LIDER/02/35/L-2/10/NCBiR/2011

Narodowe Centrum
Badań i Rozwoju

Solution of evolution equations in resummed form

Dawid Toton

Instytut Fizyki Jądrowej
im. Henryka Niewodniczańskiego
Polskiej Akademii Nauk

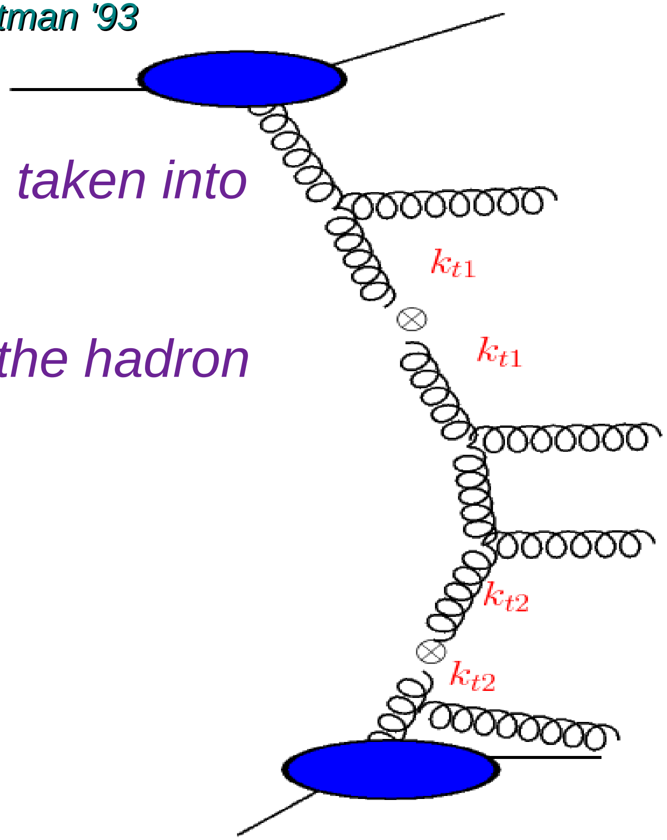
Report on research by/with:

Krzysztof Bozek
Krzysztof Kutak
Wieslaw Placzek
Magdalena Slawinska

QCD at high energies

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,b,c,d} \int \frac{d^2k_{1t}}{\pi}\frac{d^2k_{2t}}{\pi}\frac{1}{16\pi^2(x_1x_2S)^2}\overline{|\mathcal{M}_{ab\to cd}|}^2\delta^2(\vec{k}_{1t}+\vec{k}_{2t}-\vec{p}_{1t}-\vec{p}_{2t})$$
$$\times\ \phi_{a/A}(x_1,k_{1t}^2,\mu^2)\,\phi_{b/B}(x_2,k_{2t}^2,\mu^2)\,\frac{1}{1+\delta_{cd}}$$

Gribov, Levin, Ryskin '81
Ciafaloni, Catani, Hautman '93



•*Longitudinal and transversal parton degrees of freedom taken into account; also hard scale*

