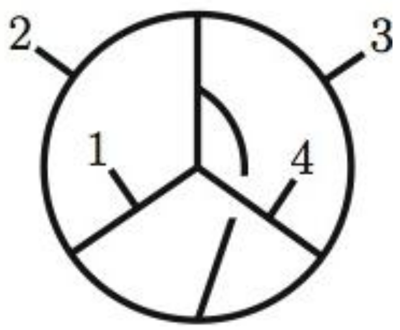
Scattering Amplitudes in Quantum Field Theory

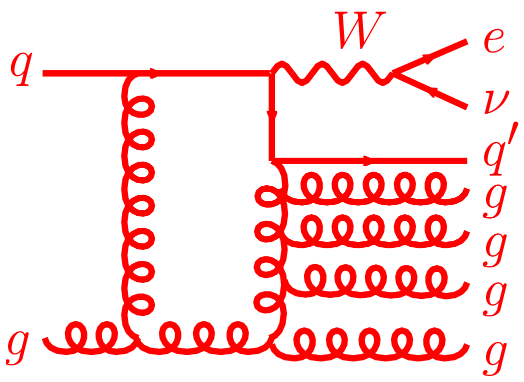
European Physical Society Meeting

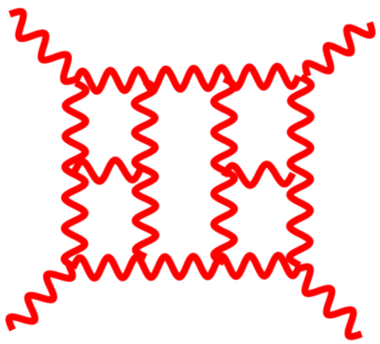
Stockholm

July 24, 2013

Zvi Bern, UCLA







Outline

1) Remarkable progress in scattering amplitudes.

2) Brief summary of new advances and ideas.

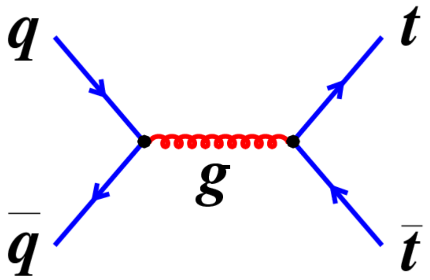
3) Example: applications to LHC physics.

4) Example: A duality between color and kinematics.

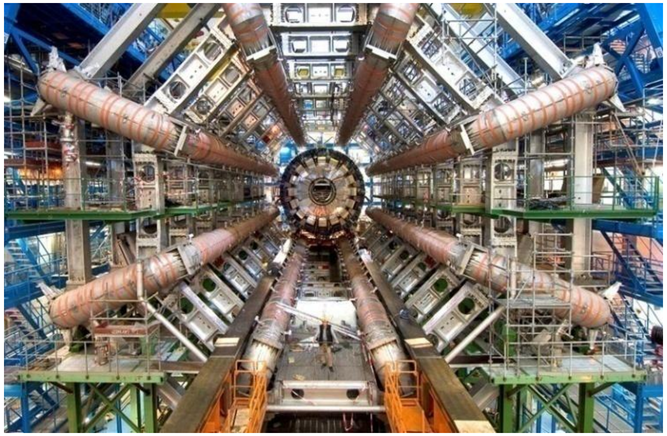
5) Example: UV surprises in supergravity theories.

Scattering amplitudes

Scattering of elementary particles is fundamental to our ability to unravel microscopic laws of nature.



Arrival of the Large Hadron Collider raises importance of collider physics and scattering amplitudes.



Here we give some examples of advances of past few years in understanding and calculating scattering in quantum field theory.

Major Advance in Scattering Amplitudes

“Impossible calculations” of scattering amplitudes in gauge and gravity theories now commonplace.

