Multi-gluon one-leg off-shell helicity amplitudes in high-energy factorization

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based on
A. van Hameren, P.K., K. Kutak, arXiv:1207.3332
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Motivation

- Off-shell amplitudes appear in k_T (or TMD – transverse-momentum dependent) factorization
- For on-shell tree-level amplitudes there are plenty of efficient tools (based on helicity method), this is not the case for off-shell ones
- One-leg off-shell amplitudes can be used in forward-central jets studies (see Krzysztof's Kutak talk on Monday)
- Two-leg off-shell amplitudes – see today's talk of Andreas van Hameren

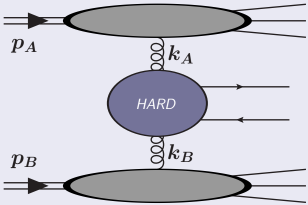
Outline

- High-energy factorization
  - Catani-Ciafaloni-Hautmann factorization approach
  - multiple gluonic final states
- Helicity method vs off-shell amplitudes
- “Gauge-restoring” amplitude
  - the Slavnov-Taylor identities
  - some less commonly used gauge issues
- Summary

High-energy factorization

CCH (Catani, Ciafaloni, Hautmann) factorization[1]

The CCH was originally stated for heavy quarks production in photo-, lepto- and hadro-production.



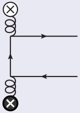
At high energies the single longitudinal components of momentum transfers dominate

$$k_A^\mu \simeq z_A p_A^\mu + k_{T\,A}^\mu, \qquad k_B^\mu \simeq z_B p_B^\mu + k_{T\,B}^\mu.$$

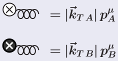
It is then argued that

$$d\sigma_{AB\to Q\overline{Q}} \simeq \int d^2 k_{T\,A} \int \frac{dz_A}{z_A} \int d^2 k_{T\,B} \int \frac{dz_B}{z_B}$$
$$\mathcal{F}\,(z_A, k_{T\,A})\, d\sigma_{g^* g^* \to Q\overline{Q}}\,(z_A, z_B, k_{T\,A}, k_{T\,B})\, \mathcal{F}\,(z_B, k_{T\,B}),$$

where $\mathcal{F}$ are unintegrated gluon structure functions undergoing BFKL evolution. The hard process $d\sigma_{g^* g^* \to Q\overline{Q}}$ is calculated by contracting an off-shell amplitude (including external off-shell propagators) with p_A, p_B:



where



It can be shown that $d\sigma_{g^* g^* \to Q\overline{Q}}$ is gauge invariant.

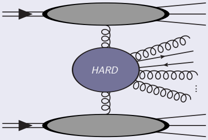
Further, we consider only tree level hard processes and asymmetric on-shell–off-shell kinematics with

$$k_A^\mu \simeq z_A p_A^\mu + k_{T\,A}^\mu, \qquad k_B^\mu \simeq z_B p_B^\mu.$$

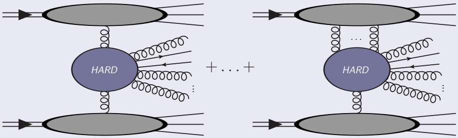
[1] S. Catani, M. Ciafaloni, F. Hautmann (1990), (1991), (1994)

High-energy factorization (cont.)

More gluons: collinear factorization

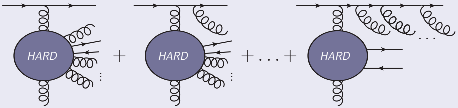


More gluons: TMD factorization



Two standard approaches to *HARD*

- Consider an on-shell process with hadron replaced by a quark and eventually perform the high-energy limit. In the axial gauge with the gauge vector p_A the following structure emerges



$\Rightarrow$ the bremsstrahlung diagrams are necessary in order to maintain gauge invariance.

- Use Lipatov's effective action and resulting Feynman rules[1].
  An off-shell gluon contracted with eikonal vector ≡ reggeon (R),
  $\Rightarrow$ *R* → particles effective vertex

Is there an alternative which is efficient for multiple final states?

[1] E. Antonov, L. Lipatov, E. Kuraev, I. Cherednikov (2005)

Off-shell amplitudes and helicity method

In what follows, we concentrate on purely gluonic amplitudes (quarks are also easily included).

Helicity method for on-shell amplitudes

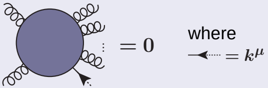
- uses the spinor representation for polarization vectors of gluons

  For a gluon with momentum k the polarization vector is defined with the help of a reference vector q, $\varepsilon_k^\mu(q)$.
- the gauge invariance is crucial

  Change of the reference momentum $q \to q'$ amounts for the transformation

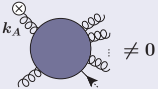
  $$\varepsilon_k^\mu(q) = \varepsilon_k^\mu(q') + k^\mu \beta_k(q, q').$$

  We can adjust q freely due to the Ward identity



- proper choice of q renders rather compact expressions for helicity amplitudes, which can be squared and summed numerically

The off-shell amplitude (without bremsstrahlung contributions) is not gauge invariant:



Let us denote our off-shell amplitude as $\mathcal{M}(\varepsilon_B, \varepsilon_1 \dots, \varepsilon_N)$.