•*Capable of taking into account finite transversal size of the hadron*

•*Realistic kinematics at lowest order*

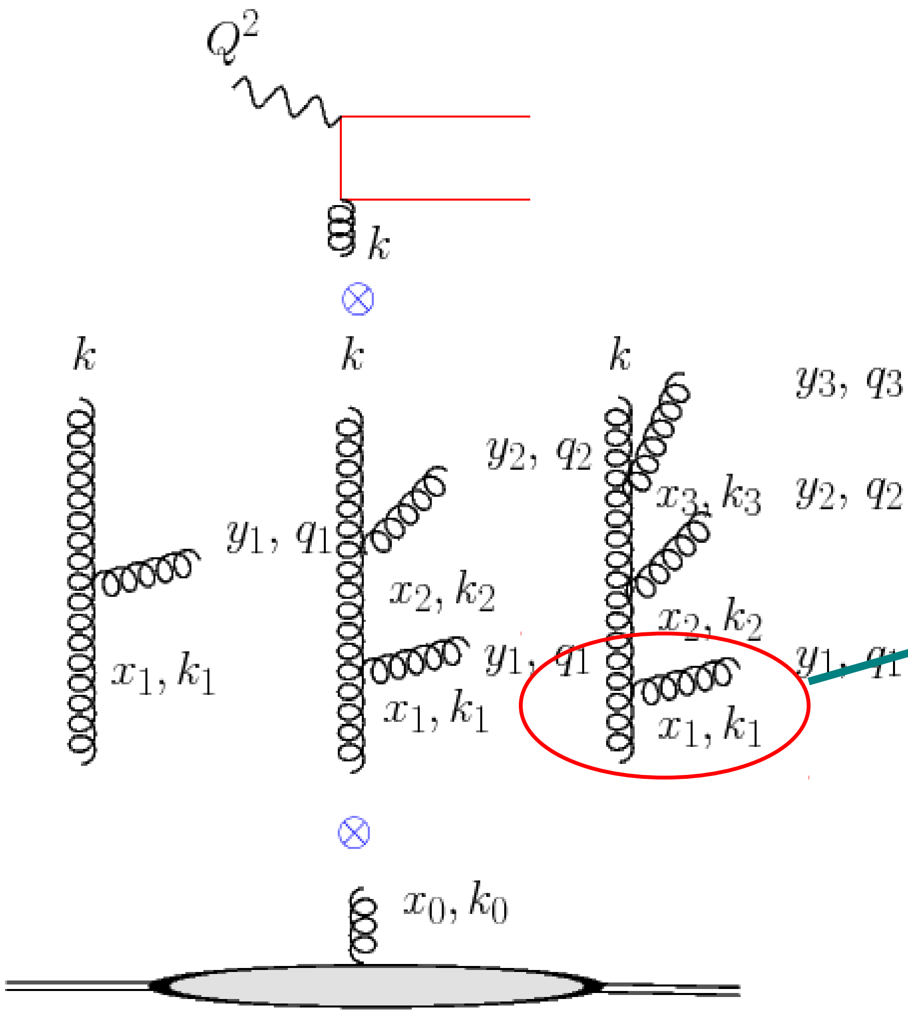
•*Gluon density depends on* k_t

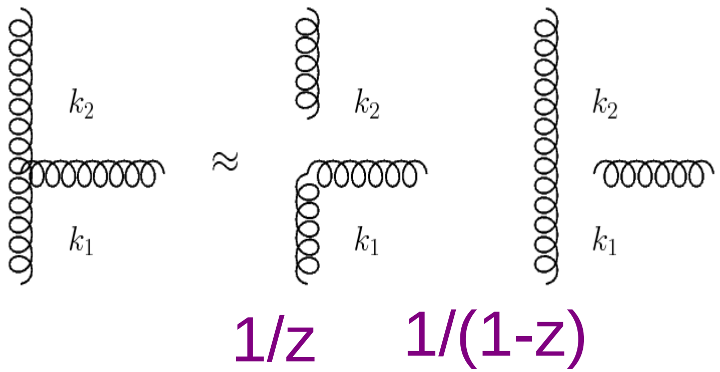
• *Gauge invariant matrix elements with off-shell gluons*

Lipatov '95

van Hameren, Kotko, Kutak 2012

CCFM evolution equation





Implemented in CASCADE
Monte Carlo ***H. Jung 02***

CCFM evolution equation

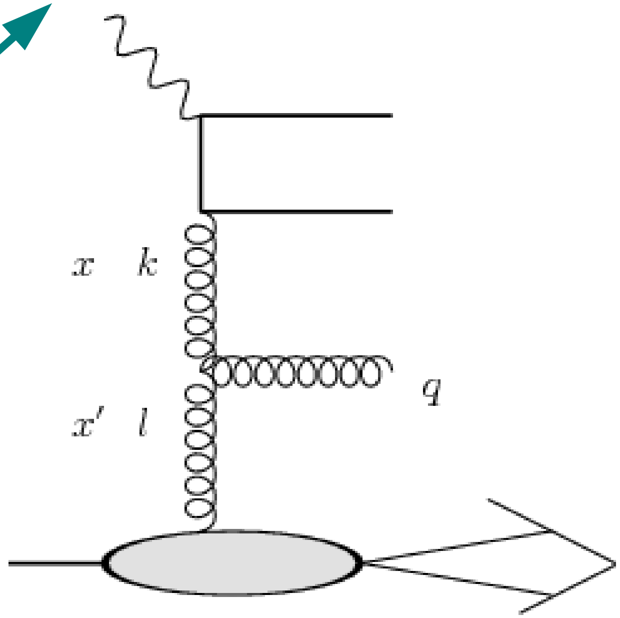
For simplicity let's consider low x limit of the CCFM

$$\mathcal{A}(x,k^2,p) = \mathcal{A}_0(x,k^2,p) + \bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}^2\mathbf{q}}{\pi q^2} \theta(p - zq) \Delta_{ns}(z,k,q) \mathcal{A}\left(\frac{x}{z}, k'^2, q\right)$$

