Resolving the puzzle of the $\gamma\gamma^* \to \pi^0$ transition form factor

Wolfgang Lucha and Dmitri Melikhov

HEPHY, Vienna & University Vienna & SINP, Moscow State University

We present the analysis of the $F_{P\gamma}(Q^2)$, $P = \pi, \eta, \eta'$ form factors and show that the recent Belle data on $\pi^0\gamma$ resolves the puzzle posed by the BaBar data on $\pi^0\gamma$. We discuss implications of these results for pion elastic form factor.

Based on works in collaboration with I. Balakireva and B. Stech

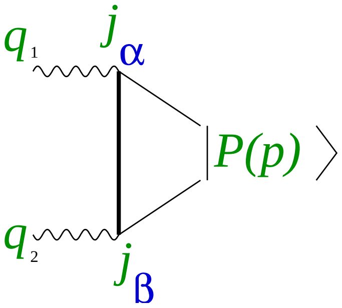
I.Balakireva, W.Lucha, DM, Phys.Rev. **D85** *(2012) 036006;*

DM, B.Stech, Phys.Rev. **D85** *(2012) 051901; Phys.Lett.* **B718** *(2012) [arXiv:1206.5764];*

W. Lucha, DM, J.Phys. **G39***, 045003 (2012); Phys.Rev.* **D86***, 016001 (2012).*

The amplitude of $\gamma\gamma^*(Q) \to P$, ($P = \pi^0, \eta, \eta', \eta_c$,) contains only one form factor:

$$\langle \gamma(q_1)\gamma^*(q_2)|P(p)\rangle = \mathrm{i}\epsilon_{\varepsilon_1\varepsilon_2 q_1 q_2} F_{P\gamma}(q_1^2 = 0, q_2^2 = -Q^2).$$



QCD factorization theorem predicts for the pion-photon transition form factor

$$Q^2 F_{\pi\gamma}(Q^2) \to \sqrt{2} f_\pi \qquad f_\pi = 0.130 \textbf{ GeV}.$$

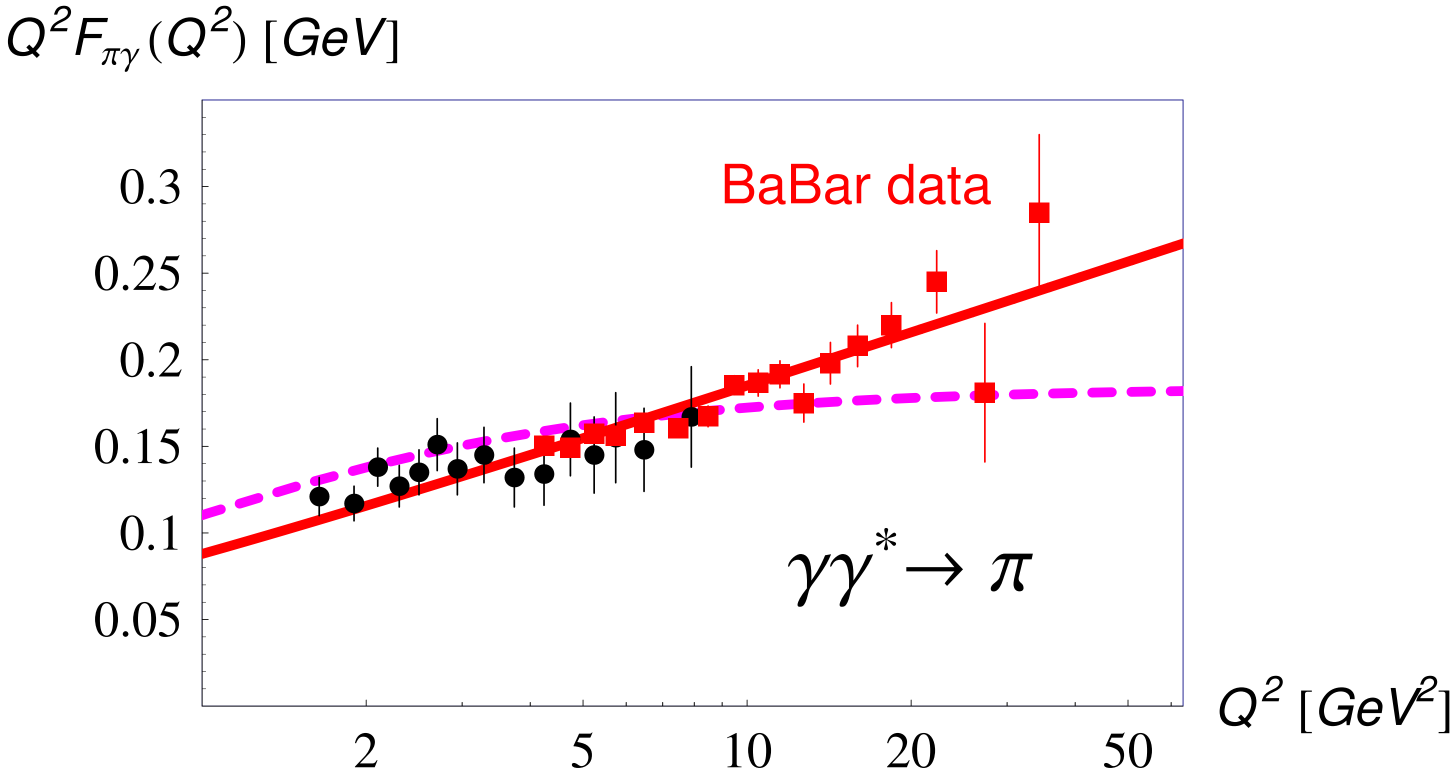
Similar scaling relations emerge for η and η' after taking into account the mixing effects.

