# Radiative decays of charmonium

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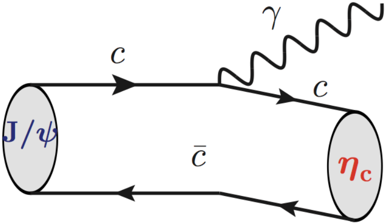
# Outline

1. Towards the solution of the $J/\psi \to \eta_c\gamma$ puzzle
2. $h_c \to \eta_c\gamma$ or "A new puzzle?"
3. Conclusions

# Why is $J/\psi \to \eta_c\gamma$ important?

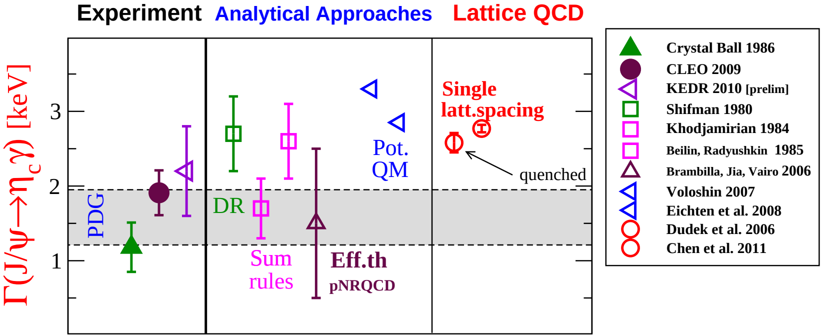


- Very soft photon: testing any method pretending to correctly estimate the non-perturbative QCD effects
- Early predictions 'impossible' to reconcile with early experimental findings
- Origin of discrepancy: (i) Non-perturbative QCD? (ii) Experiment?
- Or..... it might be that $\eta_c$ mixes with $A$ [a very light CP-odd Higgs boson – ingredient of a general 2HDM].

# What the experimenters say about $J/\psi \to \eta_c \gamma$?

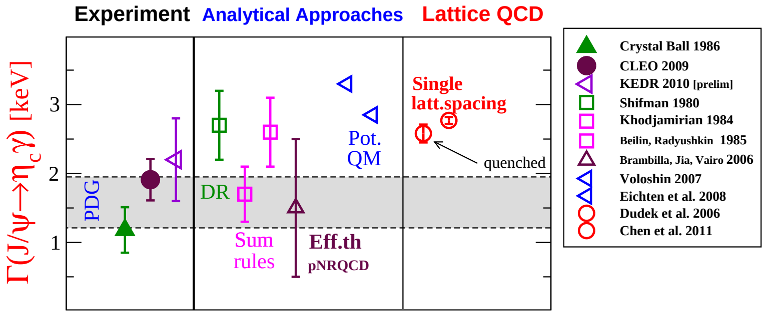
## Experimental situation – unclear

- PDG value: $\Gamma\left(J/\psi \to \eta_c\gamma\right)_{\rm PDG} = (\mathbf{1.58} \pm \mathbf{0.37})\,\mathbf{keV}$
- Low result by Crystal Ball (1986): $\Gamma\left(J/\psi \to \eta_c\gamma\right)_{\rm PDG} = (\mathbf{1.18} \pm \mathbf{0.33})\,\mathbf{keV}$
- However, CLEO (2009): $\Gamma\left(J/\psi \to \eta_c\gamma\right)_{\rm PDG} = (\mathbf{1.91} \pm \mathbf{0.28} \pm \mathbf{0.03})\,\mathbf{keV}$
- Preliminary, KEDR(2010): $\Gamma\left(J/\psi \to \eta_c\gamma\right)_{\rm PDG} = (\mathbf{2.17} \pm \mathbf{0.14} \pm \mathbf{0.37})\,\mathbf{keV}$
- Could BESSIII step in and clarify the issue?! Final KEDR?!