A few highlights from past year:

- **Constructing large chunks of the scattering amplitudes of $N = 4$ super-Yang-Mills theory, towards a full construction.**

  Alday, Arkani-Hamed, Basso, Bourjaily, Cachazo, Caron-Huot, Dixon, Duhr, Gehrmann, Golden, Goncharov, He, Henn, Heslop, Huber, Johansson, Kosower, Larsen, Lipstein, Lipatov, Maldacena, Mason, Pennington, Postnikov, Sikorowski, Sever, Spradlin, Trnka, Vergu, Vieira, Volovich and many others
- **New remarkable representations of gravity amplitudes inspired by twistor string theory.**

  Adamo, Cheung, Hodges, Cachazo, Geyer, Mason, Skinner, etc
- **Advances in constructing string theory scattering amplitudes with large numbers of external legs.**

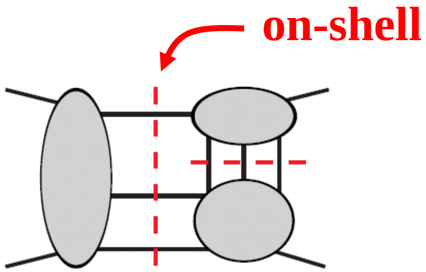
  Broedel, Drummond, Green, Mafra, Schlotterer, Stieberger, Taylor, Ragousy, Terasoma, etc
- **Relations between gravity and gauge theory amplitudes.**

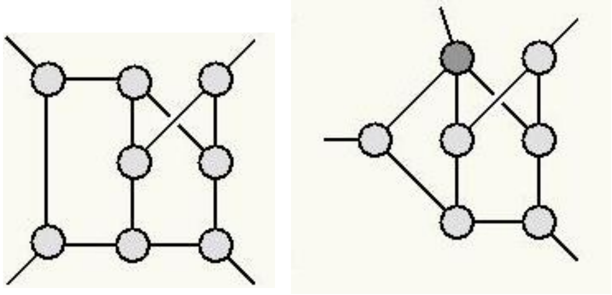
  ZB, Bjerrum-Bohr, Carrasco, Davies, Dennen, O’Connell, Huang, Johansson ,Monteiro, Roiban , etc
- **NLO QCD multijet processes for LHC physics.** See talk from de Florian

  ZB, Badger, Dixon, Febres Cordero, Hoeche, Ita,Kosower, Maitre, Ozeren, Uwer, Yundin, etc

Constructing Multiloop Amplitudes

We do have powerful tools for computing amplitude.
The ideas include:





- **Unitarity Method.**

  ZB, Dixon, Dunbar, Kosower
  ZB, Carrasco, Johansson , Kosower

- **On-shell recursion.**

  Britto, Cachazo Feng and Witten; Arkani-Hamed et al

- **Duality between color and kinematics.**

  ZB, Carrasco and Johansson

- **Advances in loop integration technology.**

Chetyrkin, Kataev , Tkachov; Vladimirov; Marcus, Sagnotti; ZB, Dixon, Kosower; V.A. Smirnov; Czakon; Gehrmann, Remifdi; A.V. Smirnov; Britto, Cachazo, Feng; Bredenstein, Dennen, Dittmaier, Pozzorini; Ossola, Papadopoulos, Pittau; Forde; Badger ; ZB, Dixon, Kosower, Forde, Ita, Maitre; Ellis, Kunszt , Giele and many others.