p – linked to some
hard scale

splitting

coherence

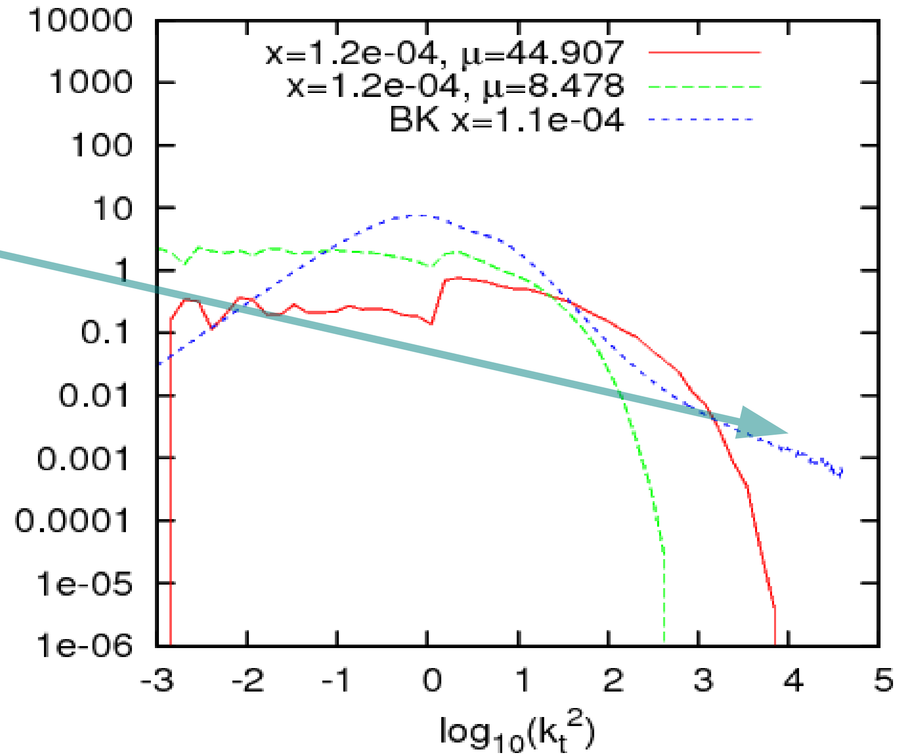


Linear equation – problems with unitarity.
Let us come back for a while to BK...

Forward physics as the way to constrain gluon both at large and small pt

- Too flat behaviour of at large kt
- Lack of saturation in CCFM small k_t

$$\mathcal{F}(x,k^2) = \frac{N_c}{\alpha_s \pi^2} k^2 \nabla_k^2 \Phi(x,k^2)$$



Needed a framework which unifies both correct behaviors

Resummed form of the BK

K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek '11

The strategy:

- *Use the equation for WW gluon density. Simple nonlinear term*
- *Split linear kernel into resolved and unresolved parts*
- *Resum the virtual contribution and unresolved ones in the linear part*
- *Postulate nonlinear CCFM by analogy*

The starting point:

$$\Phi(x,k^2) = \Phi_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^{1} \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2\Phi(x/z,l^2) - k^2\Phi(x/z,k^2)}{|k^2-l^2|} + \frac{k^2\Phi(x/z,k^2)}{\sqrt{(4l^4+k^4)}} \right] - \frac{\overline{\alpha}_s}{\pi R^2} \int_{x/x_0}^{1} \frac{dz}{z} \Phi^2(x/z,k^2)$$

$$\pi R^2 = 1$$

BK equation in the resummed exclusive form

K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek '11

$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 dz \int \frac{d^2\mathbf{q}}{\pi q^2}\,\theta(q^2-\mu^2)\,\frac{\Delta_R(z,k,\mu)}{z}\Big[\Phi(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2) - q^2\delta(q^2-k^2)\,\Phi^2(\frac{x}{z},q^2)\Big] \quad (1$$

$$\Delta_R(z,k,\mu) \equiv \exp\left(-\overline{\alpha}_s \ln\frac{1}{z}\ln\frac{k^2}{\mu^2}\right)$$

- The same resumed piece for linear and nonlinear
- Initial distribution also gets multiplied by the Regge form factor
- New scale introduced to equation. One has to check dependence of the solution on it
- Suggestive form to promote the CCFM equation to nonlinear equation which is more suitable for description of final states

Equation for exclusive states and saturation

Original formulation of BK or BFKL- difficult to address final state problem. One of possible solutions is to combine physics of BK with CCFM