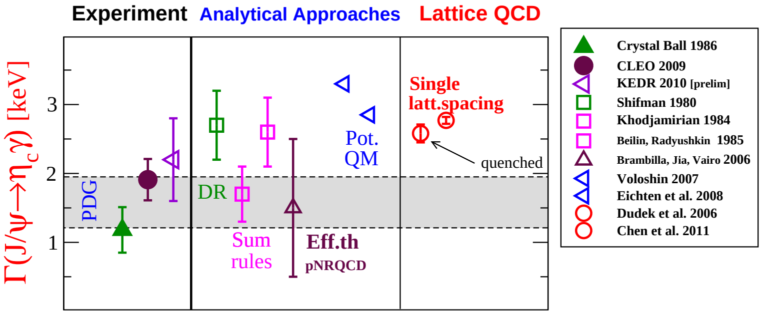
# Theory – prior to mid-2012

- Relate $\eta_c \to 2\gamma$ with $J/\psi \to \eta_c\gamma$ by using dispersion relations:
  $\Gamma\left(J/\psi \to \eta_c\gamma\right) = \mathbf{(2.2 \div 3.2)\ keV}$ `[M.A. Shifman, Z. Phys. C 6 (1980)]`
- Two QCD sum rule calculations – two different results:
  - $\sim \mathbf{(1.7 \pm 0.4)\ keV}$ `[A.Y. Khodjamirian, Sov. J. Nucl. Phys. 39 (1984)]`
  - $\sim \mathbf{(2.6 \pm 0.5)\ keV}$ `[Beilin and Radyushkin, Nucl. Phys. B 260 (1985)]`
- Effective Theory (pNRQCD): $\mathbf{(1.5 \pm 1.0)\ keV}$ `[N.Brambilla et al, PRD73 (2006)]`
- Constituent Quark Models:
  - $\sim \mathbf{3.3\,keV}$ `[M.B Voloshin, Prog.Part.Nucl.Phys. 61 (2007)]`
  - $\sim \mathbf{2.85\,keV}$ `[E. Eichten et al., Rev.Mod.Phys 80 (2008)]`



# Theory – prior to mid-2012

- Quenched lattice QCD at single lattice spacing **2.51(8) keV**
  [ J.J Dudek et al., PRD 79 (2009)]
- Unquenched QCD with $N_f = 2$ (single lattice spacing) **2.77(5) keV**
  [ Y. Chen et. al., PRD 84 (2011)]



# Lattice QCD

## What we want to achieve

- **Continuum limit**: Need simulations at several fine lattice spacings (several cut-off scales) and take the continuum limit
- **Renormalization**: Non-perturbative matching of a lattice bilinear quark operator to its local continuum counterpart [keep the same Ward identities]
- **Momentum dependence**: Need hadronic matrix element at $q^2 = 0$ (*on-shell photon*) and avoid extrapolating from unphysical $q^2$'s to $q^2 = 0$
- **Unquenching**: Need to include the effects of quark loops ($u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$) in the background gauge field configurations

## What we have now

- **Continuum limit**: Simulations at 4 lattice spacings $a \in [0.054, 0.100]$ fm ✓
- **Renormalization**: non-perturbative in the RI-MOM scheme ✓
- **Momentum dependence**: $q^2 = 0$ by using twisted boundary conditions ✓
- **Unquenching**: Included $N_\mathrm{f} = 2$ dynamical quarks ($m_\pi \in [280, 500]$ MeV) ✓

All good except perhaps for the last item: $s\bar{s}$ and $c\bar{c}$ pairs are missing!

# Computation of $\langle\eta_c|J_\mu^{\rm em}|J/\psi\rangle$

We use

- Wilson regularization of QCD on the lattice with twisted mass term (maximally twisted QCD) [Frezzotti and Rossi, JHEP 0408 (2004)]
- gauge field configurations produced by **ETMC** [publicly available via ILDG]
- compute charm quark propagators $S_c^{(\theta)}(z_1;z_2)$

and combine them in the three-point correlation functions:

$$C_{ij}^{(3)}(t) = \langle \mathrm{Tr}\left[S_c(y;0)\gamma_i S_c(0,x)\gamma_j S_c^{\vec{p}}(x,y)\gamma_5\right]\rangle$$
$$\underset{0\ll t\ll T}{\simeq} Z^i_{J/\psi} Z_{\eta_c} \exp\left[\left(E_{\eta_c} - M_{J/\psi}\right)t\right] \langle\eta_c|J_j^{\rm em}|(J/\psi)_i\rangle$$

To get rid of the sources, also need the two-point correlation functions:

$$C_{55}^{(2)}(t) = \langle \mathrm{Tr}\left[S_c(0,0;\vec{x},t)\gamma_5 S_c(\vec{x},t;\vec{0},0)\gamma_5\right]\rangle \underset{t\to\infty}{\simeq} {Z_{\eta_c}}^2 \exp\left(-M_{\eta_c}t\right)$$
$$C_{ii}^{(2)}(t) = \langle \mathrm{Tr}\left[S_c(0,0;\vec{x},t)\gamma_i S_c(\vec{x},t;\vec{0},0)\gamma_i\right]\rangle \underset{t\to\infty}{\simeq} {Z^i_{J/\psi}}^2 \exp\left(-M_{J/\psi}t\right)$$