Brodsky, Lepage combined pQCD at large Q^2 with axial anomaly at $Q^2 = 0$ and proposed

$$F_{\pi\gamma}(Q^2) \simeq \frac{\sqrt{2} f_\pi}{4\pi^2 f_\pi^2 + Q^2}.$$

No surprizes were expected, but in 2009 BaBar presented $F_{\pi\gamma}(Q^2)$ at Q^2 up to 40 GeV^2 [PRD80,052002(2009), 187 cites in INSPIRE]

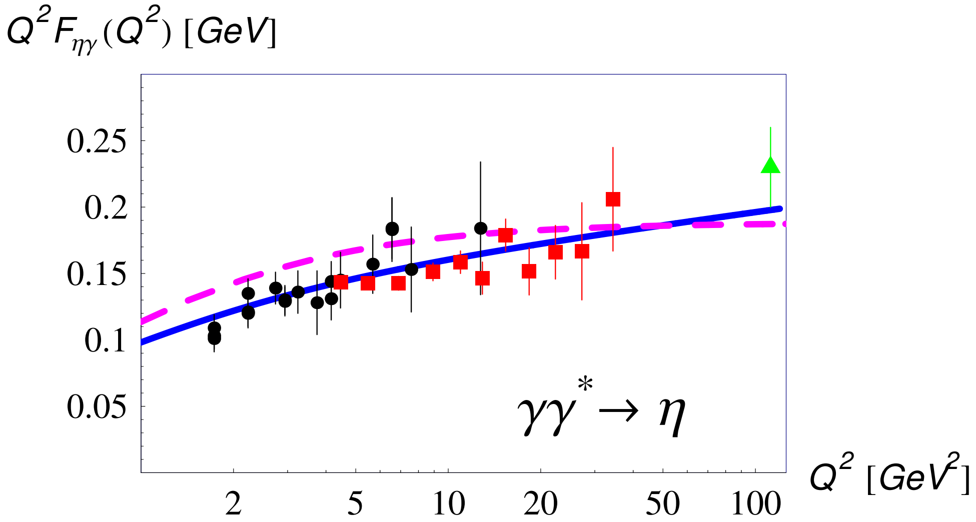
PUZZLE 1:

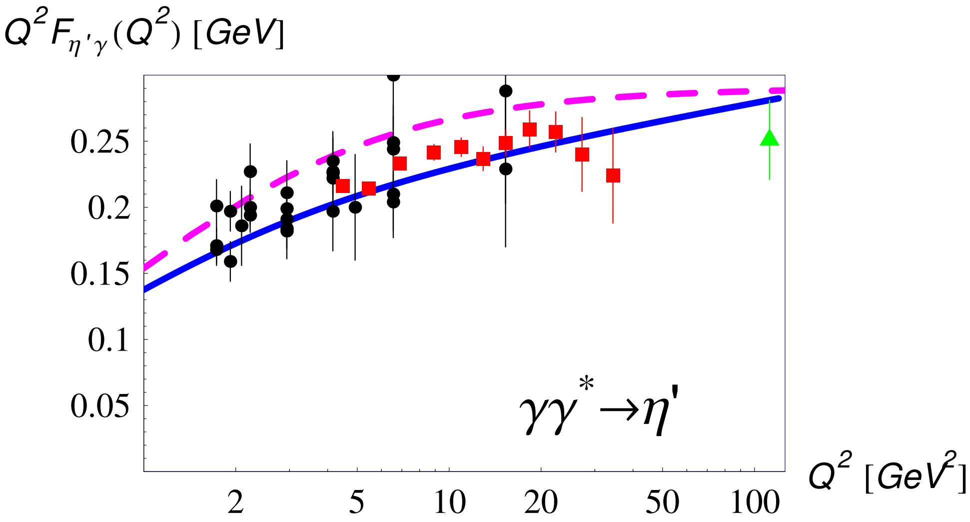


The BaBar pion form factor seems more compatible with $Q^2F_{\pi\gamma}(Q^2) \sim \log(Q^2)$.

QCD factorization theorem seems violated (or at least in danger)

PUZZLE 2:





The η and η' data is not in contradiction with saturation $Q^2 F(Q^2) \sim$ const

Why nonstrange components in η, η' and π^0 should behave so much differently?

<u>THEORY:</u>

- **OPE for 3-point function $\langle VVA \rangle$ in QCD**
- **Quark-hadron duality as a low-energy cut on the spectral representation**

$$\boxed{\pi f_\pi F_{\pi\gamma}(Q^2) = \int\limits_{4m^2}^{s_{\rm eff}(Q^2)} ds\, \rho_{\rm pQCD}(s, Q^2)}$$

Nonperturbative power corrections do not appear explicitly (implicitly hidden is $s_{\rm eff}(Q^2)$).

The effective threshold :

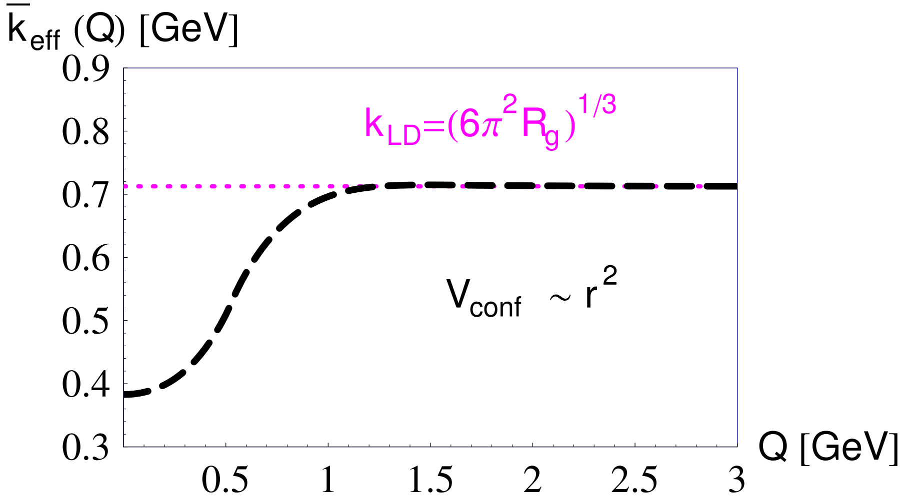
- $s_{\rm eff}(Q^2)$ **for all** Q^2 **remains bounded in the "soft" region** $s_{\rm eff}(Q^2) \sim 0.5 \div 1{\rm GeV}^2$
- **QCD factorization theorem requires** $s_{\rm eff}(Q^2 \to \infty) \to 4\pi^2 f_\pi^2$

(finding $s_{\rm eff}$ for correlators is equivalent to solving full QCD)

One can calculate $s_{\rm eff}$ in quantum mechanics:

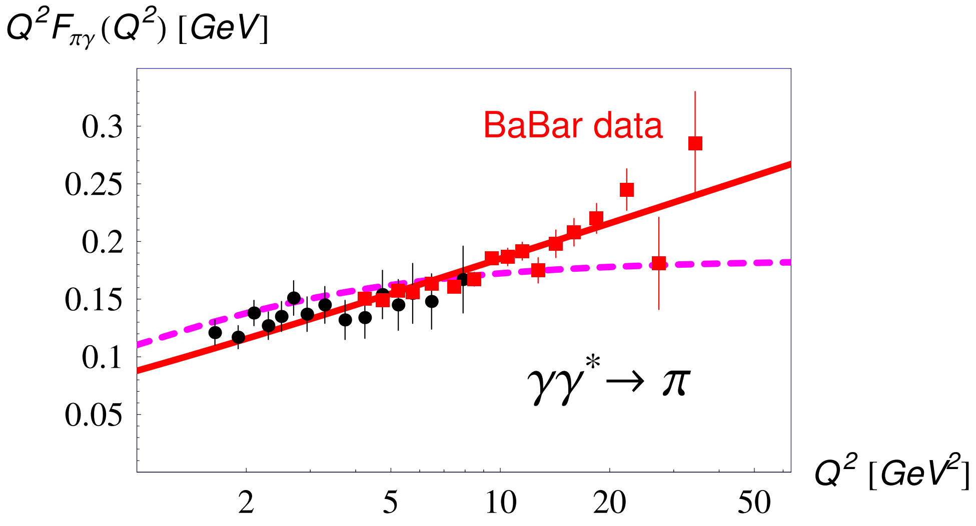
For $V(r) = V_{\rm conf}(r) - \frac{\alpha}{r}$:

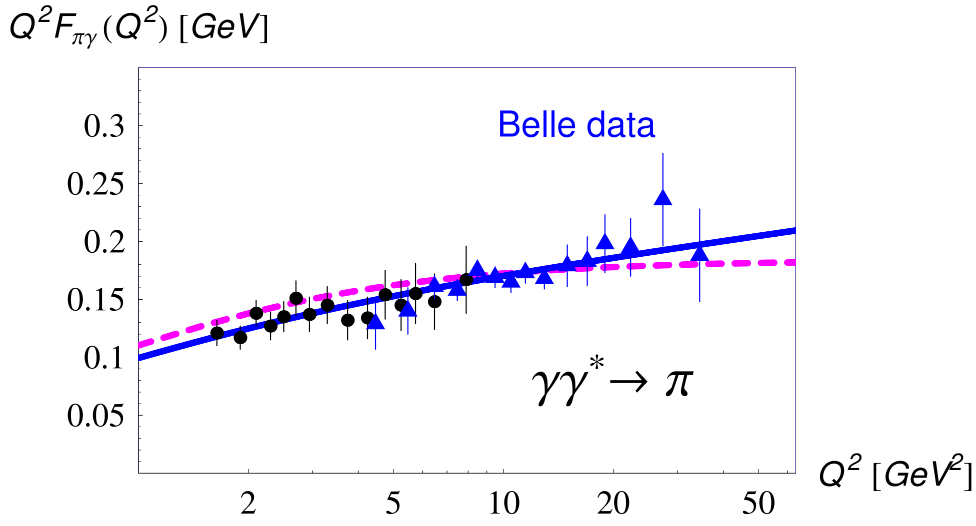
(in this case the form factors satisfy factorization theorem like in QCD)

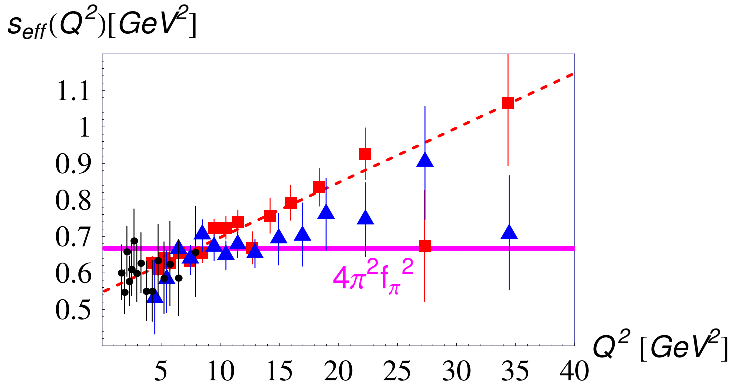


The effective threshold "saturates" at Q^2 = a few $\rm GeV^2$.