Kutak 2012, JHEP

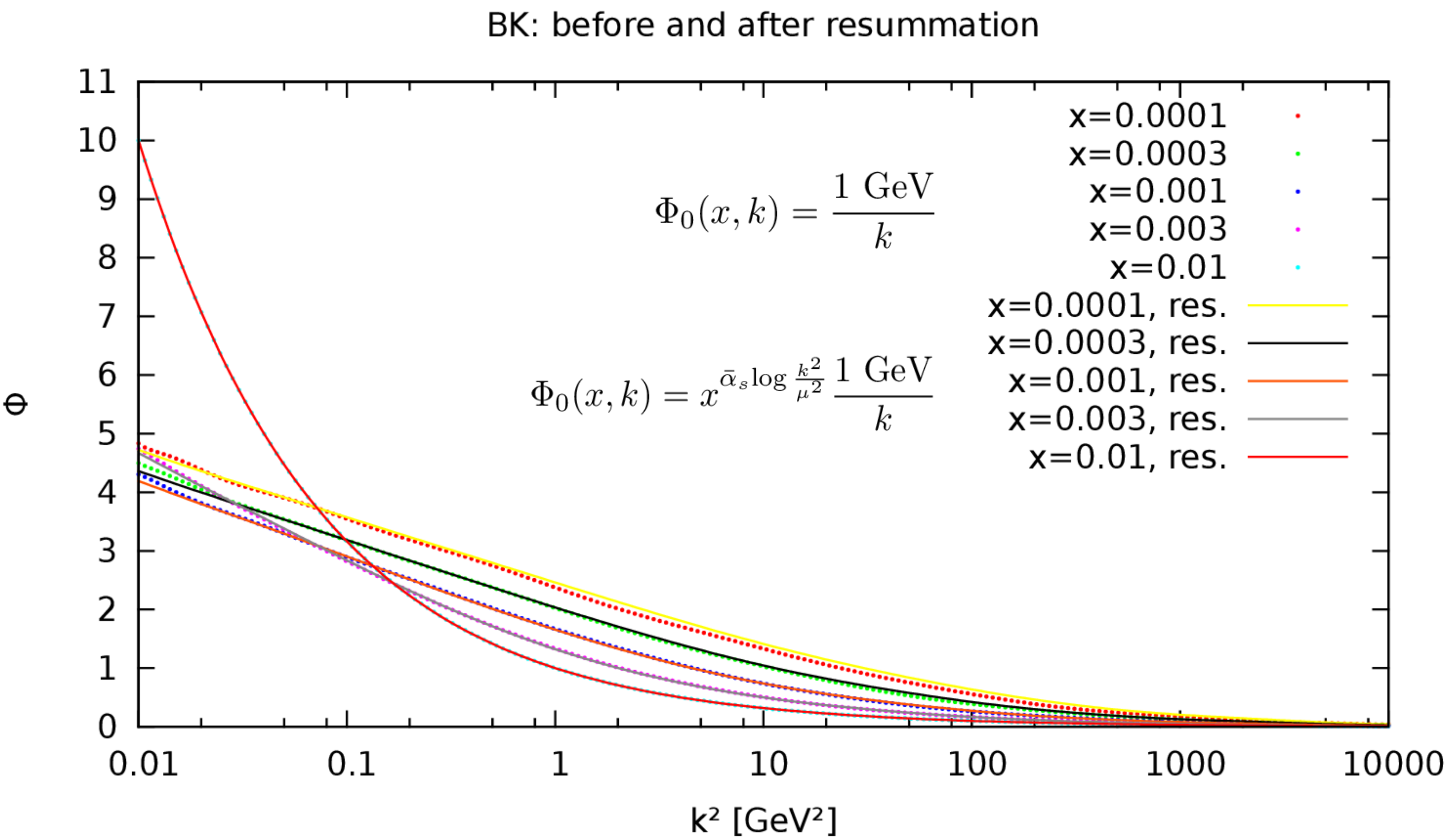
$$\mathcal{E}(x,k^2,p) = \mathcal{E}_0(x,k^2,p)$$
$$+\bar{\alpha}_s \int_x^1 \frac{\mathrm{d}z}{z} \int \frac{\mathrm{d}^2\mathbf{q}}{\pi q^2}\theta(p-zq)\Delta_{ns}(z,k,q)\left[\mathcal{E}\left(\frac{x}{z},k'^2,q\right) - q^2\delta(q^2-k^2)\mathcal{E}^2\left(\frac{x}{z},q^2,q\right)\right]$$

