Double parton distributions - bounds and evolution

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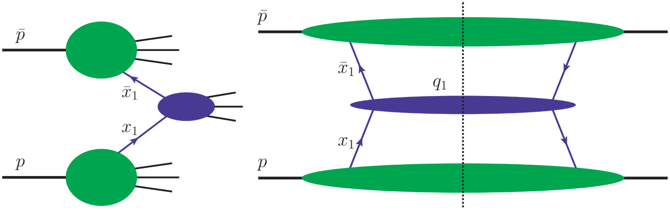
In collaboration with M. Diehl

What is double parton scattering?



- Single parton scattering cross section

$$\sigma = \sum_{ij} \int dx_1 \int d\bar{x}_1 f_i(x_1) f_j(\bar{x}_1) \hat{\sigma}_{ij}$$

(Assuming factorization of hard scattering)

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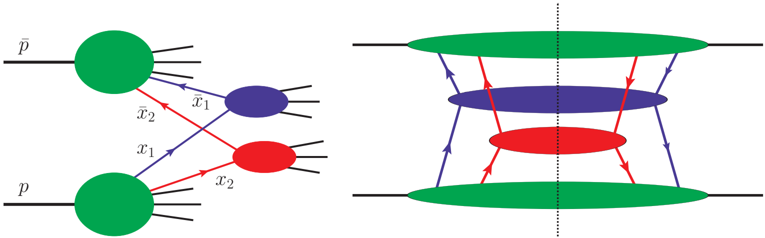

- Double parton scattering (DPS) cross section

$$\sigma_{DPS} \sim \sum_{ijkl} \int dx_1 dx_2 \int d\bar{x}_1 d\bar{x}_2 \int d^2\boldsymbol{y} f_{ik}(x_1, x_2, \boldsymbol{y}) f_{jl}(\bar{x}_1, \bar{x}_2, \boldsymbol{y}) \hat{\sigma}_{ij} \hat{\sigma}_{kl}$$

(Assuming factorization of hard scatters)

notice $\boldsymbol{y}$ dependence

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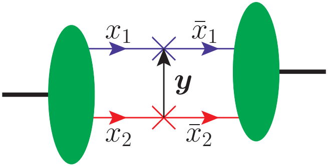

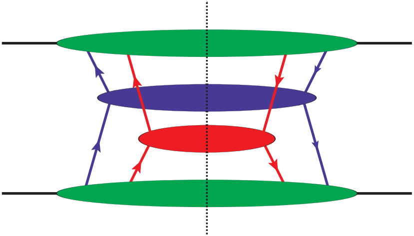

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- Inclusive DPS cross section power suppressed compared to SPS cross section
  $\sigma_{DPS}/\sigma_{SPS} \sim \Lambda^2/Q^2$

- Why study DPS?

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- Why study DPS:

- Differential cross section (in $\boldsymbol{q}_i$) not power suppressed
- $\sigma_{DPS}/\sigma_{SPS}$ expected to increase with collider energy
- Proton structure
- Background
- Multiparton interactions and Monte Carlo generators
- Experimental evidence found at ISR, SPS, Tevatron and LHC
- Many new effects, not present in single parton scattering
- No systematic theory

Polarized double parton distributions

- Spin correlations, between partons in same proton

(M. Mekhfi, 1985; M. Diehl, A. Schäfer, 2011)

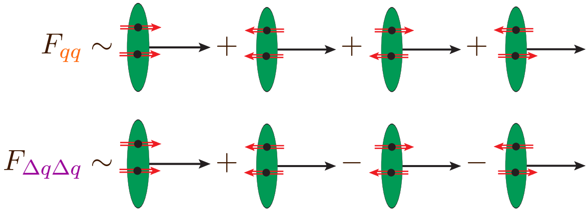
- Correlations described by parton distributions

$$F_{q_1 q_2} \sim \langle p | (\bar{q}_1 \Gamma_1 q_1)(\bar{q}_2 \Gamma_2 q_2) | p \rangle$$

- $\Gamma_{1/2} = \; \Gamma_q, \; \Gamma_{\Delta q}, \; \Gamma^j_{\delta q}$ projection operators
  - Unpolarized (q), longitudinally- (Δq) and transversely-polarized (δq) quarks
  - Index $j = 1, 2$ corresponds to the transverse spin-vector

Remember: q, Δq and δq

- Unpolarized and longitudinally polarized quarks



- Transverse polarization ($F^i_{q\delta q}, F^i_{\delta q q}, F^{ij}_{\delta q \delta q}$) - helicity interference