BaBar'2009 vs Belle'2012







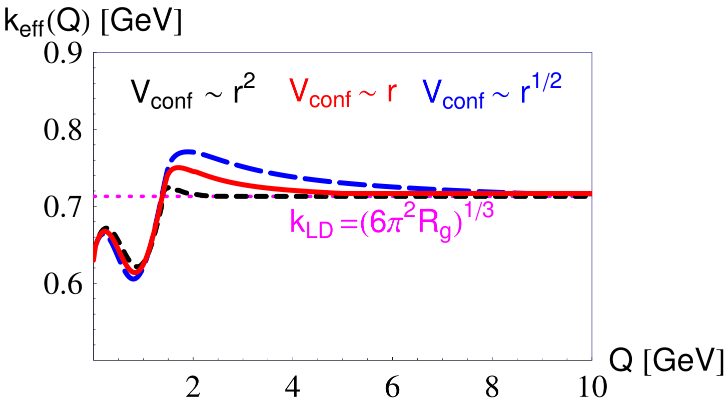
Belle data (i) is fully compatible with factorization (and with η and η' results)

and (ii) the corresponding effective threshold is fully compatible with theoretical expectations

Elastic pion form factor:

$$F_\pi(Q^2) = F_0(Q^2) + \alpha_s(Q^2) F_1(Q^2) + \ldots, \quad F_0(Q^2) \propto 1/Q^4, \quad F_1(Q^2) \propto 1/Q^2$$

Effective threshold:



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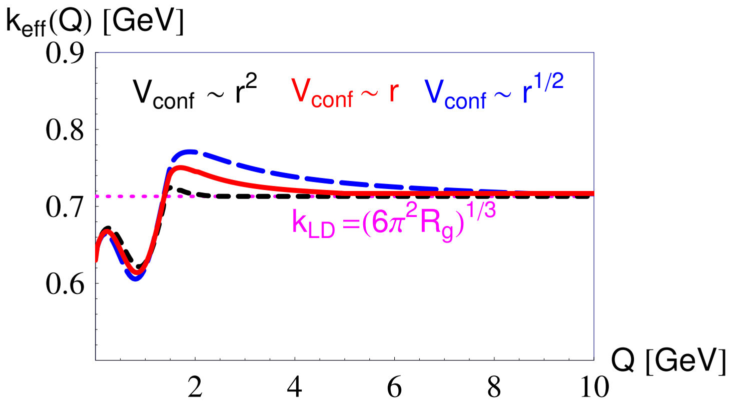