p – linked to some
hard scale

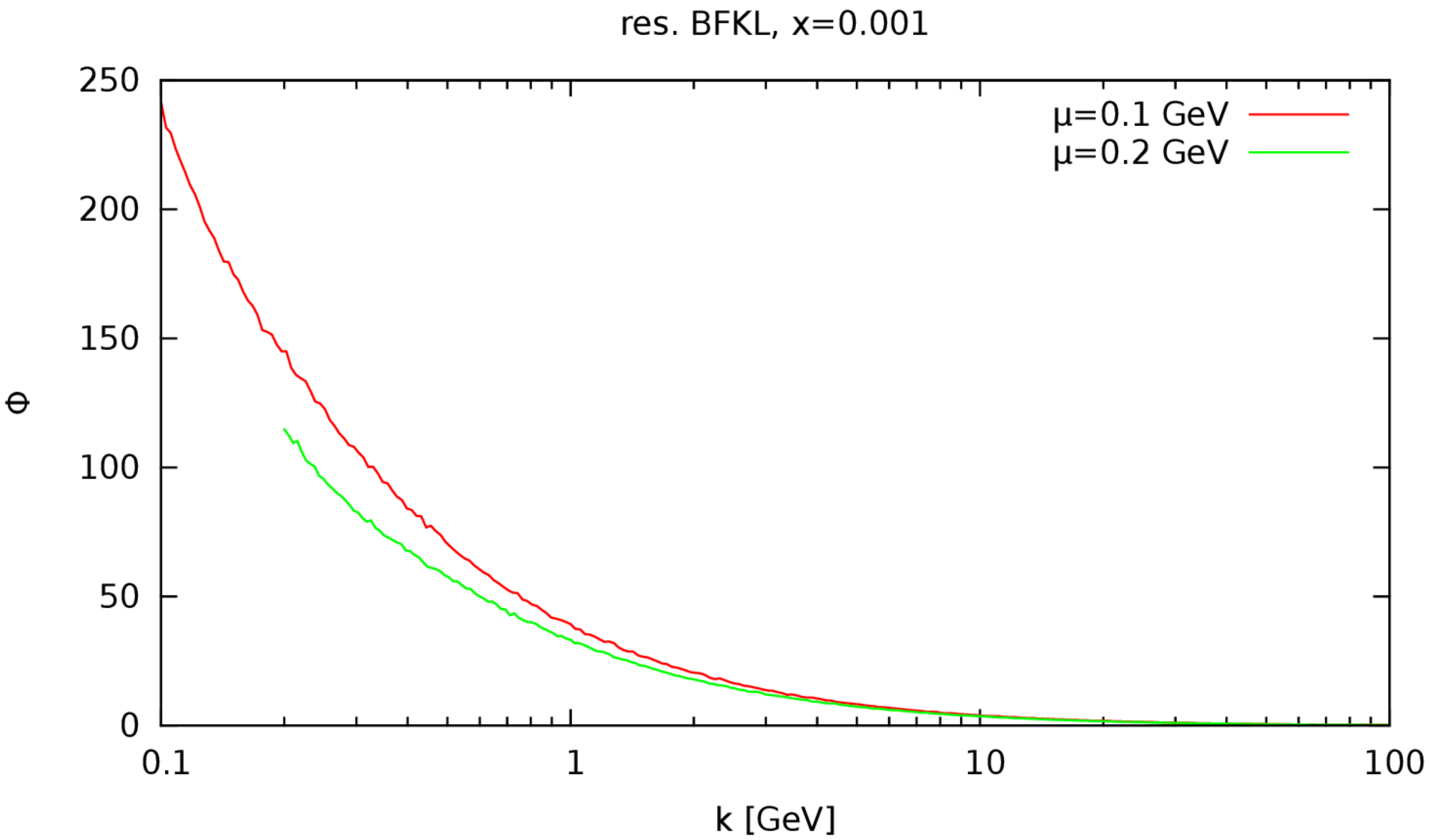
coherence

saturation

Resummation of BK – numerical results

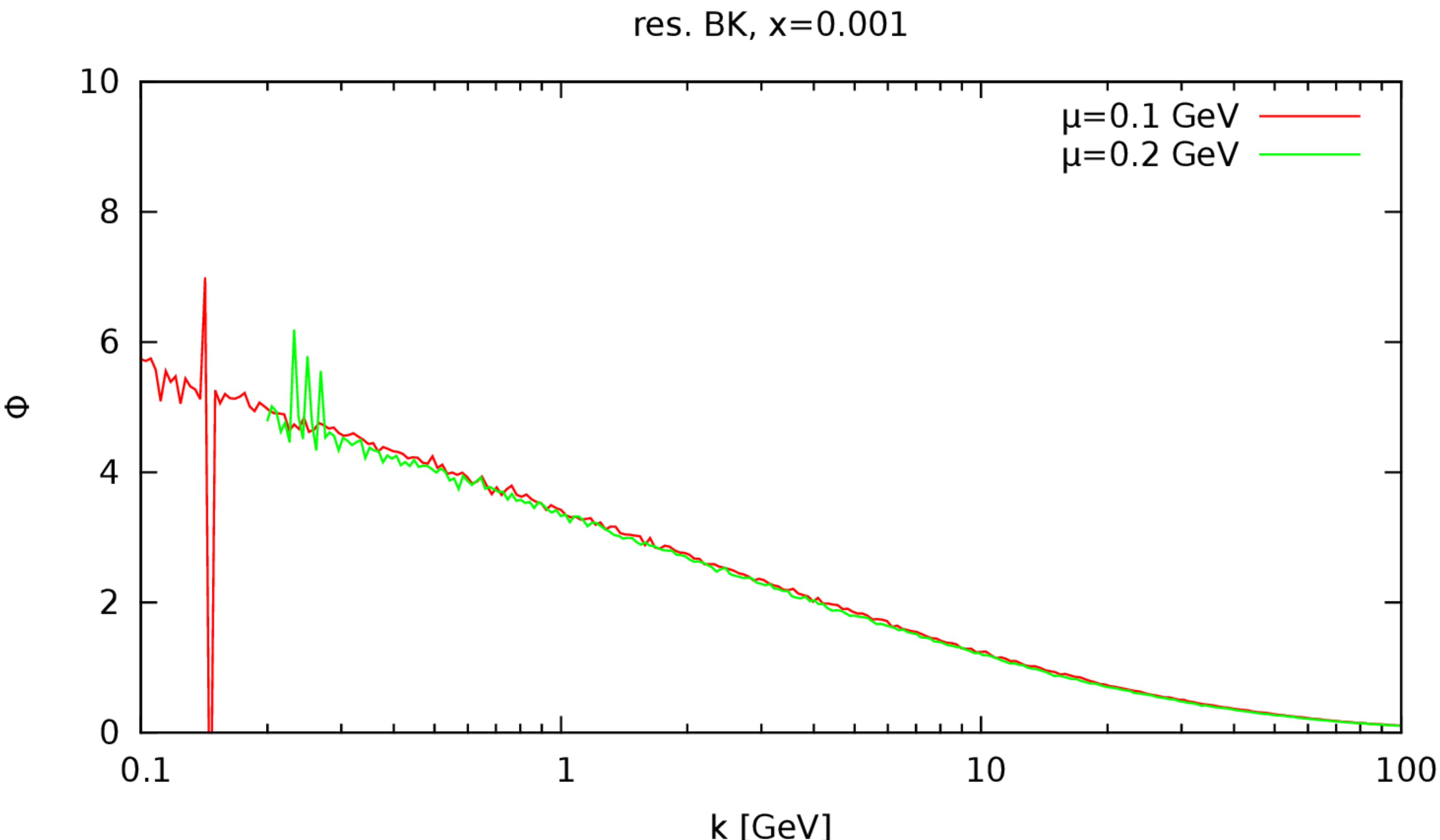


