Higher twist effects in diffractive DIS at HERA

Mariusz Sadzikowski
Jagiellonian University, Cracow

HEP 2013, Stockholm
Outline

• DGLAP description of DDIS (at HERA)

• DGLAP breakdown

• Estimation of higher twists: saturation model

• Higher twists and the data

• Conclusions

Diffractive DIS: process and variables

\[ W^2 = (P + q)^2 \quad x_P = \frac{Q^2 + M_X^2}{Q^2 + W^2} \quad x = \beta x_P \]

\[ \beta = \frac{Q^2}{Q^2 + M_X^2} \quad t = (P' - P)^2 \]

Domain of interest:

\[ W \gg M_X^2 > Q^2 \gg |t| \]

\[ \sigma_r(\beta, Q^2, x_P) = F_2^{D(3)} - \frac{y^2}{1 + (1 - y)^2} F_L^{D(3)} \]

\[ \frac{d\sigma^{e p \to e X P}}{d\beta dQ^2 dx_P} = \frac{2\pi\alpha^2}{\beta Q^4} \left[ 1 + (1 - y)^2 \right] \sigma_r(\beta, Q^2, x_P) \]
DGLAP fit to DDIS data (ZEUS, 2009)

- LRG ZEUS data
- $2.0 < Q^2 < 305$ GeV$^2$
- DGLAP (NLO) fit (LRG + LPS)
- 265 d.o.f
DGLAP breakdown: ”critical scale”

- DGLAP fits below 5 GeV$^2$ fail
- Strong DGLAP breaking effects below 3 GeV$^2$

**DGLAP breakdown: closer look**

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$Q^2$ (GeV$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.038</td>
<td>2.5</td>
</tr>
<tr>
<td>0.091</td>
<td>3.5</td>
</tr>
<tr>
<td>0.217</td>
<td>4.5</td>
</tr>
<tr>
<td>0.052</td>
<td></td>
</tr>
<tr>
<td>0.123</td>
<td></td>
</tr>
<tr>
<td>0.280</td>
<td></td>
</tr>
<tr>
<td>0.066</td>
<td></td>
</tr>
<tr>
<td>0.153</td>
<td></td>
</tr>
<tr>
<td>0.333</td>
<td></td>
</tr>
</tbody>
</table>

- Rapid deviation of the fit from data with decreasing $Q^2$
- Indication of higher twists?

Problematic region: low $x_P$, low $Q^2$
Beyond DGLAP: dipole picture

Cross-section:

$$\sigma \sim \int d^2r \, dz \, |\Psi(z, Q^2 r^2)|^2 \sigma_d(x, r^2)$$

GBW parametrization

$$\sigma_d(x, r^2) = \sigma_0 \left[ 1 - \exp \left( -\frac{r^2}{4R^2(x)} \right) \right]$$

$$R(x) = \frac{1}{Q_0} \left( \frac{x}{x_0} \right)^{\lambda/2}$$


Inclusive scattering: large energy factorization+ eikonal colour dipole scattering
Diffraction: quark – antiquark and quark – antiquark – gluon

\[ \sigma_{\text{diff}} \sim \int d^2r \, dz \, |\Psi(z, Q^2r^2)|^2 \sigma_d^2(x, r^2) \]

- An exponential t-dependence of the amplitude
- Strongly suppressed at small \( \beta \)

\[ \sigma_{2d} \sim \alpha_s \int d^2r_{02} \, K(r_{01}, r_{02}) \left[ N_{02} + N_{12} - N_{01} - N_{02}N_{12} \right]^2 \]

- Subleading in the \( \alpha_s \) constant, but enhanced at small \( \beta \) – due to the dipole size
- Effectively 2 dipoles at large \( N_c \) limit

M.L. Good and W.D. Walker, Phys. Rev. 120 (1960) 1857
Tuning the model

• The dipole cross-section fixed by the GBW fit to inclusive data (massless quarks, no charm)

• Phase space improvement following GBW calculation:

\[ F_2^{D(3)}(\beta, Q^2, x_p) = F_2^{D(3), LL(1/\beta)}(Q^2, x_p) \frac{F_2^{GBW}(\beta, Q^2, x_p)}{F_2^{GBW}(\beta = 0, Q^2, x_p)} \]