and therefore,

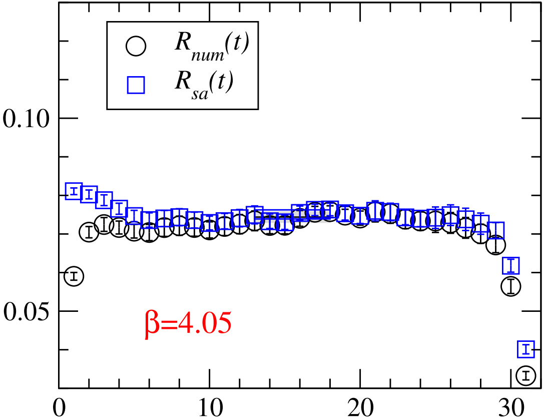
$$R(t) \equiv \frac{C_{ij}^{(3)}(t)}{Z^i_{J/\psi} Z_{\eta_c} \exp\left[\left(E_{\eta_c} - M_{J/\psi}\right)t\right]} \underset{0\ll t\ll T}{\simeq} \left\langle\eta_c\left|J_j^{em}\right|(J/\psi)_i\right\rangle$$

# Computation of $\langle \eta_c | J_\mu^{\mathrm{em}} | J/\psi \rangle$

$$R(t) \equiv \frac{C_{ij}^{(3)}(t)}{Z_{J/\psi}^{i} Z_{\eta_c} \exp\left[\left(E_{\eta_c} - M_{J/\psi}\right) t\right]} \underset{0 \ll t \ll T}{\simeq} \langle \eta_c \left| J_j^{em} \right| (J/\psi)_i \rangle$$

N.B.: Very high precision computations $\Delta = m_{J/\psi} - m_{\eta_c} = 112(3)$ MeV

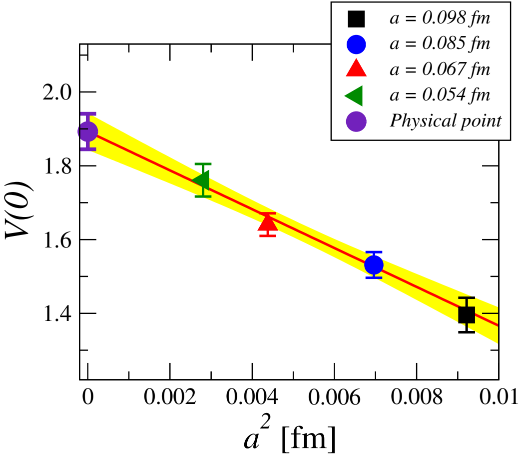
Matrix element – an example:



# $J/\psi \to \eta_c\gamma$ form factor depends on lattice spacing (hard cut-off)

$$\langle \eta_c(\vec{k}) | \mathcal{Q}_c \bar{c}\gamma_\mu c | J/\psi(\vec{p}=0,\epsilon_\lambda)\rangle \Big|_{|\vec{k}|=\frac{m_\psi^2 - m_{\eta_c}^2}{2m_\psi}} = \frac{2e}{3}\, \varepsilon_{\mu\nu\alpha\beta}\, \epsilon_\lambda^{*\nu} p^\alpha k^\beta \, \frac{2\, V(0)}{m_{J/\psi} + m_{\eta_c}}$$

Continuum limit:



but **does not** depend on the light "*sea*" quark mass!

$\boxed{V(0) = 1.92(3)(2)}$

# $J/\psi \to \eta_c\gamma$ – New developments

- Two months after our results appeared on arXiv, HPQCD published theirs.
  - totally different regularization scheme ('staggered quarks')
  - included also $N_\mathrm{f} = 2+1$ (i.e. $s\bar{s}$ pairs)

  Result in continuum limit agrees with ours [G.C.Donald et al., PRD86 ('12)]

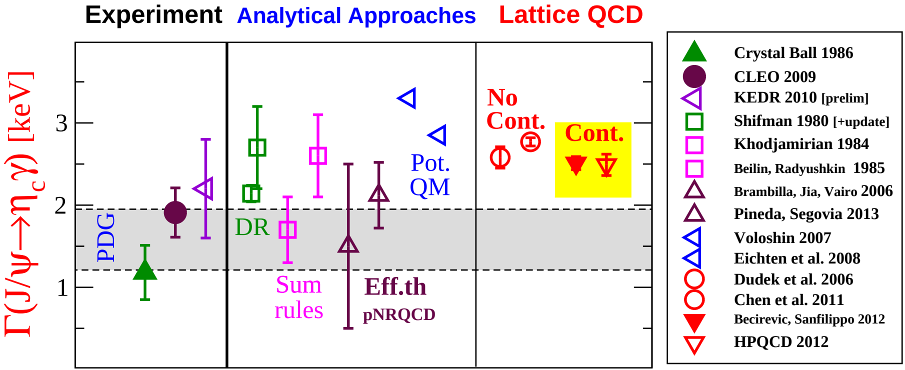
$$\boxed{V(0)_{\mathrm{Orsay}} = 1.92(3)(2)} \qquad \boxed{V(0)_{\mathrm{HPQCD}} = 1.90(7)(1)}$$

- Update old Shifman's result with modern exp. result for $\Gamma(\eta_c \to \gamma\gamma)$
  $\Rightarrow$ more accurate $\Gamma(J/\psi \to \eta_c\gamma)$ – larger than the PDG value!

