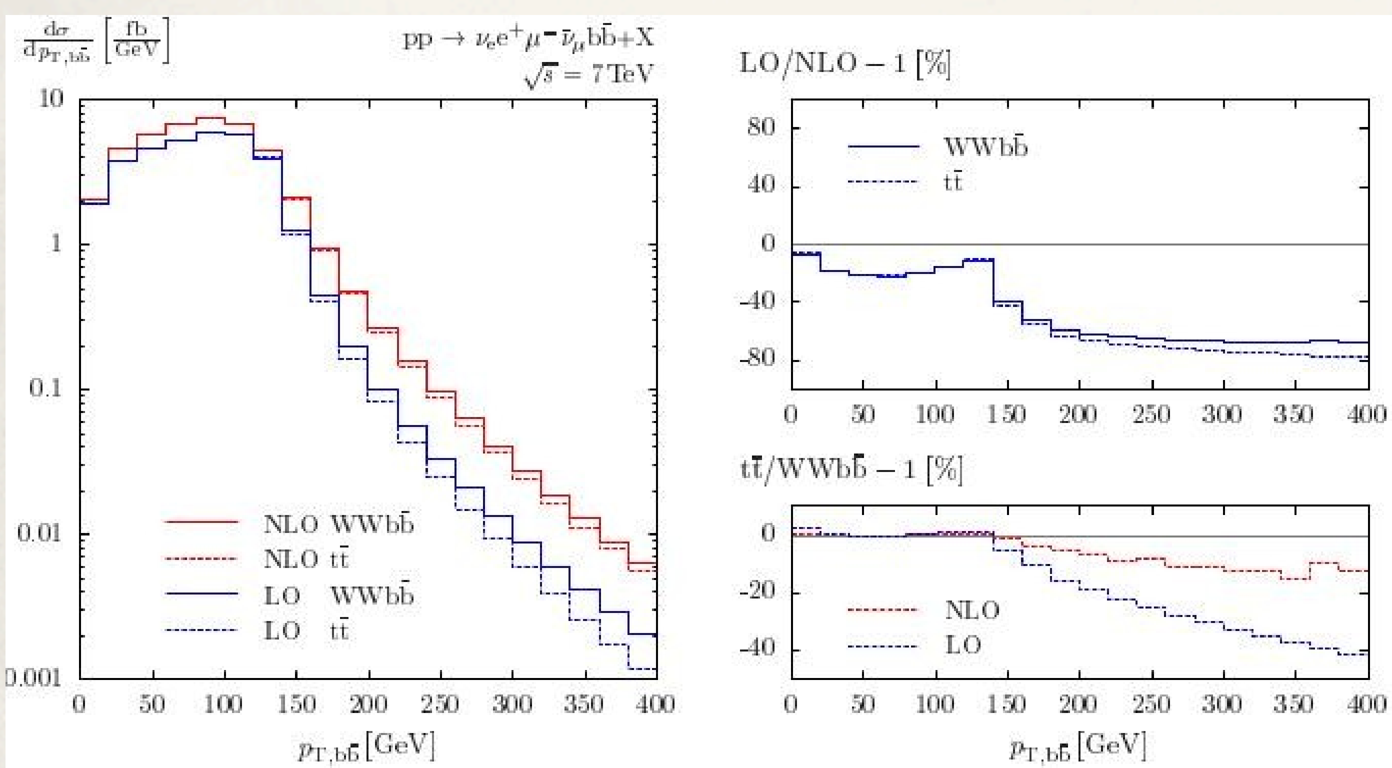


Off-shell effects in top quark pair production

- How well does the narrow width approximation work for the LHC physics?
- We can answer this question by comparing top quark pair production computed in the narrow width approximation with complete computation for WWbb final state



Collider	$\sqrt{s}$ [TeV]	approx.	$\sigma_{t\bar{t}}$ [fb]	$\sigma_{WWb\bar{b}}$ [fb]	$\sigma_{t\bar{t}}/\sigma_{WWb\bar{b}} - 1$	Ref. [25]
Tevatron	1.96	LO	$44.691(8)^{+19.81}_{-12.58}$	$44.310(3)^{+19.68}_{-12.49}$	$+0.861(19)\%$	$+0.8\%$
		NLO	$42.16(3)^{+0.00}_{-2.91}$	$41.75(5)^{+0.00}_{-2.63}$	$+0.98(14)\%$	$+0.9\%$
LHC	7	LO	$659.5(1)^{+261.8}_{-173.1}$	$662.35(4)^{+263.4}_{-174.1}$	$-0.431(16)\%$	-0.4%
		NLO	$837(2)^{+42}_{-87}$	$840(2)^{+41}_{-87}$	$-0.41(31)\%$	-0.2%
LHC	14	LO	$3306.3(1)^{+1086.8}_{-763.6}$	$3334.6(2)^{+1098.5}_{-771.2}$	$-0.849(7)\%$	$---$
		NLO	$4253(3)^{+282}_{-404}$	$4286(7)^{+283}_{-407}$	$-0.77(19)\%$	$---$



