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Theory Predictions for Higgs Boson Production

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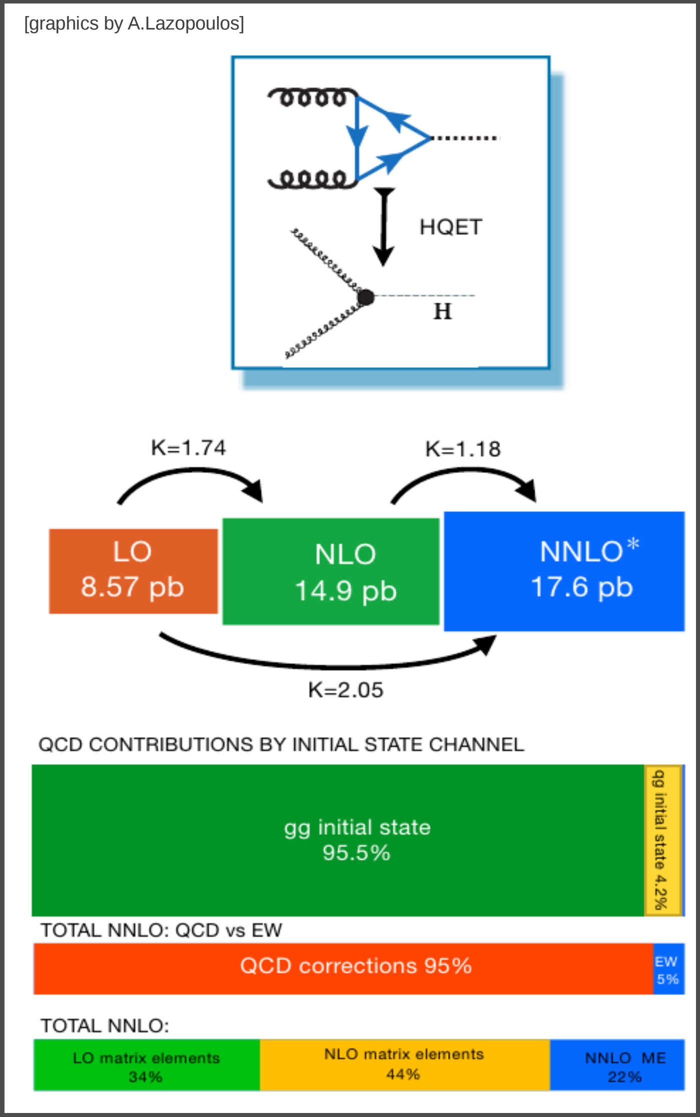
With the 14 TeV LHC we are about to enter a new era of Higgs precision physics!

We should therefore revise our best theoretical predictions and their uncertainties.

Here focus on Higgs production in gluon-fusion:

- yields the largest Higgs cross section
- is one of the worst perturbative observables
  that is is known in Collider Physics.

Higgs boson cross section: The Gluon Fusion channel



Perturbative Corrections:

- NLO QCD corrections known exactly (with top-bottom interference)
- NNLO QCD corrections (in HQET)
- subleading terms in the m_H/(2m_t) expansion
- EW corrections known
- mixed QCD EW corrections

Resummation:

- Soft gluon NNLL, Pi^2
- Transverse momentum resummation to NNLL (now also with exact top and bottom dependence)
- Jet transverse momentum and cone size R to NNLL
- Approximate N3LO

Tools:

- mc@nlo, powheg, higlu, HNNLO, Hres, Hqt, Fehip, FehiPro (code was never published), Hpro, Ihixs, **Ehixs (next slide)**

EHIXS

FH, A. Lazopoulos

A soon to appear C++ tool for simulating fully differential Higgs production at NNLO!

