Will Planck Observe Gravity Waves?

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Outline

- Inflationary Cosmology
- Tree Level Gauge Singlet Higgs Inflation
- Quantum Smearing
- SUSY Higgs (Hybrid) Inflation
- Summary

[Guth, Linde, Albrecht & Steinhardt, Starobinsky, Mukhanov, Hawking, . . .]

Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for n_s, r, $dn_s/d\ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Physics Beyond the SM?

Slow-roll Inflation

- Inflation is driven by some potential $V(\phi)$:
- Slow-roll parameters:

$$\epsilon = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2, \ \eta = m_p^2\left(\frac{V''}{V}\right).$$

- The spectral index n_s and the tensor to scalar ratio r are given by

$$n_s - 1 \equiv \frac{d\ln\Delta_{\mathcal{R}}^2}{d\ln k}, \ r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2},$$

where Δ_h^2 and $\Delta_{\mathcal{R}}^2$ are the spectra of primordial gravity waves and curvature perturbation respectively.

- Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index n_s and the tensor to scalar ratio r are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \ r \simeq 16\epsilon.$$

- The tensor to scalar ratio r can be related to the energy scale of inflation via

$$V(\phi_0)^{1/4} = 3.3 \times 10^{16}\, r^{1/4} \text{ GeV}.$$

- The amplitude of the curvature perturbation is given by

$$\Delta_{\mathcal{R}}^2 = \frac{1}{24\pi^2}\left(\frac{V/m_p^4}{\epsilon}\right)_{\phi=\phi_0} = 2.43 \times 10^{-9} \text{ (WMAP7 normalization)}.$$

- The spectrum of the tensor perturbation is given by

$$\Delta_h^2 = \frac{2}{3\,\pi^2}\left(\frac{V}{m_P^4}\right)_{\phi=\phi_0}.$$

- The number of e-folds after the comoving scale $l_0 = 2\,\pi/k_0$ has crossed the horizon is given by

$$N_0 = \frac{1}{m_p^2}\int_{\phi_e}^{\phi_0}\left(\frac{V}{V'}\right) d\phi.$$

Inflation ends when $\max[\epsilon(\phi_e), |\eta(\phi_e)|] = 1$.

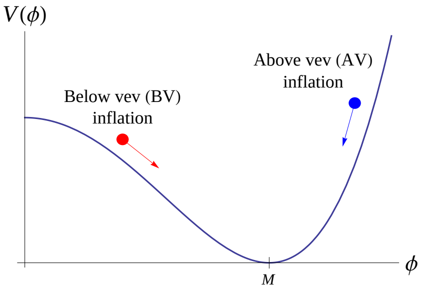
Tree Level Gauge Singlet Higgs Inflation

[Kallosh and Linde, 07; Rehman, Shafi and Wickman, 08]

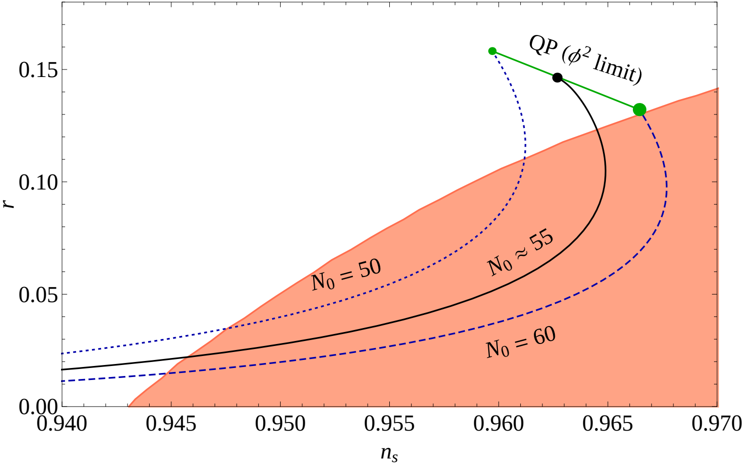
- Consider the following Higgs Potential:

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{M}\right)^2\right]^2 \quad \longleftarrow \text{(tree level)}$$

Here ϕ is a gauge singlet field.



