Will Planck Observe Gravity Waves?

Qaisar Shafi

Bartol Research Institute
Department of Physics and Astronomy
University of Delaware

in collaboration with N. Okada, K.Pallis, M. Rehman, N. Senoguz, J. Wickman.

EPSHEP 2013
Stockholm, Sweden,
Outline

- Inflationary Cosmology
- Tree Level Gauge Singlet Higgs Inflation
- Quantum Smearing
- SUSY Higgs (Hybrid) Inflation
- Summary
Inflationary Cosmology

[Guth, Linde, Albrecht & Steinhardt, Starobinsky, Mukhanov, Hawking, ...]

Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for $n_s$, $r$, $dn_s/d\ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Physics Beyond the SM?
Inflation is driven by some potential $V(\phi)$:

- **Slow-roll parameters:**
  
  \[ \epsilon = \frac{m_p^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = m_p^2 \left( \frac{V''}{V} \right). \]

- The spectral index $n_s$ and the tensor to scalar ratio $r$ are given by
  
  \[ n_s - 1 \equiv \frac{d\ln \Delta^2_R}{d\ln k}, \quad r \equiv \frac{\Delta^2_h}{\Delta^2_R}, \]

  where $\Delta^2_h$ and $\Delta^2_R$ are the spectra of primordial gravity waves and curvature perturbation respectively.

- Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index $n_s$ and the tensor to scalar ratio $r$ are given by
  
  \[ n_s \simeq 1 - 6\epsilon + 2\eta, \quad r \simeq 16\epsilon. \]
- The tensor to scalar ratio $r$ can be related to the energy scale of inflation via
  \[ V(\phi_0)^{1/4} = 3.3 \times 10^{16} \, r^{1/4} \text{ GeV}. \]
- The amplitude of the curvature perturbation is given by
  \[ \Delta^2_R = \frac{1}{24\pi^2} \left( \frac{V/m_p^4}{\epsilon} \right)_{\phi=\phi_0} = 2.43 \times 10^{-9} \text{ (WMAP7 normalization)}. \]
- The spectrum of the tensor perturbation is given by
  \[ \Delta^2_h = \frac{2}{3\pi^2} \left( \frac{V}{m_p^4} \right)_{\phi=\phi_0}. \]
- The number of $e$-folds after the comoving scale $l_0 = 2\pi/k_0$ has crossed the horizon is given by
  \[ N_0 = \frac{1}{m_p^2} \int_{\phi_e}^{\phi_0} \left( \frac{V}{V'} \right) d\phi. \]
  Inflation ends when $\max[\epsilon(\phi_e), |\eta(\phi_e)|] = 1$. 
Consider the following Higgs Potential:

\[ V(\phi) = V_0 \left[ 1 - \left( \frac{\phi}{M} \right)^2 \right]^2 \]  

Here \( \phi \) is a gauge singlet field.

WMAP/Planck data favors BV inflation.
Quantum Smearing

\[ V = \frac{1}{4} \lambda (\phi^2 - v^2)^2 + \frac{C}{16\pi^2} \phi^4 \log \left[ \frac{\phi}{v} \right] + \text{const.}, \]
\[ V(\phi) = \frac{\kappa}{2} \left( 2 \kappa \phi^2 + \log \left( \frac{\phi}{\phi_p} \right) \right) \]
Supersymmetry

- Resolution of the gauge hierarchy problem
- Predicts plethora of new particles which LHC should find
- Unification of the SM gauge couplings at
  \[ M_{\text{GUT}} \sim 2 \times 10^{16} \text{ GeV} \]
- Cold dark matter candidate (LSP)
- Radiative electroweak breaking
- String theory requires supersymmetry (SUSY)

Alas, SUSY not yet seen at LHC
MSSM

\[ \alpha_i^{-1} \]

\[ \text{Log}_{10}[\Lambda/\text{GeV}] \]

\[ \alpha_1^{-1}, \alpha_2^{-1}, \alpha_3^{-1} \]
Attractive scenario in which inflation can be associated with symmetry breaking $G \rightarrow H$

Simplest inflation model is based on

$$W = \kappa S (\Phi \Phi - M^2)$$

$S =$ gauge singlet superfield, $(\Phi, \Phi)$ belong to suitable representation of $G$

Need $\Phi, \Phi$ pair in order to preserve SUSY while breaking $G \rightarrow H$ at scale $M \gg$ TeV, SUSY breaking scale.