Great Increase of speed for NNLO real emissions:

- Integrand reduction with symmetric Groebner basis
- Non-linear mappings on phase space variables for efficient factorisation of all infra-red singularities

Includes:

- All known fixed order corrections
- Anomalous standard model couplings
- All important decays
- Also bb → H fully differential at NNLO

NNLO Theory Uncertainty

IHixs@8TeV:[Anastasiou, Buehler, FH, Lazopoulos]
(all known perturb. corr.)

m_H(GeV)	MSTW08 $\sigma(pb)$	$\%\delta_{PDF}$	$\%\delta_{\mu_F}$
125	20.69	$^{+7.79}_{-7.53}$	$^{+8.37}_{-9.26}$

Central scale $\mu = \mu_R = \mu_F = \frac{m_H}{2}$

Scale variation $\frac{m_H}{4} \leq \mu \leq m_H$

The pdf uncertainty is computed at 90%CL.

De Florian & Grazzini @ 8TeV:
(all known pert. Corr. + NNLL soft resummation)

m_H (GeV)	σ (pb)	scale(%)	PDF+α_S(%)
125.0	19.31	+7.2 -7.8	+7.5 -6.9

Central scale $\mu = \mu_R = \mu_F = m_H$

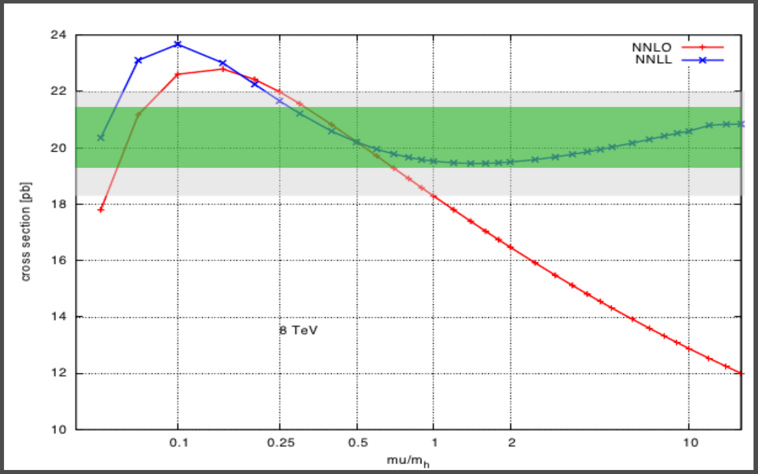
Scale variation $\frac{m_H}{2} \leq \mu_R, \mu_F \leq 2m_H$

The pdf uncertainty is computed at 68%CL. acc. To PDF4LHC

Note: dominant uncertainty is from μ_R variation

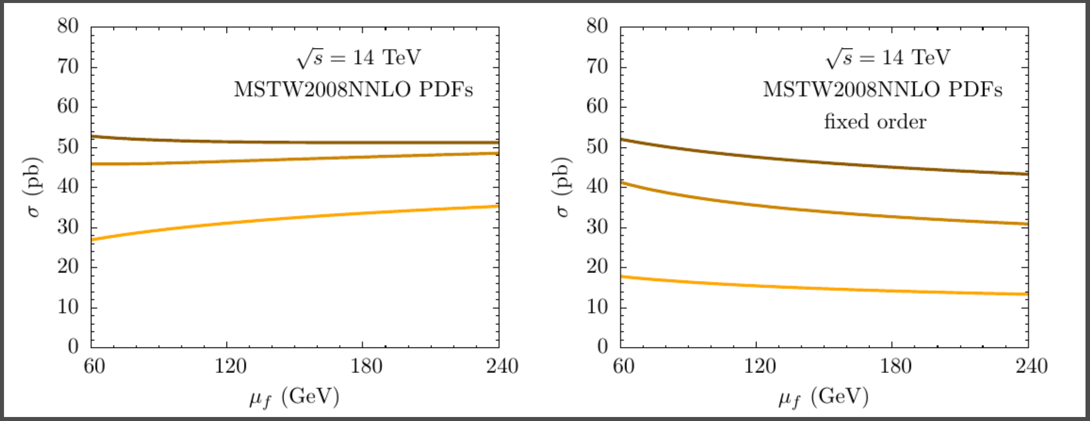
How can one reduce perturbative uncertainties?

