HIGHER SPIN ALGEBRA and HOLOGRAPHIC FLUID in $AdS_5 \times S^5$

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- Higher spin fields constitutute an essential ingredient of AdS/CFT correspondence as they are AdS duals of huge class of operators on the CFT side.

- AdS/hydrodynamics correspondence is another, seemingly unrelated development, suggesting that equations of hydrodynamics and relations between various transport coefficients can be obtained by studying deformations of AdS_5 gravity, such as deformed boosted black hole with velocity u^m and temperature T. The dual gravity models, however, have issues with unitarity and do not appear to be fundamental theories but rather effective theories, with some d.o.f.

(such as higher spins) integrated out.

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The purpose of this talk (based on the recent work arxiv:1304.0898) is to relate these developments using the string theory approach

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We analyze the noncritical RNS string model in AdS_5 background. The physical vertex operators in this model are the elements of nonzero ghost cohomologies $H_N \sim H_{-N-2}$ (definition will be given below) that, in particular, describe the following massless fields in space-time:

- $u_m(p)V^m$ $(m = 0, 1, 2, 3)$- spin 1 open string excitation; $V^m \in H_1 \sim H_{-3}$

- $G_{mn}V^{mn}$ - spin 2 (graviton), $V^{mn} \in H_1 \otimes \bar{H}_1 \sim H_{-3} \otimes H_{-3}$

- $\omega^{a_1 \ldots a_{s-1}|b_1 \ldots b_t} \equiv \omega^{s|t}V_{s|t}$ - massless spin S excitations in Vasiliev's formalism; $V_{s|t} \in H_{s-2} \sim H_{-s}$

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MAIN RESULT:

- The low-energy limit of this model is AdS gravity coupled to higher spins.

- The graviton's β-function in this model is given by:

$$\beta_{mn} = R_{mn} + 8g_{mn} - 4T_{mn} \qquad (1)$$

where T_{mn} is the stress-energy tensor of the $d=4$ conformal fluid , evaluated at the gauge with the temperature value $\tau = \frac{1}{\pi}$, given by:

$$T_{mn} = g_{mn} + 4u_m u_n + \sum_N T_{mn}^{(N)}$$

is the "ideal fluid" contribution and $T_{mn}^{(N)}$ are dissipative terms of order N in derivatives of u.

- The velocity constraint $u^2 = u_m u^m = -1$ results from vanishing of the β_{u_m} in the leading order, as well as from the Weyl invariance constraints on the spin 3 vertex operator (see below)

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Calculation of the first-derivative contribution to the graviton's β-function gives

$$T^{(1)}_{mn} = \rho_{mn}$$

$$\rho^{mn}(p) = \int d^4k\{\frac{i}{3}u^m(k-p)u^n(k-p)$$
$$u_k(k-p)g^{kl}(k+p)(Q^2_{Liouville}p_l + k_l)$$
$$+\frac{i}{2}(u^m(k-p)(k_l-p_l)g^{ln}(k+p)$$
$$+u^n(k-p)(k_l-p_l)g^{lm})(k+p)$$

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In $d = 5$, where $Q_{Liouville} = \sqrt{2}$, $T^{(1)}$ can be transformed to the following expression in the position space:

$$\rho^{mn}(p) = \frac{1}{2}\Pi^{mk}\Pi^{nl}(\partial_k u_l + \partial_l u_k) - \frac{1}{3}\Pi^{mn}\partial_l u^l$$

with

$$\Pi^{mn} = g^{mn} + u^m u^n \qquad (2)$$

which coincides with the first order dissipative contribution to T^{mn} (leading to Navier-Stokes equation). Note that the structure of this tensor (leading to relative normalizations of sheer and bulk viscosities, as well as to well-known $\frac{1}{4\pi}$ sheer viscosity to entropy density ratio) crucially depends on the value of the Liouville background charge