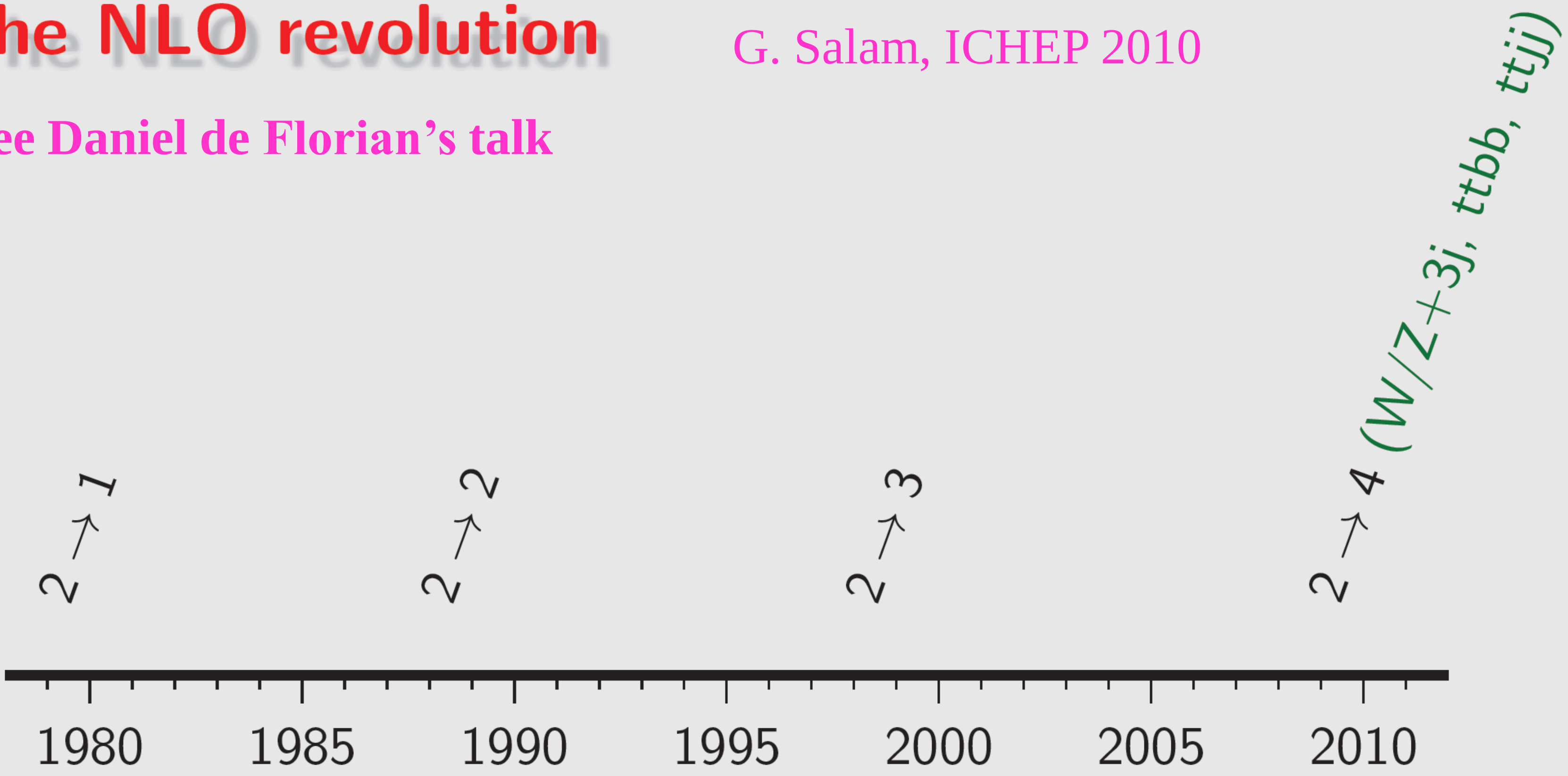
In this talk we will show you some applications of these ideas.