Standard top-like selection cuts are applied to WWbb final state for cross-section calculation

Kinematic distributions agree very well when on-shell kinematics is allowed but may show larger deviations when this is not the case

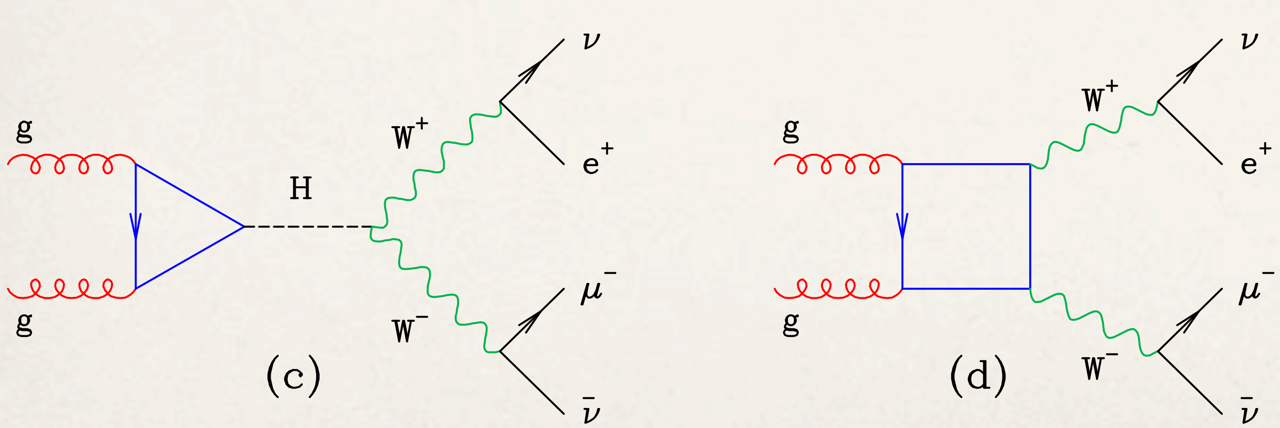
Denner, Dittmaier, Kallweit, Pozzorini, Schulze

Interference effects in Higgs production

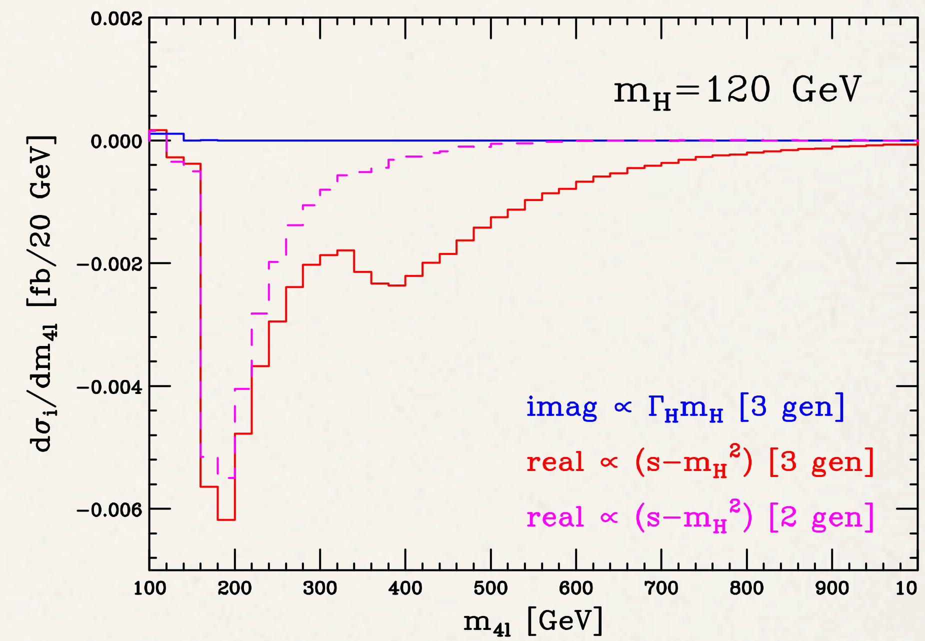
- The Higgs boson that is produced in the gluon fusion and decays to two W bosons can interfere with direct production of two W bosons