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Next, the second order contribution to T^{mn} in the β-function is

$$T^{mn(2)} = \sum_{k=1}^{5} T_k^{mn(2)}$$

$$T_1^{mn(2)} = Q^2_{Liouv}(\rho^{ml}\rho_l^n - \frac{1}{3}\Pi^{mn}\rho_{kl}\rho^{kl})$$

$$T_2^{mn(2)} = \frac{2Q^2_{Liouv}}{c_{Liouv}-1}\rho^{mn}\partial_l u^l$$

$$T_3^{mn(2)} = (Q^2_{Liouv}-1)(u^k u^l \partial_k u^m \partial_l u^n)$$

$$-\frac{Q^2_{Liouv}}{c_{Liouv}-1}\Pi^{mn}u^k u^l \partial_k u_p \partial^l u^p$$

$$T_4^{mn(2)} = (Q^2_{Liouv}-1)\Pi^{km}\Pi^{ln}$$

$$-\frac{1}{c_{Liouv}-1}\Pi^{mn}\Pi^{kl}u^p(\partial_p\partial_l u_m + \partial_p\partial_m u_l)$$

$$T_5^{mn(2)} = -\epsilon_{l_1 l_2 l_3 l}\epsilon^{lkp(m}\rho_p^{n)}$$

$$u^{l_1}\partial^{l_2}u^{l_3}(\partial_k u_l + \partial_l u_k)$$

2. AdS String Sigma-Model in RNS Formalism

The AdS string sigma-model (Regime 2) is described by the the generating functional

$$Z(u^m, g^{mn}, \Omega_m^{a_1 \dots a_{s-1}|b_1 \dots b_t}) = \int D[X, \psi, \varphi, \lambda, ghosts$$

$$exp\{-S_{RNS} + \int d^4 p u_m(p) V^m(p) + g_{mn}(p) V^{mn}(p)$$

$$+ \sum_{s\geq 3; 0\leq t\leq s-1} \Omega_m^{A_1 \dots A_{s-1}|B_1 \dots B_t}(p) \times V^m_{A_1 \dots A_{s-1}|B_1 \dots B_t}(p)\}$$

where

$$S_{RNS} = S_{matter} + S_{bc} + S_{\beta\gamma} + S_{Liouville}$$

$$S_{matter} = -\frac{1}{4\pi}\int d^2z(\partial X_m\bar\partial X^m + \psi_m\bar\partial\psi^m + \bar\psi_m\partial\bar\psi^m)$$

$$S_{bc} = \frac{1}{2\pi}\int d^2z(b\bar\partial c + \bar b\partial\bar c)$$

$$S_{\beta\gamma} = \frac{1}{2\pi}\int d^2z(\beta\bar\partial\gamma + \bar\beta\partial\bar\gamma)$$

$$S_{Liouville} = -\frac{1}{4\pi}\int d^2z(\partial\varphi\bar\partial\varphi + \bar\partial\lambda\lambda + \partial\bar\lambda\bar\lambda + \mu_0 e^{B\varphi}(\lambda\bar\lambda + F))$$

where S_{RNS} is the full d-dimensional RNS superstring action; $X^m(m = 0, ...d-1)$ are the space-time coordinates; φ, λ, F are components of super Liouville field and the Liouville background charge is

$$Q = B + B^{-1} = \sqrt{\frac{9-d}{2}}$$

VERTEX OPERATORS:

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Spin 1:

$$V^m = K \circ \oint dz e^{\phi+\bar{\phi}}$$
$$\times\{\bar{\lambda}\bar{\partial}^2 X^m - 2\bar{\partial}\bar{\lambda}\bar{\partial}X^m$$
$$+ip^m(\frac{1}{2}\bar{\partial}^2\bar{\lambda} + \frac{1}{q}\partial\varphi\partial\lambda$$
$$-\frac{1}{2}\lambda(\partial\varphi)^2 +$$
$$(1+3q^2)\lambda(3\partial\psi_p\psi^p - \frac{1}{2q}\partial^2\varphi))\}$$

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This massless open string operator is the element of $H_1 \sim H_{-3}$ and must not be confused with usual photon excitation in flat string theory (the element of H_0)