Resummation may reduce scale uncertainties:



Treshhold resummation
[Grazzini's Online calculator]

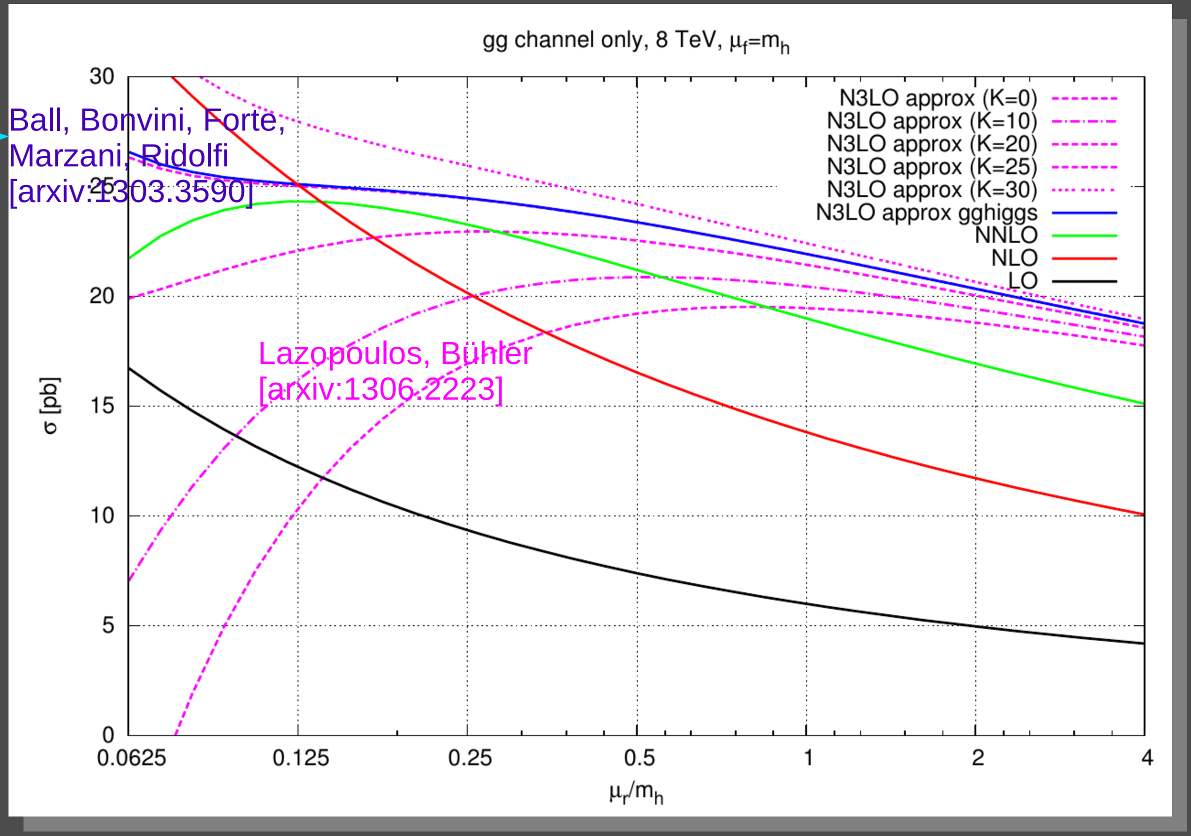
Treshhold with SCET and π² – resummation
[0809.4283]



Another way is to compute the next order:

$$\sigma^{\mathrm{N3LO}}_{PP\to H+X} = \alpha_s(\mu_R)^5\left[K + f(\sigma^{\text{lower orders}}_{PP\to H+X}, \log\mu_R)\right]$$

