Radiative decays of charmonium

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1. Towards the solution of the $J/\psi \rightarrow \eta_c \gamma$ puzzle

2. $h_c \rightarrow \eta_c \gamma$ or "A new puzzle?"

3. Conclusions
Why is $J/\psi \to \eta_c \gamma$ important?

- Very soft photon: testing any method pretending to correctly estimate the non-perturbative QCD effects
- Early predictions 'impossible' to reconcile with early experimental findings
- Origin of discrepancy: (i) Non-perturbative QCD? (ii) Experiment?
- Or..... it might be that $\eta_c$ mixes with $A$ [a very light CP-odd Higgs boson – ingredient of a general 2HDM].
What the experimenters say about $J/\psi \rightarrow \eta_c \gamma$?

Experimental situation – unclear

- PDG value: $\Gamma (J/\psi \rightarrow \eta_c \gamma)_{PDG} = (1.58 \pm 0.37) \text{ keV}$
- Low result by Crystal Ball (1986): $\Gamma (J/\psi \rightarrow \eta_c \gamma)_{PDG} = (1.18 \pm 0.33) \text{ keV}$
- However, CLEO (2009): $\Gamma (J/\psi \rightarrow \eta_c \gamma)_{PDG} = (1.91 \pm 0.28 \pm 0.03) \text{ keV}$
- Preliminary, KEDR(2010): $\Gamma (J/\psi \rightarrow \eta_c \gamma)_{PDG} = (2.17 \pm 0.14 \pm 0.37) \text{ keV}$
- Could BESSIII step in and clarify the issue?! Final KEDR?!
Theory – prior to mid-2012

- Relate $\eta_c \rightarrow 2\gamma$ with $J/\psi \rightarrow \eta_c \gamma$ by using dispersion relations:
  \[ \Gamma (J/\psi \rightarrow \eta_c \gamma) = (2.2 \div 3.2) \text{ keV} \] [M.A. Shifman, Z. Phys. C 6 (1980)]

- Two QCD sum rule calculations – two different results:
  - $\sim (1.7 \pm 0.4) \text{ keV}$ [A.Y. Khodjamirian, Sov. J. Nucl. Phys. 39 (1984)]
  - $\sim (2.6 \pm 0.5) \text{ keV}$ [Beilin and Radyushkin, Nucl. Phys. B 260 (1985)]

- Effective Theory (pNRQCD): $(1.5 \pm 1.0) \text{ keV}$ [N. Brambilla et al, PRD73 (2006)]

- Constituent Quark Models:
  - $\sim 3.3 \text{ keV}$ [M.B Voloshin, Prog.Part.Nucl.Phys. 61 (2007)]
  - $\sim 2.85 \text{ keV}$ [E. Eichten et al., Rev.Mod.Phys 80 (2008)]
Theory – prior to mid-2012

- Quenched lattice QCD at single lattice spacing $2.51(8) \text{ keV}$
  [ J.J Dudek et al., PRD 79 (2009)]

- Unquenched QCD with $N_f = 2$ (single lattice spacing) $2.77(5) \text{ keV}$
  [ Y. Chen et. al., PRD 84 (2011)]
What we want to achieve

- **Continuum limit**: Need simulations at several fine lattice spacings (several cut-off scales) and take the continuum limit
- **Renormalization**: Non-perturbative matching of a lattice bilinear quark operator to its local continuum counterpart [keep the same Ward identities]
- **Momentum dependence**: Need hadronic matrix element at $q^2 = 0$ (on-shell photon) and avoid extrapolating from unphysical $q^2$'s to $q^2 = 0$
- **Unquenching**: Need to include the effects of quark loops ($u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$) in the background gauge field configurations

What we have now

- **Continuum limit**: Simulations at 4 lattice spacings $a \in [0.054, 0.100]$ fm ✓
- **Renormalization**: non-perturbative in the RI-MOM scheme ✓
- **Momentum dependence**: $q^2 = 0$ by using twisted boundary conditions ✓
- **Unquenching**: Included $N_f = 2$ dynamical quarks ($m_\pi \in [280, 500]$ MeV) ✓