Example: Applications to NLO QCD and LHC Physics

The NLO revolution

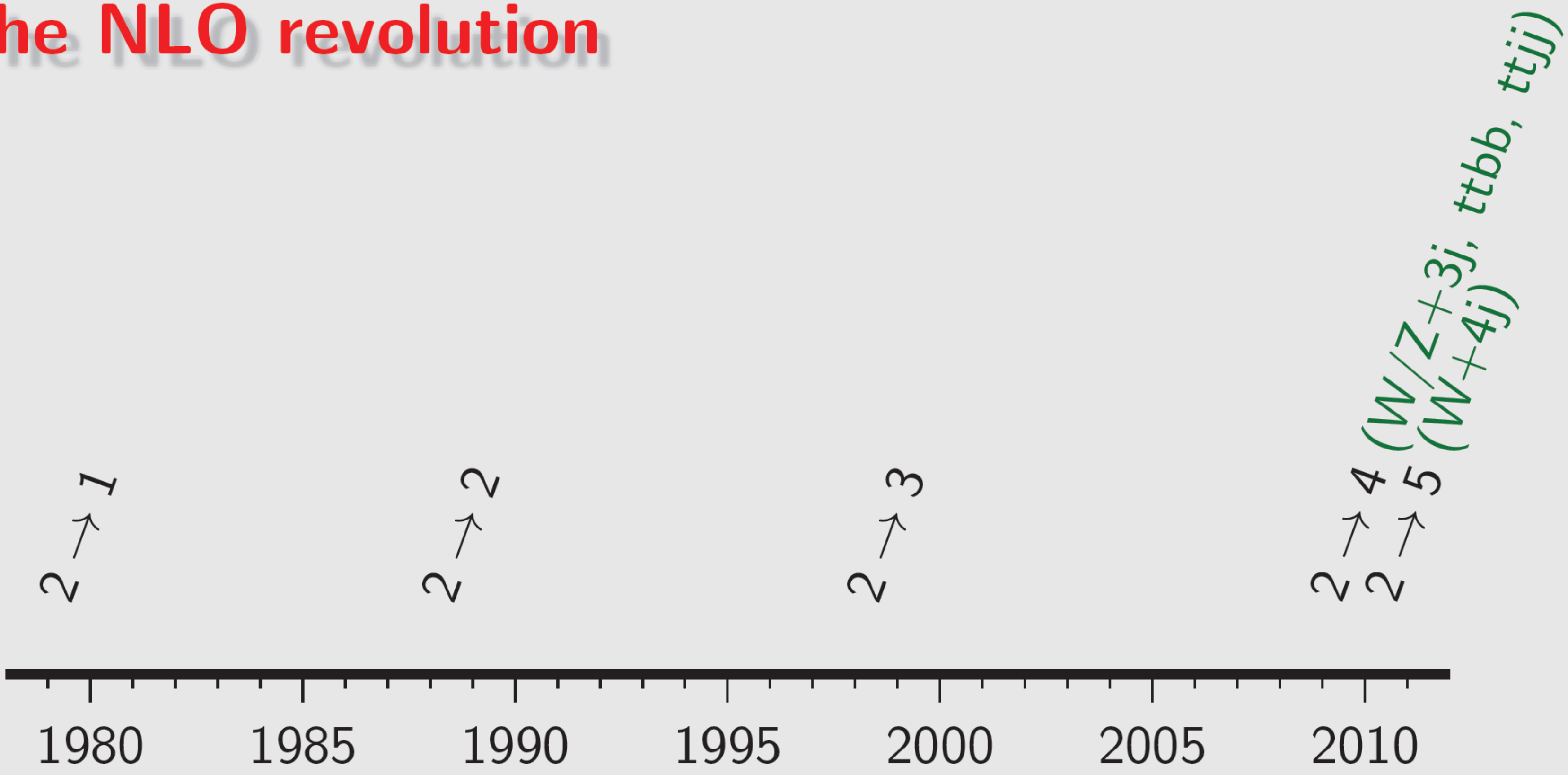
G. Salam, ICHEP 2010

See Daniel de Florian's talk



2009: NLO $W{+}3j$ [Rocket: Ellis, Melnikov & Zanderighi] [unitarity]
2009: NLO $W{+}3j$ [BlackHat: Berger et al] [unitarity]
2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al] [traditional]
2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al] [unitarity]
2009: NLO $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth et al] [traditional]
2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al] [unitarity]
2010: NLO $Z{+}3j$ [BlackHat: Berger et al] [unitarity]