Numerical studies – BFKL



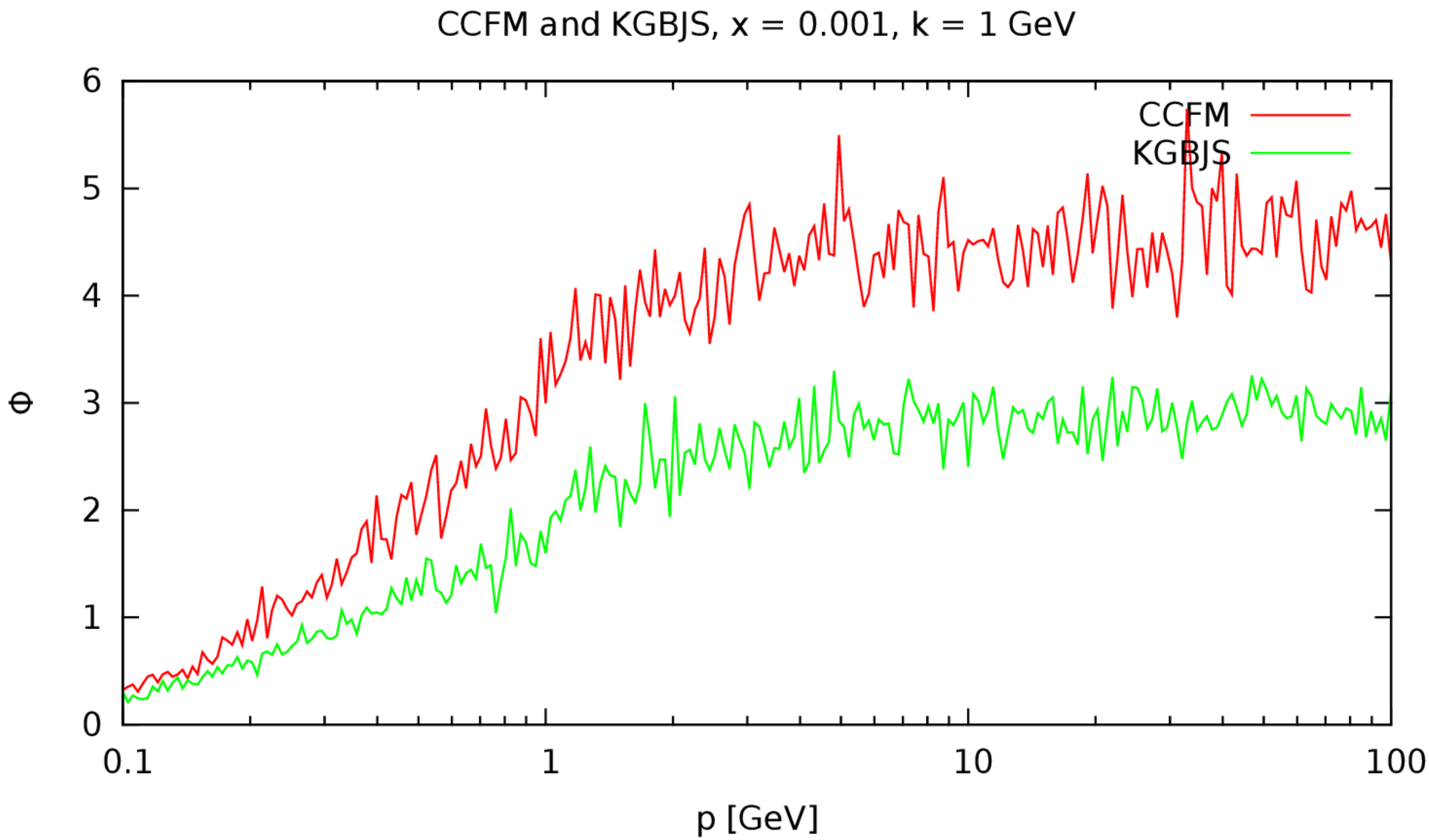
$$\Phi_0(x,k) = x^{\bar{\alpha}_s \log \frac{k^2}{\mu^2}} \frac{1\ \mathrm{GeV}}{k}$$