- Polarized double parton distributions
  - Unpolarized and longitudinally polarized quarks

$$F_{qq} = f_{qq}(x_1, x_2, \boldsymbol{y})$$
$$F_{\Delta q \Delta q} = f_{\Delta q \Delta q}$$

  - Singly transversely polarized quarks ($\tilde{y}^i = y^j \epsilon^{ij}$)

$$F^i_{q\delta q} = M \tilde{y}^i f_{q\delta q}$$
$$F^i_{\delta q q} = M \tilde{y}^i f_{\delta q q}$$

  - Doubly transversely polarized quarks

$$F^{ij}_{\delta q \delta q} = \delta^{ij} f_{\delta q \delta q} + \left(2y^i y^j - y^2 \delta^{ij}\right) M^2 f^t_{\delta q \delta q}$$

6 different polarized distributions

- Double Drell-Yan cross section ($W^\pm$, Z, γ^*)
  - Longitudinal polarization change both magnitude and angular modulation
  - Azimuthal correlations

$$d\sigma^{(2)} \sim \sin^2\theta_1 \sin^2\theta_2 \Big[A\cos 2(\varphi_1 - \varphi_2) - B\sin 2(\varphi_1 - \varphi_2)\Big]$$
$$\times \int d^2\boldsymbol{y}\, f_{\delta q_1 \delta q_2}(x_1, x_2, \boldsymbol{y}) \bar{f}_{\delta\bar{q}_1 \delta\bar{q}_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

(transversely polarized quarks in both interactions)

(TK, M. Diehl, 2012)

- Similar features in other cross sections (such as double dijet)
- Double gluon and mixed quark-gluon distributions
- For gluon g unpolarized, Δg longitudinally polarized and δg linearly polarized.

polarized distributions have profound effect on cross section

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$$d\sigma^{(2)} \sim \sin^2\theta_1 \sin^2\theta_2 \Big[A\cos 2(\varphi_1 - \varphi_2) - B \sin 2(\varphi_1 - \varphi_2) \Big] \times \int d^2\boldsymbol{y}\, f_{\delta q_1 \delta q_2}(x_1, x_2, \boldsymbol{y}) \bar{f}_{\delta \bar{q}_1 \delta \bar{q}_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

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how big can the polarized distributions be?

Positivity bounds

- Many distributions, poorly known $f_{ab}(x_1, x_2, \boldsymbol{y})$
- Derive constraints on their sizes
- Constraints from probabilistic interpretation
- Can derive positivity bounds, similar to single parton distributions

(A. Bacchetta, M. Boglione, P.J. Mulders, 1999; M. Diehl, Ph. Hägler, 2005)

- Positivity bounds on DPD's:

$$f_{ab} + h_{\delta a \delta b} - h^t_{\delta a \delta b} \pm \sqrt{(h_{\delta a b} + h_{a \delta b})^2 + (f_{\Delta a \Delta b} - h_{\delta a \delta b} - h^t_{\delta a \delta b})^2} \geq 0$$

$$f_{ab} - h_{\delta a \delta b} + h^t_{\delta a \delta b} \pm \sqrt{(h_{\delta a b} - h_{a \delta b})^2 + (f_{\Delta a \Delta b} + h_{\delta a \delta b} + h^t_{\delta a \delta b})^2} \geq 0$$

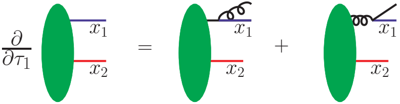
(M. Diehl, TK, 2013)

- Bounds can aid construction of double parton distributions
- Sets an upper limit on the size of spin correlations

- How are the bounds affected by evolution?
- For single parton in polarized proton (Soffer bound), first argued to be broken by evolution but later shown to hold

(C. Bourrely, J. Soffer and O. Teryaev, 1998)

- Evolution of unpolarized double parton distribution (x_1 parton)



$$\frac{\partial f_{qq}(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2)}{\partial \log {\mu_1}^2} \sim P_{qq} \otimes_1 f_{qq} + P_{qg} \otimes_1 f_{gq},$$

where the P_{ab}'s are the usual splitting kernels (without inhomogeneous term)

- Analogous equations for polarized partons
- Bounds stable under leading order evolution

(M.Diehl, TK, 2013)

Numerical solution of evolution equations

- Numerically study the evolution
- Evolution code:
  - Modified version of evolution code written by J. Gaunt and J. Stirling

(J. Gaunt, J. Stirling 2009)