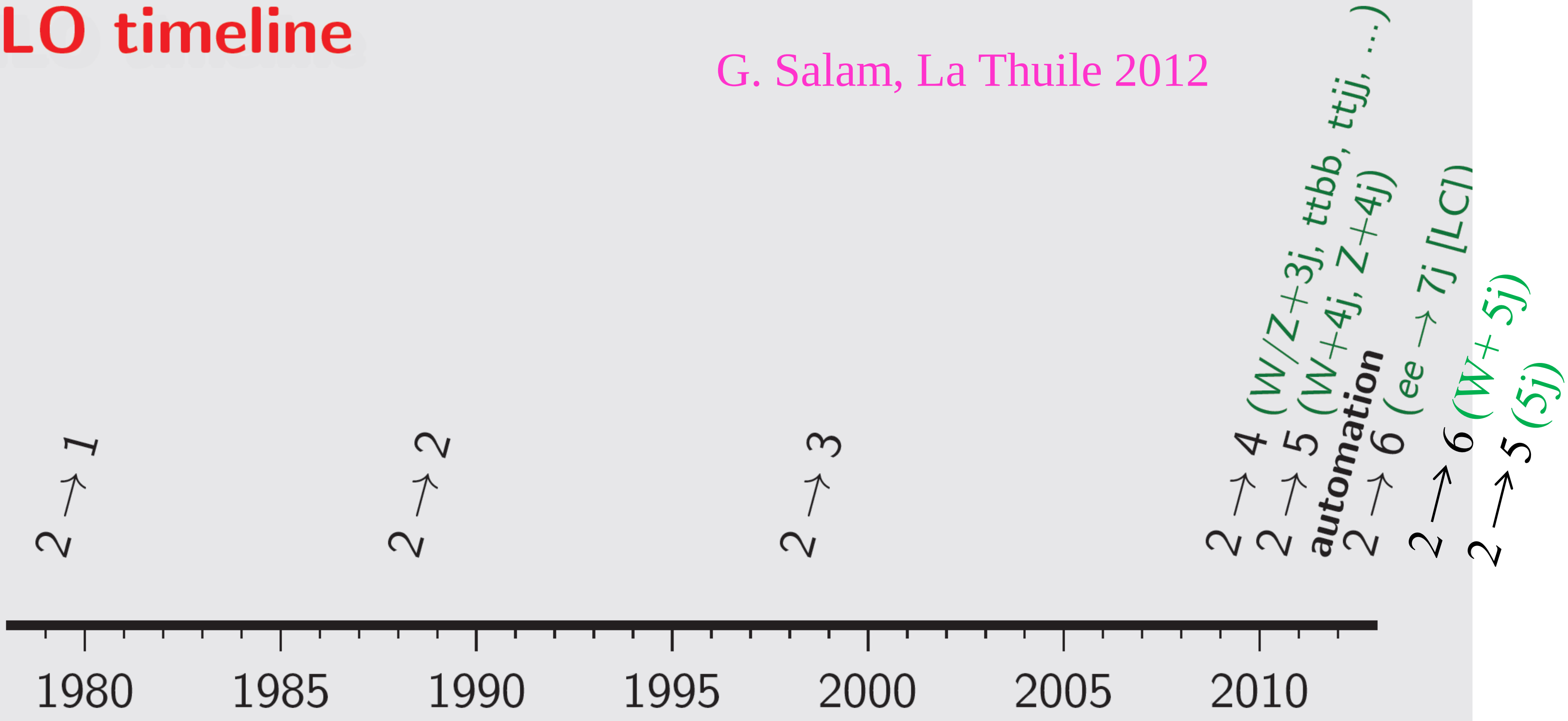
The NLO revolution



2010: NLO W+4j [BlackHat: Berger et al, preliminary] [unitarity]

