Resolving the puzzle of the $\gamma\gamma^* \rightarrow \pi^0$ transition form factor

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We present the analysis of the $F_{P\gamma}(Q^2)$, $P = \pi, \eta, \eta'$ form factors and show that the recent Belle data on $\pi^0\gamma$ resolves the puzzle posed by the BaBar data on $\pi^0\gamma$. We discuss implications of these results for pion elastic form factor.

Based on works in collaboration with I. Balakireva and B. Stech

The amplitude of $\gamma \gamma^*(Q) \rightarrow P, (P = \pi^0, \eta, \eta', \eta_c,)$ contains only one form factor:

$$\langle \gamma(q_1)\gamma^*(q_2)|P(p)\rangle = i\varepsilon_1\varepsilon_2 q_1q_2 F_{\gamma P}(q_1^2 = 0, q_2^2 = -Q^2).$$

QCD factorization theorem predicts for the pion-photon transition form factor

$$Q^2 F_{\pi\gamma}(Q^2) \rightarrow \sqrt{2} f_\pi \quad f_\pi = 0.130 \text{ GeV}.$$

Similar scaling relations emerge for $\eta$ and $\eta'$ after taking into account the mixing effects.

Brodsky, Lepage combined pQCD at large $Q^2$ with axial anomaly at $Q^2 = 0$ and proposed

$$F_{\pi\gamma}(Q^2) \approx \frac{\sqrt{2} f_\pi}{4\pi^2 f_\pi^2 + Q^2}.$$

No surprizes were expected, but in 2009 BaBar presented $F_{\pi\gamma}(Q^2)$ at $Q^2$ up to 40 GeV$^2$ [PRD80,052002(2009), 187 cites in INSPIRE]
The BaBar pion form factor seems more compatible with $Q^2 F_{\pi\gamma}(Q^2) \sim \log(Q^2)$.

QCD factorization theorem seems violated (or at least in danger)
The $\eta$ and $\eta'$ data is not in contradiction with saturation $Q^2 F(Q^2) \sim \text{const}$

Why nonstrange components in $\eta$, $\eta'$ and $\pi^0$ should behave so much differently?
THEORY:

- OPE for 3-point function $\langle VVA \rangle$ in QCD
- Quark-hadron duality as a low-energy cut on the spectral representation

\[
\pi f_\pi F_{\pi\gamma}(Q^2) = \int_{4m^2}^{s_{\text{eff}}(Q^2)} ds \rho_{\text{pQCD}}(s, Q^2)
\]

Nonperturbative power corrections do not appear explicitly (implicitly hidden is $s_{\text{eff}}(Q^2)$).

The effective threshold:

- $s_{\text{eff}}(Q^2)$ for all $Q^2$ remains bounded in the “soft” region $s_{\text{eff}}(Q^2) \sim 0.5 \div 1 \text{GeV}^2$
- QCD factorization theorem requires $s_{\text{eff}}(Q^2 \rightarrow \infty) \rightarrow 4\pi^2 f_\pi^2$

(finding $s_{\text{eff}}$ for correlators is equivalent to solving full QCD)
One can calculate $s_{\text{eff}}$ in quantum mechanics:

For $V(r) = V_{\text{conf}}(r) - \frac{\alpha}{r}$:

(in this case the form factors satisfy factorization theorem like in QCD)

The effective threshold "saturates" at $Q^2 = \text{a few GeV}^2$. 
BaBar’2009 vs Belle’2012

Belle data (i) is fully compatible with factorization (and with $\eta$ and $\eta'$ results) and (ii) the corresponding effective threshold is fully compatible with theoretical expectations
Elastic pion form factor:

\[ F_\pi(Q^2) = F_0(Q^2) + \alpha_s(Q^2)F_1(Q^2) + \ldots, \quad F_0(Q^2) \propto 1/Q^4, \quad F_1(Q^2) \propto 1/Q^2 \]

Effective threshold:
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Effective threshold:

\[ k_{\text{eff}}(Q) \quad [\text{GeV}] \]

\[ s_{\text{eff}}(Q) \quad [\text{GeV}^2] \]

Some recent theoretical predictions:
Summary and conclusions

• Meson-photon transition form factors:
The Belle data resolves the puzzle of the $\pi^0\gamma$ form factor: the results on $\pi^0\gamma$ from Belle is fully compatible with the results on $\eta\gamma$ and $\eta'\gamma$. Moreover, all three form factors are fully compatible with the pQCD asymptotic formula at $Q^2 \geq 10 - 15 \text{ GeV}^2$.

• Pion elastic form factor:
We predict the asymptotic regime for the effective threshold $s_{\text{eff}}(Q^2) = 4\pi^2 f_\pi^2$ (NOT for the form factor!) to be reached at $Q^2 \sim 5 - 6 \text{ GeV}^2$. (For the form factor this yields unambiguous predictions for separate contributions in the perturbative expansion). This is testable at JLab.

• Is there still room for violation of factorization?
A better fit to the full set of the meson-photon transition form factors might prefer a small universal logarithmic rise of $Q^2 F(Q^2)$. If established experimentally, this rise would mean violation of factorization.