- Structurally, the photon operator has the form

$$V^m(p) = K \circ \oint dz L^m(z,p)$$

In the limit of $p = 0$ the operators L^m are the global symmetry generators of the RNS action, inducing the global space-time symmetries that are realized nonlinearly, mixing ghost and matter degrees of freedom. These symmetry generators form the AdS isometry algebra, given by

$$[T^m, T^n] = -T^{mn}$$
$$T^m \equiv K \circ \oint dz L^m(z, p = 0)$$
$$T^{mn} = K \circ \oint dz \psi^m \psi^n(z)$$
$$[T^{m_1 n_1}, T^{m_2 n_2}]$$
$$\eta^{m_1 m_2} T^{n_1 n_2} + \eta^{n_1 n_2} T^{m_1 m_2}$$
$$-\eta^{m_1 n_2} T^{n_1 m_2} - \eta^{m_2 n_1} T^{m_1 n_2}$$

Just as the usual photon reduces to translation operator in flat space at $p = 0$, the massless $s = 1$ operator in our model is reduced to AdS transvection generator at zero momentum. Next, V^{mn} is the spin 2 (graviton) vertex operator at ghost cohomology $H_1 \otimes \bar{H}_1 \sim H_{-3} \otimes \bar{H}_{-3}$ (not to be confused with the "standard" string theory graviton, that is the element of $H_0 \otimes \bar{H}_0$) and describes fluctuations around the flat vacuum Just as the standard graviton is the object bilinear in flat space translations, the $H_1 \otimes \bar{H}_1$ graviton in our case is bilinear in AdS transvections.

The explicit expression for V^{mn} is

$$V^{mn} = [K \otimes \bar{K}] \circ \int d^2 z e^{\phi + \bar{\phi}}$$
$$\times \{ \bar{\lambda} \bar{\partial}^2 X^m - 2 \bar{\partial} \bar{\lambda} \bar{\partial} X^m$$
$$+ i p^m (\frac{1}{2} \bar{\partial}^2 \bar{\lambda} + \frac{1}{Q_L} \partial \varphi \partial \lambda - \frac{1}{2} \lambda (\partial \varphi)^2 +$$
$$(1 + 3 q^2) \lambda (3 \partial \psi_p \psi^p - \frac{1}{2 Q_L} \partial^2 \varphi)) \}$$
$$\{ \bar{\lambda} \bar{\partial}^2 X^n - 2 \bar{\partial} \bar{\lambda} \bar{\partial} X^n$$
$$+ i p^n (\frac{1}{2} \bar{\partial}^2 \bar{\lambda} + \frac{1}{Q_L} \bar{\partial} \bar{\varphi} \bar{\partial} \bar{\lambda}$$
$$- \frac{1}{2} \bar{\lambda} (\bar{\partial} \bar{\varphi})^2 + (1 + 3 {q_L}^2) \bar{\lambda} (3 \bar{\partial} \bar{\psi}_q \bar{\psi}^q$$
$$- \frac{1}{2 Q_L} \bar{\partial}^2 \bar{\varphi})) \} e^{ipX}$$

where the holomorphic homotopy transform of an operator V $K \circ V$ is defined according to

$$K \circ V = V$$
$$+\frac{(-1)^N}{N!}\oint \frac{dz}{2i\pi}(z-w)^N : K\partial^N W : (z)$$
$$+\frac{1}{N!}\oint \frac{dz}{2i\pi}\partial_z^{N+1}[(z-w)^N K(z)]K\{Q_{brst}, U\}$$

where w is some arbitrary point on the worldsheet, U and W are the operators defined according to

$$[Q_{brst}, V(z)] = \partial U(z) + W(z)$$

$$K = ce^{2\chi - 2\phi}$$

is the homotopy operator satisfying

$$\{Q_{brst}, K\} = 1$$

and N is the leading order of the operator product

$$K(z_1)W(z_2) \sim (z_1 - z_2)^N Y(z_2) \\ +O((z_1 - z_2)^{N+1})$$

and similarly for antiholomorphic transformation $\bar{K} \circ \oint$ ... The important property of the K transformation is the homomorphism relation preserving the OPE structure constants.