Numerical studies – BK



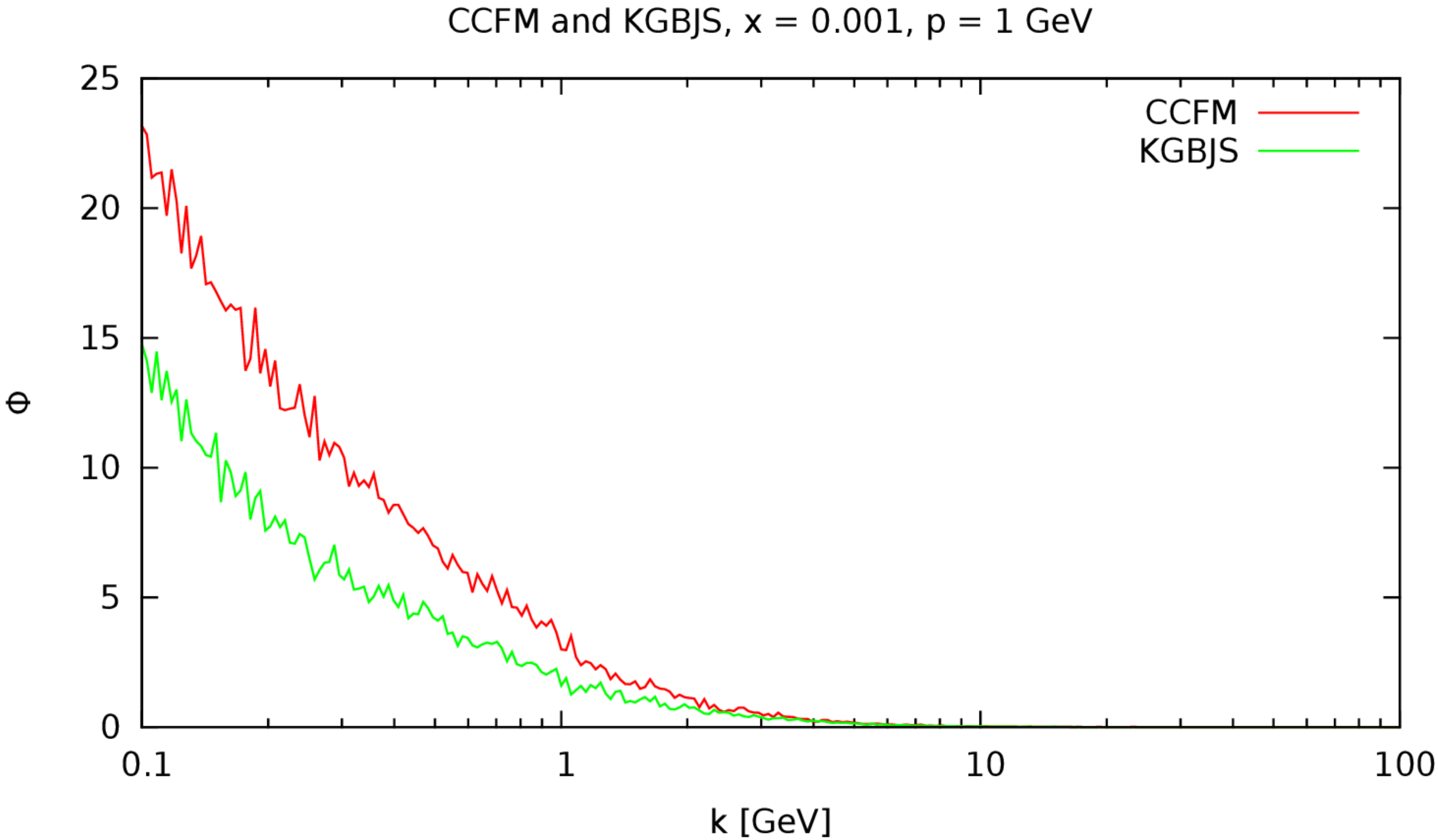
$$\Phi_0(x,k) = x^{\bar{\alpha}_s \log \frac{k^2}{\mu^2}} \frac{1\ \mathrm{GeV}}{k}$$