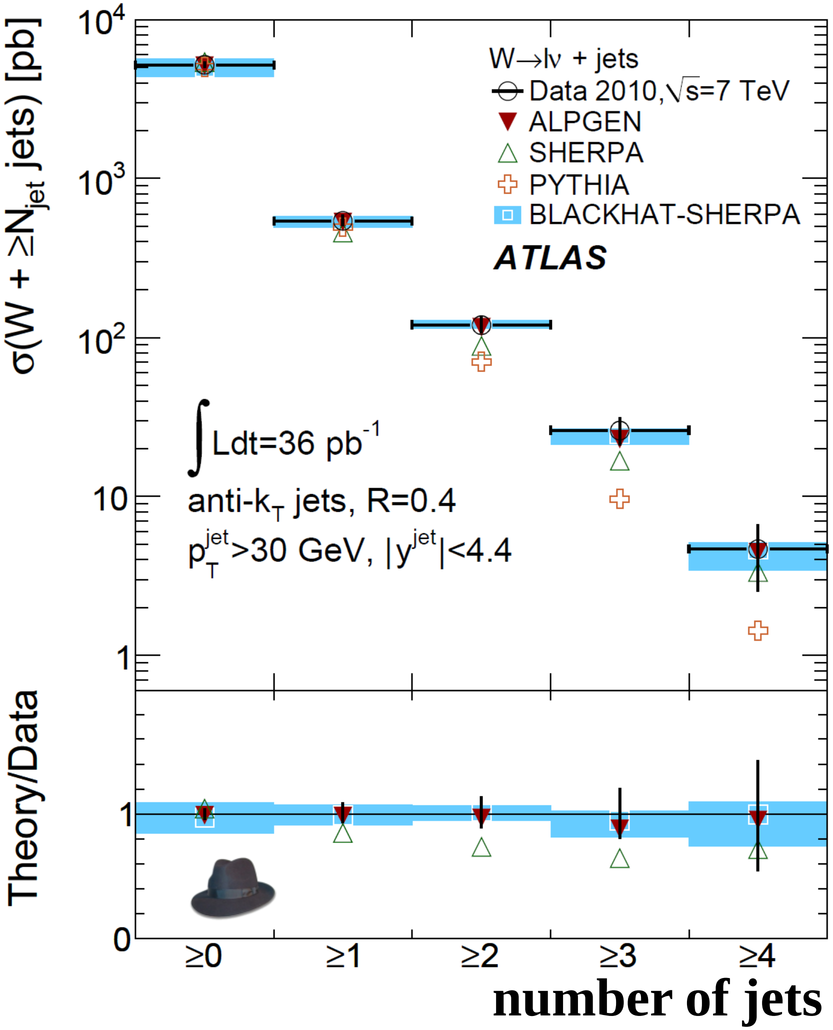
NLO timeline

G. Salam, La Thuile 2012



2010: NLO $W+4j$ [BlackHat+Sherpa: Berger et al]	[unitarity]
2011: NLO $WWjj$ [Rocket: Melia et al]	[unitarity]
2011: NLO $Z+4j$ [BlackHat+Sherpa: Ita et al]	[unitarity]
2011: NLO $4j$ [BlackHat+Sherpa: Bern et al]	[unitarity]
2011: first automation [MadNLO: Hirschi et al]	[unitarity + feyn.diags]
2011: first automation [Helac NLO: Bevilacqua et al]	[unitarity]
2011: first automation [GoSam: Cullen et al]	[feyn.diags(+unitarity)]
2011: $e^+e^- \to 7j$ [Becker et al, leading colour]	[numerical loops]
2013: NLO W+5j [BlackHat+Sherpa: Bern et al]	[unitarity]
2013: NLO 5j [Badger et al, Preliminary]	[unitarity]

ATLAS Comparison against NLO QCD



***W* +1, 2, 3, 4 jets inclusive**

ATLAS compared data against NLO theoretical predictions

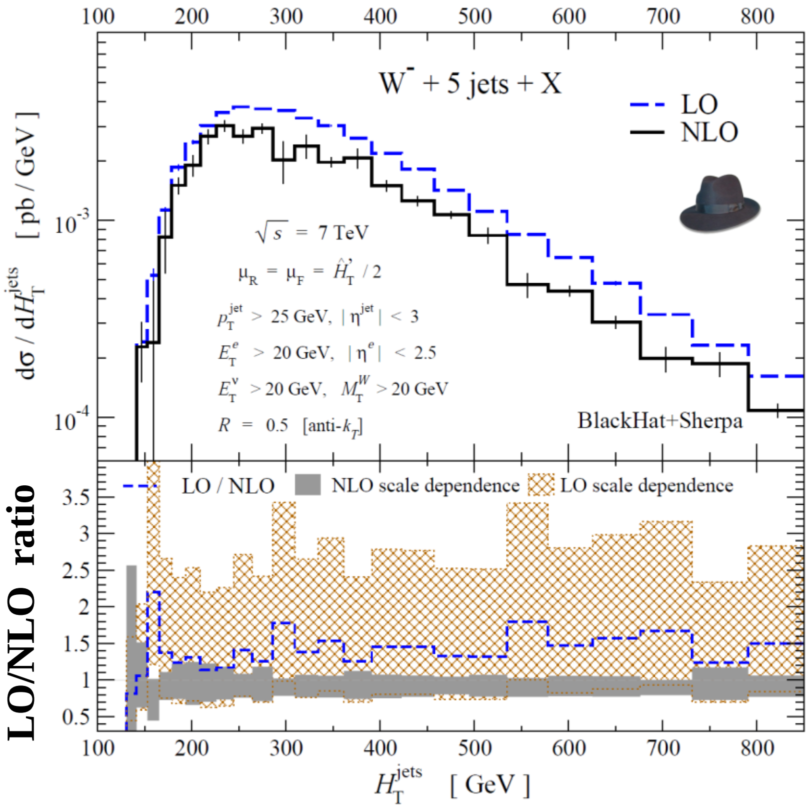
Powerful experimental confirmation of NLO approach. No tuning!