- **pNRQCD** description of M1-decays improved [Pineda & Segovia, PRD87 (2013)]
  - exact inclusion of the static potential in the LO Hamiltonian
  - resummation of large logs by means of RGE

  cancellation of renormalon ambiguity → smaller uncertainty in $\Gamma(J/\psi \to \eta_c\gamma)$

# Theory 2013 better than experiment! Need help from BESIII and KEDR



Problem is solved as far as theory is concerned. Ball is thrown back to experimenters.

# Why is $h_c \to \eta_c\gamma$ important?

Facts:

- $h_c(1P)$ –elusive for many years– finally observed in 2005 at CLEO
- Confirmation in 2008 by BaBar in $B \to X_{\bar{c}c}K^{(*)}$ through the dominant $X_{\bar{c}c} = h_c(1P) \to \eta_c\gamma$
- BESIII in 2010 observes $\psi' \to h_c(\to \eta_c\gamma)\pi^0$
- CLEO in 2011 improves accuracy of $m_{h_c}$ from $e^+e^- \to \pi^+\pi^- h_c$
- PDG summarized:

$$J^{PC} = 1^{+-} \qquad m_{h_c} = 3525.4(1)\ \text{MeV} \quad \Gamma(h_c) = (0.7 \pm 0.4)\ \text{MeV}$$

$$\mathcal{B}(h_c \to \eta_c\gamma) = (51 \pm 6)\%$$

$$\Rightarrow \Gamma(h_c \to \eta_c\gamma)_{\text{PDG}} = (0.36 \pm 0.21)\ \text{MeV}$$

- What does Lattice QCD has to say about $h_c$?

# $h_c \to \eta_c \gamma$ on the lattice

- We find its mass in continuum limit to be

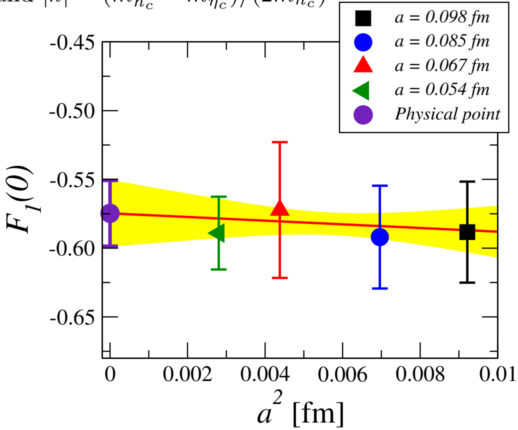
$$m_{h_c} = 3542(32)\ \text{MeV} \qquad [3525.4(1)\ \text{MeV}]_{\text{PDG}} \qquad \text{OK}$$

- E1-transition form factor

$$\langle \eta_c(k)|J_\mu^{\text{em}}|h_c(p,\epsilon_\lambda)\rangle = ie\frac{2}{3}\left[m_{h_c}F_1(q^2)\left(\epsilon_\mu^{\lambda *} - \frac{\epsilon_\lambda^* \cdot q}{q^2}q_\mu\right) + \dots\right]$$

on-shell photon for $|\vec{p}| = 0$ and $|\vec{k}| = (m_{h_c}^2 - m_{\eta_c}^2)/(2m_{h_c})$

$$\boxed{F_1(0) = -0.57(2)(1)}$$



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- E1-transition form factor

$$F_1(0) = -0.57(2)(1)\ \longrightarrow \Gamma(h_c \to \eta_c\gamma) = (0.72 \pm 0.05 \pm 0.02)\ \text{MeV}$$

which is larger than $\Gamma(h_c \to \eta_c\gamma)_{\text{PDG}} = (0.36 \pm 0.21)$ MeV [sic!]

- More experimental study needed to elucidate the origin of this discrepancy.
- Would be great if other methods were employed as well:
  - improve pNRQCD for E1-transitions
  - QCD sum rule estimate is missing (need revisiting $J/\psi \to \eta_c\gamma$ as well)

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2. Our result for $\Gamma(J/\psi \to \eta_c \gamma)$ solves the theory part of the puzzling discrepancy between theory and experiment and exposes a problem on experimental side: our results agree with modern-day experiment. New experimental result for $\Gamma(J/\psi \to \eta_c \gamma)$ with accuracy comparable with Crystal Ball (86) is indispensable.

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6. More theoretical and experimental work needed to clarify the $h_c \to \eta_c\gamma$ problem.