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Next,

$$V_m^{a_1 \ldots a_{s-1}|b_1 \ldots b_t}(p) \equiv V^{s|t}(p)(0 \leq t \leq s-1)$$

are the vertex operators for massless higher spin s gauge fields

$$\Omega_m^{a_1 \ldots a_{s-1}|b_1 \ldots b_t} \equiv \Omega^{s|t}$$

in Vasiliev's frame-like formalism. In this work, we restrict ourselves to open string excitations (sufficient for symmetric one-row higher spin fields) For a general spin value s the explicit expressions for the $V^{s|t}$ vertex operators are complicated. However, these expressions pleasantly simplify for $t = s-3$ with the vertex operators for $\Omega^{s|s-3}$ extra gauge fields given by the homotopy transforms:

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$$V_m^{a_1 \dots a_{s-1}|b_1 \dots b_{s-3}} = K \circ \oint dz e^{(s-2)\phi} \psi_m$$
$$\times \partial X^{a_1} \dots \partial X^{a_{s-1}} \partial \psi^{(b_1} \dots \partial^{(s-3)} \psi^{b_{s-3})} e^{ipX}(z)$$

Thus the operators for the spin s are the elements of the ghost cohomologies $H_{s-2} \sim H_{-s}$. The OPE structure of the operators in the different cohomologies is described by the fusion rules:

$$[H_{s_1}]\otimes[H_{s_2}] = \sum_{s=|s_1-s_2+1|}^{s_1+s_2} [H_s]$$
$$(s_1 \geq s_2; s_{1,2} \neq 0)$$

similar to the general structure of higher spin algebras in AdS. In other words, the vertex operators from different cohomologies form the operator algebra realizatons of HS algebras. The rest of auxiliary fields along with the dynamical field $\Omega^{s|0}$ can be related to $\Omega^{s|s-3}$ through generalized zero curvature relations in terms of ghost cohomology conditions.

This concludes the list of vertex operators contributing to the graviton's β-

function, considered in this talk.

CONTRIBUTIONS to the β-FUNCTION.

In the leading order, the linearized contributions follow from the Weyl invariance constraints on the vertex operators. The calculations are the standard ones,e.g., using the ϵ-expansion techniques. However, the crucial novelty in our case is the appearance of the cosmological term. The cosmological term for β_{mn} appears as a result of nontrivial ghost dependence of V^{mn}, i.e. as a result of V^{mn} being an element of nonzero cohomology $H_1 \otimes \bar{H}_1$. Namely, the Weyl invariance constraints can be conveniently deduced from the OPE:

$$\sim \int d^2z \int d^2w T_{z\bar{z}}(z, \bar{z}) V_{grav}(w, \bar{w})$$

by expanding around the midpoint and evaluating the coefficient in front of

$$\sim \frac{V_{grav}(\frac{z+w}{2}, \frac{\bar{z}+\bar{w}}{2})}{|z-w|^2}$$

(note that the trace $T_{z\bar{z}}$ of the stress-energy tensor, generating the Weyl transformation, is nonzero in the underlying ϵ-expansion). For a usual graviton operator

$$\sim G_{mn}(p) \int d^2w \partial X^m \bar{\partial} X^n e^{ipX}(w, \bar{w})$$

in the bosonic string this procedure leads, after simple calculation, to the standard β-function contribution, quadratic in momentum, given by the linearized part of the Ricci tensor plus the second derivative of the dilaton $\sim R_{mn}^{lin} - 2p_m p_n D$ with the dilaton $D \sim tr(G_{mn})$.