All good except perhaps for the last item: $s\bar{s}$ and $c\bar{c}$ pairs are missing!
We use
- Wilson regularization of QCD on the lattice with twisted mass term (maximally twisted QCD) [Frezzotti and Rossi, JHEP 0408 (2004)]
- gauge field configurations produced by ETMC [publicly available via ILDG]
- compute charm quark propagators $S^c_{\theta}(z_1; z_2)$

and combine them in the three-point correlation functions:

$$C_{ij}^{(3)}(t) = \langle \text{Tr} \left[ S_c(y; 0) \gamma_i S_c(0, x) \gamma_j S^\dagger_c(x, y) \gamma_5 \right] \rangle \simeq Z_{\eta_c}^i Z_{J/\psi}^j \exp \left[ \left( E_{\eta_c} - M_{J/\psi} \right) t \right] \langle \eta_c | J^{em}_{j} | (J/\psi)_i \rangle$$

To get rid of the sources, also need the two-point correlation functions:

$$C_{55}^{(2)}(t) = \langle \text{Tr} \left[ S_c(0, 0; \vec{x}, t) \gamma_5 S_c(\vec{x}, t; \vec{0}, 0) \gamma_5 \right] \rangle \underset{t \to \infty}{\simeq} Z_{\eta_c}^2 \exp (-M_{\eta_c} t)$$

$$C_{ii}^{(2)}(t) = \langle \text{Tr} \left[ S_c(0, 0; \vec{x}, t) \gamma_i S_c(\vec{x}, t; \vec{0}, 0) \gamma_i \right] \rangle \underset{t \to \infty}{\simeq} Z_{J/\psi}^i \exp (-M_{J/\psi} t)$$

and therefore,

$$R(t) \equiv \frac{C_{ij}^{(3)}(t)}{Z_{J/\psi}^i Z_{\eta_c}^j \exp \left[ \left( E_{\eta_c} - M_{J/\psi} \right) t \right]} \simeq \langle \eta_c | J^{em}_{j} | (J/\psi)_i \rangle$$
Computation of $\langle \eta_c | J_{\mu}^{\text{em}} | J/\psi \rangle$

$$R(t) \equiv \frac{C_{ij}^{(3)}(t)}{Z_{J/\psi}^i Z_{\eta_c} Z^{\text{exp}} \left[ (E_{\eta_c} - M_{J/\psi}) t \right]} \sim \langle \eta_c | J_{\mu}^{\text{em}} | (J/\psi)_i \rangle$$

N.B.: Very high precision computations $\Delta = m_{J/\psi} - m_{\eta_c} = 112(3)$ MeV

Matrix element – an example:

![Graph](image-url)
$J/\psi \rightarrow \eta_c \gamma$ form factor depends on lattice spacing (hard cut-off)

$$\langle \eta_c(\vec{k})|Q_c\bar{c}\gamma_\mu c|J/\psi(\vec{p} = 0, \epsilon_\lambda)\rangle \bigg|_{|\vec{k}| = \frac{m^2_\psi - m^2_{\eta_c}}{2m_\psi}} = \frac{2e}{3} \varepsilon_{\mu \nu \alpha \beta} \epsilon^*_\lambda \bar{p}^\alpha k^\beta \frac{2V(0)}{m_{J/\psi} + m_{\eta_c}}$$

Continuum limit:

but does not depend on the light "sea" quark mass!

$V(0) = 1.92(3)(2)$
Two months after our results appeared on arXiv, HPQCD published theirs.

- totally different regularization scheme (‘staggered quarks’)
- included also $N_f = 2+1$ (i.e. $s\bar{s}$ pairs)

Result in continuum limit agrees with ours [G.C.Donald et al., PRD86 ('12)]

\[ V(0)_{\text{Orsay}} = 1.92(3)(2) \quad \text{and} \quad V(0)_{\text{HPQCD}} = 1.90(7)(1) \]

- Update old Shifman’s result with modern exp. result for $\Gamma(\eta_c \to \gamma\gamma)$
  ⇒ more accurate $\Gamma(J/\psi \to \eta_c\gamma)$ – larger than the PDG value!