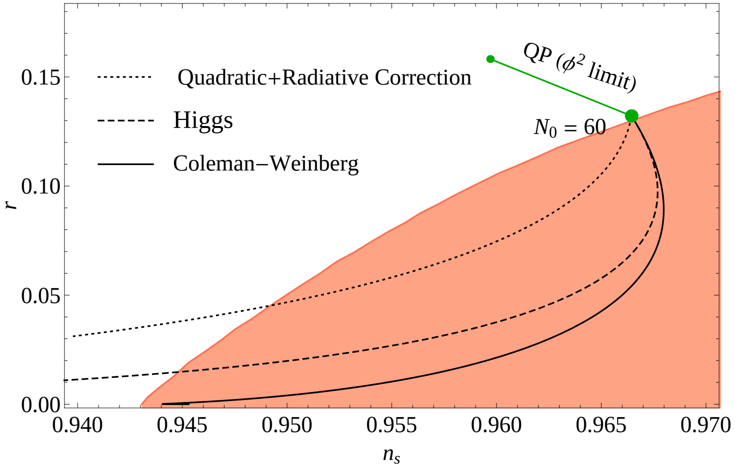
- WMAP/Planck data favors BV inflation.

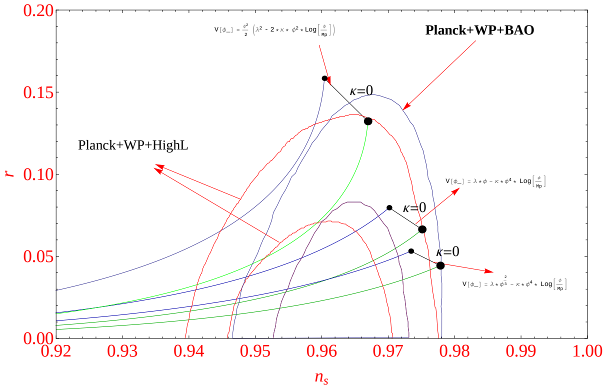


QP (ϕ^2 limit)
$N_0 = 50$
$N_0 \approx 55$
$N_0 = 60$
0.15
0.10
0.05
0.00
r
0.940
0.945
0.950
0.955
0.960
0.965
0.970
n_s

Quantum Smearing

$$V = \frac{1}{4}\lambda(\phi^2 - v^2)^2 + \frac{C}{16\pi^2}\phi^4 \log\left[\frac{\phi}{v}\right] + \text{const.},$$





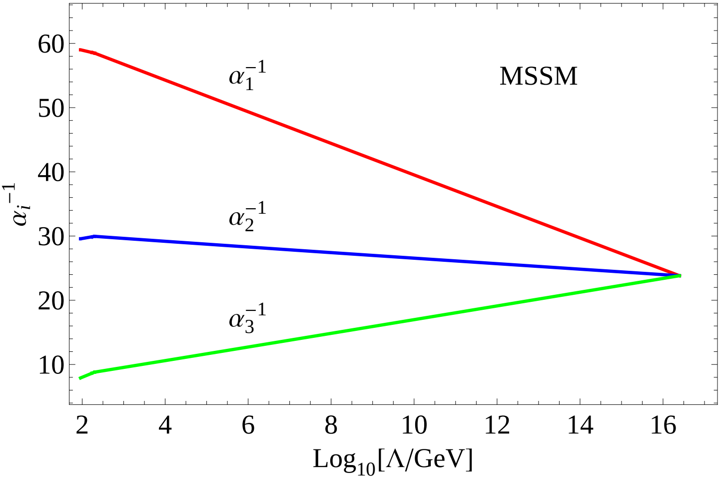
$V[\phi\_] = \frac{\phi^2}{2}\left(\lambda^2 - 2 * \kappa * \phi^2 * \mathrm{Log}\left[\frac{\phi}{Mp}\right]\right)$
Planck+WP+BAO
$\kappa=0$
Planck+WP+HighL
$V[\phi\_] = \lambda * \phi - \kappa * \phi^4 * \mathrm{Log}\left[\frac{\phi}{Mp}\right]$
$\kappa=0$
$\kappa=0$
$V[\phi\_] = \lambda * \phi^{\frac{2}{3}} - \kappa * \phi^4 * \mathrm{Log}\left[\frac{\phi}{Mp}\right]$
0.20
0.15
0.10
0.05
0.00
r
0.92
0.93
0.94
0.95
0.96
0.97
0.98
0.99
1.00
n_s