The calculation, leading to the identical result, is similar in superstring theory. The graviton operator must then be taken at canonical ghost picture (unintegrated $b - c$ picture and $(-1,-1)$ $\beta - \gamma$ ghost picture), so

$$V_{grav} = c\bar{c}e^{-\phi-\bar{\phi}}\psi^m\psi^n e^{ipX}$$

and

$$\begin{aligned} T_{z\bar{z}} \equiv & T_{z\bar{z}}^{matter} \\ & + T_{z\bar{z}}^{b-c} + T_{z\bar{z}}^{\beta-\gamma} \\ = -\frac{1}{2}(\partial X_m \bar{\partial} X^m & - \bar{\partial}\psi_m \psi^m \\ & - \partial\bar{\psi}_m \bar{\psi}^m + \partial\sigma\bar{\partial}\sigma \\ & + \partial\chi\bar{\partial}\chi - \partial\phi\bar{\partial}\phi) \end{aligned}$$

The OPE of V_{grav} with $T^{matter}_{z\bar{z}}$ then contributes the term $\sim p^2 G_{mn}$ to the graviton's beta-function (which is the gauge-fixed linearized part of the Ricci tensor, with the gauge condition $\sim p^m G_{mn} = 0$), while the contribution stemming from the OPE with $T^{b-c}_{z\bar{z}}$ cancels the one from the OPE with $T^{\beta-\gamma}_{z\bar{z}}$ since

- $$\begin{aligned} &\partial\sigma\bar{\partial}\sigma(z,\bar{z})c\bar{c}(w,\bar{w}) \\ &\sim \frac{1}{|z-w|^2}c\bar{c}(w,\bar{w}) \\ &\partial\phi\bar{\partial}\phi(z,\bar{z})e^{-\phi-\bar{\phi}}(w,\bar{w}) \\ &\sim \frac{1}{|z-w|^2}e^{-\phi-\bar{\phi}}(w,\bar{w}) \end{aligned}$$

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and σ and ϕ-terms of $T_{z\bar{z}}$ have opposite signs.

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It is this cancellation that ensures the absence of "cosmological terms" in the β-function of the graviton with the conventional vertex operator leading to Einstein gravity around the flat vacuum. In case of the vertex operator (), the OPE of $T_{z\bar{z}}^{matter}$ with $V_{grav}^{H_{-3}\otimes H_{-3}}$ still leads to the linear part of the Ricci tensor.However, since this operator is the element of $H_{-3}\otimes H_{-3}$, and its canonical ϕ-ghost picture is $(-3,-3)$ (refs), the contributions from $T_{z\bar{z}}^{b-c}$ and $T_{z\bar{z}}^{\beta-\gamma}$ no longer cancel each other:

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$$(T^{b-c}_{z\bar{z}} + T^{\beta-\gamma}_{z\bar{z}})(z,\bar{z})V^{H_{-3}\otimes H_{-3}}_{grav}(w,\bar{w}) \sim \frac{\frac{1}{2}(1-3^2)V^{H_{-3}\otimes H_{-3}}_{grav}}{|z-w|^2}$$

leading to the cosmological term proportional to $\sim 4G_{mn}$ in the β-function. Thus the Weyl invariance condition brings the piece proportional to

$$\sim R^{linearized}_{mn} + 4g_{mn}$$

to the β-function (assuming that the dilaton is switched off). The higher order (quadratic) terms in β_{mn} are given by the appropriate 3-point functions, resulting in the quadratic part of the Ricci tensor, plus the matter tensor describing the holographic hydrodynamics.

4. 3-point CORRELATORS AND QUADRATIC CONTRIBUTIONS

In this work, instead of considering the higher spin vertex operators as sources of the fundamental HS fields, we regard them as the sources of excitations given by the polynomial expressions in u^m. The expressions are constructed so as to satisfy the BRST on-shell constraints, identical to those for higher spin vertex operators (degree s of polynomial equals to the spin of the underlying fundamental field in space-time, emitted by the corresponding vertex operator). As a result, the polynomial contributions to the graviton's β-function are controled by the structure constants of the HS operator algebra. In this talk, we restrict ourselves to $s = 3$ contributions. The vertex operator for the dynamical $\Omega^{2|0}$ field is given by

$$V^{2|0}_{s=3} = K \circ \oint dz e^{\phi} \psi_m \partial X^a \partial X^b e^{ipX}(z)$$

with the corresponding on-shell and Weyl invariance conditions on $\Omega^m_{ab}(p)$ given by

$$\eta_{ab}\Omega^{ab}_m = \eta^m_a \Omega^{ab}_m = 0$$
$$p_a \Omega^{ab}_m(p) = 0$$

