Monte Carlo for solving CCFM equation

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Motivation

Features of CCFM evolution equation

- interpolating framework for large to low x
- account for interference effects to LO approximations due to color coherence (angular ordering)
- enables to match PDF with a hard process matrix element given the scale of the last emission (better than BFKL)
- recently extended to non-linear equation $\rightarrow$ saturation (?)

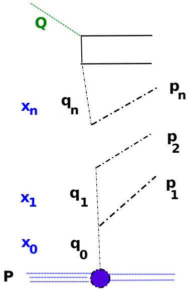
The existing programs

- CASCADE (based on SMALLX)
- programs on the grid

Why a new Monte Carlo?

- fully exclusive (all partons' four momenta)
- simulates complete CCFM as well as approximations
- cleaner environment (one evolution variable instead of the interplay of two)

Kinematics in initial state radiation



$x_{i+1} = z_{i+1} x_i$

$z \longrightarrow 0$ hard emission; $z \longrightarrow 1$ soft emission

p_{T_i} – transverse momenta of emitted gluons

$\tilde{p}_i = \frac{p_{T_i}}{1 - z_i}$

$q_{T_i} = |q_{T_0} - \sum_{j=1}^{i} p_{T_j}|$

– accumulated transverse momentum

angular ordering:

$\tilde{p}_{i+1} \geq z_i \tilde{p}_i \Leftrightarrow \ln \frac{p_i^+}{p_{T_i}} \equiv \eta_i < \eta_{i+1}$

Catani, Fiorani, Marchesini, *Small-x behaviour of initial state radiation in perturbative QCD*, Nucl. Phys B336 1990.

Catani, Fiorani, Marchesini, *QCD coherence in initial state radiation*, Phys. Letters B234 1990.

Sketch of derivation

1. recursive relations for n-gluon amplitude $\mathcal{F}_n$ in terms of $\mathcal{F}_{n-1}$ by means of soft gluons insertions

$$\mathcal{F}_n(x, q_T, z, p) = \int dz' \int \frac{d^2p'}{\pi p'^2} \Delta_S(p, z'p') P(z, p, q_T) \mathcal{F}_{n-1}\left(\frac{x}{z}, q'_T, z', p'\right)$$

2. imposing angular ordering; Q - maximal allowed angle

$$F_n(x, q_T, Q) = \int_x^1 dz \int \frac{d^2p}{\pi p^2} \Theta(Q - zp) \Delta_S(Q, zp) \mathcal{F}_n(x, q_T, z, p)$$

Kwiecinski, Martin, Sutton, *The gluon distribution at small x obtained from a unified evolution equation*, Phys.Rev. D52 (1995), 1445-1458

CCFM equation

$$F(x, q_T, Q) = \sum_{n=0}^{\infty} F_n(x, q_T, Q),$$

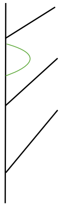
iterative form convenient for a Monte Carlo implementation

$$F(x, q_T, Q) = F(x_0, q_{T_0}, Q_0)\Delta_S(Q, Q_0)\delta(x - x_0)\delta^2(q_T - q_{T_0}) + \sum_{n=1}^{\infty} \Delta_S(Q, z_n p_n)\times$$

$$\times \prod_{i=1}^{n} \int \frac{dz_i}{z_i} \int \frac{d^2 p_i}{\pi p_i^2} \Theta(Q - z_n p_n)\Delta_S(p_{i+1}, z_i p_i)P(z_i, p_i, p_{T_{i+1}})F(x_0, q_{T_0}, Q_0)$$

CCFM splitting function $\quad P(z, p, q_T) = \alpha_S \frac{N_C}{\pi}\left(\frac{1}{1-z} + \frac{\Delta_{NS}(p,z,q_T)}{z}\right)$

compare with DGLAP $\quad P(z) = \alpha_S \frac{N_C}{\pi}\left(\frac{1}{1-z} + \frac{1}{z} + z(1-z) - 2\right)$



Sudakov form factor is given by

$$\Delta_S(q, z\bar{p}) = \exp\left(-\int_{(z\bar{p})^2}^{q^2} \frac{d\tilde{p}^2}{\tilde{p}^2} \int_0^{1-p_0/p} \frac{dz}{1-z} \frac{N_C \alpha_S}{\pi}\right)$$

scale-dependent infrared cutoff

cutoff on the minimal angle

resums soft virtual corrections

Non-Sudakov

$$\Delta_{NS}(p, z, q^2) = \exp\left\{ -N_C \frac{\alpha_S}{\pi} \int_z^1 \frac{dz'}{z'} \int_{z'^2 p_T^2}^{q_T^2} \frac{dp_T^2}{p_T^2} \right\}$$

$$= \exp\left\{ -N_C \frac{\alpha_S}{\pi} \ln\frac{1}{z} \ln\frac{q_T^2}{z p_T^2} \right\},$$