N3LO approx
Soft + BFKL



The ultimate precision at N3LO

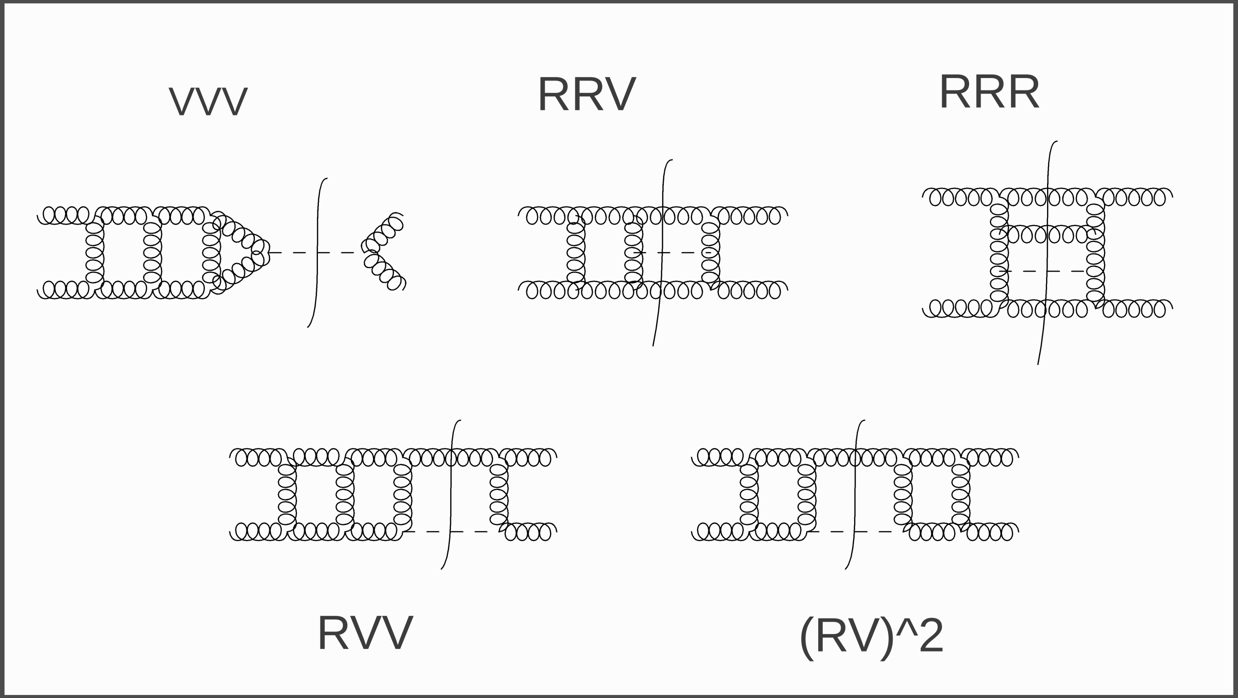
Order	Cross section [pb]	$\sigma/\sigma_{\text{NNLO}}$	$\sigma/\sigma_{\text{LO}}$
LO	$10.31\,^{+26.9\%}_{-16.6\%}$	0.51	1.00
NLO	$17.41\,^{+20.8\%}_{-12.7\%}$	0.86	1.69
NNLO	$20.27\,^{+8.3\%}_{-7.1\%}$	1.00	1.97
N^3LO (K=0)	$18.53\,^{+1.2\%}_{-7.9\%}$	0.91	1.80
N^3LO (K=5)	$19.23\,^{+0.3\%}_{-5.1\%}$	0.95	1.87
N^3LO (K=10)	$19.92\,^{+0.0\%}_{-2.6\%}$	0.98	1.93
N^3LO (K=15)	$20.62\,^{+0.4\%}_{-2.2\%}$	1.02	2.00
N^3LO (K=20)	$21.31\,^{+2.0\%}_{-3.1\%}$	1.05	2.07
N^3LO (K=30)	$22.70\,^{+6.0\%}_{-4.9\%}$	1.12	2.20
N^3LO (K=40)	$24.09\,^{+9.6\%}_{-6.5\%}$	1.19	2.34