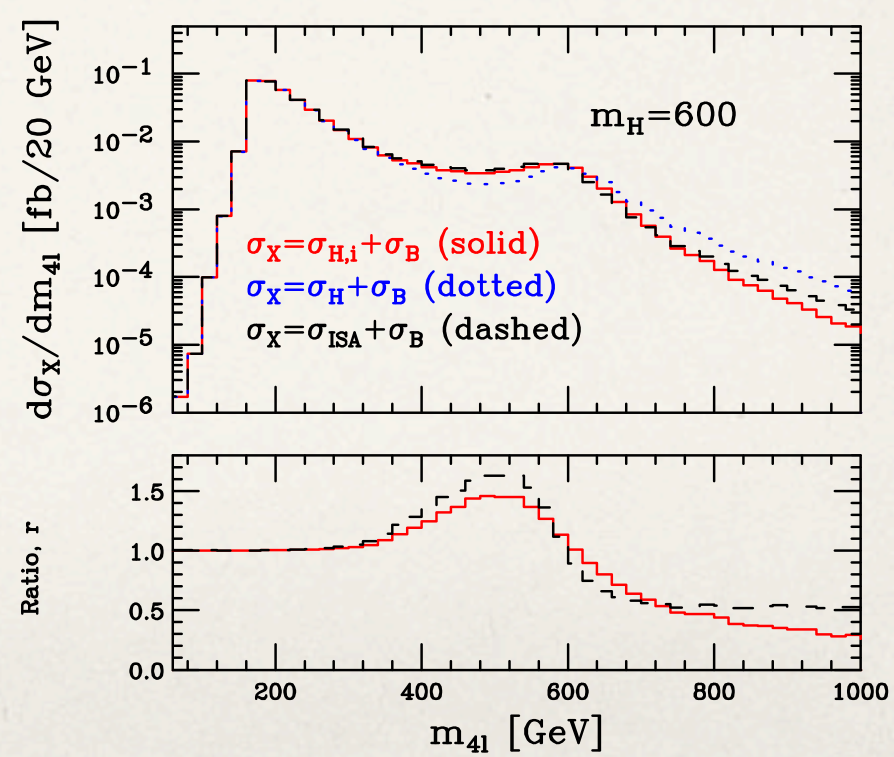
- This effect is not important for the light Higgs boson but may change shapes of kinematic distributions for the heavier one

- A success of soft approximation in describing the Higgs production suggests that a similar approach will work for the interference



Campbell, Ellis, Williams





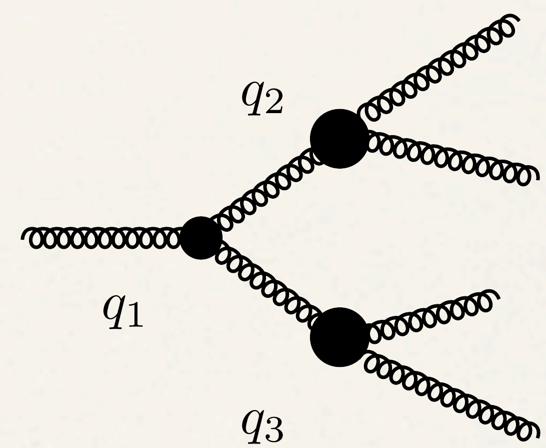
Choosing the factorization and renormalization scales

- Since NLO computations are more scale-independent than LO computations, we can ask the question -- what is the right scale choice at LO ?

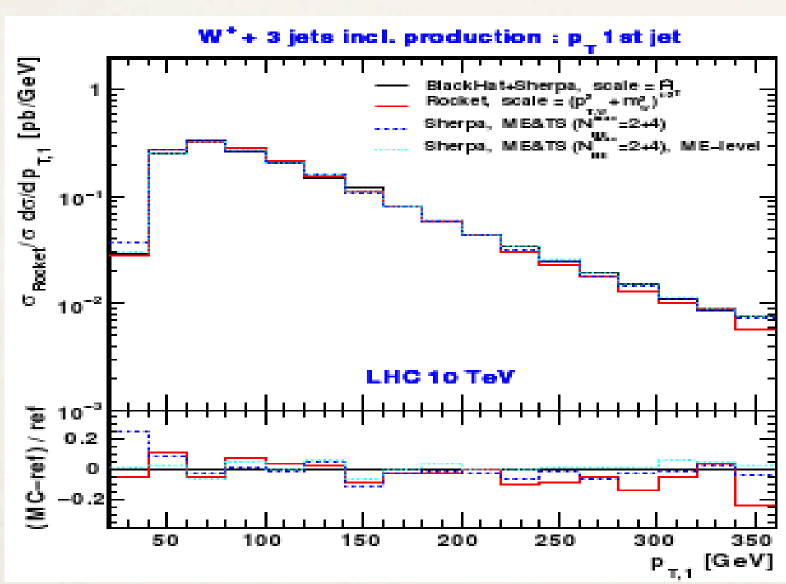
- Of course, no universal answer exists since the coupling constant runs; the ``right scale'' is determined by the local dynamics

