Automated NLO calculations with OpenLoops

Stefano Pozzorini
Zürich University

based on

F. Cascioli, P. Maierhöfer and S.P.
PRL **108** (2012) 111601 [arXiv:1111.5206]

and work in collaboration with

F. Krauss, F. Siegert and J.Thompson

“The first three years of the LHC”, Mainz, March 20, 2013

Outline of the talk

NLO automation

OpenLoops algorithm

OpenLoops+Sherpa

Irreducible background to $\mathrm{H} \to \mathrm{WW}$ + 0,1 jets

Multi-particle processes at NLO

Various general techniques to perform NLO calculations

- exist since decades but badly fail when applied to multi-particle processes

Important $2 \to 4$ **processes at the LHC** (2005/2007 Les Houches priority list)

$$\mathrm{pp} \to \quad \mathrm{t\bar{t}b\bar{b}}, \qquad \mathrm{t\bar{t}}jj, \qquad VV\mathrm{b\bar{b}}, \qquad VVjj, \qquad Vjjj, \qquad \mathrm{b\bar{b}b\bar{b}}$$

Multi-particle challenges (mostly in virtual one-loop part!)

- *AUTOMATION:* min man power
- *FLEXIBILITY:* max applicability
- *SPEED:* avoid big clusters
- *STABILITY:* avoid Gram determinants

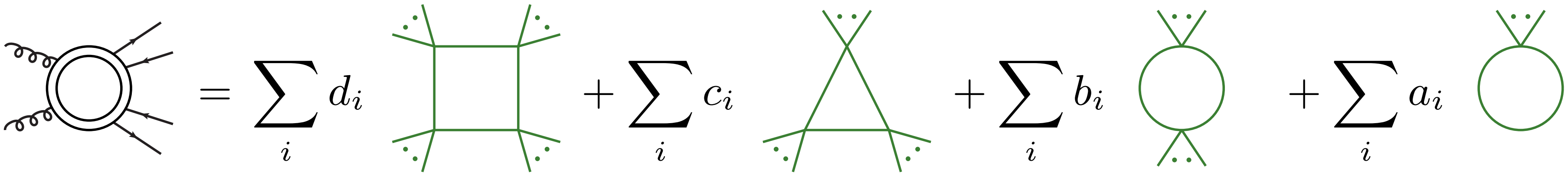
“NLO revolution”

$\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$	[Bredenstein, Denner, Dittmaier, S.P. ‘09] [Bevilacqua, Czakon, Papadopoulos, Pittau, Worek ‘09]
$\mathrm{pp} \to \mathrm{t\bar{t}t\bar{t}}$	[Bevilacqua, Worek ‘12]
$\mathrm{pp} \to \mathrm{t\bar{t}}jj$	[Bevilacqua, Czakon, Papadopoulos, Pittau, Worek ‘10]
$\mathrm{pp} \to \mathrm{WWb\bar{b}}$	[Denner, Dittmaier, Kallweit, S.P. ‘11] [Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek ‘11]
$\mathrm{pp} \to \mathrm{b\bar{b}b\bar{b}}$	[Greiner, Guffanti, Reiter, Reuter ‘11]
$\mathrm{pp} \to \mathrm{WW}jj$	[Melia, Melnikov, Rontsch, Zanderighi ‘10] [Greiner, Heinrich, Mastrolia, Ossola, Reiter, Tramontano ‘12]
$\mathrm{pp} \to \mathrm{W/Z} + 3j$	[Ellis, Melnikov, Zanderighi ‘09] [Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre ‘09–‘10]
$\mathrm{pp} \to \mathrm{W/Z} + 4j$	[Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre ‘11]
$\mathrm{pp} \to 4j$	[Bern, Diana, Dixon, Febres Cordero, Hoeche, Kosower, Ita, Maitre, Ozeren ‘11] [Badger, Biedermann, Uwer, Yundin ‘12]
$\mathrm{pp} \to W\gamma\gamma\, j$	[Campanario, Englert, Rauch, Zeppenfeld ‘11]
$\mathrm{e^+e^-} \to 7j$	[Becker, Goetz, Reuschle, Schwan, Weinzierl ‘11]

NLO Automation & Co

CutTools	[Ossola, Papadopoulos, Pittau '08]
Samurai	[Mastrolia, Ossola, Reiter, Tramontano '10]
FormCalc	[Agrawal, Hahn, Mirabella '11]
HELAC-NLO	[Bevilacqua, Czakon, Garzelli, van Hameren, Kardos, Papadopoulos, Pittau, Worek '11]
MadLoop	[Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau '11]
GoSam	[Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano '11]
BlackHat	[Bern, Diana, Dixon, Febres Cordero, Kosower, Ita, Maitre, Ozeren]
NGluon	[Badger, Biedermann, Uwer, Yundin '11]
Recola	[Actis, Denner, Hofer, Scharf, Uccirati '12]
MCFM	[Campbell, Ellis, Williams]
VBFNLO	[Arnold, Bähr, Bozzi, Campanario, Englert, Figy, Greiner, Hakstein, Hankele, Jäger, Klämke, Kubocz, Oleari, Plätzer, Prestel, Worek, Zeppenfeld '08]
aMC@NLO	[Frederix, Frixione, Maltoni, Stelzer, Torielli]
POWHEG	[Alioli, Nason, Oleari, Re]
Sherpa	[Hoeche, Hoeth, Krauss, Schoenherr, Schumann, Siegert, Zapp]

Main one-loop approaches vs OpenLoops



	Standard	**OpenLoops**	**On-shell**
Reduction to SIs	tensor integrals	tensor int. & OPP	on-shell/OPP
Amplitudes	loop diagrams	"open loops"	tree amplitudes
	mostly algebraic	numerical recursion	mostly numerical
Speed for $n_{\text{part}} \leq 6$	very high	very high	slower
Automation ($n_{\text{part}} > 5$)	very large codes	highly automatic	highly automatic
Stability	stable expansions	stable with TIs	quadruple precision

OpenLoops algorithm

Diagrams and colour

Tree and loop amplitudes as *sums of diagrams*

$$\mathcal{M} = \sum_d \mathcal{M}^{(d)} \qquad \text{(up to } 10^4 \text{ loops)}$$

Factorisation of colour factors from colour-stripped diagrams

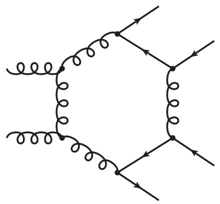
$$\mathcal{M}^{(d)} = \mathcal{A}^{(d)} \mathcal{C}^{(d)}$$

Colour reduction and sums are done algebraically (zero CPU cost)

Everything else is done numerically

Loop-momentum dependence

Structure of colour-stripped loop diagram



$$= \int \frac{\mathrm{d}^D q \, \mathcal{N}(q)}{D_0 D_1 \dots D_{n-1}} = \sum_{r=0}^{R} \mathcal{N}_{\mu_1 \dots \mu_r} \int \frac{\mathrm{d}^D q \; q^{\mu_1} \dots q^{\mu_r}}{D_0 D_1 \dots D_{n-1}}$$

$$D_i = (q + p_i)^2 - m_i^2$$

Treatment of $\mathcal{N}(q)$ polynomial

- separate loop momenta from helicity-dependent coefficients $\mathcal{N}_{\mu_1 \dots \mu_r}$

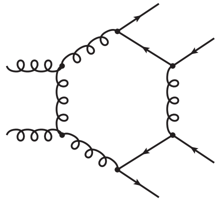
$\Rightarrow$ very efficient helicity sums!

- recursive numerical algorithm* for $\mathcal{N}_{\mu_1 \dots \mu_r}$ (see later)

*first proposed within Dyson-Schwinger framework [van Hameren '09]

Loop-momentum dependence

Structure of colour-stripped loop diagram



$$= \int \frac{\mathrm{d}^D q \, \mathcal{N}(q)}{D_0 D_1 \dots D_{n-1}} = \sum_{r=0}^{R} \mathcal{N}_{\mu_1 \dots \mu_r} \int \frac{\mathrm{d}^D q \; q^{\mu_1} \dots q^{\mu_r}}{D_0 D_1 \dots D_{n-1}}$$

Usual $\mathcal{N}(q)$ from Feynman rules

$$\mathcal{N}(q) = g_{\mathrm{S}}^6 \, [\bar{u}_3 \gamma^{\nu_1} (\not{q} + \not{k}_4 + m_{\mathrm{t}}) \gamma^{\alpha} v_4] \, [\bar{u}_5 \gamma^{\nu_2} (\not{q} + \not{k}_2 - m_{\mathrm{b}}) \gamma^{\alpha} v_6] \, g^{\rho_1 \rho_2}$$
$$\times \, \varepsilon_1^{\mu_1} \, [g_{\mu_1 \nu_1} (p_1 - q - k_5)_{\rho_1} + g_{\nu_1 \rho_1} (2q + k_5)_{\mu_1} - g_{\rho_1 \mu_1} (p_1 + q)_{\nu_1}]$$
$$\times \, \varepsilon_2^{\mu_2} \, [g_{\mu_2 \nu_2} (p_2 + q + k_1)_{\rho_2} - g_{\nu_2 \rho_2} (2q + k_1)_{\mu_2} - g_{\rho_2 \mu_2} (p_2 - q)_{\nu_2}]$$

Separation of q-monomials

$$\mathcal{N}(q) = \sum_{r=0}^{R} \mathcal{N}_{\mu_1 \dots \mu_r} \; q^{\mu_1} \dots q^{\mu_r}$$

$$= \sum_{\substack{n_0 \dots n_3 = 0 \\ n_0 + n_1 + n_2 + n_3 \le R}}^{R} \mathcal{N}_{n_0 \dots n_3} \; \underbrace{q^0 \dots q^0}_{n_0} \dots \underbrace{q^3 \dots q^3}_{n_3}$$

Systematic symmetrisation

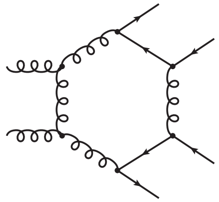
only $\binom{R+4}{4}$ symm. $\mathcal{N}_{n_0 \dots n_3}$ components

R	0	1	2	3	4	5	6
$\binom{R+4}{4}$	1	5	15	35	70	126	210

6 particles (brace under $R = 4$–5)

Loop-momentum dependence

Structure of colour-stripped loop diagram



$$= \int \frac{\mathrm{d}^D q\, \mathcal{N}(q)}{D_0 D_1 \dots D_{n-1}} = \sum_{r=0}^{R} \mathcal{N}_{\mu_1 \dots \mu_r} \underbrace{\int \frac{\mathrm{d}^D q\, q^{\mu_1} \dots q^{\mu_r}}{D_0 D_1 \dots D_{n-1}}}_{T^{\mu_1 \cdots \mu_r} =}$$

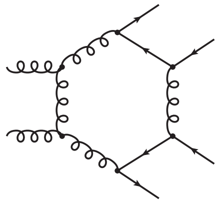
(A) Recursive reduction of **tensor integrals** to scalar integrals [Denner/Dittmaier '05]

$$= \int \mathrm{d}^D q \left[\sum_{i_1} \frac{a_{i_1}^{\mu_1 \cdots \mu_r}}{D_{i_1}} + \sum_{i_1, i_2} \frac{b_{i_1 i_2}^{\mu_1 \cdots \mu_r}}{D_{i_1} D_{i_2}} + \sum_{i_1, i_2, i_3} \frac{c_{i_1 i_2 i_3}^{\mu_1 \cdots \mu_r}}{D_{i_1} D_{i_2} D_{i_3}} + \sum_{i_1, i_2, i_3, i_4} \frac{d_{i_1 i_2 i_3 i_4}^{\mu_1 \cdots \mu_r}}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} \right] + R_1^{\mu_1 \cdots \mu_r}$$

- **avoid instabilities** with Gram-determinant (and other) expansions
- D-dim reduction but $\mathcal{N}_{\mu_1 \dots \mu_r}$ with $0 \le \mu_1 \dots \mu_r \le 3$ indices ...