Numerical results – CCFM vs. KGBJS



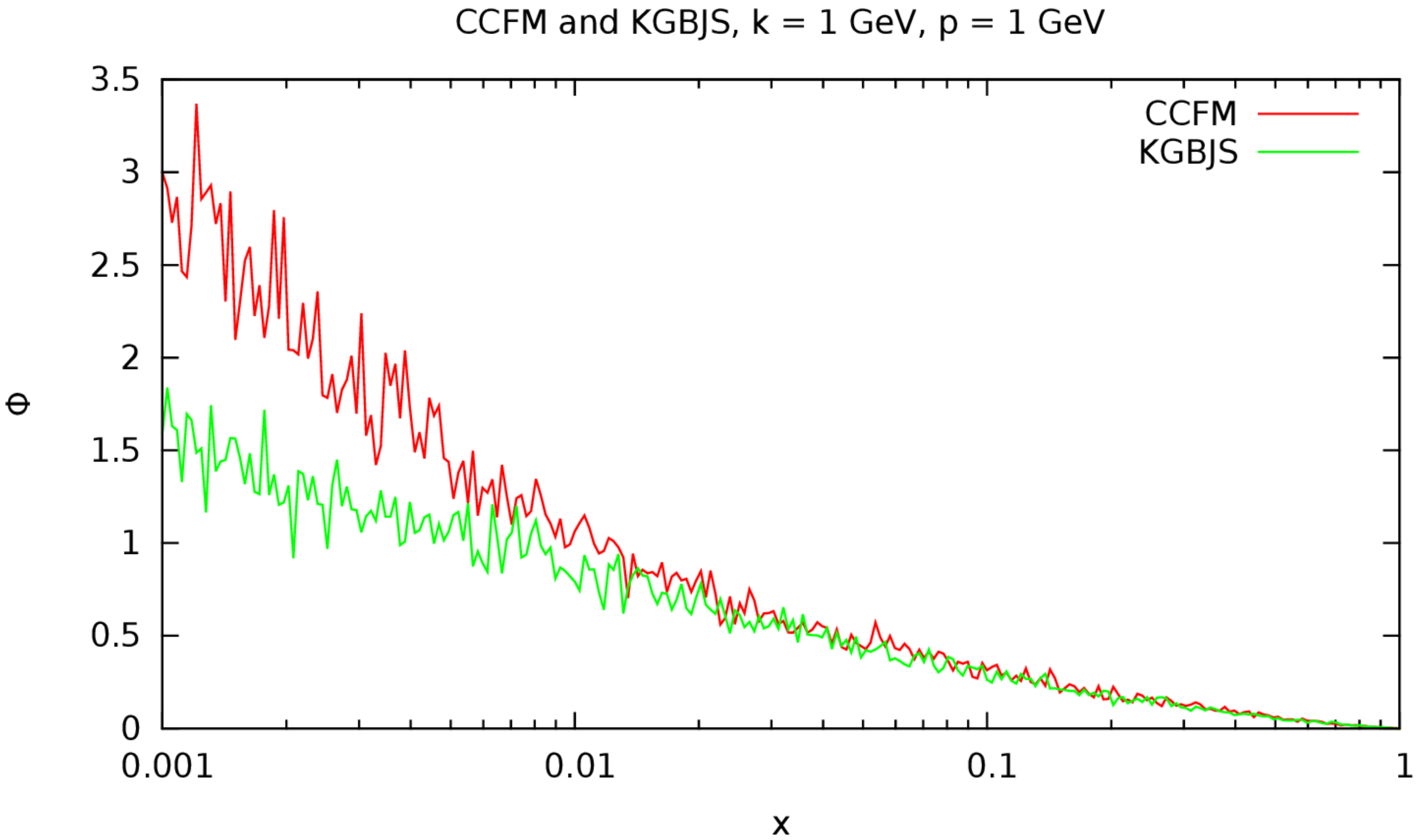
$$\mathcal{A}_0 = \theta\left(p - k\right) \mathrm{e}^{\left(\frac{k}{1\ \mathrm{GeV}}\right)^2}$$

Numerical results – CCFM vs. KGBJS



$$\mathcal{A}_0 = \theta\,(p - k)\,\mathrm{e}^{\left(\frac{k}{1\ \mathrm{GeV}}\right)^2}$$

Numerical results – CCFM vs. KGBJS



$$\mathcal{A}_0 = \theta\,(p-k)\,\mathrm{e}^{\left(\frac{k}{1\ \mathrm{GeV}}\right)^2}$$

Conclusions and outlook

- *With help of LHC there comes opportunity to test parton densities both when the parton density is probed at low x and low enough $\boldsymbol{k_t}$.*

- *HERA and RHIC data further gives some hints for saturation*

- *Results based on BK/DGLAP approach predicts saturation in p-Pb and suggest its presence in p-p*