- CKKW/MLM procedure seems to capture important kinematics features in many cases

$$\mathrm{Prob}(a \to b) \sim \alpha_s(p_\perp)$$



$$|\mathcal{M}|^2 \sim \prod_{i=1}^{N} \alpha_s(q_i) \prod_{ij} \frac{\Delta(q_i)}{\Delta(q_j)}$$



S. Hoche, J. Huston, D. Maitre, J. Winter and G. Zanderighi

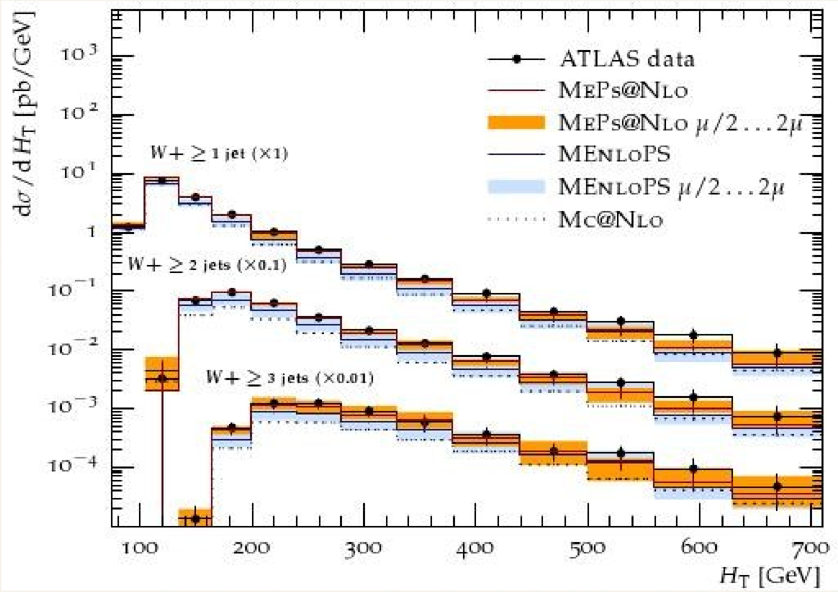
It is possible to extend the scale-setting prescription of CKKW to NLO by choosing the geometric mean of nodal scales to compute the virtual corrections (MINLO)

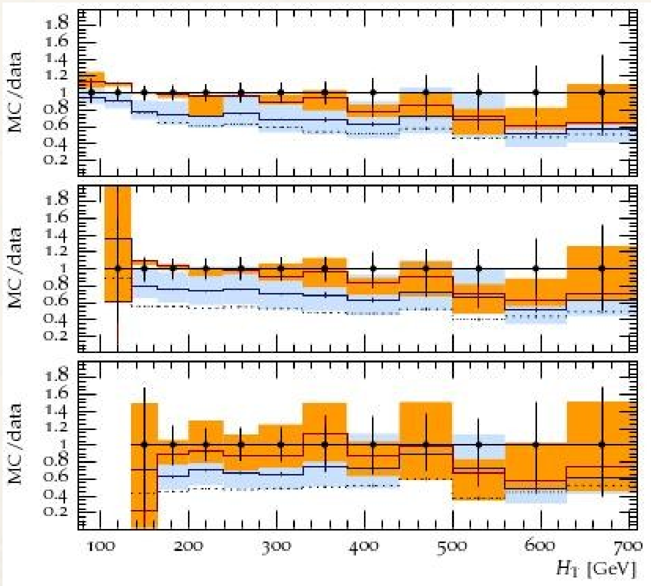
Hamilton, Nason, Zanderighi

Can MINLO be a poor-man solution to exact NNLO rates and shapes? It will be interesting to see that.

Improving on NLO results

- It is possible to improve on some shortcomings of NLO computations (they fail close to kinematic boundaries)
  - combine NLO computations with parton showers (MC@NLO, POWHEG, SHERPA) -- an ever increasing number of processes is being implemented in these programs
  - understand how to merge NLO QCD predictions for processes with different multiplicities (MEPS@NLO)
  - include decays, with NLO QCD corrections
  - develop NNLO technology for multi-particle processes