The only suitable degree 3 polynomial in u satisfying these on-shell constraints is given by

$$\Omega_{mab}(p) = \int d^4k \int d^4q u_m(k+q)u_a(k-p)u_b(q-p) + \frac{1}{2}\delta_{ab}u_m(p) - \frac{1}{2}(\delta_{ma}u_b(p) + \delta_{mb}u_a(p))$$

provided that u_a satisfies $u_a u^a = -1$ with zero vorticity condition $p_{[a}u_{b]}(p) = 0$ and incompressibility $p_a u^a(p) = 0$. However, since the β-function is the object that must be calculated off-shell, in the calculations below we shall keep the terms, that are both non-transverse and have nonzero vorticities, as they only vanish in the on-shell limit.

The overall result is given by (with $Q_{Liouv} = \sqrt{\frac{9-d}{2}} = \sqrt{2}$):

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\begin{aligned}
\beta_{mn} = R_{mn} + 8g_{mn} - 4T_{mn} \\
T_{mn} = g_{mn} + 4u_m u_n \\
+\Pi^{mk}\Pi^{nl}(\partial_k u_l + \partial_l u_k) \\
-\frac{2}{3}\Pi^{mn}\partial_l u^l \\
+\sum_{k=1}^{5} T_k^{mn(2)}
\end{aligned}
$$

with

$$T_1^{mn(2)} = 2(\rho^{ml}\rho_l^n - \frac{1}{3}\Pi^{mn}\rho_{kl}\rho^{kl})$$

$$T_2^{mn(2)} = \frac{\sqrt{2}}{3}\rho^{mn}\partial_l u^l$$

$$T_3^{mn(2)} = 2u^k u^l \partial_k u^m \partial_l u^n$$

$$-\frac{2}{3}\Pi^{mn}u^k u^l \partial_k u_p \partial^l u^p$$

$$T_4^{mn(2)} = \sqrt{2}\Pi^{km}\Pi^{ln}$$

$$-\frac{\sqrt{2}}{3}\Pi^{mn}\Pi^{kl}u^p(\partial_p\partial_l u_m + \partial_p\partial_m u_l)$$

$$T_5^{mn(2)} = -\frac{1}{\sqrt{2}}\epsilon_{l_1 l_2 l_3 l}\epsilon^{lkp(m}\rho_p^{n)}u^{l_1}$$

$$\times\partial^{l_2}u^{l_3}(\partial_k u_l + \partial_l u_k)$$

The cosmological and ideal fluid terms are contributed by the ϵ-expansion (Weyl invariance constraint on the AdS graviton) and $< 2-1-1 >$ correlator.

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The first derivative terms (contributing to sheer and bulk viscosities) stem from the Weyl invariance constraints on spin 3 vertex operator $V^{2|1}$ for $\Omega^{2|1}$ extra field.

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The second derivative terms are contributed by $< 2-1-1 >$ and $< 2-3-3 >$ correlators. The spin 3 contribution is crucial for the structure of the stress tensor to be conformally invariant in $d=4$.

5. Conclusions and outlook

- The β-function of the graviton operator describing the gravitational fluctuations around AdS vacuum is given by equations of gravity with cosmological constant and with the matter

- The matter stress tensor is the one of the second order hydrodynamics; it is conformally invariant in $d = 4$ and this invariance is crucially ensured by the spin 3 contributions to the graviton's β-function

- Further checks are needed to establish the relation between gradient expansion in hydrodynamics and contribudions from spins $s \geq 4$ at higher orders; the conjecture is that the expansion order in holographic hydrodynamics is related to spin values in the higher spin algebra

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The separate, very interesting issue is the appearance of the emergent AdS space as a result of Weyl invariance condition for operators at higher cohomologies. It appears that the cohomology structure, scale invariance and emerging space-time geometry in string theory are closely related (work in progress)