- Generally evolution washes out spin correlations

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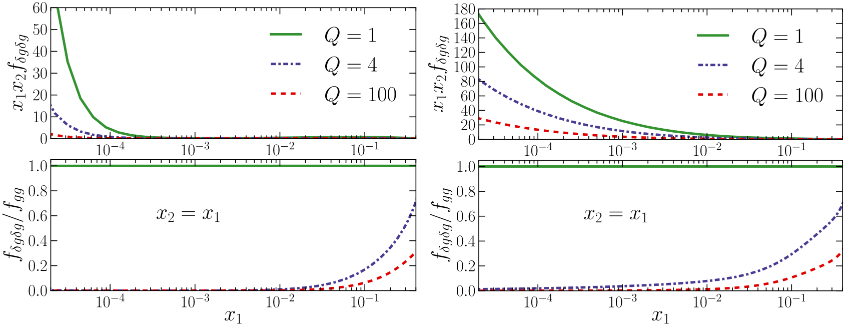
Need ansatz for DPDs at starting scale

- Build on products of single parton distributions, $f_{ab} \sim f_a f_b$
- Ansatz saturating the positivity bounds:
  - polarized distributions equal the unpolarized at starting scale

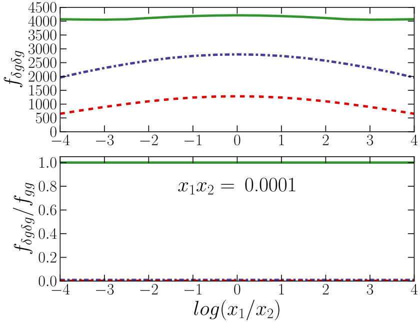
- Linearly polarized gluons ($f_{\delta g \delta g}$)
- Starting from leading order distributions from MSTW (left) or GJR (right)

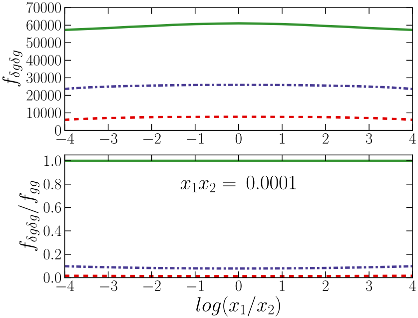
(A. Martin, W. Stirling, R. Thorne, G. Watt 2009, M. Gluck, P. Jimenez-Delgado, E. Reya 2007)



- Quickly washed out by evolution

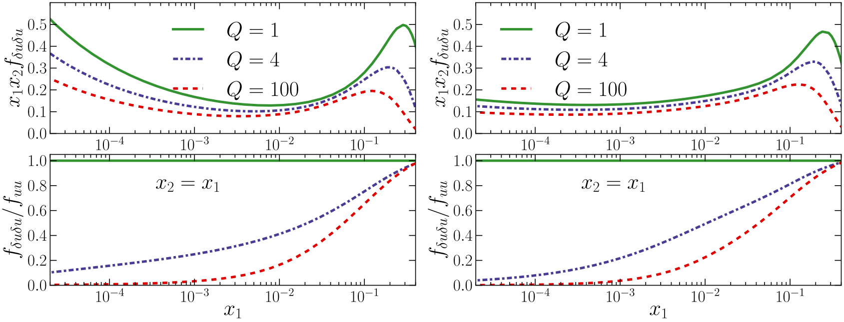
- $f_{\delta g \delta g}$ at fixed $x_1 x_2$





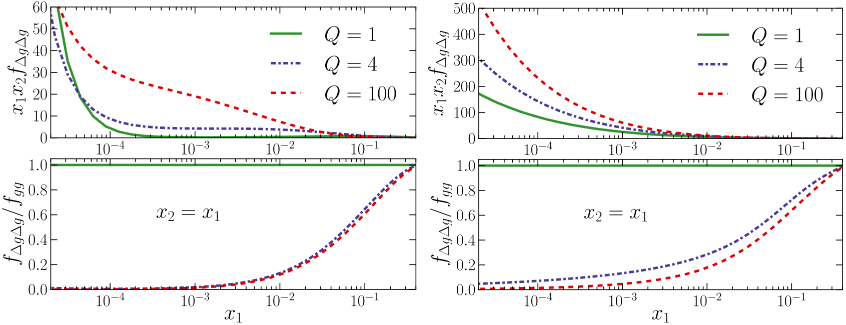
- $\log x_1/x_2 \sim$ rapidity difference between interactions
- Quickly washed out $\Rightarrow$ negligible already at medium scales
- Differences depending on starting single pdfs