MC@NLO: n-jets NLO, n+1 jets at LO, rest -- PS

MENLOPS: n-jets NLO, n+1, n+2...jets LO matched to parton shower

MEPS@NLO: n, n+1, n +2...jets at NLO merged and matched to parton shower

Hoche, Krauss, Schonherr, Siegert

NNLO QCD computations

- Development of a working method for generic NNLO calculations proved to be an intellectually challenging problem
- In 1999, Smirnov and Tausk started a breakthrough in the computation of two-loop virtual corrections for 2-to-2 processes; many results followed shortly after that
- Suitable subtraction terms for real emission processes were in the making for the next decade and there is a feeling that we finally have them
- Two primary approaches (plus SCET-based slicing)
  - antenna subtraction
  - sector decomposition/FKS

NNLO computations: singular limits

- A part of a NNLO computation that proved to be highly non-trivial, is the construction of subtraction counter-terms that make real-emission contributions finite

- If we consider NNLO for an n-jet process, we must deal with up to n+2 particles in the final state, two of which can become unresolved and cause singularities. These singularities can be double-soft, double-collinear, single-soft and single-collinear. In these limits amplitudes factorize into universal factors and lower-multiplicity amplitudes, as shown in the double soft limit

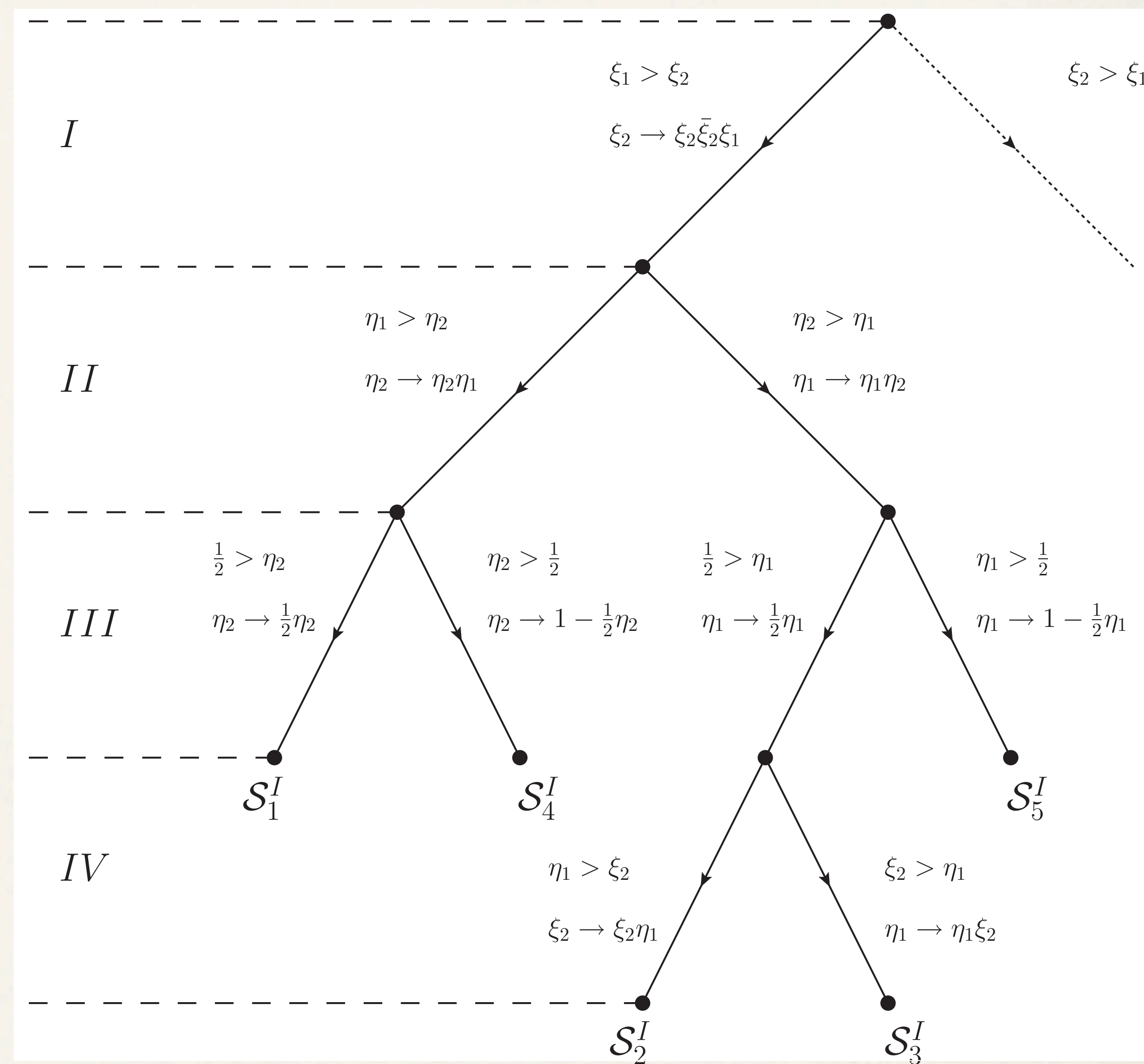
$$\mathcal{M}_{n+2}(p_1, ..p_n, g_1, g_2) \approx S^{ab}\left(\{p\}, g_1, g_2\right) \mathcal{M}_n^{ab}(p_1, \ldots, p_n)$$

- It is remarkable that all singular limit required for the NNLO computations were available for quite some time already, yet they were not used in the computations ! The problem, as always, are overlapping singularities that can be approached from two separate limits...