Towards exact N3LO

- No calculation has been done before at this order for a hadron collider
- Challenges our current methodology for higher order calculations
- Problem of infra-red divergences even more pronounced! Most singular limits unknown! IR poles up to $\frac{1}{\epsilon^6}$.
- Nevertheless an analytical evaluation should be feasible, since partonic cross depends only on a single parameter

$$z = \frac{m_H^2}{\hat{s}}$$

Status of N3LO



+ UV and collinear counter terms

VVV:

- Known [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]

RVV: in progress

- 2-loop amplitude known up to O(ε) [...]
- Further singularities in phase space integral

(RV)^2: in progress

- 1-loop amplitude known to all orders in ε

RRV: in progress

- Not even 1-loop known to all orders in ε

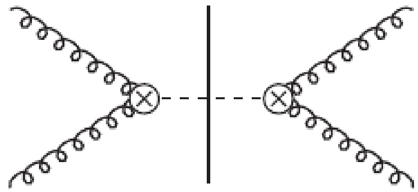
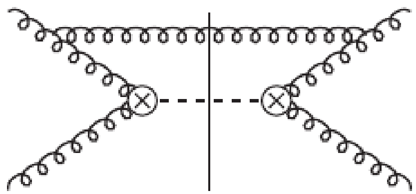
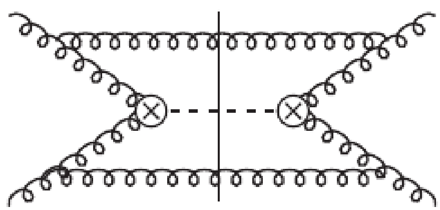
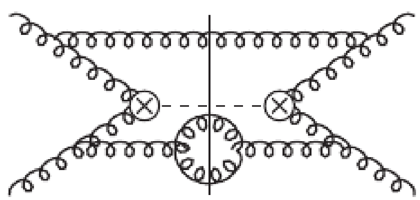
RRR:

- Now know first two terms in soft expansion [Anasatsaiou, Duhr, Dulat, Mistlberger]

Collinear/UV counterterms:

- known [Pak, Rogal, Steinhauser; Anastasiou, Buehler, Duhr, FH; Höschele, Hoff, Pak, Steinhauser, Ueda; Buehler, Lazopoulos]

Growth of Complexity of real emissions

LO		1 diagram	1 integral
NLO		10 diagrams	1 integral
NNLO		381 diagrams	18 integrals
N3LO		26565 diagrams	~200 integrals

[Graphics by Claude Duhr]

Conclusions

- A new challenge for theory is to match the next generation of Higgs precision data expected from the LHC 14 and beyond!
- Theoretical predictions for Higgs production are in good shape, but higher precision is now required!
- N3LO approx. is pointing towards a possible under estimation of the current uncertainties.
- New tools for the evaluation of the exact N3LO correction are emerging
- The N3LO frontier may be reached in the coming year!

Backup

Reverse Unitarity

- A technique developed for inclusive Higgs @ NNLO:

$$\delta_+(q^2) \to \left(\frac{1}{q^2}\right)_c \equiv \frac{1}{2\pi i}\,\mathrm{Disc}\,\frac{1}{q^2} = \frac{1}{2\pi i}\left(\frac{1}{q^2+i0} - \frac{1}{q^2-i0}\right)$$

- Allows to establish differentiation properties:

$$\frac{\partial}{\partial q_\mu}\left[\left(\frac{1}{q^2}\right)_c\right]^\nu = -\nu\left[\left(\frac{1}{q^2}\right)_c\right]^{\nu+1} 2q^\mu$$

$$\left[\left(\frac{1}{q^2}\right)_c\right]^{-\nu} \to 0, \quad \forall\,\nu = 0,1,2,\ldots$$