Supersymmetry

- Resolution of the gauge hierarchy problem
- Predicts plethora of new particles which LHC should find
- Unification of the SM gauge couplings at

  $M_{GUT} \sim 2 \times 10^{16}$ GeV
- Cold dark matter candidate (LSP)
- Radiative electroweak breaking
- String theory requires supersymmetry (SUSY)

Alas, SUSY not yet seen at LHC



MSSM
α_1^{-1}
α_2^{-1}
α_3^{-1}
α_i^{-1}
10
20
30
40
50
60
2
4
6
8
10
12
14
16
$\mathrm{Log}_{10}[\Lambda/\mathrm{GeV}]$

SUSY Higgs (Hybrid) Inflation

[Dvali, Shafi, Schaefer; Copeland, Liddle, Lyth, Stewart, Wands '94]

[Lazarides, Schaefer, Shafi '97][Senoguz, Shafi '04; Linde, Riotto '97]

- Attractive scenario in which inflation can be associated with symmetry breaking $G \longrightarrow H$
- Simplest inflation model is based on

$$W = \kappa\, S\, (\Phi\, \overline{\Phi} - M^2)$$

S = gauge singlet superfield, $(\Phi\,, \overline{\Phi})$ belong to suitable representation of G

- Need $\Phi\,, \overline{\Phi}$ pair in order to preserve SUSY while breaking $G \longrightarrow H$ at scale $M \gg$ TeV, SUSY breaking scale.
- R-symmetry

$$\Phi\, \overline{\Phi} \to \Phi\, \overline{\Phi}, \;\; S \to e^{i\alpha}\, S, \;\; W \to e^{i\alpha}\, W$$

$\Rightarrow$ W is a unique renormalizable superpotential

- Some examples of gauge groups:

$$G = U(1)_{B-L},\ \text{(Supersymmetric superconductor)}$$

$$G = SU(5) \times U(1), \quad (\Phi = 10), \quad \text{(Flipped } SU(5))$$

$$G = 3_c \times 2_L \times 2_R \times 1_{B-L},\ (\Phi = (1, 1, 2, +1))$$

$$G = 4_c \times 2_L \times 2_R,\ (\Phi = (\overline{4}, 1, 2)),$$

$$G = SO(10),\ (\Phi = 16)$$

- At renormalizable level the SM displays an ‘accidental’ global $U(1)_{B-L}$ symmetry.

- Next let us ‘gauge’ this symmetry, so that $U(1)_{B-L}$ is now promoted to a local symmetry. In order to cancel the gauge anomalies, one may introduce 3 SM singlet (right-handed) neutrinos.

  This has several advantages:

- See-saw mechanism is automatic and neutrino oscillations can be understood.

- RH neutrinos acquire masses only after $U(1)_{B-L}$ is spontaneously broken; Neutrino oscillations require that RH neutrino masses are $\lesssim 10^{14}\mathrm{GeV}$.

- RH neutrinos can trigger leptogenesis after inflation, which subsequently gives rise to the observed baryon asymmetry;

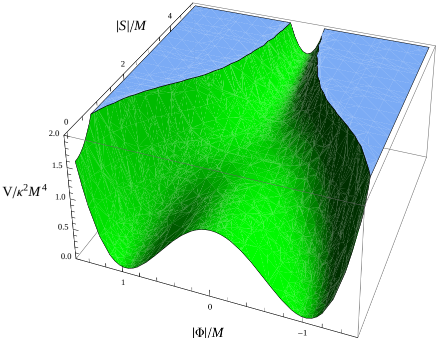
- Last but not least, the presence of local $U(1)_{B-L}$ symmetry enables one to explain the origin of Z_2 'matter' parity of MSSM. (It is contained in $U(1)_{B-L} \times U(1)_Y$, if $B-L$ is broken by a scalar vev, with the scalar carrying two units of $B-L$ charge.)