- At NLO, the problem is solved by either phase-space slicing, or by Catani-Seymour construction of the dipole terms that interpolate between soft and collinear singular limits, or by the phase-space partitioning as pointed out by Frixione, Kunszt and Signer (FKS)

FKS@NNLO

- Kinematic regions that may lead to potential singularities can be separated by partitioning the phase-space (sectors must cover full phase-space)
    - in each sector we must be able to identify particles that may ``produce singularities''
        - at NLO: one and only one particle can become soft and one and only one pair of particles -- collinear
        - at NNLO: for each sector two well-defined particles are allowed to become soft and at most three well-defined particles -- collinear
- In each sector natural variables to factorize phase-space are energies of (potentially) soft particles and angles for their emission relative to their hard collinear partners



As pointed out by Czakon, at NNLO further universal decomposition into five sub-sectors is required, to extract all the singularities

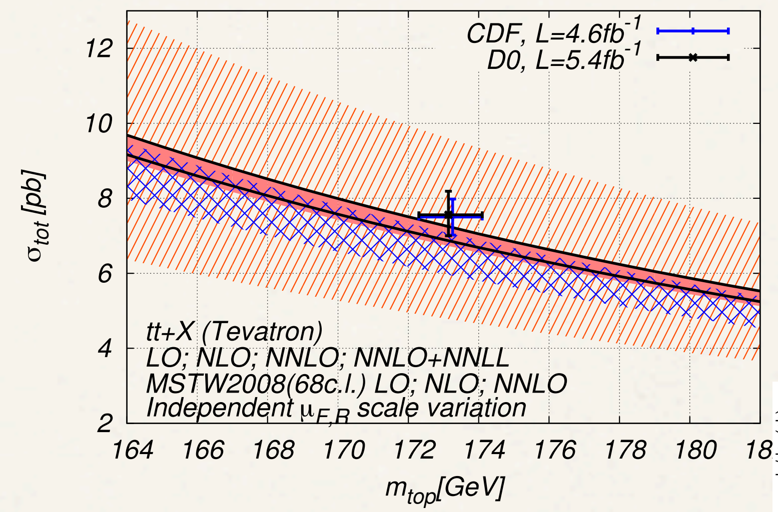
First NNLO results for 2 $\longrightarrow$ 2 processes at hadron colliders

- First NNLO QCD results for top quark pair production, dijet production and H+jet production in hadron collisions appeared recently

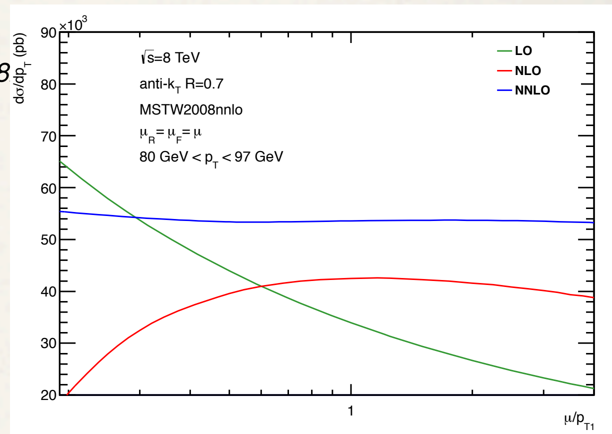
- While these results are still not complete, there is no question that they represent a breakthrough in perturbative QCD that will, eventually, provide important phenomenological insights

- One thing that we learned from first NNLO results is that -- no matter how they are obtained -- they require significant computational effort where speed and numerical stability are important.