- Hence one can use the usual IBP relations originally developed for loop computations to find a set of independent master integrals

Differential equations

- IBP relations allow to set up a system of differential equations for the masters:

$$\frac{\partial}{\partial z}\mathbf{F}_i(z,\epsilon) = \sum_j c_{ij}(z,\epsilon)\,\mathbf{F}_j(z,\epsilon)$$

- Can solve masters, after diagonalisation, needing just the soft limit $z \mapsto 1$ of each master
- To get the higher epsilon terms we had recompute the NNLO master soft integrals to higher orders and we noticed something striking. There appeared to be only 2 independent soft master integrals.

Soft master integrals for NNLO

- We were able to show that all but one master integral were directly proportional to the soft phase space volume

$$\mathbf{S}_1(z,\epsilon)=\mathbf{S}_7(z,\epsilon)=\mathbf{S}_{11a}(z,\epsilon)=1\,,$$

$$\mathbf{S}_2(z,\epsilon)=\frac{2(3-4\epsilon)}{(1-2\epsilon)\,(1-z)^2}\,,$$

$$\mathbf{S}_3(z,\epsilon)=\mathbf{S}_8(z,\epsilon)=\mathbf{S}_9(z,\epsilon)=\mathbf{S}_{10}(z,\epsilon)=2\frac{(1-2\epsilon)(3-4\epsilon)}{\epsilon^2(1-z)^2}\,,$$

$$\mathbf{S}_4(z,\epsilon)=\mathbf{S}_5(z,\epsilon)=-2\,\frac{(1-2\epsilon)\,(3-4\epsilon)\,(1-4\epsilon)}{\epsilon^3(1-z)^4}\,,$$

$$\mathbf{S}_6(z,\epsilon)=-8\,\frac{(1-2\epsilon)\,(3-4\epsilon)\,(1-4\epsilon)}{\epsilon^3(1-z)^4}\,,$$

$$\mathbf{S}_{12}(z,\epsilon)=\frac{1}{2}\,,$$

$$\mathbf{S}_{13}(z,\epsilon)=-\frac{3-4\epsilon}{(1-2\epsilon)\,(1-z)}\,,$$

$$\mathbf{S}_{14}(z,\epsilon)=\frac{3-4\epsilon}{\epsilon\,(1-z)}\,,$$

$$\mathbf{S}_{15}(z,\epsilon)=\frac{(1-2\epsilon)(3-4\epsilon)}{\epsilon^2(1-z)^2}\,,$$

$$\mathbf{S}_{16}(z,\epsilon)=\frac{(1-2\epsilon)\,(3-4\epsilon)\,(1-4\epsilon)}{\epsilon^3(1-z)^3}\,,$$

$$\mathbf{S}_{17}(z,\epsilon)=-2\,\frac{(1-2\epsilon)\,(3-4\epsilon)\,(1-4\epsilon)}{\epsilon^3(1-z)^4}\,,$$

$$\mathbf{S}_{18}(z,\epsilon)=-4\frac{(1-2\epsilon)(3-4\epsilon)(1-4\epsilon)}{\epsilon^3(1-z)^4}\,{}_3F_2\left(1,1,-\epsilon;1-\epsilon,1-2\epsilon;1\right)$$

The master integrals needed for the soft are only a tiny subset of the full set of masters!

A new technique for soft integrals

- Expand propagators around z=1:

$$\frac{1}{\left[p_{3\ldots N} - p_{12}\right]^2 - z\, p_{3\ldots N}^2} = \sum_{k=0}^{\infty} \bar{z}^k \frac{\left(-p_{3\ldots N}^2\right)^k}{\left[p_{12}\cdot\left(p_{12} - 2p_{3\ldots N}\right)\right]^{k+1}}$$

- For the RRR found only 10 master integrals!
- Even better can expand cross section around z=1 to any order using just these 10 masters!!