- Tree Level Potential

$$V_F = \kappa^2 \left(M^2 - |\Phi^2|\right)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- SUSY vacua

$$|\langle\overline{\Phi}\rangle| = |\langle\Phi\rangle| = M, \ \langle S\rangle = 0$$



Take into account radiative corrections (because during inflation $V \neq 0$ and SUSY is broken by $F_S = -\kappa\, M^2$)

- Mass splitting in $\Phi - \overline{\Phi}$

$$m_\pm^2 = \kappa^2\, S^2 \pm \kappa^2\, M^2, \quad m_F^2 = \kappa^2\, S^2$$

- One-loop radiative corrections

$$\Delta V_{\text{1loop}} = \frac{1}{64\pi^2} \text{Str}[\mathcal{M}^4(S)(\ln \frac{\mathcal{M}^2(S)}{Q^2} - \frac{3}{2})]$$

- In the inflationary valley ($\Phi = 0$)

$$V \simeq \kappa^2\, M^4 \left(1 + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x)\right)$$

where $x = |S|/M$ and

$$F(x) = \frac{1}{4}\left(\left(x^4+1\right)\ln\frac{\left(x^4-1\right)}{x^4} + 2x^2 \ln\frac{x^2+1}{x^2-1} + 2\ln\frac{\kappa^2 M^2 x^2}{Q^2} - 3\right)$$

Full Story

Also include supergravity corrections + soft SUSY breaking terms

- The minimal Kähler potential can be expanded as

$$K = |S|^2 + |\Phi|^2 + \left|\overline{\Phi}\right|^2$$

- The Sugra scalar potential is given by

$$V_F = e^{K/m_p^2}\left(K_{ij}^{-1} D_{z_i} W D_{z_j^*} W^* - 3m_p^{-2}|W|^2\right)$$

where we have defined

$$D_{z_i} W \equiv \frac{\partial W}{\partial z_i} + m_p^{-2}\frac{\partial K}{\partial z_i} W;\; K_{ij} \equiv \frac{\partial^2 K}{\partial z_i \partial z_j^*}$$

and $z_i \in \{\Phi, \overline{\Phi}, S, ...\}$

[Senoguz, Shafi '04; Jeannerot, Postma '05]

- Take into account sugra corrections, radiative corrections and soft SUSY breaking terms:

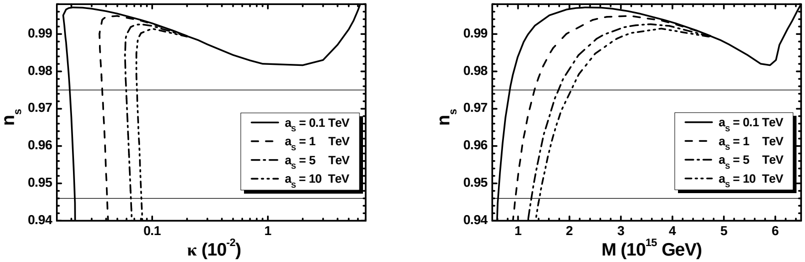
$$V \simeq \kappa^2 M^4 \left(1 + \left(\frac{M}{m_p}\right)^4 \frac{x^4}{2} + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x) + a_s \left(\frac{m_{3/2} x}{\kappa M}\right) + \left(\frac{m_{3/2} x}{\kappa M}\right)^2\right)$$

where $a_s = 2\,|2 - A| \cos[\arg S + \arg(2 - A)]$, $x = |S|/M$ and $S \ll m_P$.