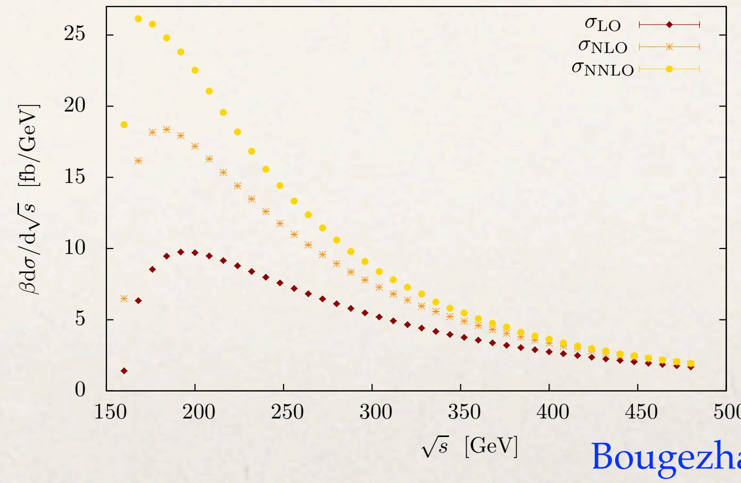
- This observation has important implications for ingredients of perturbative computations that we rely upon



Czakon, Mitov, Barnreuther



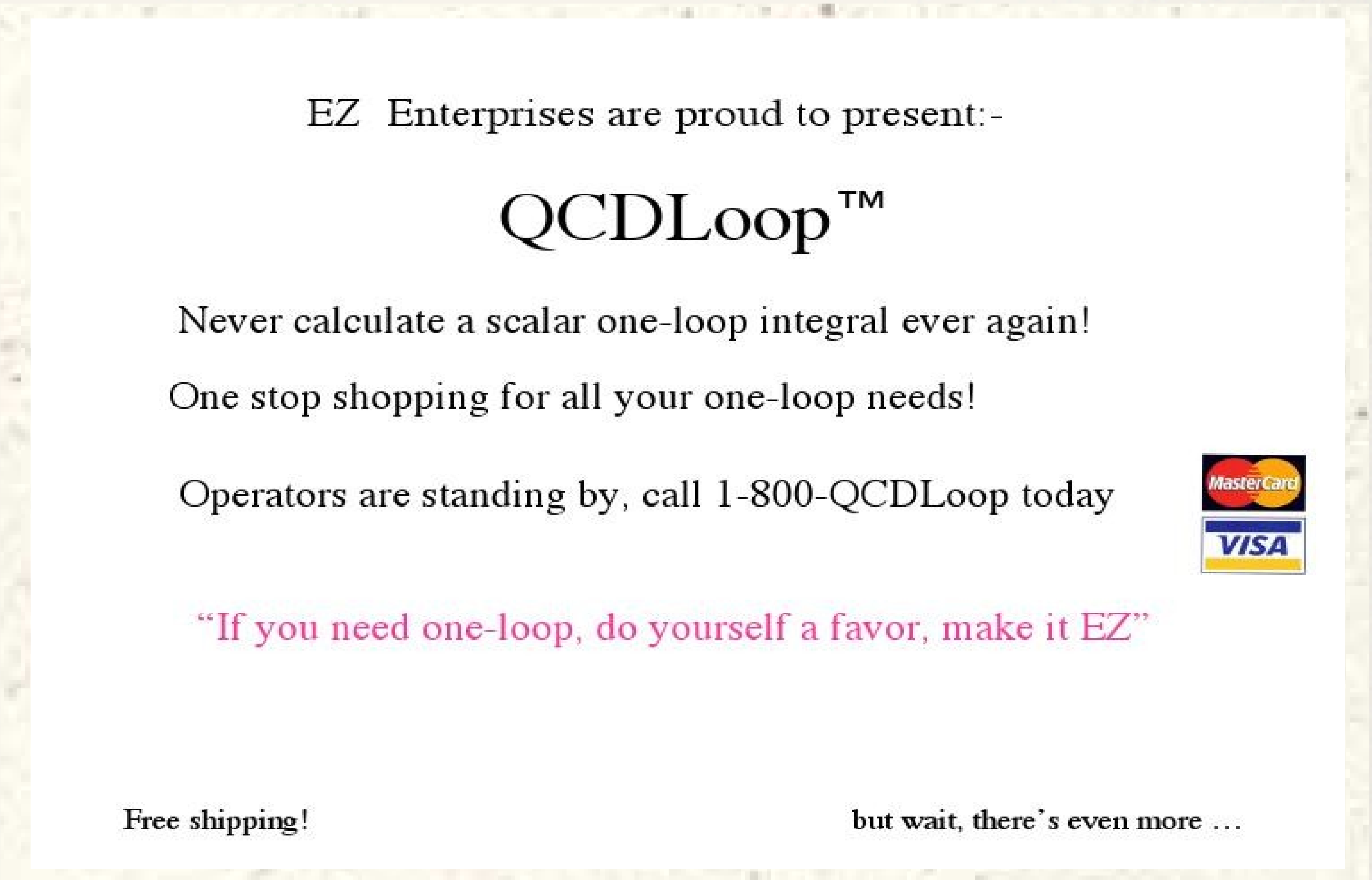
Gehrmann-der Ridder, Gehrmann, Glover, Pires