R-symmetry

$$\Phi \Phi \rightarrow \Phi \Phi, \quad S \rightarrow e^{i\alpha} S, \quad W \rightarrow e^{i\alpha} W$$

$\Rightarrow$ $W$ is a unique renormalizable superpotential
Some examples of gauge groups:

\[ G = U(1)_{B-L}, \text{ (Supersymmetric superconductor)} \]

\[ G = SU(5) \times U(1), \quad (\Phi = 10), \quad \text{(Flipped \: SU(5))} \]

\[ G = 3c \times 2L \times 2R \times 1_{B-L}, \quad (\Phi = (1, 1, 2, +1)) \]

\[ G = 4c \times 2L \times 2R, \quad (\Phi = (\overline{4}, 1, 2)), \]

\[ G = SO(10), \quad (\Phi = 16) \]
At renormalizable level the SM displays an ‘accidental’ global $U(1)_{B-L}$ symmetry.

Next let us ‘gauge’ this symmetry, so that $U(1)_{B-L}$ is now promoted to a local symmetry. In order to cancel the gauge anomalies, one may introduce 3 SM singlet (right-handed) neutrinos.

This has several advantages:

- See-saw mechanism is automatic and neutrino oscillations can be understood.
• RH neutrinos acquire masses only after $U(1)_{B-L}$ is spontaneously broken; Neutrino oscillations require that RH neutrino masses are $\lesssim 10^{14}$ GeV.

• RH neutrinos can trigger leptogenesis after inflation, which subsequently gives rise to the observed baryon asymmetry;

• Last but not least, the presence of local $U(1)_{B-L}$ symmetry enables one to explain the origin of $Z_2$ 'matter' parity of MSSM. (It is contained in $U(1)_{B-L} \times U(1)_Y$, if $B - L$ is broken by a scalar vev, with the scalar carrying two units of $B - L$ charge.)
Tree Level Potential

\[ V_F = \kappa^2 (M^2 - |\Phi|^2)^2 + 2\kappa^2 |S|^2 |\Phi|^2 \]

SUSY vacua

\[ |\langle \Phi \rangle| = |\langle \Phi \rangle| = M, \quad \langle S \rangle = 0 \]
Take into account radiative corrections (because during inflation $V \neq 0$ and SUSY is broken by $F_S = -\kappa M^2$)

- **Mass splitting in $\Phi - \bar{\Phi}$**
  
  $$m_{\pm}^2 = \kappa^2 S^2 \pm \kappa^2 M^2, \quad m_F^2 = \kappa^2 S^2$$

- **One-loop radiative corrections**
  
  $$\Delta V_{1\text{loop}} = \frac{1}{64\pi^2} \text{Str}[M^4(S)(\ln \frac{M^2(S)}{Q^2} - \frac{3}{2})]$$

- **In the inflationary valley ($\Phi = 0$)**
  
  $$V \simeq \kappa^2 M^4 \left(1 + \frac{\kappa^2 N}{8\pi^2} F(x)\right)$$

where $x = |S|/M$ and

$$F(x) = \frac{1}{4} \left((x^4 + 1) \ln \frac{x^4 - 1}{x^4} + 2x^2 \ln \frac{x^2 + 1}{x^2 - 1} + 2 \ln \frac{\kappa^2 M^2 x^2}{Q^2} - 3\right)$$
Also include supergravity corrections + soft SUSY breaking terms

- The minimal Kähler potential can be expanded as

$$K = |S|^2 + |\Phi|^2 + |\bar{\Phi}|^2$$

- The Sugra scalar potential is given by

$$V_F = e^{K/m_p^2} \left( K_{ij}^{-1} D_{z_i} W D_{z_j}^* W^* - 3m_p^{-2} |W|^2 \right)$$

where we have defined

$$D_{z_i} W \equiv \frac{\partial W}{\partial z_i} + m_p^{-2} \frac{\partial K}{\partial z_i} W; \quad K_{ij} \equiv \frac{\partial^2 K}{\partial z_i \partial z_j^*}$$