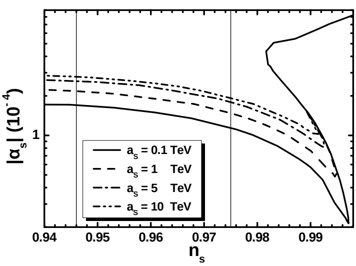
Note: No 'η problem' with minimal (canonical) Kähler potential !

Results

[Pallis, Shafi, 2013; Rehman, Shafi, Wickman, 2010]

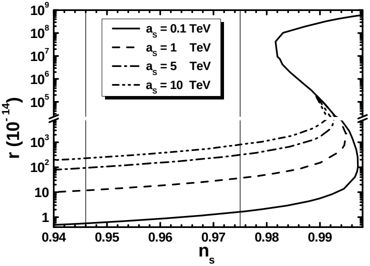


(a) (b)



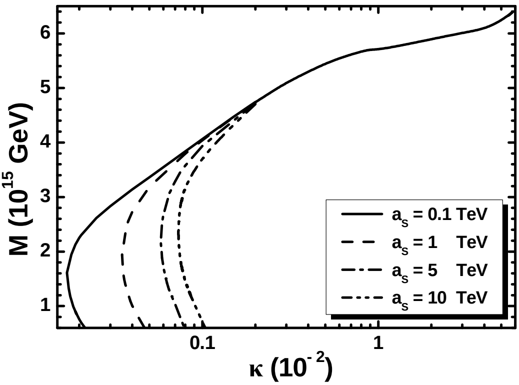
|α_s| (10^-4)
1
a_s = 0.1 TeV
a_s = 1 TeV
a_s = 5 TeV
a_s = 10 TeV
0.94
0.95
0.96
0.97
0.98
0.99
n_s

(a)

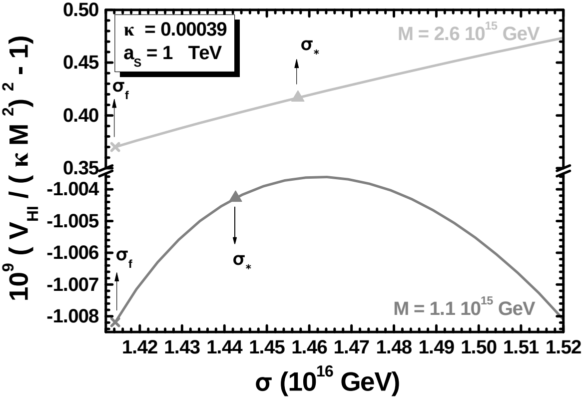


r (10^-14)
10^9
10^8
10^7
10^6
10^5
10^3
10^2
10
1
a_s = 0.1 TeV
a_s = 1 TeV
a_s = 5 TeV
a_s = 10 TeV
0.94
0.95
0.96
0.97
0.98
0.99
n_s

(b)



6
5
4
3
2
1
M (10^15 GeV)
0.1
1
κ (10^-2)
a_s = 0.1 TeV
a_s = 1 TeV
a_s = 5 TeV
a_s = 10 TeV



κ = 0.00039
a_S = 1 TeV
M = 2.6 10^{15} GeV
M = 1.1 10^{15} GeV
σ_*
σ_f
σ_*
σ_f
0.50
0.45
0.40
0.35
-1.004
-1.005
-1.006
-1.007
-1.008
1.42 1.43 1.44 1.45 1.46 1.47 1.48 1.49 1.50 1.51 1.52
σ (10^{16} GeV)
10^9 (V_{HI} / (κ M^2) ^{2} - 1)

Non-Minimal SUSY Hybrid Inflation and Tensor Modes

- Minimal SUSY hybrid inflation model yields tiny r values $\lesssim 10^{-10}$
- A more general analysis with a non-minimal Kähler potential can lead to larger r-values;
- The Kähler potential can be expanded as:

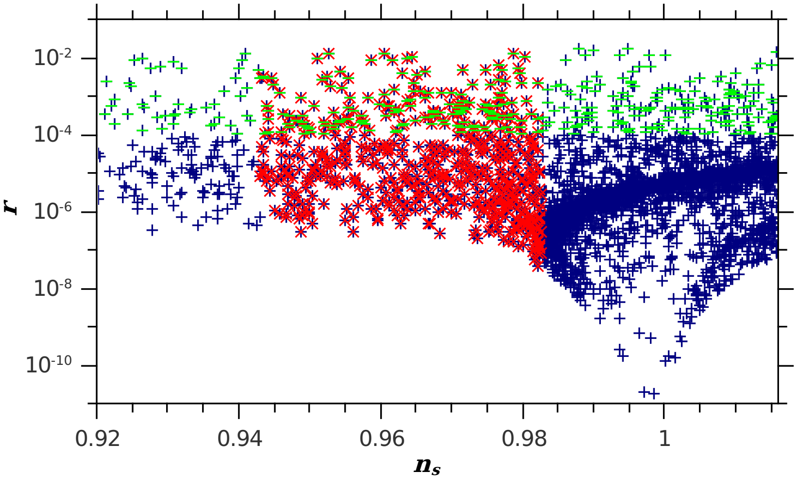
$$K = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2 + \frac{\kappa_S}{4}\frac{|S|^4}{m_P^2} + \frac{\kappa_\Phi}{4}\frac{|\Phi|^4}{m_P^2} + \frac{\kappa_{\overline{\Phi}}}{4}\frac{|\overline{\Phi}|^4}{m_P^2} + \kappa_{S\Phi}\frac{|S|^2|\Phi|^2}{m_P^2} + \kappa_{S\overline{\Phi}}\frac{|S|^2|\overline{\Phi}|^2}{m_P^2} + \kappa_{\Phi\overline{\Phi}}\frac{|\Phi|^2|\overline{\Phi}|^2}{m_P^2} + \frac{\kappa_{SS}}{6}\frac{|S|^6}{m_P^4} + \cdots ,$$

The scalar potential becomes

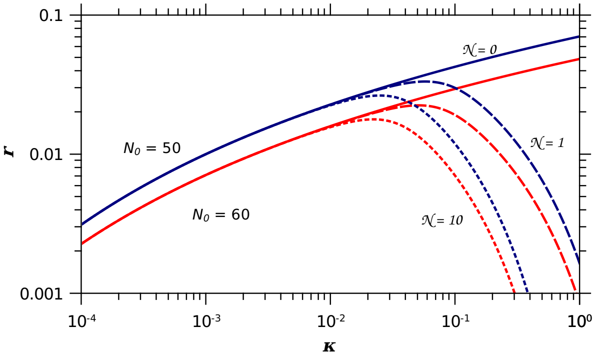
$$V \simeq \kappa^2 M^4 \left(1 - \kappa_S \left(\frac{M}{m_P}\right)^2 x^2 + \gamma_S \left(\frac{M}{m_P}\right)^4 \frac{x^4}{2} + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x) + a \left(\frac{m_{3/2}\, x}{\kappa\, M}\right) + \left(\frac{M_S\, x}{\kappa\, M}\right)^2\right)$$

with (leading order) non-minimal Kähler, SUGRA, radiative, and soft SUSY-breaking corrections, and where

$$\gamma_S \equiv 1 - \frac{7}{2}\kappa_S + 2\kappa_S^2 - 3\kappa_{SS}$$



10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
r
0.92
0.94
0.96
0.98
1
n_s



While radiative corrections are subdominant at large r, they play a crucial role in limiting the size of r. This limiting behavior comes in *indirectly* via the number of e-foldings N_0.

Summary

- The predictions of r (primordial gravity waves) for various models of inflation are as follows:
  - Gauge Singlet Higgs Inflation:

$$r \gtrsim 0.02 \quad \text{for} \quad n_s \geq 0.96$$

  - SUSY Higgs (Hybrid) Inflation:

$$r \lesssim 10^{-10} \text{ (minimal)}, \qquad r \lesssim 0.03 \text{ (non-minimal)}$$

- Updated results for r, $\frac{dn_s}{d\ln k}$, n_s are eagerly awaited.