Loop-momentum dependence

Structure of colour-stripped loop diagram

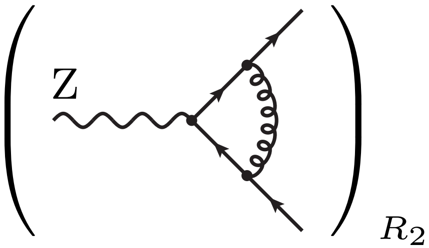


$$= \int \frac{\mathrm{d}^D q\, \mathcal{N}(q)}{D_0 D_1 \dots D_{n-1}} = \sum_{r=0}^{R} \mathcal{N}_{\mu_1\dots\mu_r} \underbrace{\int \frac{\mathrm{d}^D q\; q^{\mu_1} \dots q^{\mu_r}}{D_0 D_1 \dots D_{n-1}}}_{T^{\mu_1\cdots\mu_r} =}$$

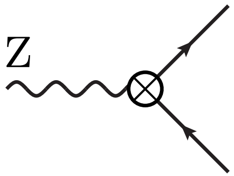
Extra rational terms from $3 < \mu_1, \dots, \mu_r \leq D-1$

$$R_2 = \sum_{\mu_1\dots\mu_r=0}^{D-1} \mathcal{N}_{\mu_1\dots\mu_r}\Big|_{D=4-2\varepsilon} T_{\mathrm{UV}}^{\mu_1\dots\mu_r} - \sum_{\mu_1\dots\mu_r=0}^{3} \mathcal{N}_{\mu_1\dots\mu_r}\Big|_{D=4} T_{\mathrm{UV}}^{\mu_1\dots\mu_r}$$

From catalogue of 2-, 3- and 4-point 1PI diagrams (depends only on model)



$=$

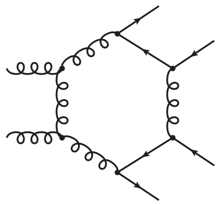


$$= -\frac{g_{\mathrm{S}}^2}{16\pi^2}\frac{N_c^2-1}{2N_c}\gamma^\mu (g_{\mathrm{V}}^{\mathrm{Z}} - g_{\mathrm{A}}^{\mathrm{Z}}\gamma_5) \qquad \text{etc.}$$

Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau '09–'11 Shao, Zhang, Chao '11

Loop-momentum dependence

Structure of colour-stripped loop diagram



$$= \underbrace{\int \frac{\mathrm{d}^D q \, \mathcal{N}(q)}{D_0 D_1 \dots D_{n-1}}}_{\delta\mathcal{A}^{(d)} =} = \sum_{r=0}^{R} \mathcal{N}_{\mu_1 \dots \mu_r} \int \frac{\mathrm{d}^D q \; q^{\mu_1} \dots q^{\mu_r}}{D_0 D_1 \dots D_{n-1}}$$

(B) Direct OPP reduction at the amplitude level [Ossola, Papadopolous, Pittau '07]

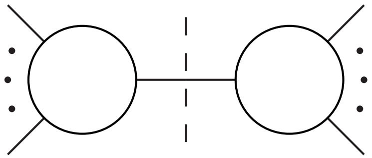
$$= \int \mathrm{d}^D q \left[\sum_{i_1} \frac{a_{i_1}}{D_{i_1}} + \sum_{i_1, i_2} \frac{b_{i_1 i_2}}{D_{i_1} D_{i_2}} + \sum_{i_1, i_2, i_3} \frac{c_{i_1 i_2 i_3}}{D_{i_1} D_{i_2} D_{i_3}} + \sum_{i_1, i_2, i_3, i_4} \frac{d_{i_1 i_2 i_3 i_4}}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} \right] + R_1$$

- **repeated $\mathcal{N}(q)$ evaluations** at on-shell multiple cuts (complex q)
- **very efficient with** $\mathcal{N}(q) = \sum \mathcal{N}_{\mu_1 \dots \mu_r} \, q^{\mu_1} \dots q^{\mu_r}$ **construction** [see also tensorial reconstruction Heinrich, Ossola, Reiter, Tramontano '10]

Trees, loops and open loops

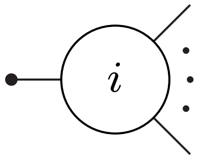
Tree generator

Colour-stripped **tree diagrams** are built by merging *sub-trees*



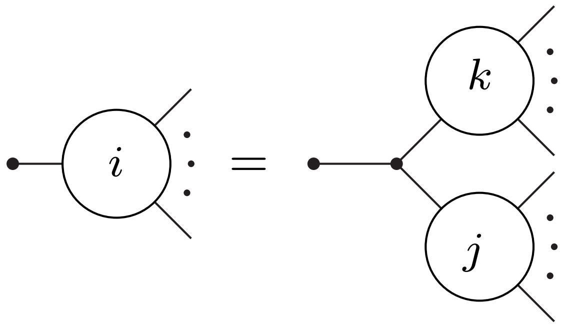
individual tree diagram

Sub-tree amplitudes are handled as *numerical* n-tuples

$w^{\beta}(i) =$ 

$\beta \leftrightarrow$ off-shell line spin

and *recursively merged* by attaching vertices and propagators



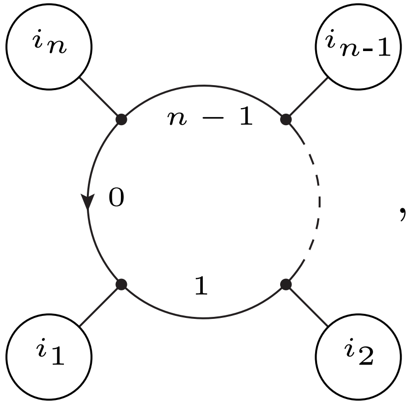
$$w^{\beta}(i) = \frac{X^{\beta}_{\gamma\delta}(i,j,k)}{p_i^2 - m_i^2}\, w^{\gamma}(j)\, w^{\delta}(k)$$

Flexible (only $\mathcal{L}_{\rm int}$ dependent) **and fast** (many diagrams share *common sub-trees*)

Cut loops and "open loops"

A colour-stripped **n-point loop diagram** is an **ordered set of n sub-trees** connected by loop propagators $D_i = (q+p_i)^2 - m_i^2 + \mathrm{i}\varepsilon$

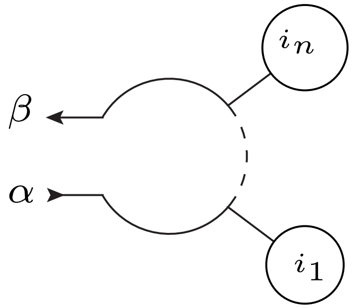
$$\int \frac{\mathrm{d}^D q\, \mathcal{N}(\mathcal{I}_n;q)}{D_0 D_1 \dots D_{n-1}} =$$



$$, \qquad \mathcal{I}_n = \{i_1,\dots,i_n\}$$

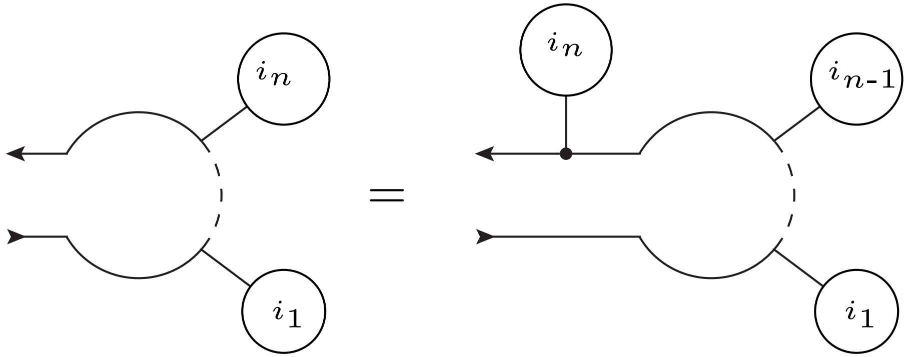
The **OPP-reduction** input $\mathcal{N}(\mathcal{I}_n;q)$ can be obtained by constructing **cut loops** with *tree-like generators*

$$\mathcal{N}_\alpha^\beta(\mathcal{I}_n;q) =$$



$$, \qquad \sum_\alpha \mathcal{N}_\alpha^\alpha(\mathcal{I}_n;q) = \mathcal{N}(\mathcal{I}_n;q)$$

Sub-trees along the loop are recursively attached to each other



Standard cut-loop construction

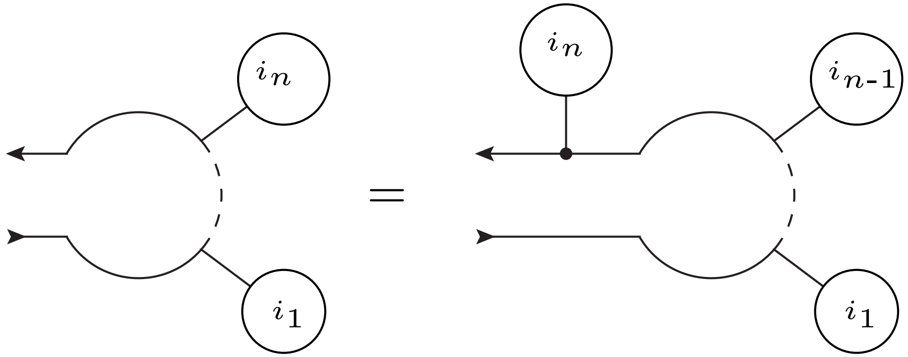
Build n-point cut loops by **merging lower-point cut loops and sub-trees**

$$\mathcal{N}_{\alpha}^{\beta}(\mathcal{I}_n;q) = X^{\beta}_{\gamma\delta}(\mathcal{I}_n, i_n, \mathcal{I}_{n-1})\, \mathcal{N}_{\alpha}^{\gamma}(\mathcal{I}_{n-1};q)\; w^{\delta}(i_n)$$

applying **conventional tree recursion**

- very high one-loop automation level in `Helac-NLO` (off-shell-current recursion), `MadLoop` (Feynman diagrams)
- CPU expensive OPP evaluations at multiple q-values since *tree algorithms conceived for fixed momenta*

Nature of loop amplitudes requires loop-momentum *functional* dependence!