- Transversely polarized up-quarks ($f_{\delta u \delta u}$)



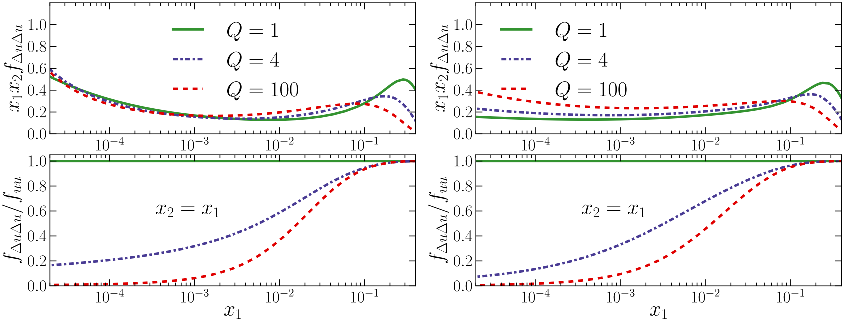
- Can be significant up to larger scales
- Smaller differences between MSTW and GJR

- Longitudinally polarized gluons ($f_{\Delta g \Delta g}$)



- Stays sizable up to larger scales
- Differences depending on starting single pdfs

- Longitudinally polarized up-quarks ($f_{\Delta u \Delta u}$)



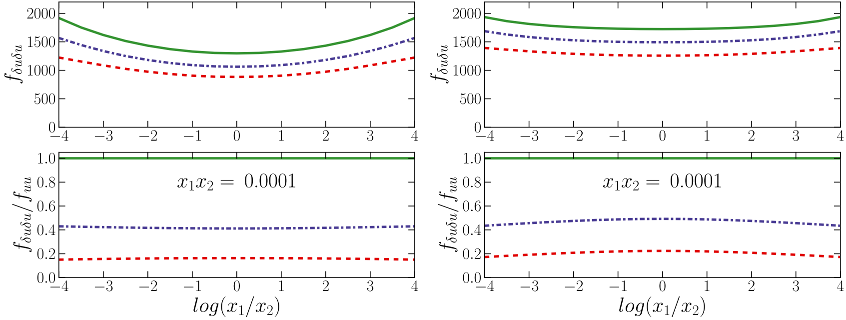
- Remains large up to larger scales

Summary

- Many unexplored effects in double parton scatterings
- Spin correlations have profound effects on cross sections
- We derived positivity bounds on distributions
- Bounds are stable under leading order evolution
- Bounds set upper limit on spin correlations between partons in the proton, at any scale
- Linear gluon polarization quickly washed out by evolution
- Transverse quark polarization significant up to high scales
- Longitudinal quark and gluon polarization significant up to high scales

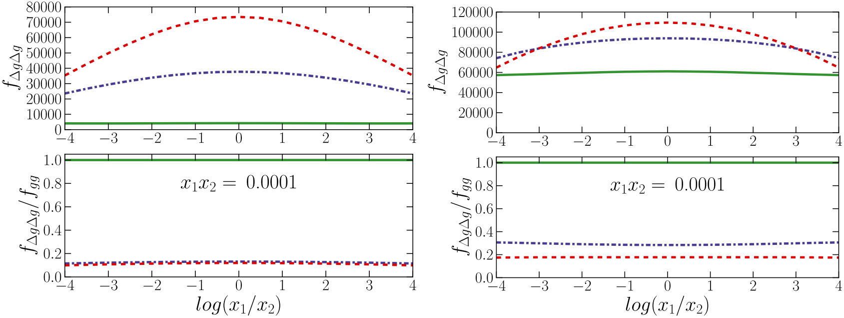
Backup Slides

- $f_{\delta u \delta u}$ at fixed $x_1 x_2$



- Can be significant up to larger scales
- Small differences between MSTW and GJR

- $f_{\Delta g \Delta g}$ at fixed $x_1 x_2$



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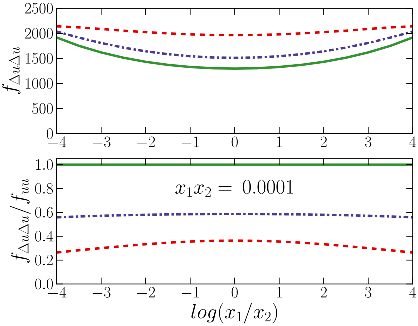

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Angles and coordinate systems