• In the gluonic term, ”x_0” parameter is rescaled by a factor of 2 (in inclusive DIS case ”x_0” relates to Bjorken x for DDIS to pomeron x_p)
Data vs DGLAP: crucial bins of low $Q^2$

Low $\beta$ region expected contributions from 2 gluons emissions from the dipol
Data vs DGLAP and twist-2 from GBW

Satisfactory consistence of twist-2 GBW and NLO DGLAP fit
Data vs DGLAP + twist-4 from GBW

- Correct sign of twist-4 but too weak effect
- Emergence of twist-4 contribution correlated with data deviations from DGLAP fit
Data vs DGLAP + twist-4 + twist-6

- Good description of data
- Dependence on $Q^2$ difficult to explain without higher twists
Data vs DGLAP + twist-4 + twist-6

- Improvement in the description of data
- Red: DGLAP fit
- Blue: DGLAP (fit) + twists
H1 data vs. + twist-4 + twist-6

- H1-LRG 2012
- H1 fit B
- + Tw 4
- + Tw 4+6

\[ \xi = 0.001 \]

\[ k = 4 \]
\[ \beta = 0.32 \]

\[ k = 3 \]
\[ \beta = 0.2 \]

\[ k = 2 \]
\[ \beta = 0.13 \]

\[ k = 1 \]
\[ \beta = 0.08 \]

\[ k = 0 \]
\[ \beta = 0.05 \]

\[ \xi \sigma_{\nu_T} D(\lambda) \times 3^k \]

Q^2 [GeV^2]
Data vs DGLAP + twist-4 + twist-6

- Warning: inclusion of all twists below DGLAP and twist-4
Constraints on higher twists

• Very weak constraints from experimental inclusive DIS data

• BFKL bootstrap (LL): only one (reggeized) gluon couples to
  one fundamental (quark) line \(\rightarrow\) eikonal multi-gluon coupling
  is unrealistic \(\rightarrow\) cut off some higher twists is reasonable

• Example: GBW couples 2 gluons at the amplitude level:
  twist-2 and twist-4 in the diffractive cross-section
Conclusions

• HERA data are consistent with discovery of the strong (up to 100%) positive higher twists effect in DDIS at, and below $Q^2$ of order 5 GeV$^2$

• The main evidence: significant, systematic deviation of DDIS data from DGLAP fits at small $x$ and $Q^2$

• The saturation model predicts correctly the DGLAP breakdown line $(x,Q^2)$ due to the emergence of higher twists

• The saturation model provides a good description of data when the twist series is cut-off at twist-6

• Experimental and theoretical exploration of higher twists may be now possible
Diffraction: quark – antiquark contribution

\[ x_\perp F_{L,T}^{D(3)}(\beta, Q^2, x_\perp) = \frac{Q^4}{4\pi^2\alpha\beta} \left( \frac{d\sigma_{L,T}^{q\bar{q}}}{dM_x^2} \right) \]

\[ \frac{d\sigma_{L,T}^{q\bar{q}}}{dM_x^2} = \frac{1}{16\pi b_D} \sum_f \int \frac{d^2 p}{(2\pi)^2} \int_0^1 dz \delta \left( \frac{p_x^2}{z\bar{z}} - M_x^2 \right) \sum_{\text{spin}} \left| \int d^2 r e^{i\vec{p} \cdot \vec{r}} \psi_{h\bar{h},\lambda}^f(Q, z, \bar{r}) \sigma(r) \right|^2 \]

\[ \sum_{\text{spin}} \psi_{h\bar{h},\lambda}^f(Q, z, \bar{r}) \psi_{h\bar{h},\lambda}^{f*}(Q, z, \bar{r}') = \frac{N_c\alpha e_f^2}{2\pi^2} \begin{cases} 4Q^2(z\bar{z})^2K_0(\varepsilon r)K_0(\varepsilon r') & (L) \\ \varepsilon^2(z^2 + (1-z)^2)^{\frac{p \cdot p'}{r \cdot r'}}K_1(\varepsilon r)K_1(\varepsilon r') & (T) \end{cases} \]