- \textbf{pNRQCD} description of M1-decays improved [Pineda & Segovia, PRD87 (2013)]
  - exact inclusion of the static potential in the LO Hamiltonian
  - resummation of large logs by means of RGE
  - cancellation of renormalon ambiguity → smaller uncertainty in $\Gamma(J/\psi \to \eta_c\gamma)$
Theory 2013 better than experiment! Need help from BESIII and KEDR

Problem is solved as far as theory is concerned. Ball is thrown back to experimenters.
Why is $h_c \rightarrow \eta_c \gamma$ important?

Facts:

- $h_c(1P)$—elusive for many years—finally observed in 2005 at CLEO
- Confirmation in 2008 by BaBar in $B \rightarrow X_{\bar{c}c}K^*$ through the dominant $X_{\bar{c}c} = h_c(1P) \rightarrow \eta_c \gamma$
- BESIII in 2010 observes $\psi' \rightarrow h_c(\rightarrow \eta_c \gamma)\pi^0$
- CLEO in 2011 improves accuracy of $m_{h_c}$ from $e^+e^- \rightarrow \pi^+\pi^- h_c$
- PDG summarized:

\[
J^{PC} = 1^{+-} \quad m_{h_c} = 3525.4(1) \text{ MeV} \quad \Gamma(h_c) = (0.7 \pm 0.4) \text{ MeV}
\]
\[
B(h_c \rightarrow \eta_c \gamma) = (51 \pm 6)\%
\]
\[
\Rightarrow \Gamma(h_c \rightarrow \eta_c \gamma)_{\text{PDG}} = (0.36 \pm 0.21) \text{ MeV}
\]

What does Lattice QCD has to say about $h_c$?
We find its mass in continuum limit to be

\[ m_{hc} = 3542(32) \text{ MeV} \quad [3525.4(1) \text{ MeV}]_{PDG} \quad \text{OK} \]

E1-transition form factor

\[
\langle \eta_c(k) | J^\text{em}_\mu | h_c(p, \epsilon_\lambda) \rangle = i e \frac{2}{3} \left[ m_{hc} F_1(q^2) \left( \epsilon^*_\mu - \frac{\epsilon^*_\lambda \cdot q}{q^2} q_\mu \right) + \ldots \right]
\]

on-shell photon for \(|\vec{p}| = 0\) and \(|\vec{k}| = (m^2_{hc} - m^2_{\eta_c})/(2m_{hc})\)

\[ F_1(0) = -0.57(2)(1) \]
We find its mass in continuum limit to be

\[ m_{hc} = 3542(32) \text{ MeV} \quad \text{[3525.4(1) MeV]}_{\text{PDG}} \quad \text{OK} \]

\[ F_1(0) = -0.57(2)(1) \quad \rightarrow \Gamma(h_c \rightarrow \eta_c \gamma) = (0.72 \pm 0.05 \pm 0.02) \text{ MeV} \]

which is larger than \( \Gamma(h_c \rightarrow \eta_c \gamma)_{\text{PDG}} = (0.36 \pm 0.21) \text{ MeV} \quad [\text{sic!}] \)

More experimental study needed to elucidate the origin of this discrepancy.

Would be great if other methods were employed as well:
- improve pNRQCD for E1-transitions
- QCD sum rule estimate is missing (need revisiting \( J/\psi \rightarrow \eta_c \gamma \) as well)
After many years of painstaking improvement of lattice QCD we are finally able to perform a precision computation of the radiative decays of charmonia.
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Our result confirmed in a totally independent lattice study by HPQCD, both being consistent with new findings based on pNRQCD. If the discrepancy between theory and experiment persists, this might be a hint for mixing of $\eta_c$ with a light CP-odd Higgs.

Solving the $J/\psi \rightarrow \eta_c \gamma$ puzzle seems to lead to a new one: $h_c \rightarrow \eta_c \gamma$. We computed the relevant hadronic matrix element and found $\Gamma(h_c \rightarrow \eta_c \gamma) = 0.72(5)(2)$ MeV, that appears larger than the value inferred from the present experimental findings (esp. BESIII). More theoretical and experimental work needed to clarify the $h_c \rightarrow \eta_c \gamma$ problem.
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