Bounding hadronic uncertainties in $c$ to $u$ decays

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Based on work in progress by AB and Brian Meadows
Overview

- From mixing to CP violation
- The experimental stage

- Isospin relations
  - $D$ to $\pi\pi$
  - $D$ to $\rho\rho$
  - $D$ to $\pi^+\pi^-\pi^0$

- Conclusions
Experimentalists have been chasing neutral meson mixing in the charm sector for decades...

- Both of the B Factories saw this in 2007 via different observables.
- More recently LHCb has produced a beautiful demonstration of this quantum mechanical effect.
From mixing to CP violation

- Experimentalists have been chasing neutral meson mixing in the charm sector for decades...
- Now that mixing is under-control... we can look for:
  - CP violation in mixing.
    \[ |q/p| \neq 1 \]
    Hard to find, but important
  - Direct CP violation.
    \[ |A|^2 \neq |\overline{A}|^2 \]
    Hard to interpret, but should be there
  - Recall the conditions required for direct CP violation place constraints on what we can learn from such observables.
    \[ A_{CP} = \frac{\Gamma - \Gamma}{\Gamma + \Gamma} \propto A_1 A_2 \sin(\phi_1 - \phi_2) \sin(\delta_1 - \delta_2) \]
    e.g. for two interfering amplitudes
  - CP violation in the interference between mixing and decay.
    \[ \lambda_f = \frac{q}{p} \frac{\overline{A}}{A} \neq 1 \]
    Hard to find, but important, test CPV in the CKM matrix
From mixing to CP violation

  - Good way to interpret current data ... but.

- Another more general approach is summarised in AB, Meadows, Inguglia PRD 84 114009 (2011). This breaks down the different contributions in terms of the CKM phase contributions to different amplitudes.

- In order to learn about the standard model from CP violation measurements we need to understand:
  - mixing: i.e. q/p;
  - decay contributions in $\overline{A}/A$;
  - hadronic uncertainties that contribute to the interference between these two terms. This means that eventually we will need to understand the CKM contributions to different amplitudes to understand the types of CP violation in charm.
The experimental stage

- LHCb, the Tevatron and B Factories have been searching for direct CP violation.
  - Focus on the $\Delta A_{\text{CP}}$ measurement: $A_{\text{KK}} - A_{\pi\pi}$

- $\Delta A_{\text{CP}}$ is a direct CP asymmetry different measurement – knowledge of interfering amplitudes and strong phase differences are required to relate measurement back to the CKM matrix, $A_{\Gamma}$ is related to $\Delta A_{\text{CP}}$ via $\lambda_f$, but measured independently (not optimal use of information)
The experimental stage

- These decays have contributions from (mixing+) tree, exchange and penguin topologies:

\[ D \rightarrow K^+ K^- \] (Use this mode to measure the phase of D mixing)

\[ D \rightarrow \pi^+ \pi^- \] (Use this mode to measure the phase of D mixing + a weak phase \( \beta_c \))
The experimental stage

Can we find new physics via $\Delta A_{\text{CP}}$?

e.g. Interpreting pion tag data from LHCb, this example assumes penguins (NP) amplitudes are 1% of the tree. Exact value of $r=A_1/A_2$ doesn't change the conclusion.

- SM solution is well defined at tree level.
- With one observable (=constraint) you can't constrain the three parameters of relevance ($r, \Delta \delta, \Delta \phi$) to search for NP.
- Need a more complicated experimental approach: time-dependent CP asymmetry measurements, including $\overline{A}/A$.
- Hadronic uncertainties from penguins play a role in interpreting the results of time-dependent $c \rightarrow d$ transitions …. that is the rest of this talk.
The experimental stage

- Can we find new physics via $\Delta A_{CP}$? Not with this observable

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Isospin relations

- As with K and B decays the decays D to $\pi\pi$ are related by isospin symmetry:

$$\frac{1}{\sqrt{2}} A^{++} = A^{+0} - A^{00},$$
$$\frac{1}{\sqrt{2}} A^{--} = A^{-0} - A^{00},$$

- Similarly one can extend to $\rho\pi$ and $\rho\rho$ decays, with the corresponding additional complexity.

- Following from this it is straightforward to constrain the shift from penguins on the weak phase measurement that can be made in D to hh decays ($h = \pi, \rho$) using existing data.

e.g. see Gronau & London Phys. Rev. Lett. 65, 3381 (1990)
As with K and B decays the decays D to $\pi\pi$ are related by Isospin symmetry:

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Following from this it is straightforward to constrain the shift from penguins on the weak phase measurement that can be made in D to $hh$ decays ($h = \pi, \rho$) using existing data.

e.g. see Gronau & London Phys. Rev. Lett. 65, 3381 (1990)
Using inputs from the PDG we can estimate the effect of penguin pollution in $D \rightarrow \pi \pi$ decays, ignoring long distance and electroweak penguin effects.