Open-loops construction

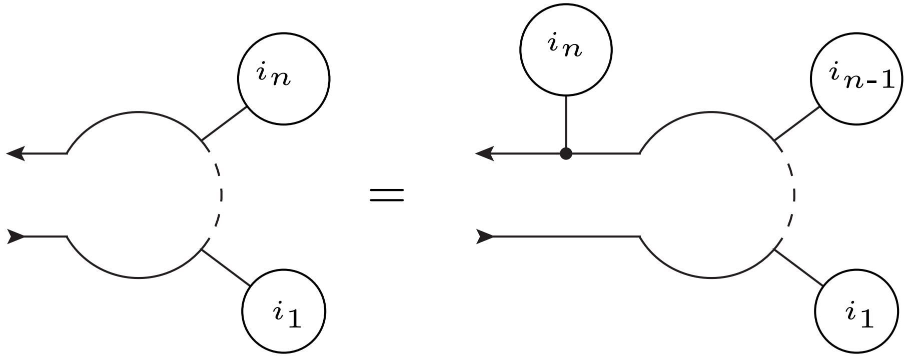
Handle building blocks of recursion as *polynomials* in the loop momentum q

$$\underbrace{\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n;q)}_{\sum_{r=0}^{n}\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)\; q^{\mu_1}\ldots q^{\mu_r}} = \underbrace{X^{\beta}_{\gamma\delta}(\mathcal{I}_n,i_n,\mathcal{I}_{n-1})}_{Y^{\beta}_{\gamma\delta}+q^{\nu}\; Z^{\beta}_{\nu;\gamma\delta}}\;\underbrace{\mathcal{N}^{\gamma}_{\alpha}(\mathcal{I}_{n-1};q)}_{\sum_{r=0}^{n-1}\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})\; q^{\mu_1}\ldots q^{\mu_r}}\; w^{\delta}(i_n)$$

and construct polynomial coefficients with

$$\underbrace{\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)}_{\text{"open loop"}} = \left[Y^{\beta}_{\gamma\delta}\,\mathcal{N}^{\gamma}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1}) + Z^{\beta}_{\mu_1;\gamma\delta}\,\mathcal{N}^{\gamma}_{\mu_2\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})\right]\; w^{\delta}(i_n)$$

encodes full q-dependence of cut loop



Open-loops construction

Handle building blocks of recursion as *polynomials* in the loop momentum q

$$\underbrace{\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n;q)}_{\sum_{r=0}^{n} \mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)\, q^{\mu_1}\ldots q^{\mu_r}} = \underbrace{X^{\beta}_{\gamma\delta}(\mathcal{I}_n,i_n,\mathcal{I}_{n-1})}_{Y^{\beta}_{\gamma\delta} + q^{\nu}\, Z^{\beta}_{\nu;\gamma\delta}} \; \underbrace{\mathcal{N}^{\gamma}_{\alpha}(\mathcal{I}_{n-1};q)}_{\sum_{r=0}^{n-1} \mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})\, q^{\mu_1}\ldots q^{\mu_r}} \; w^{\delta}(i_n)$$

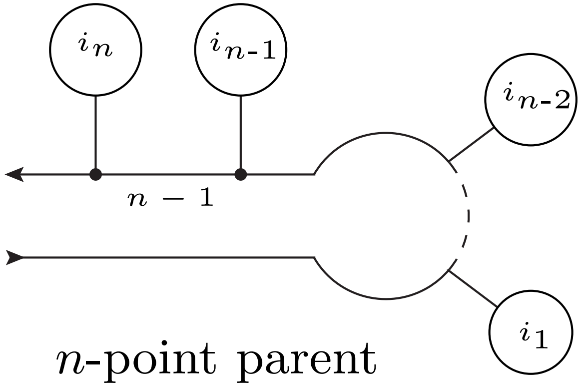
and construct polynomial coefficients with

$$\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n) = \left[Y^{\beta}_{\gamma\delta}\, \mathcal{N}^{\gamma}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1}) + Z^{\beta}_{\mu_1;\gamma\delta}\, \mathcal{N}^{\gamma}_{\mu_2\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})\right] w^{\delta}(i_n)$$

- fast OPP reduction of $\mathcal{N}(q)$ or tensor integrals with $\mathcal{N}_{\mu_1\ldots\mu_r}$
- combines *tree-recursion* and *tensor-integral* language
- simple concept but entirely new generator (fully flexible and automatic)

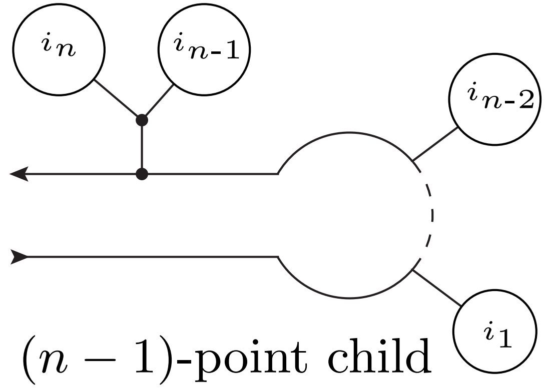
Recursive construction of higher-point open loops

Construct n-point *"parent diagrams"* from pre-computed parts of $(n-1)$-point *"child diagrams"* using **pinch relations**



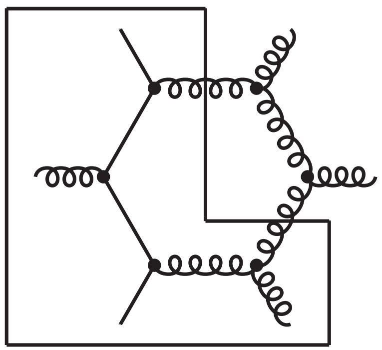
n-point parent

recycle $\mathcal{I}_{n-2}$ open loop $\Longleftarrow$

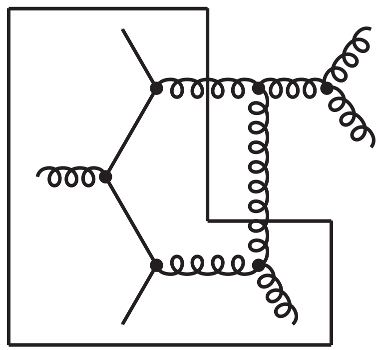


$(n-1)$-point child

Example



6-point parent



5-point child

Complicated diagrams require only "last missing piece" (always works in QCD!)

Implementation and technical performance

OpenLoops Implementation

Full automation of 1-loop QCD corrections to SM processes

- process-definition file $\Rightarrow$ `Fortran 90` tree & 1-loop matrix elements

Internal structure of the program

(1)	Topologies, field insertions	`FeynArts`
(2)	Skeleton of recursion, colour, code generation	`OpenLoops` (`Mathematica`)
(3)	Merging of sub-trees and open loops	`OpenLoops` (`Fortran 90`)
(4)	Tensor-integral reduction	`COLLIER` (Denner/Dittmaier/Hofer)
	OPP reduction	`CutTools 1.6.5` / `Samurai 1.0`

Performance studies

Four families of $2 \to 2, 3, 4$ reactions with $n = 0, 1, 2$ gluons

- $\mathrm{u\bar{u}} \to \mathrm{t\bar{t}} + n\mathrm{g}$
- $\mathrm{u\bar{u}} \to \mathrm{W^+W^-} + n\mathrm{g}$
- $\mathrm{u\bar{d}} \to \mathrm{W^+g} + n\mathrm{g}$
- $\mathrm{gg} \to \mathrm{t\bar{t}} + n\mathrm{g}$

Aim

- performance for non-trivial LHC processes
- scaling with particle multiplicity

Flexibility and Automation

Process	size [MB]	t_{code} [s]
$u\bar{u} \to t\bar{t}$	0.1	2.2
$u\bar{u} \to W^+W^-$	0.1	7.2
$u\bar{d} \to W^+g$	0.1	4.2
$gg \to t\bar{t}$	0.2	5.4
$u\bar{u} \to t\bar{t}g$	0.4	12.8
$u\bar{u} \to W^+W^-g$	0.4	39.8
$u\bar{d} \to W^+gg$	0.5	22.9
$gg \to t\bar{t}g$	1.2	52.9
$u\bar{u} \to t\bar{t}gg$	3.6 (200)*	236 ($\sim 10^6$)*
$u\bar{u} \to W^+W^-gg$	2.5 (1000)*	381.7 ($\sim 10^6$)*
$u\bar{d} \to W^+ggg$	4.2	366.2
$gg \to t\bar{t}gg$	16.0	3005

Compact code

- 100 kB to few MB object files
- $\mathcal{O}(10^2\text{–}10^3)$ compression in $2 \to 4$