There exists an “amplitude” $\mathcal{W}(\varepsilon_B, \varepsilon_1, \dots, \varepsilon_N)$ such that

$$\widetilde{\mathcal{M}}(\varepsilon_B, \varepsilon_1, \dots, \varepsilon_N) = \mathcal{M}(\varepsilon_B, \varepsilon_1, \dots, \varepsilon_N) + \mathcal{W}(\varepsilon_B, \varepsilon_1, \dots, \varepsilon_N)$$

satisfies

$$\widetilde{\mathcal{M}}(\varepsilon_B, \varepsilon_1, \dots, k_i, \dots, \varepsilon_N) = 0.$$

Gauge-restoring amplitude

Reduction formula for CCH factorization

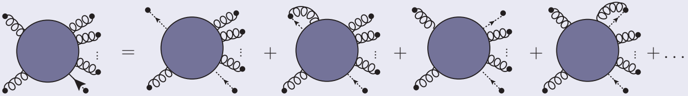
The high-energy amplitude with the proper kinematics can be implemented via

$$\mathcal{M}(\varepsilon_B, \varepsilon_1, \ldots, \varepsilon_N) = \lim_{k_A \cdot p_A \to 0} \lim_{k_B^2 \to 0} \lim_{k_1^2 \to 0} \ldots \lim_{k_N^2 \to 0} \left(\left| \vec{k}_A \right| p_A^{\mu_A} \right) \left(k_B^2 \varepsilon_B^{\mu_1} \right) \left(k_1^2 \varepsilon_1^{\mu_1} \right) \ldots \left(k_N^2 \varepsilon_N^{\mu_N} \right) \tilde{G}_{\mu_A \mu_B \mu_1 \cdots \mu_N} (k_A, k_B, k_1, \ldots, k_N),$$

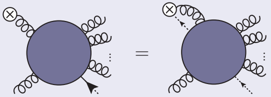
where $\tilde{G}$ is the momentum-space Green function.

Slavnov-Taylor (S-T) identity

We apply the S-T identity to $\tilde{G}$:



After applying the reduction formula most of the terms vanish, except one



The r.h.s term is precisely the amount of gauge-invariance violation and can be calculated (note however, this is not the “gauge-restoring” amplitude yet, as it contains the external ghost line).

Gauge-restoring amplitude (cont.)

Remarks concerning gauges and ghosts

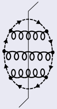
- It is allowed to use two different gauges for on-shell lines and internal off-shell lines.
- Ghosts do exist in the axial gauge (but usually decouple)[1]

  A ghost-gluon coupling in the axial gauge is  $= ig\, f_{abc}\, n^{\mu}$, where n is a gauge vector.
- Usually, when squaring an amplitude one uses sum over *physical* gluon polarization

$$\sum_{\lambda} \varepsilon_k^{(\lambda)\,\mu}(q)\, \varepsilon_k^{(\lambda)\,\nu\,*}(q) = -g^{\mu\nu} + \frac{q^{\mu} k^{\nu} + q^{\nu} k^{\mu}}{q \cdot k},$$

  with some light-like momentum q.

  Alternatively, one can use external gluons in the Feynman gauge and *cut ghost loops*. $\longrightarrow$



- The last remark allows us to trade an external ghost with momentum k to a gluon projected onto some light-like momentum q

$$\text{external ghost} \to \frac{\varepsilon_k \cdot q}{k \cdot q}$$

[1] e.g. G. Leibrandt, *Rev. Mod. Phys.* (1987)

Gauge-restoring amplitude (cont.)

It turns out that the gauge-restoring amplitude can be easily obtained by using axial gauge with gauge vector p_A and summing all the gauge contributions with proper replacements of external ghosts.

For instance, for a fixed color ordering $(A, B, 1, \ldots, N)$ the sum collapses into a single term

$$\mathcal{W}_{\mathrm{ord}}\left(\varepsilon_B, \varepsilon_1, \ldots, \varepsilon_N\right) = -\left(\frac{-g}{\sqrt{2}}\right)^N \frac{\left|\vec{k}_{TA}\right| \, \varepsilon_B \cdot p_A \, \varepsilon_1 \cdot p_A \ldots \varepsilon_N \cdot p_A}{k_B \cdot p_A \, \left(k_B - k_1\right) \cdot p_A \ldots \left(k_B - \ldots - k_{N-1}\right) \cdot p_A}$$

- Those amplitudes correspond to bremsstrahlung diagrams in “embedding approach”.
- The full gauge invariant amplitude $\tilde{\mathcal{M}} = \mathcal{M} + \mathcal{W}$ does satisfy ordinary collinear and soft behaviour
- It corresponds to Lipatov’s $R \to (N+1)\, G$ effective vertex
- If we choose the reference momentum for polarization vector of any of the external gluons to be p_A the amplitude $\mathcal{W}$ vanishes due to the property of polarization vectors $\varepsilon_k\,(q) \cdot q = 0$.
- We have explicit analytical expressions for helicity amplitudes for $G^*G \to GG$, $G^*G \to GGG$, $G^*G \to GQ\overline{Q}$
- Approach tested numerically up to $N = 10$

Summary

- Our task – develop some methods/tools for efficient calculations of multi-partonic tree-level amplitudes relevant for high-energy factorization
- First step – one-leg off-shell amplitudes
- We have reconstructed a gauge-restoring contribution using just the Slavnov-Taylor identities
- The gauge-restoring amplitude allows for using the helicity method and existing tools for matrix elements