Monte Carlo algorithm

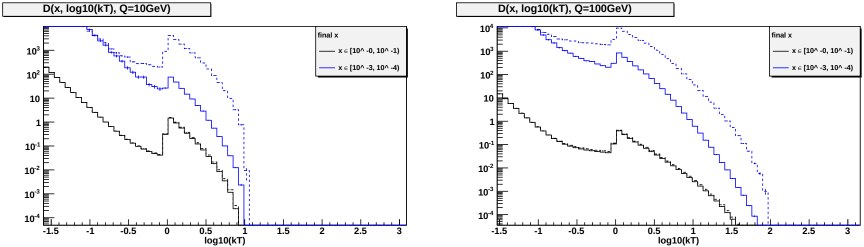
- interpret evolution equation as a Markov chain in (η_i, x_i)
- use DGLAP evolution as a lighthouse...
  - use Sudakow form factor as a probability distribution in generating each emissions and as a correcting weight for the last emission
  - employ well-exercised methodology to include additional effects (running α_S) and improve efficiency
- ... and modify to account for CCFM properties
  - generate full particles' kinematics
  - impose angular ordering of emissions
  - include the non-Sudakov form factor as a correcting weight for a "DGLAP-wise" generated distributions

Jadach et al., *Constrained MC for QCD evolution with rapidity ordering and minimum k_T*, CPC 180 (2009) 675-698.

Monte Carlo algorithm

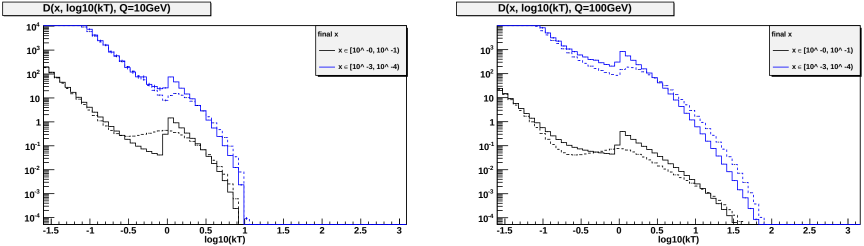
- keeping the constrain: $\eta_i < \eta_{i+1}; \eta = \ln \frac{p_i^+}{p_i^-}$, $\eta \in \left[\ln \frac{2Ex_0}{\Lambda_{QCD}}, 0\right]$
- keeping real and virtual soft contributions $\rightarrow$ cutoff on minimal $k_T = 1GeV$
- η_i, x_i are mapped into four-momenta of on-shell massless gluons
  $p_{j_T} = \exp(-\eta_j)\sqrt{s}(x_{j-1} - x_j),$
  $p_j^+ = \sqrt{s}(x_{j-1} - x_j), \;\; p_j^- = \frac{p_{j_T}^2}{p_j^+},$
- stopping rule $\ln Q_{max} = \sqrt{s\bar{\xi}} = \frac{\bar{p}}{x}$
  $\bar{\xi}, \bar{p}$ - max rapidity, angle in the CM system

Results – comparison with DGLAP



DGLAP (dashed line) and CCFM (solid line) distributions.
$Q_max = 10, 100$
Initial condition: $1/k_T^2 x^0$.
$x \in [1, 0.1)$ (black) and $x \in [10^{-3}, 10^{-4})$

Results – neglecting soft emissions



Complete CCFM distributions (solid line) with removed Sudakov and $1/(1-z)$) (dashed line), constant α_S. Evolution time Q= 10 (left), 100 (right). Initial condition: $1/k_T^2 x^0$.

Comparisons with other programs – underway

Different choices of the maximal scale

- q_T in Marchesini, Webber, NPB349(1991) 617-634
- maximal angle (CASCADE, Avsar, Stasto, JHEP06 (2010) 112)

evolution in rapidity, stopping rule in maximal angle
In order to make quantitative comparisons, a need to modify stopping rule in the MC.
Problem for Monte Carlo! evolution scale not given a priori

Conclusions

- We extended the scope of a program simulating DGLAP evolution into CCFM
- The program models a complete CCFM evolution
- Obtained a qualitatively good description of k_T distributions
- Comparisons with other programs, simulating data reaquires translating a stopping rule into maximal angle (underway).