Fast code generation/compilation

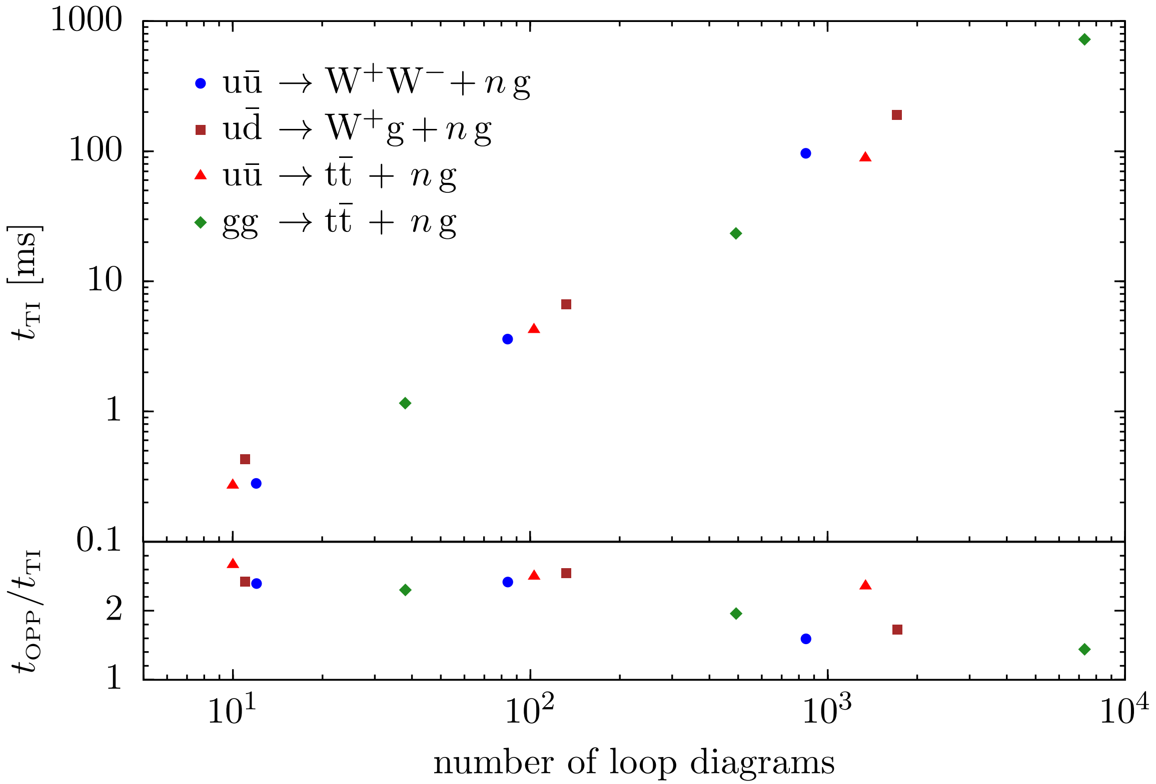
- few seconds to minutes
- $\mathcal{O}(10^3)$ speed-up in $2 \to 4$

Large-scale applicability!

*$\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ & $\mathrm{WWb\bar{b}}$ (Bredenstein, Denner, Dittmaier, Kallweit and S.P. ‘09–‘11)

High CPU efficiency for multi-particle LHC processes

Timings including col/hel sums (`single Intel i5-750` core)



$2 \to 4$ amplitudes

- $n_{\mathrm{diag}} = \mathcal{O}(10^3$–$10^4)$
- $t_{2\to 4} \lesssim 0.1$–$1\,\mathrm{s/point}$

Extrapolating linear scaling

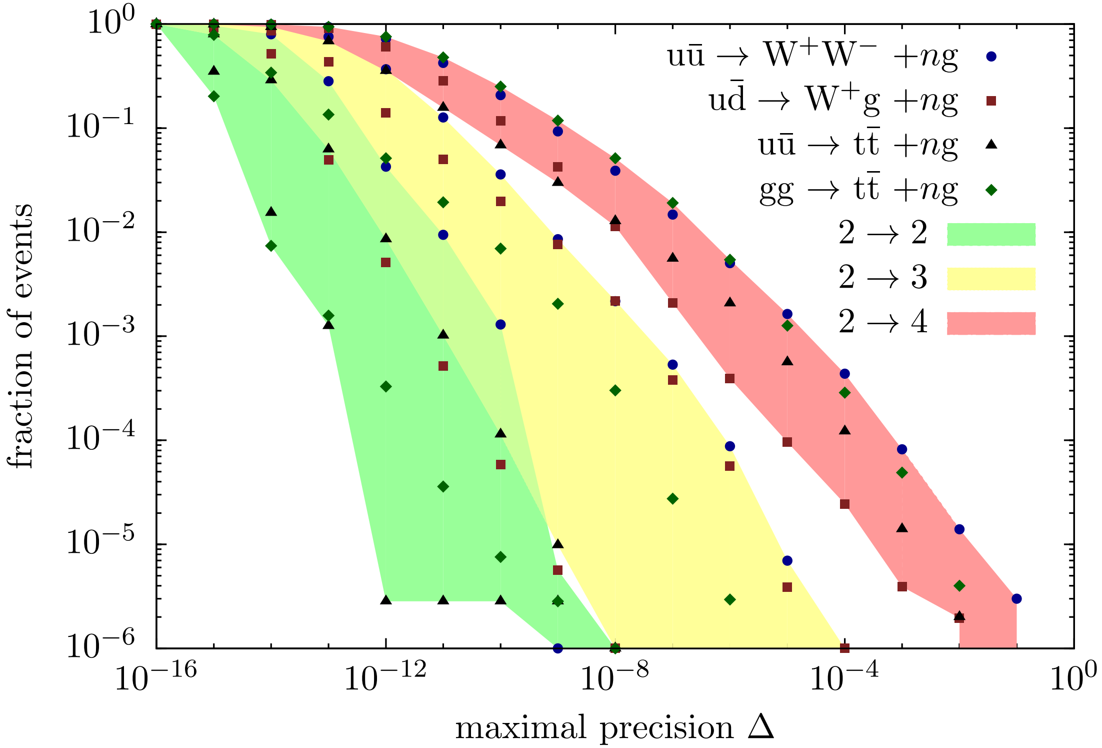
- $n_{\mathrm{diag}} = \mathcal{O}(10^5)$ and $2 \to 5$ feasible

$t_{\mathrm{OPP}} \simeq t_{\mathrm{TI}}$ with open loops!

Numerical stability with **tensor integrals** in double precision

Stability Δ_{S} in samples of 10^6 points ($\sqrt{\hat{s}} = 1\,\mathrm{TeV}$, $p_{\mathrm{T}} > 50\,\mathrm{GeV}$, $\Delta R_{ij} > 0.5$)

$$\{p_i, m_j, \mu\} \to \{\xi p_i, \xi m_j, \xi\mu\}, \qquad \delta\mathcal{W} \to \delta\mathcal{W}' = \xi^K \delta\mathcal{W}, \qquad \Delta_{\mathrm{S}} = \frac{\xi^{-K}\delta\mathcal{W}' - \delta\mathcal{W}}{\delta\mathcal{W}}$$



Average number of digits

- 11-15

Cross section accuracy

- depends on tails
- stability issues grow with n_{part}

$2 \to 4$ processes very stable

- $\lesssim 0.01\%$ prob. that $\Delta_{\mathrm{S}} < 10^{-3}$

High stability thanks to Gram-determinant (and other) expansions in `COLLIER`

Process-by-process validation

One-loop amplitudes carefully checked before pheno applications

(1) Self-consistency checks (necessary but insufficient!)

- UV/IR pole cancellations, Ward identities, OpenLoops vs standard trees

(2) Precision check against independent calculation

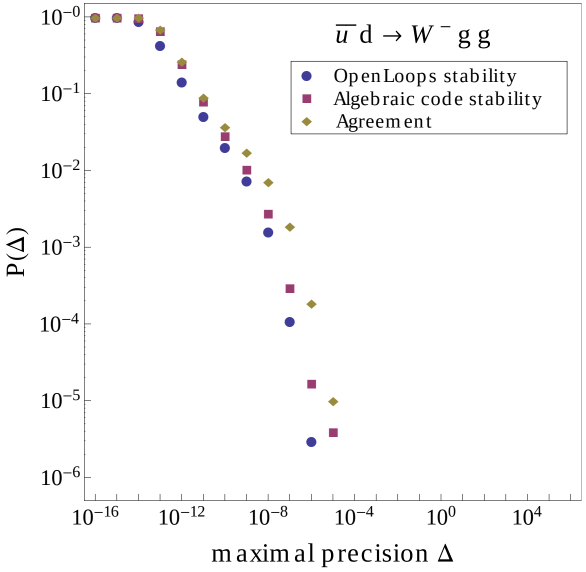
Process	OpenLoops	CAG	agreement
$\bar{u}d \to \mathrm{e}^- \bar{\nu}_\mathrm{e}$	$1.75911512013164631 \cdot 10^{-5}$	$1.75911512013164970 \cdot 10^{-5}$	$2 \cdot 10^{-15}$
$\bar{u}d \to \mathrm{e}^- \bar{\nu}_\mathrm{e}\, g$	$1.44264278623861696 \cdot 10^{-6}$	$1.44264278623860193 \cdot 10^{-6}$	$1 \cdot 10^{-14}$
$\bar{u}d \to \mathrm{e}^- \bar{\nu}_\mathrm{e}\, gg$	$1.46590850889200081 \cdot 10^{-9}$	$1.46590850889179753 \cdot 10^{-9}$	$1 \cdot 10^{-13}$
$u\bar{u} \to \mathrm{e}^+\mathrm{e}^-$	$3.39939790956756674 \cdot 10^{-5}$	$3.39939790956757419 \cdot 10^{-5}$	$2 \cdot 10^{-15}$
$u\bar{u} \to \mathrm{e}^+\mathrm{e}^-\mathrm{g}$	$7.00209271987758522 \cdot 10^{-7}$	$7.00209271987766145 \cdot 10^{-7}$	$1 \cdot 10^{-14}$
$u\bar{u} \to \mathrm{e}^+\mathrm{e}^-\mathrm{gg}$	$6.93325400698801091 \cdot 10^{-9}$	$6.93325400699020542 \cdot 10^{-9}$	$3 \cdot 10^{-13}$

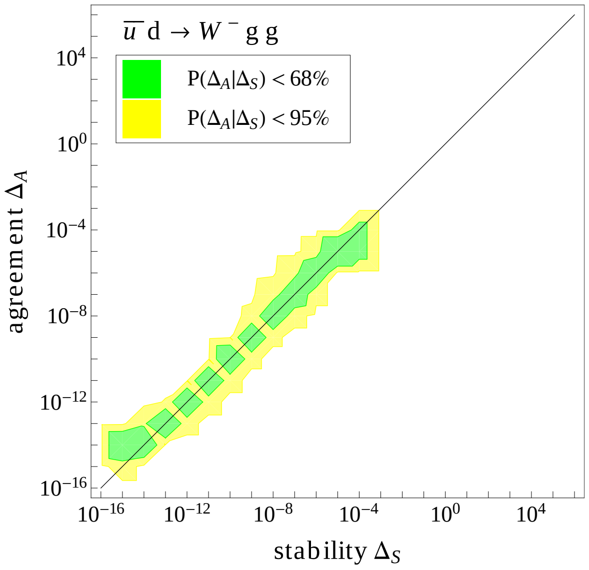
(CAG = computer-algebra generator developed for $\mathrm{t\bar{t}b\bar{b}}$ & $\mathrm{WWb\bar{b}}$)

similar agreement for more than 100 partonic reactions!