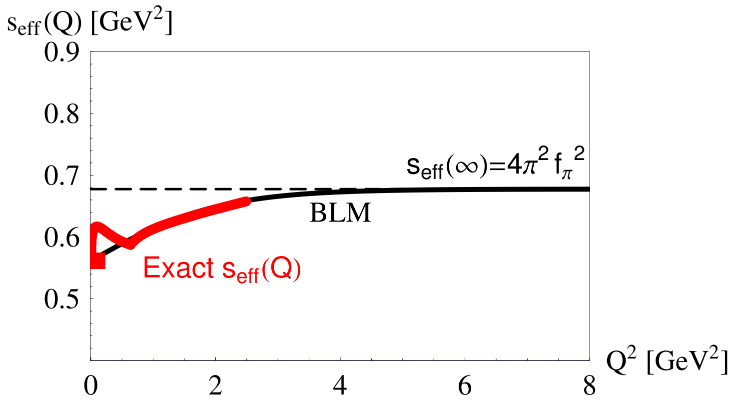

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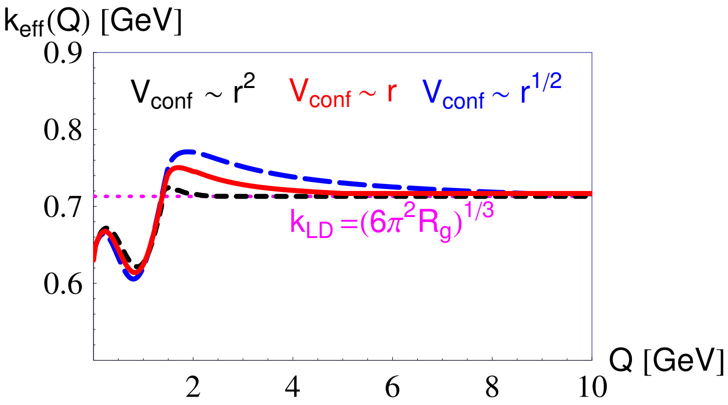
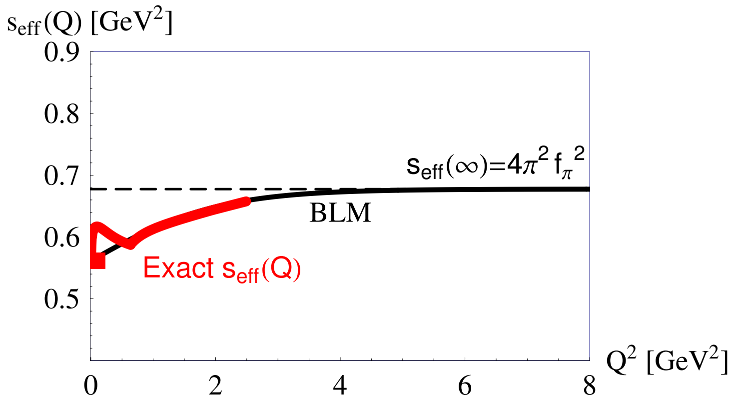
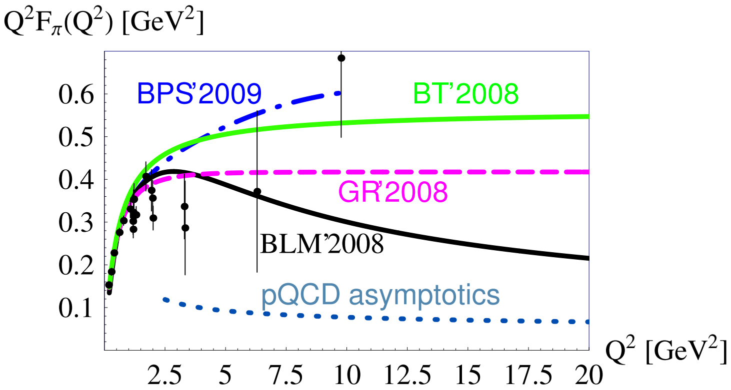
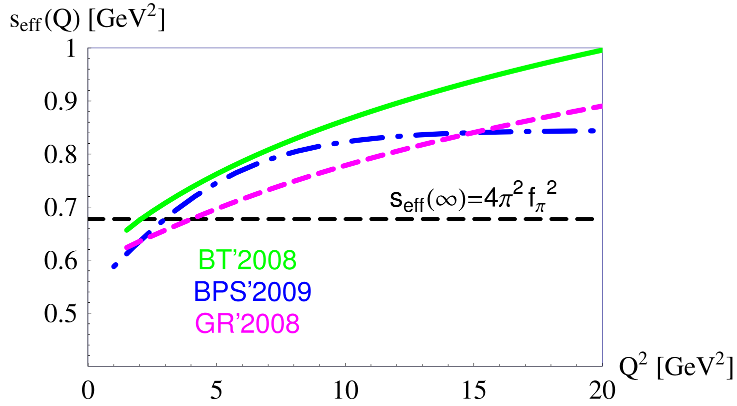




Some recent theoretical predictions:





Summary and conclusions

• Meson-photon transition form factors:

The Belle data resolves the puzzle of the $\pi^0\gamma$ form factor: the results on $\pi^0\gamma$ from Belle is fully compatible with the results on $\eta\gamma$ and $\eta'\gamma$. Moreover, all three form factors are fully compatible with the pQCD asymptotic formula at $Q^2 \geq 10 - 15$ GeV^2.

• Pion elastic form factor:

We predict the asymptotic regime for the effective threshold $s_{\rm eff}(Q^2) = 4\pi^2 f_\pi^2$ (NOT for the form factor!) to be reached at $Q^2 \sim 5-6\mathrm{GeV}^2$. (For the form factor this yields unambiguous predictions for separate contributions in the perturbative expansion). This is testable at JLab.

• Is there still room for violation of factorization?

A better fit to the full set of the meson-photon transition form factors might prefer a small universal logarithmic rise of $Q^2F(Q^2)$. If established experimentally, this rise would mean violation of factorization.