- Require branching ratios and CP asymmetries for $\pi\pi$ decays (assume $A_{CP}=0$).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(D^0 \rightarrow \pi^+\pi^-) \times 10^{-3}$</td>
<td>$1.400 \pm 0.026$</td>
</tr>
<tr>
<td>$B(D^\pm \rightarrow \pi^+\pi^0) \times 10^{-3}$</td>
<td>$1.19 \pm 0.06$</td>
</tr>
<tr>
<td>$B(D^0 \rightarrow \pi^0\pi^0) \times 10^{-3}$</td>
<td>$0.80 \pm 0.05$</td>
</tr>
</tbody>
</table>

Experimentally limited by modes with neutrals in the final state.

Current data limits $\delta\beta_c$ to 2.7°.

Can't distinguish between mirror solutions here and in [-180, 0].
Taking a similar approach for $D \to \rho\rho$ decays, and ignoring possible $I=1$ amplitudes one can try to estimate penguin pollution in these decays (needs an angular analysis of the data!).

PDG data highlight one clear point:

- There is a lack of good experimental data in this set of decays.

CLEO measured $D^0 \to \rho^0\rho^0$ explicitly – but measurements of the other modes have not been done (they were treated as an inclusive 4 body final state).

Here we assume that the 4 body final state decays to $\pi^+\pi^0\pi^-\pi^0$ and $\pi^+\pi^-\pi^+\pi^0$ are dominated by $\rho^+\rho^-$ and $\rho^+\rho^0$.

Also assume naive factorisation to estimate $f_L \sim 0.83$ for these decays, and analyse only the longitudinal polarisation.
$D \rightarrow \rho\rho$

- Under these assumptions we get:

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<tr>
<td>$B(D^0 \rightarrow \rho^+\rho^-) \times 10^{-3}$</td>
<td>$10.0 \pm 0.9^\dagger$</td>
</tr>
<tr>
<td>$B(D^\pm \rightarrow \rho^+\rho^0) \times 10^{-3}$</td>
<td>$11.3 \pm 0.8^\dagger$</td>
</tr>
<tr>
<td>$B(D^0 \rightarrow \rho^0\rho^0) \times 10^{-3}$</td>
<td>$1.82 \pm 0.13$</td>
</tr>
<tr>
<td>$f_L(D^0 \rightarrow \rho^+\rho^-) \times 10^{-3}$</td>
<td>$0.83 \times$ (Ref. [3])</td>
</tr>
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<td>$0.83 \times$ (Ref. [3])</td>
</tr>
<tr>
<td>$f_L(D^0 \rightarrow \rho^0\rho^0) \times 10^{-3}$</td>
<td>$0.69 \pm 0.08$</td>
</tr>
</tbody>
</table>

- Current data limits $\delta \beta c$ to $4.6^\circ$.
- It is important that BES III and the B Factories perform angular analyses of these decays to understand if this estimate is robust.

Can't distinguish between mirror solutions here and in $[-180, 0]$.

Ambiguities can be resolved by combining modes!
The D to $3\pi$ Dalitz structure is well known from BaBar and CLEO.

This means one can use the Snyder and Quinn approach to constrain penguins in these decays.

$I = 2$ is $T$ only.

$I = 0, 1$ are $T$ and $P$.

Amplitude subscripts relate to the $\Delta I$ value ($I = 1/2$ etc.).

$\Delta I = 1/2$ transitions are $I = 0, 1$.

$\Delta I = 3/2$ transitions are $I = 1, 2$, and SM penguins can't contribute to these.

\[
D \rightarrow \pi^+ \pi^- \pi^0
\]

\[
A^{+-} = A_3 + B_3 + \frac{1}{\sqrt{2}} A_1 + B_1
\]

\[
= T^{+-} + P_1 + P_0
\]

\[
A^{-+} = A_3 - B_3 - \frac{1}{\sqrt{2}} A_1 + B_1
\]

\[
= T^{-+} - P_1 + P_0
\]

\[
A^{00} = 2A_3 - B_1
\]

\[
= [T^{+-} + T^{-+} + T^{00} - T^{0+}] / 2 - P_0
\]

\[
A^{+0} = \frac{3}{\sqrt{2}} A_3 - \frac{1}{\sqrt{2}} B_1 + A_1
\]

\[
= [T^{+0} + 2P_1] / \sqrt{2}
\]

\[
A^{0+} = \frac{3}{\sqrt{2}} A_3 + \frac{1}{\sqrt{2}} B_1 - A_1
\]

\[
= [T^{0+} - 2P_1] / \sqrt{2}
\]
So instead of triangles one has pentagons.

\[ D \rightarrow \pi^+ \pi^- \pi^0 \]

Here the charged modes are observed in different Dalitz plots and don't bring anything to the table.

A priori one doesn't know the relative strong phase between charged and neutral modes.

- Current data limits \(2\delta\beta_c\) to \(1^\circ\).
- We estimate a time-dependent measurement from BaBar would yield \(\sigma(\beta_{c,\text{eff}}) \sim 1^\circ\).
Conclusions

- Hadronic uncertainties arising from penguin pollution in D decays to hh (h=π, ρ), and the π^+π^-π^0 Dalitz plot can be controlled to the (few) degree level.

- A step toward understanding precision of CKM angle tests that can be done with time-dependent CP asymmetry measurements in the charm sector.