(3) Stability studies with large phase-space samples

OpenLoops vs CAG agreement (Δ_A) and intrinsic stability (Δ_S) well consistent (1-2 digit correlation at 95% CL)





(4) Stability monitor at runtime

- 20% of points with largest K-factor recomputed with 2$^{\text{nd}}$ tensor reduction
- wild instabilities with $K \gg 1$ extremely rare ($P < 10^{-4}$ in highly nontrivial processes) $\Rightarrow$ set $K \to 1$

OpenLoops+Sherpa

Goals and status

Goals

- full NLO automation from process definition to analysis output
- cover wide range of $2 \to 2, 3, 4$ processes
- automatic interface to `Sherpa, aMC@NLO, POWHEG`

Predictions

- fixed order (NLO)
- matching to shower (MC@NLO or POWHEG matching)
- merging different jet multiplicities (MEPS@NLO)

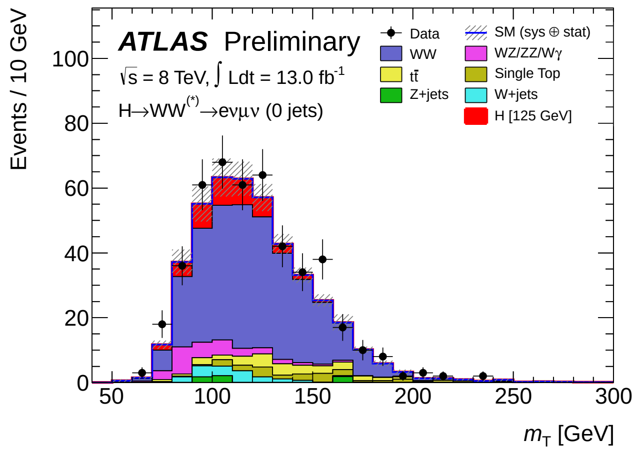
Status of `Sherpa+OpenLoops`

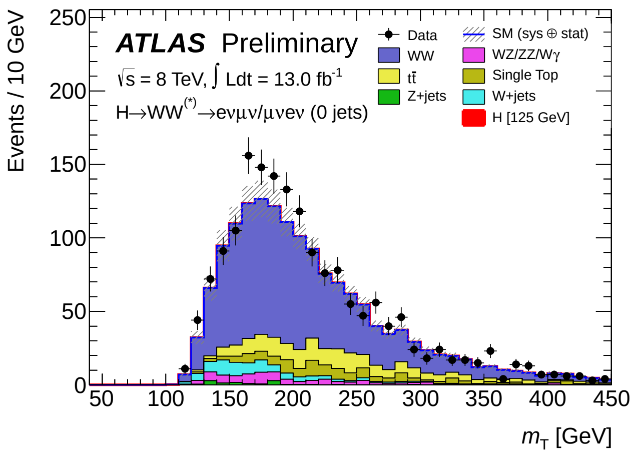
- fully automated: NLO calculations steered via Sherpa Runcards
- see online demonstration for $\mathrm{pp} \to \mathrm{e}^- \bar{\nu}_\mathrm{e} \mu^+ \nu_\mu$

Irreducible Background to $\mathrm{H} \to \mathrm{WW} \to \mathrm{e}^-\bar{\nu}_\mathrm{e}\mu^+\nu_\mu$

SIGNAL: $\Delta\phi_{e\mu} < 1.8,\ m_{e\mu} < 50\,\mathrm{GeV}$ CONTROL: $m_{e\mu} > 80\,\mathrm{GeV}$

0j

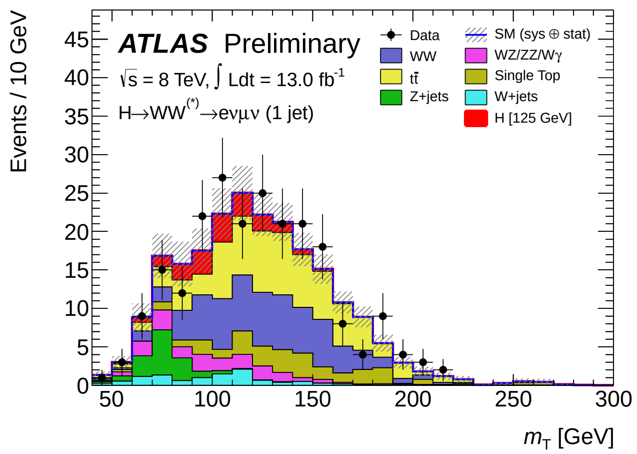


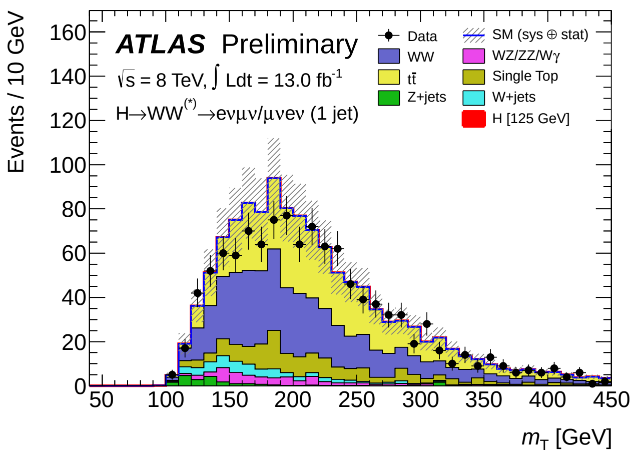


ATLAS analysis

- 0/1/2 jet bins
- WW+tt backg. from data+`POWHEG`
- $\mu_\mathrm{H} = 1.5 \pm 0.6$ $\Rightarrow 1.0 \pm 0.3$

1j





Theory issues

- jet-bin systematics?
- improve predictions in 1-jet bin!

Theory predictions for $\mathrm{pp} \to \mathrm{e}^- \bar{\nu}_\mathrm{e} \mu^+ \nu_\mu$+jets

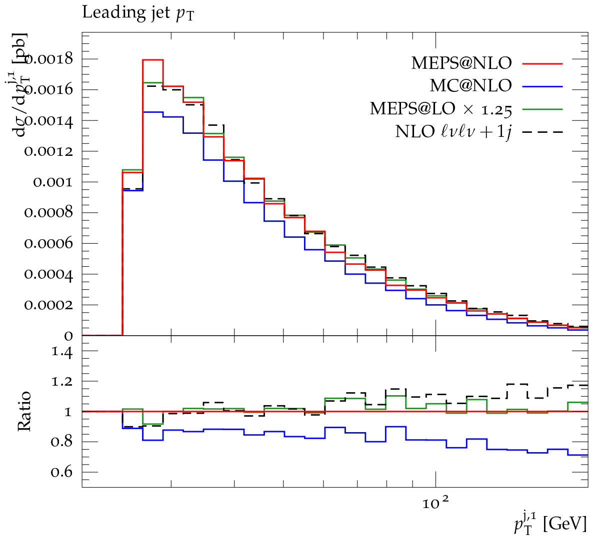
	NLO	gg-induced	NLO+PS
0 jets	Campbell ‘11	Binoth et al. ‘05 Campbell ‘11	Melia et al. ‘11 Frederix et al. ‘11
1 jet	Dittmaier et al. ‘07 (OS) Campbell et al. ‘07 (OS)	Melia et al. ‘12 Agrawal,Shivaji ‘12	
2 jets	Jäger et al. ‘06 (EW) Melia et al. ‘12 GoSam ‘12		Jäger,Zanderighi ‘13 (EW)

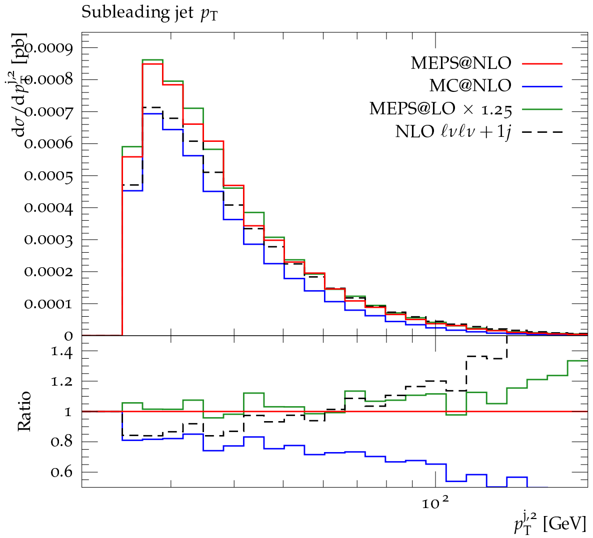
New predictions with Sherpa+OpenLoops (very preliminary!)

- MEPS@NLO for $ll\nu\nu + 0,1$ jets for ATLAS/CMS analysis
- all off-shell, interference and spin-correlation effects
- $gg \to ll\nu\nu + 0,1$ jets in progress

$\mathrm{pp} \to \mathrm{e}^-\bar{\nu}_\mathrm{e}\mu^+\nu_\mu$+jets with Sherpa+OpenLoops

$p_{\mathrm{T},1}$ $p_{\mathrm{T},2}$





ATLAS analysis cuts

Various approximations

NLO for $ll\nu\nu$+0,1 jets ⇒ no resummation

MC@NLO for $ll\nu\nu$ (incl) ⇒ LO in 1-jet bin

MEPS@NLO for $ll\nu\nu$ + 0,1 jets ⇒ NLO in 0- and 1-jet bins

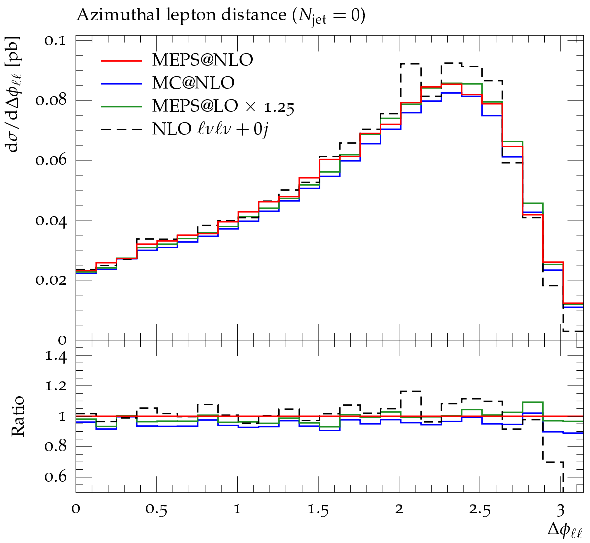
Theory uncertainties (not shown - work in progress)