and $$z_i \in \{\Phi, \bar{\Phi}, S, \ldots\}$$
Take into account sugra corrections, radiative corrections and soft SUSY breaking terms:

\[
V \simeq \kappa^2 M^4 \left( 1 + \left( \frac{M}{m_p} \right)^4 \frac{x^4}{2} + \frac{\kappa^2 N}{8\pi^2} F(x) + a_s \left( \frac{m_{3/2} x}{\kappa M} \right) + \left( \frac{m_{3/2} x}{\kappa M} \right)^2 \right)
\]

where \( a_s = 2|2 - A| \cos[\arg S + \arg(2 - A)] \), \( x = |S|/M \) and \( S \ll m_P \).

Note: No ‘\( \eta \) problem’ with minimal (canonical) Kähler potential!
Results

[Pallos, Shafi, 2013; Rehman, Shafi, Wickman, 2010]
\[ a_s = 0.1 \text{ TeV} \]
\[ a_s = 1 \text{ TeV} \]
\[ a_s = 5 \text{ TeV} \]
\[ a_s = 10 \text{ TeV} \]

\[ \kappa (10^{-2}) \]

\[ M (10^{15} \text{ GeV}) \]
$10^9 \left( \frac{V_{HI}}{\kappa M^2} - 1 \right)$

- $\kappa = 0.00039$
- $a_s = 1$ TeV
- $M = 1.1 \times 10^{15}$ GeV
- $M = 2.6 \times 10^{15}$ GeV

$\sigma$ (10$^{16}$ GeV)
Minimal SUSY hybrid inflation model yields tiny $r$ values
\[ \lesssim 10^{-10} \]

A more general analysis with a non-minimal Kähler potential can lead to larger $r$-values;

The Kähler potential can be expanded as:

\[
K = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2 + \frac{\kappa_S}{4} \frac{|S|^4}{m_P^2} + \frac{\kappa_\Phi}{4} \frac{|\Phi|^4}{m_P^2} + \frac{\kappa_{\Phi\Phi}}{4} \frac{|\overline{\Phi}|^4}{m_P^2} + \\
\kappa_S \Phi \frac{|S|^2 |\Phi|^2}{m_P^2} + \kappa_{S\overline{\Phi}} \frac{|S|^2 |\overline{\Phi}|^2}{m_P^2} + \kappa_{\Phi\overline{\Phi}} \frac{|\Phi|^2 |\overline{\Phi}|^2}{m_P^2} + \frac{\kappa_{SS}}{6} \frac{|S|^6}{m_P^4} + \cdots,
\]
The scalar potential becomes

\[ V \simeq \kappa^2 M^4 \left( 1 - \kappa_S \left( \frac{M}{m_P} \right)^2 x^2 + \gamma_S \left( \frac{M}{m_P} \right)^4 \frac{x^4}{2} + \right. \]

\[ \frac{\kappa^2 N}{8\pi^2} F(x) + a \left( \frac{m_{3/2} x}{\kappa M} \right) + \left( \frac{M_S x}{\kappa M} \right)^2 \left. \right) \]

with (leading order) non-minimal Kähler, SUGRA, radiative, and soft SUSY-breaking corrections, and where

\[ \gamma_S \equiv 1 - \frac{7}{2} \kappa_S + 2\kappa_S^2 - 3\kappa_{SS} \]
The figure illustrates a scatter plot with the $n_s$ axis ranging from 0.92 to 1.0 and the $r$ axis ranging from $10^{-10}$ to $10^{-2}$. The data points are color-coded to differentiate various categories or conditions.
While radiative corrections are subdominant at large $r$, they play a crucial role in limiting the size of $r$. This limiting behavior comes in indirectly via the number of e-foldings $N_0$. 
The predictions of $r$ (primordial gravity waves) for various models of inflation are as follows:

- **Gauge Singlet Higgs Inflation:**
  \[ r \gtrsim 0.02 \text{ for } n_s \geq 0.96 \]

- **SUSY Higgs (Hybrid) Inflation:**
  \[ r \lesssim 10^{-10} \text{ (minimal), } r \lesssim 0.03 \text{ (non-minimal)} \]

- Updated results for $r$, $\frac{d n_s}{d \ln k}$, $n_s$ are eagerly awaited.