\[ \frac{d\sigma_{L,T}^{q\bar{q}}}{dM_x^2} = \frac{N_c\alpha}{8\pi^2 b_D Q^2} \sum_f e_f^2 \int_0^\infty dp d\rho' \rho K_{0,1}(\rho)J_{0,1}(w\rho) \rho' K_{0,1}(\rho')J_{0,1}(w\rho') \]

\[ \left[ h_{L,T} \left( \frac{\rho^2}{Q^2R^2} \right) + h_{L,T} \left( \frac{\rho'^2}{Q^2R^2} \right) - h_{L,T} \left( \frac{\rho^2 + \rho'^2}{Q^2R^2} \right) \right] \]

\[ h_L(v) = \frac{1}{8} \left[ \frac{4}{3} - \sqrt{\pi}G_{1,2}^{2,0} \left( v \left| \begin{array}{c} \frac{5}{2} \\ 0, 2 \end{array} \right. \right) \right] \]

\[ h_T(v) = \frac{1}{8} \left[ \frac{4}{3} - \sqrt{\pi}G_{1,2}^{2,0} \left( v \left| \begin{array}{c} \frac{3}{2} \\ 0, 1 \end{array} \right. \right) + \sqrt{\pi}G_{1,2}^{2,0} \left( v \left| \begin{array}{c} \frac{5}{2} \\ 0, 2 \end{array} \right. \right) \right] \]
Diffraction: quark – antiquark – gluon contribution

\[
\frac{d\sigma^{q\bar{q}\gamma}}{dM_x^2} = \frac{1}{16\pi b_D} \frac{N_c \alpha_s}{2\pi^2} \frac{\sigma_0^2}{M_x^2} \sum_f \int d^2r_{01} \int_0^1 dz \sum_{\text{spin}} |\psi_{h\bar{h},\lambda}(Q, z, r_{01})|^2 \sigma_{2d}(r_{01})
\]

\[
\sigma_{2d}(r_{01}) = \int d^2r_{02} K(01|2) \left[ N_{02} + N_{12} - N_{02}N_{12} - N_{01} \right]^2
\]

\[
\frac{d\sigma^{q\bar{q}\gamma}}{dM_x^2} = \frac{1}{16\pi b_D} \frac{N_c \alpha_s}{2\pi^2} \frac{\sigma_0^2}{M_x^2} \sum_f \int ds \frac{4Q_0^2}{Q^2} \left( \frac{4Q_0^2}{Q^2} \right)^{-s} \tilde{H}_{L,T}^f(-s) \tilde{\sigma}_{2d}(s)
\]

\[
K(01|2) = \frac{r_{01}^2}{r_{02}^2 r_{12}^2}
\]

\[
\tilde{\sigma}_{2d}(s) = I_1 - I_2
\]

\[
I_1 = \left( \frac{Q_0^2}{\pi} \right)^s \int d^2r_{01} (r_{01}^2)^{s-1} \int d^2r_{02} K(01|2) \left[ (N_{02} + N_{12} - N_{02}N_{12})^2 - N_{01}^2 \right]
\]

\[
I_2 = \left( \frac{Q_0^2}{\pi} \right)^s \int d^2r_{01} (r_{01}^2)^{s-1} \int d^2r_{02} K(01|2) 2N_{01} \left[ N_{02} + N_{12} - N_{02}N_{12} - N_{01} \right]
\]

\[
I_1 = \pi (Q_0 R)^{2s} 2^{1+s}(2^{1+s} - 1) \Gamma(s) \left[ H_s - 3F_2(1, 1, 1 - s; 2, 2; -1) \right]
\]

\[
I_2 = \pi (Q_0 R)^{2s} 2^{1+2s} \Gamma(s) \left\{ 1 - 2^{1-s} + 3^{-s} + \frac{2^{-s}s}{1+s} \left[ 1 - 2F_1 \left( 1+s, 1+s; 2+s; -1/2 \right) \right] \right\}
\]

\[
H_s = \sum_{k=1}^{s} \frac{1}{k}
\]
**BFKL bootstrap equation**

Bootstrap:

\[ H_{\text{BFKL}} = \beta \implies \text{Reduction} \]

- BFKL bootstrap (LL) → only one (reggeized) gluon couples to one fundamental (quark) line
- Common double logarithmic limit of BFKL nad DGLAP evolutions → eikonal multi-gluon coupling is unrealistic → cut off of some higher twists