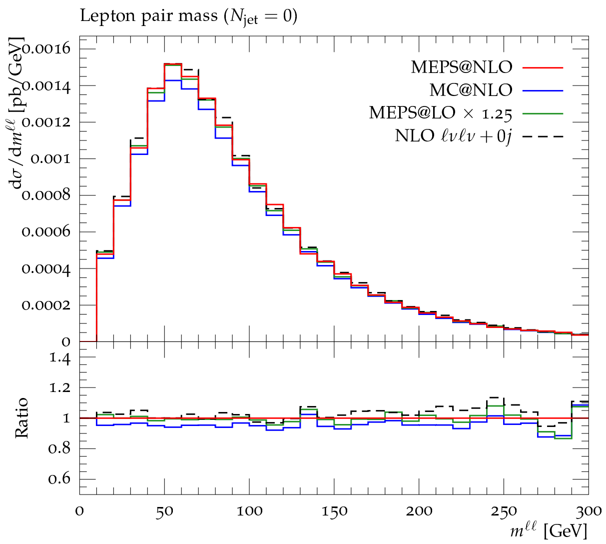
- QCD scales $\mu_\mathrm{R}, \mu_\mathrm{F}$ & resummation scale $\mu_\mathrm{Q} \Leftrightarrow \ln(p_\mathrm{T}^\mathrm{veto}) \Leftrightarrow$ realistic error in jet bins

Lepton observables in 0-jet and 1-jet bins (preselection cuts)

$\Delta\phi_{e\mu}$ | $m_{e\mu}$

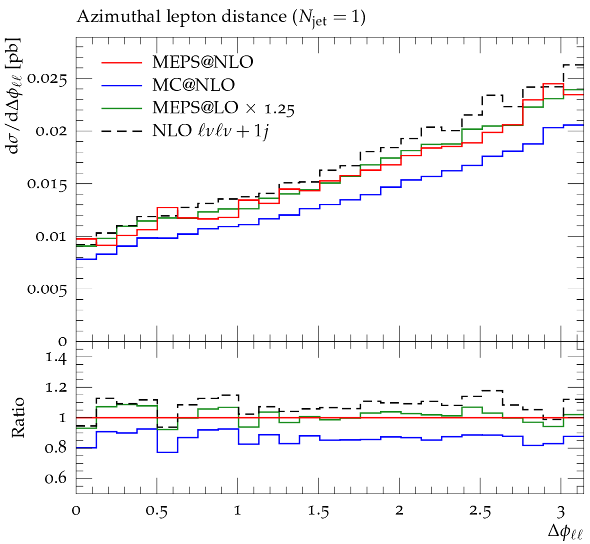
0j

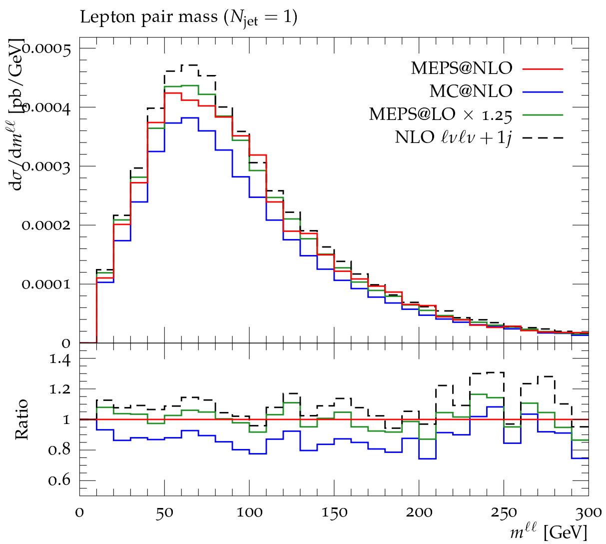




few-% agreement

1j



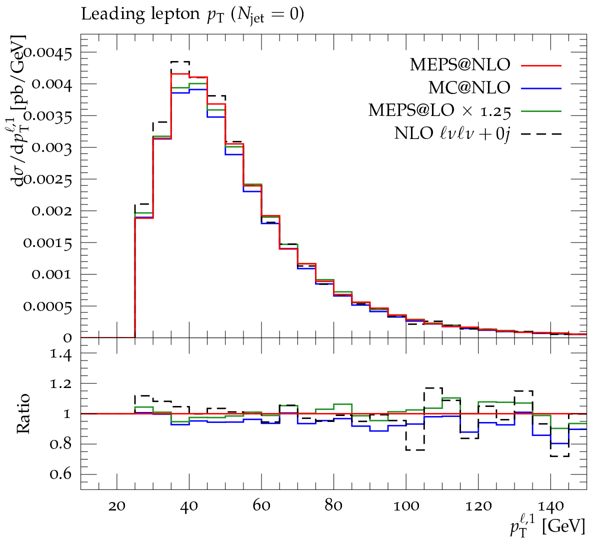


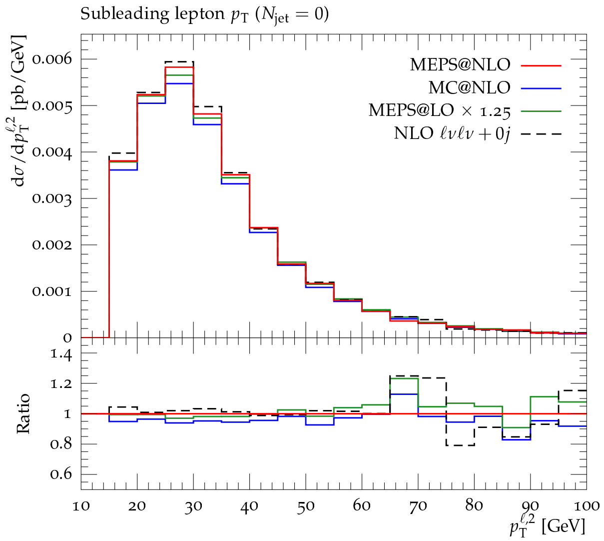
$\sim 10\%$ difference

Lepton p_{T} in 0-jet and 1-jet bins (preselection cuts)

p_{T,l_1} | p_{T,l_2}

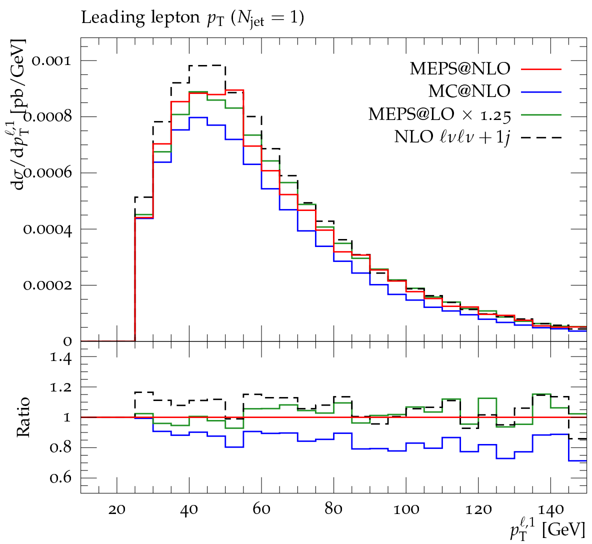
0j

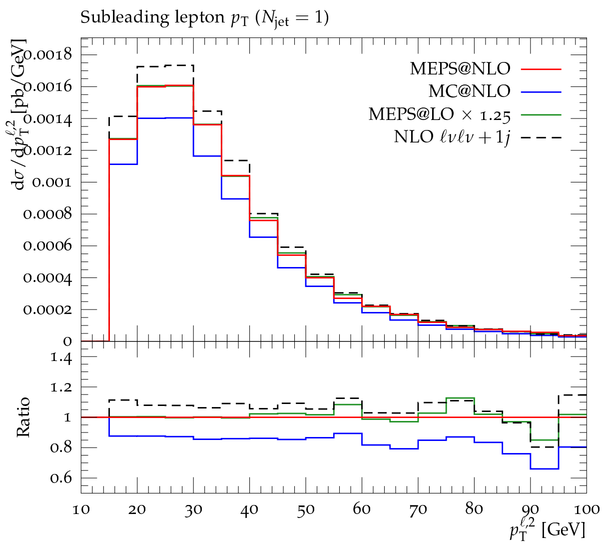




few-% agreement

1j



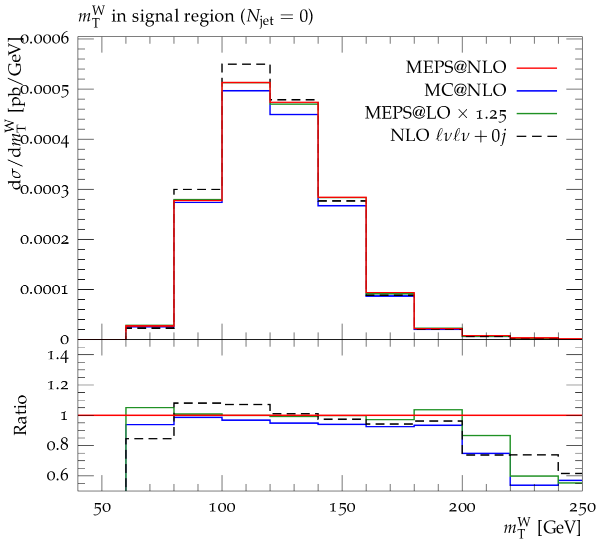


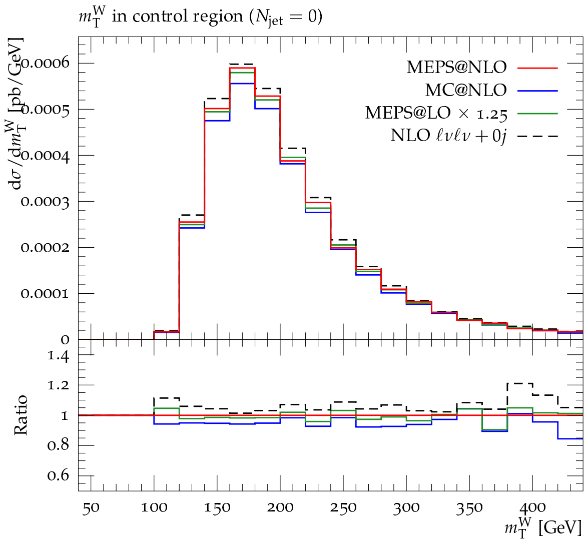
$\sim$ 10% difference
(shape distortion)