Bougezhal, Caola, K.M., Petriello, Schulze

Things we have learned

- It is important to have compact and numerically stable expressions for tree and one-loop helicity amplitudes
- Can numerical methods developed for one-loop computations be used in the NNLO context?
- Stable results for one-loop master integrals are necessary since one-loop calculations are taken to very singular regions
- MCFM is a fantastic resource for scattering amplitudes -- tree and one-loop -- that have ever been computed



R.K.Ellis, talk at the Buffalo Loopfest, 2008

Which NNLO results can be expected

- A NNLO calculation for a tree process X can not be done without a one-loop calculation for a process X+jet and a two-loop correction to X
- Ignoring the two-loop amplitudes, any process which is currently known at NLO defines a process that may, eventually, become known at NNLO
- From this perspective, we appear to be doing really well since we have extremely advanced NLO calculations available (MCFM --> MCFM@NNLO)
- However, the numerical methods developed for most advanced one-loop computations may not be up to the task; this is a field where continuous improvements of one-loop computational algorithm may, eventually, pay off

4jets@NLO	3jets@NNLO
W+4jets@NLO	W+3jets@NNLO
tt+2jets@NLO	tt+1jet@NNLO
WW+2jets@NLO	WW+1jet@NNLO

Two-loop amplitudes

- The next big problem with NNLO computations are virtual corrections to multi-jet processes. Both, reduction to and evaluation of master integrals, are difficult and, probably, will require new ideas

- Even with traditional methods for loop computations, we see significant differences between NLO and NNLO -- at NLO Passarino-Veltman reduction is algebraic but at NNLO no algebraic reduction is possible and integration-by-parts is needed for a complete solutions

- We should expect similar issues with extensions of OPP to two-loops -- parametric integration of spurious terms is the key at NLO but ``real'' integration will likely be needed at NNLO

- Indeed, recent attempts to extend the on-shell methods and the OPP procedure to compute two-loop diagrams for multi-jet production showed significant increase in the number of irreducible scalar products. Planar N=4 SYM remains a spectacular extension but it is not so clear how to benefit from it for N=0.

Two-loop computations

- An interesting alternative direction is provided by pure numerical methods
- Weinzierl et al. showed that one can formulate the integration procedure directly in momentum space, both at one-loop and beyond
- One-loop integrals are made finite by subtraction terms and contour deformation
- At one-loop, the method was used to obtain large-N cross-sections for the production of up to seven (!) jets in electron-positron annihilation

y_{cut}	$\frac{N_c^4}{8} A_{5,\mathrm{lc}}$	$\frac{N_c^5}{16} B_{5,\mathrm{lc}}$
0.002	$(5.0529 \pm 0.0004) \cdot 10^3$	$(4.275 \pm 0.006) \cdot 10^5$
0.001	$(1.3291 \pm 0.0001) \cdot 10^4$	$(1.050 \pm 0.026) \cdot 10^6$
0.0006	$(2.4764 \pm 0.0002) \cdot 10^4$	$(1.84 \pm 0.15) \cdot 10^6$

y_{cut}	$\frac{N_c^5}{16} A_{6,\mathrm{lc}}$	$\frac{N_c^6}{32} B_{6,\mathrm{lc}}$
0.001	$(1.1470 \pm 0.0002) \cdot 10^5$	$(1.46 \pm 0.04) \cdot 10^7$
0.0006	$(2.874 \pm 0.002) \cdot 10^5$	$(3.88 \pm 0.18) \cdot 10^7$

y_{cut}	$\frac{N_c^6}{32} A_{7,\mathrm{lc}}$	$\frac{N_c^7}{64} B_{7,\mathrm{lc}}$
0.0006	$(2.49 \pm 0.08) \cdot 10^6$	$(5.4 \pm 0.3) \cdot 10^8$

$$R_n = \left(\frac{\alpha_s}{2\pi}\right)^{n-2} A_n + \left(\frac{\alpha_s}{2\pi}\right)^{n-1} B_n$$

Conclusions

- Perturbative QCD provides good description of the wealth of data on hard scattering collected at the Tevatron and the LHC

- Important for this success, is the recent progress with NLO QCD as it allows us to make realistic and accurate description of complex final states

- Same progress drives theoretical developments in matching fixed order computations to parton showers and merging theoretical predictions for various jet multiplicities

- A working technology to perform complex NNLO QCD computations finally appeared. We now have the time to consolidate the NNLO technology and move on to the NNLO phenomenology

- All the developments in pQCD that so many people worked on diligently through the past decay are becoming key for detailed understanding the properties of the Higgs boson and for looking for clues about BSM in the SM-like data