- These hadronic uncertainties also play a role in understanding approximate observables such as A_Γ in D to ππ.

- BES III, LHCb and the other e^+e^- machines can help improve experimental constraints in this area.
  - Measure rates and direct CP asymmetries of D decays to hh and 3π final states (h=π, ρ).
  - Perform time-dependent measurements to access information from λ_f.

- It will be interesting to see what LHCb and BES III can bring to the table with data already in hand.
  - We're looking forward to finding out what we can learn about CP violation and the CKM matrix from charm in the coming decade.
CKM

- CKM > $O(\lambda^3)$ is model dependent: we choose Buras et al.

$$V_{CKM} = \left( \begin{array}{ccc}
1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} & \frac{\lambda}{A\lambda^3[1 - \bar{\rho} - i\bar{\eta}]} & A\lambda^3(\bar{\rho} - i\bar{\eta}) + A\lambda^5(\bar{\rho} - i\bar{\eta})/2 \\
-\lambda + \frac{A^2\lambda^5[1 - 2(\bar{\rho} + i\bar{\eta})]}{2A\lambda^3[1 - \bar{\rho} - i\bar{\eta}]} & 1 - \frac{\lambda^2}{2} - \frac{\lambda^4(1 + 4A^2)}{8} & A\lambda^2 \\
-A\lambda^2 + A\lambda^4[1 - 2(\bar{\rho} + i\bar{\eta})]/2 & 1 - A^2\lambda^4/2 & 1
\end{array} \right) + O(\lambda^6)$$

- Note:
  - At this order in $\lambda$, $V_{cd}$ is complex
  - Correspondence between $\beta$ and $\arg(V_{td})$ and $\gamma$ and $\arg(V_{ub})$ is not exact any more
  - Direct angle measurements from B decays, and indirect constraints lead to a prediction of the $cu$ triangle:

  $$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0$$

  $\alpha_c = (111.5 \pm 4.2)^\circ$
  $\beta_c = (0.0350 \pm 0.0001)^\circ$
  $\gamma_c = (68.4 \pm 0.1)^\circ$

- Can try to constrain this.
Formalism: $D^*$ tagged events

$\pi$

$D^*$

$D^0 \bar{D}^0$

$f_{CP}$, or CP admixture

The time dependence is familiar

$$\Gamma(P^0 \rightarrow f) \propto e^{-\Gamma_{1t}} \left[ \frac{1 + e^{\Delta \Gamma t}}{2} + \frac{Re(\lambda_f)}{1 + |\lambda_f|^2} (1 - e^{\Delta \Gamma t}) + e^{\Delta \Gamma t/2} \left( \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos \Delta M t - \frac{2Im(\lambda_f)}{1 + |\lambda_f|^2} \sin \Delta M t \right) \right]$$

$$\Gamma(\bar{P}^0 \rightarrow f) \propto e^{-\Gamma_{1t}} \left[ \frac{1 + e^{\Delta \Gamma t}}{2} + \frac{Re(\lambda_f)}{1 + |\lambda_f|^2} (1 - e^{\Delta \Gamma t}) + e^{\Delta \Gamma t/2} \left( -\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos \Delta M t + \frac{2Im(\lambda_f)}{1 + |\lambda_f|^2} \sin \Delta M t \right) \right]$$

Recall $\Delta \Gamma \neq 0$ as

$$x = \frac{\Delta M}{\Gamma}, \text{ and } y = \frac{\Delta \Gamma}{2\Gamma}$$

$\lambda_f = \frac{\sqrt{q} \ e^{i\phi_{MIX}}}{\sqrt{p}} $ $\bar{A} \ e^{i\phi_{CP}}$

The aim is to understand what mixing AND decay can teach us.
Formalism: threshold tagged events

The time dependence is familiar

\[
\Gamma(P^0 \rightarrow f) \propto e^{-\Gamma_1 |\Delta t|} \left[ \frac{h_+ + \text{Re}(\lambda_f)}{1 + |\lambda_f|^2} h_- + e^{\Delta \Gamma |\Delta t|/2} \left( \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos \Delta M \Delta t - \frac{2\text{Im}(\lambda_f)}{1 + |\lambda_f|^2} \sin \Delta M \Delta t \right) \right],
\]

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\]

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Formalism: threshold events

- Can construct an asymmetry from these rates in the usual way:

\[
A(\Delta t) = \frac{\Gamma(\Delta t) - \Gamma(\Delta t)}{\Gamma(\Delta t) + \Gamma(\Delta t)} = 2e^{\Delta \Gamma|\Delta t|/2} \left( |\lambda_f|^2 - 1 \right) \cos \Delta M \Delta t + 2i \text{Im} \lambda_f \sin \Delta M \Delta t \\
\frac{1 + |\lambda_f|^2}{(1 + |\lambda_f|^2)h_+ + 2h_- \text{Re} \lambda_f}
\]

The asymmetry is slowly varying.

Won't see oscillation maxima as in the case of B decays.

Effect of \( \Delta \Gamma \) is seen in the difference between solid (\( \Delta \Gamma \neq 0 \)) and dashed (\( \Delta \Gamma = 0 \)) curves.