Transverse WW mass in signal and control regions

m_T, SIGNAL | m_T, CONTROL

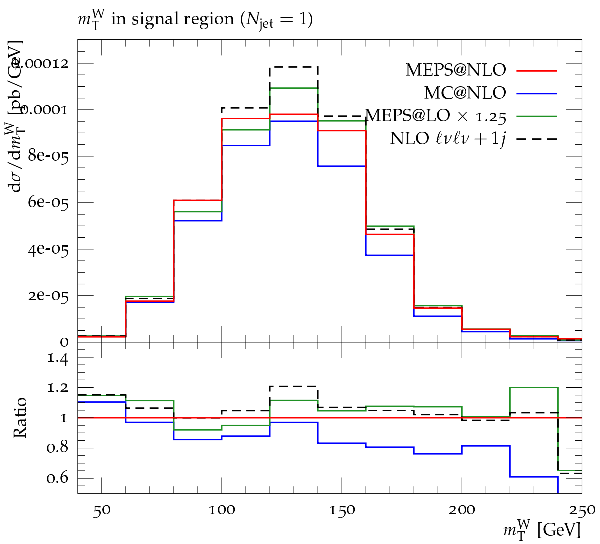
0j

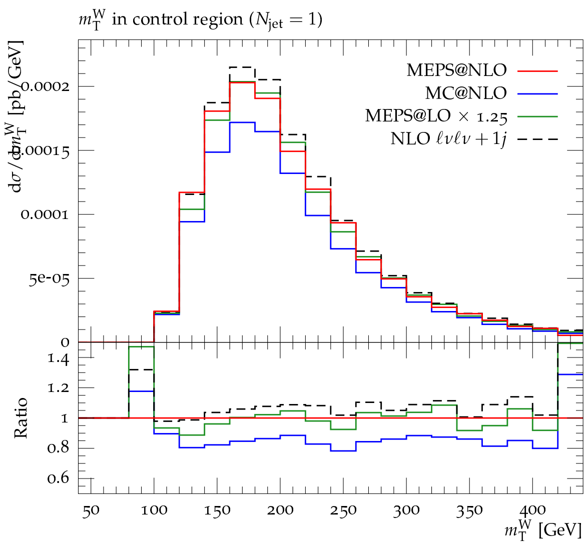




few-% agreement

1j





$\sim$ 10% difference
(shape distortion)

systematics must be understood at percent level for H $\rightarrow$ WW analysis!

Exclusive 0/1-jet XS in Signal (S) and Control (C) regions

0 jets	**NLO** $\pm\Delta_{\rm QCD}$	**MC@NLO** $\pm\Delta_{\rm QCD}\pm\Delta_{\rm res}$	**MEPS@NLO** $\pm\Delta_{\rm QCD}\pm\Delta_{\rm res}$
$\sigma_{\rm S}$ [pb]	34.9	32.6	34.1
$\sigma_{\rm C}$ [pb]	70.8	64.3	67.7
$\sigma_{\rm S}/\sigma_{\rm C}$	0.494	0.506	0.504
1 jet	**NLO** $\pm\Delta_{\rm QCD}$	**MC@NLO** $\pm\Delta_{\rm QCD}\pm\Delta_{\rm res}$	**MEPS@NLO** $\pm\Delta_{\rm QCD}\pm\Delta_{\rm res}$
$\sigma_{\rm S}$ [pb]	9.44	7.66	8.76
$\sigma_{\rm C}$ [pb]	28.9	23.0	27.3
$\sigma_{\rm S}/\sigma_{\rm C}$	0.326	0.333	0.321

NLO (no resum.) vs **MC@NLO** (LO for ≥ 1 j) vs **MEPS@NLO** (best)

- 0-jet bin: 5% differences in $\sigma_{\rm S,C}$ $\Rightarrow$ 1-2% in $\sigma_{\rm S}/\sigma_{\rm C}$
- 1-jet bin: 5–15% differences in $\sigma_{\rm S,C}$ $\Rightarrow$ 2–3% in $\sigma_{\rm S}/\sigma_{\rm C}$

Todo list

- **study QCD+resummation scale uncertainties** with MEPS@NLO
- consider theory-driven (alternatively to data-driven) WW estimate

Sherpa–OpenLoops process library for ATLAS/CMS

Status and available processes

- careful process-by-process validation (most processes ready)
- full set of NLO QCD diagrams, full colour
- off-shell leptonic W/Z decays: interferences, complex masses
- on-shell top quarks with LO decays

W/Z	γ	jets	HQ pairs	single-top	Higgs
V+3j	γ+3j	3(4)j	$\mathrm{t\bar{t}}$+1j	tb+1j	(H+2j)
VV+2j	$\gamma\gamma$+1(2)j		$\mathrm{t\bar{t}}V$+0(1)j	t+1(2)j	VH+1j
gg $\to VV$+1j	$V\gamma$+2j		$\mathrm{b\bar{b}}V$+0(1)j	tW+0(1)j	$\mathrm{t\bar{t}H}$
VVV+0(1)j					$qq \to \mathrm{H}qq$+0(1)j

lower jet multiplicities implicitly understood

Summary

Key features of OpenLoops

- numerical, recursive, diagrammatic
- trees replaced by "open loops" (loop-momentum polynomials)

Automation, flexibility and speed

- fully automated: input file $\Rightarrow$ compact code within $\mathcal{O}$(sec/min)
- flexible: QCD corrections to any SM process
- very fast up to $2 \to 4$ with $\mathcal{O}(10^4)$ diagrams

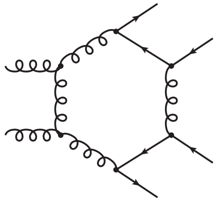
Sherpa+OpenLoops

- fully automated NLO, MC@NLO, MEPS@NLO generation
- library with lots of SM processes for ATLAS/CMS
- first pheno studies in progress

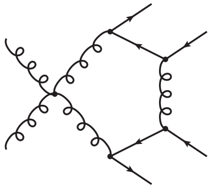
BACKUP SLIDES

Example: $gg \to t\bar{t}b\bar{b}$

Factorisation of colour structures (3 contributions per quartic vertex)



$$= \mathcal{A}^{(d)} f^{a_1 bd} f^{a_2 cd} \left(T^b T^e\right)_{i_3 i_4} \left(T^c T^e\right)_{i_5 i_6}$$



$$= \left[\mathcal{A}^{(d_1)} f^{a_1 bd} f^{a_2 cd} + \mathcal{A}^{(d_2)} f^{a_1 cd} f^{a_2 bd} + f^{a_1 a_2 d} f^{bcd} \mathcal{A}^{(d_3)}\right] \left(T^b T^e\right)_{i_3 i_4} \left(T^c T^e\right)_{i_5 i_6}$$

Fully automatic colour reduction with well-known SU(N) identities

$$f^{abc} T^c = -\mathrm{i}[T^a, T^b], \qquad T^a_{ij} T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N_\mathrm{c}}\delta_{ij}\delta_{kl}\right), \qquad \text{etc.}$$

Colour interference and summation once and for all at the level of colour basis

$$\begin{aligned}
\mathcal{C}_1 &= T^{a_1}_{i_3 i_4} T^{a_2}_{i_5 i_6}, \quad \mathcal{C}_2 = \delta_{i_3 i_4} \left(T^{a_1} T^{a_2}\right)_{i_5 i_6}, \quad \mathcal{C}_3 = \delta_{i_3 i_4} \left(T^{a_2} T^{a_1}\right)_{i_5 i_6}, \\
\mathcal{C}_4 &= T^{a_1}_{i_5 i_6} T^{a_2}_{i_3 i_4}, \quad \mathcal{C}_5 = \delta_{i_5 i_6} \left(T^{a_1} T^{a_2}\right)_{i_3 i_4}, \quad \mathcal{C}_6 = \delta_{i_5 i_6} \left(T^{a_2} T^{a_1}\right)_{i_3 i_4}, \\
\mathcal{C}_7 &= T^{a_1}_{i_3 i_6} T^{a_2}_{i_5 i_4}, \quad \mathcal{C}_8 = \delta_{i_3 i_6} \left(T^{a_1} T^{a_2}\right)_{i_5 i_4}, \quad \mathcal{C}_9 = \delta_{i_3 i_6} \left(T^{a_2} T^{a_1}\right)_{i_5 i_4}, \\
\mathcal{C}_{10} &= T^{a_1}_{i_5 i_4} T^{a_2}_{i_3 i_6}, \quad \mathcal{C}_{11} = \delta_{i_5 i_4} \left(T^{a_1} T^{a_2}\right)_{i_3 i_6}, \quad \mathcal{C}_{12} = \delta_{i_5 i_4} \left(T^{a_2} T^{a_1}\right)_{i_3 i_6}, \\
\mathcal{C}_{13} &= \delta^{a_1 a_2} \delta_{i_3 i_4} \delta_{i_5 i_6}, \quad \mathcal{C}_{14} = \delta^{a_1 a_2} \delta_{i_3 i_6} \delta_{i_5 i_4}.
\end{aligned}$$

Commuting helicity summation and tensor reduction

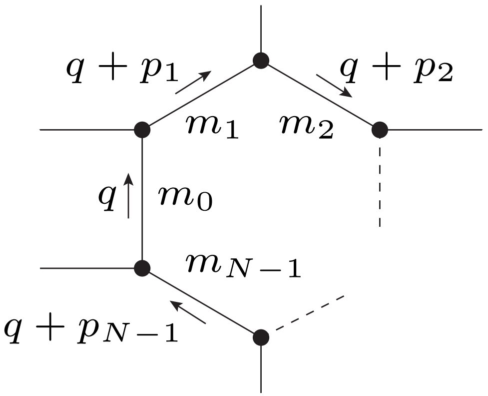
Seperation of helicity and loop-momentum dependence very useful

$$\delta\mathcal{M}^{(d')} = \mathcal{C}^{(d')} \int \frac{\mathrm{d}^D q}{D_0 D_1 \dots D_{n-1}} \left\{ \sum_{r=0}^{R} \mathcal{N}_{\mu_1 \dots \mu_r} \, q^{\mu_1} \dots q^{\mu_r} \right\}$$

No CPU expensive helicity-by-helicity reduction: first sum and then reduce only once

$$\sum_{\mathrm{hel,col}} \left[\mathcal{M}^* \delta\mathcal{M}^{(d')} \right] = \int \frac{\mathrm{d}^D q}{D_0 D_1 \dots D_{n-1}} \left\{ \sum_{r=0}^{R} \sum_{\mathrm{hel,col}} \left[\mathcal{M}^* \mathcal{C}^{(d)} \mathcal{N}_{\mu_1 \dots \mu_r} \right] q^{\mu_1} \dots q^{\mu_r} \right\}$$

Saves lot of CPU, works also with OPP reduction $\Rightarrow$ further strong OPP speed-up!



$$T_N^{\mu_1\dots\mu_P} = \int \mathrm{d}^D q \frac{q^{\mu_1}\dots q^{\mu_P}}{D_0\dots D_{N-1}}, \qquad D_i = (q+p_i)^2 - m_i^2 + \mathrm{i}\varepsilon$$

$$T_{1,2,3,4,\dots} = A, B, C, D, \dots$$

Recursive reduction to scalar integrals ($N \leq 4,\ P = 0$)

Collection of methods developed for $e^+e^- \to 4f$ [Denner/Dittmaier '05]

(A) **Space-time 4-dimensionality ($N \geq 5$)** — Melrose '65, Denner/Dittmaier '02–'05, Binoth/Guillet/Heinrich/Pilon/Schubert '05

(B) **Lorentz invariance ($N \leq 4$)** — Passarino/Veltman '79

(C) **Expansions to cure $\det(Z)$-instabilities** — Denner/Dittmaier '05

(A) Space-time 4-dimensionality for $N \geq 5$ [Binoth et al. '05; Denner, Dittmaier '05]

Linear dependence in $D = 4$ of *hexagon* external momenta $p_1, \ldots, p_5$ yields

$$\mathcal{F}^\mu = \begin{vmatrix} q^\mu & 2qp_1 & \ldots & 2qp_5 \\ p_1^\mu & 2p_1p_1 & \ldots & 2p_1p_5 \\ \vdots & \vdots & \ddots & \vdots \\ p_4^\mu & 2p_4p_1 & \ldots & 2p_4p_5 \\ 0 & f_1 & \ldots & f_5 \end{vmatrix} = 0, \qquad 2qp_i = D_i - D_0 - f_i$$

Rank-$(P+1)$ hexagon $\rightarrow$ **rank-P pentagon reduction** expanding along 1st row

$$\int \frac{q^{\mu_1} \ldots q^{\mu_P}}{D_0 D_1 \cdots D_5} \mathcal{F}^\mu = \begin{vmatrix} F^{\mu\mu_1\cdots\mu_P} & \Delta E^{\mu_1\cdots\mu_P}(1) & \ldots & \Delta E^{\mu_1\cdots\mu_P}(5) \\ p_1^\mu & 2p_1p_1 & \ldots & 2p_1p_5 \\ \vdots & \vdots & \ddots & \vdots \\ p_4^\mu & 2p_4p_1 & \ldots & 2p_4p_5 \\ 0 & f_1 & \ldots & f_5 \end{vmatrix} = \mathcal{O}(D-4)$$

- qualitatively similar reduction for all $N \geq 5$
- simultaneous N, P reduction until $N \leq 4$ and $P \leq 4$

(B) Tensor integrals with $N = 4, 3$ [Passarino/Veltman '79; Denner '93]

Rank reduction via contractions

$$2p_i^{\mu} q_{\mu} = -f_i + D_i - D_0, \qquad g^{\mu\nu} q_{\mu} q_{\nu} = m_0^2 + D_0$$

Covariant decomposition

$$T_N^{\mu_1 \dots \mu_P} = \sum_{i_1 \dots i_P = 0}^{N-1} T_{i_1 \dots i_P}^{(P)} \{g \dots p\}_{i_1 \dots i_P}^{\mu_1 \dots \mu_P}$$

General solution

$$2(D + P - N - 1)\, T_{00 i_3 \dots i_P}^{(P)} = \sum_{k=1}^{N-1} f_k\, T_{k i_3 \dots i_P}^{(P-1)} + 2m_0^2\, T_{i_3 \dots i_P}^{(P-2)} + \text{lower-point}$$

$$\sum_{n=1}^{N-1} Z_{mn}\, T_{n i_2 \dots i_P}^{(P)} = -2 \sum_{r=2}^{P} \delta_{m i_r}\, T_{00 i_2 \dots \hat{i}_r \dots i_P}^{(P)} - f_m\, T_{i_2 \dots i_P}^{(P-1)} + \text{lower-point}$$

- R_1 **rational terms** from **catalogue of UV residues**

$$(D - 4)\, T_{00 i_3 \dots i_P}^{(P)} = R_{00 i_3 \dots i_P}^{(P)}$$

- **unstable Gram-matrix inversion** in $\det(Z) \to 0$ regions

$$Z_{kl} = 2p_k p_l, \qquad (Z)_{kl}^{-1} = \frac{\tilde{Z}_{lk}}{\det(Z)}$$

$$\tilde{X}_{0j} T^{(P)}_{i_1\ldots i_P} = \det(Z)\, T^{(P+1)}_{j i_1\ldots i_P} + 2\sum_{n=1}^{N-1} \tilde{Z}_{jn} \sum_{r=1}^{P} \delta_{n i_r} T^{(P+1)}_{00 i_1\ldots \hat{i}_r \ldots i_P} + \text{lower-point}$$

$$2\tilde{Z}_{kl} T^{(P+1)}_{00 i_2\ldots i_P} = \Big\{ -\det(Z)\, T^{(P+1)}_{kl i_2\ldots i_P} + 2m_0 \tilde{Z}_{kl} T^{(P-1)}_{i_2\ldots i_P} + \sum_{n,m=1}^{N-1} \Big[f_n f_m T^{(P-1)}_{i_2\ldots i_P} + 2\sum_{r=2}^{P} (f_n \delta_{m i_r} + f_m \delta_{n i_r})$$

$$\times T^{(P)}_{00 i_2\ldots \hat{i}_r \ldots i_P} + 4 \sum_{\substack{r,s=2 \\ r\neq s}}^{P} \delta_{n i_r} \delta_{m i_s} T^{(P+1)}_{0000 i_2\ldots \hat{i}_r \ldots \hat{i}_s \ldots i_P} \Big] \tilde{\tilde{Z}}_{(kn)(lm)} + \text{lower-point} \Big\} \Big(D+1+P-N+\sum_{r=2}^{P} \bar{\delta}_{i_r 0}\Big)^{-1}$$

(C) $\det(Z)$**-expansion for** *tensor* **integrals with** $N = 4, 3$ [Denner/Dittmaier '05]

Iterative expansion with K adapted to $\det(Z)$ and target precision

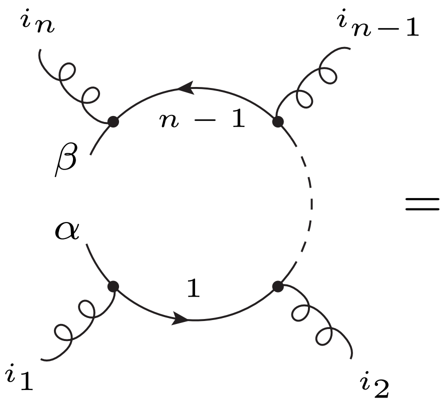
$$T^{(P)} = T^{(P)}_K + \mathcal{O}\left[\det(Z)^{K+1}\right]$$

Various alternative methods: further expansions ($\tilde{Z}_{kl}$ or $\tilde{X}_{0j} = -\sum_k \tilde{Z}_{jk} f_k$ small); modified set of MIs ($T_0 \to T_{00\ldots00}$); solutions of PV-identities with $\det(Z) \to \det(Y)$

General & robuts solution to instability problems (important for $2 \to 4$!)

Example of Open Loops construction: fermion loop

$$\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n;q) =$$



$$= g_{\mathrm{s}}\left[(\not{q} + \not{p}_n + m)\gamma^{\nu}\right]_{\beta\gamma}\, \mathcal{N}^{\gamma}_{\alpha}(\mathcal{I}_{n-1};q)\, \varepsilon^{*}_{\nu}(p_n,\lambda_n)$$

- n-point open-loop coefficients of rank $r = 0, 1, \ldots, n$

$$\begin{aligned}
\mathcal{N}^{\beta}_{;\alpha}(\mathcal{I}_n) &= g_{\mathrm{s}}\left[(\not{p}_n + m)\gamma^{\nu}\right]_{\beta\gamma} \mathcal{N}^{\gamma}_{;\alpha}(\mathcal{I}_{n-1})\, \varepsilon^{*}_{\nu}(p_n,\lambda_n)\\
\mathcal{N}^{\beta}_{\mu_1;\alpha}(\mathcal{I}_n) &= g_{\mathrm{s}}\left\{\left[(\not{p}_n + m)\gamma^{\nu}\right]_{\beta\gamma} \mathcal{N}^{\gamma}_{\mu_1;\alpha}(\mathcal{I}_{n-1}) + \left[\gamma_{\mu_1}\gamma^{\nu}\right]_{\beta\gamma} \mathcal{N}^{\gamma}_{;\alpha}(\mathcal{I}_{n-1})\right]\ \varepsilon^{*}_{\nu}(p_n,\lambda_n)\\
&\text{etc.}
\end{aligned}$$

- initial condition for 0-point rank-0 open loop

$$\mathcal{N}^{\gamma}_{;\alpha}(\mathcal{I}_0) = \delta^{\gamma}_{\alpha}$$

- rank, i.e. complexity, increases with n $\Rightarrow$ symmetrised $\mu_1 \ldots \mu_r$ components!

Speed of one-loop amplitudes with tensor integrals

Process	$t^{\mathrm{TI}}_{\mathrm{pol}}$ [ms]	n_{hel}	$t^{\mathrm{TI}}_{\mathrm{unpol}}$ [ms]
$u\bar{u} \to t\bar{t}$	0.25	2	0.27
$u\bar{u} \to W^+W^-$	0.25	2	0.28
$u\bar{d} \to W^+g$	0.39	2	0.43
$gg \to t\bar{t}$	0.89	4	1.16
$u\bar{u} \to t\bar{t}g$	3.5	4	4.2
$u\bar{u} \to W^+W^-g$	2.7	4	3.6
$u\bar{d} \to W^+gg$	5.3	4	6.7
$gg \to t\bar{t}g$	13.6	8	23.4
$u\bar{u} \to t\bar{t}gg$	56.2	8	88.4 (180)*
$u\bar{u} \to W^+W^-gg$	65.6	8	96.4 (180)*
$u\bar{d} \to W^+ggg$	134.5	8	190.5
$gg \to t\bar{t}gg$	335.0	16	725.0

(W/t decays to massless fermions)

Timings including col/hel sums

- $2 \to 2$: $t_{\mathrm{unpol}} \lesssim 1\,\mathrm{ms/point}$
- $2 \to 4$: $t_{\mathrm{unpol}} \lesssim 0.1\text{–}1\,\mathrm{s/point}$

very fast!

Efficient helicity summation

- for $2 \to 4$ processes saves factor

$$\frac{n_{\mathrm{hel}} t_{\mathrm{pol}}}{t_{\mathrm{unpol}}} \simeq 5\text{–}7$$

*pp → $t\bar{t}b\bar{b}$ & WW$b\bar{b}$ (Bredenstein, Denner, Dittmaier, Kallweit and S.P '09/'10)