Validate in regions where no new physics is expected and look for discrepancies on tails of distributions where new physics may be hiding.

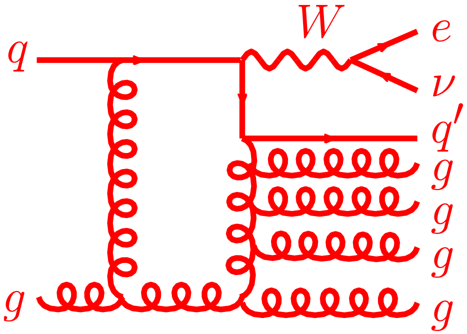
NLO predictions from BlackHat used to aid CMS search for supersymmetry by giving reliable theory uncertainties.

W + 5 Jets in NLO QCD

transverse hadronic energy spectrum



Transverse hadronic energy



A triumph for on-shell methods.

Recently finished and accepted in PRD

arXiv:1304.1253

A new level for "state-of-the-art" LHC theory: First NLO QCD 2→6 process.

See Daniel de Florian's talk

Example: A new understanding of gravity scattering amplitudes

Gravity vs Gauge Theory

Consider the gravity Lagrangian

$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R$$

$$\kappa^2 = 32\pi\, G_{\text{Newton}}$$

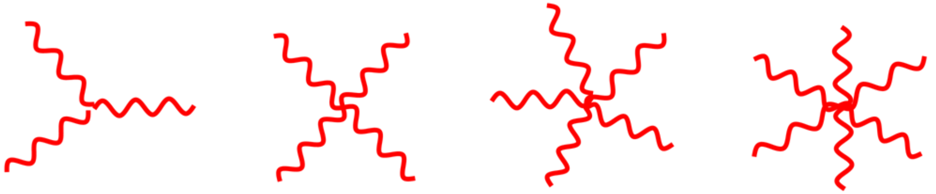
flat metric

graviton field

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

metric

Infinite number of complicated interactions

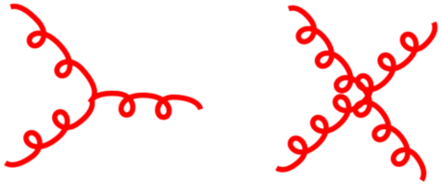


$+\cdots$

terrible mess

Compare to Yang-Mills Lagrangian on which QCD is based

$$L_{\text{YM}} = \frac{1}{g^2}F^2$$

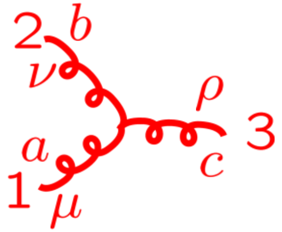


Only three and four point interactions

Gravity seems so much more complicated than gauge theory.

Three Vertices

Standard Feynman diagram approach.



Three-gluon vertex:

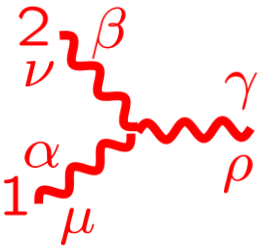
$$V_{3\,\mu\nu\sigma}^{abc} = -g f^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho + \eta_{\nu\rho}(k_1-k_2)_\mu + \eta_{\rho\mu}(k_1-k_2)_\nu)$$

$$k_i^2 = E_i^2 - \vec{k}_i^2 \neq 0$$

Three-graviton vertex:

$$G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) =$$

$$\begin{aligned}
&\mathrm{sym}[-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma})\\
&\quad + P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma})\\
&\quad + P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma})\\
&\quad + 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})]
\end{aligned}$$



About 100 terms in three vertex

Naïve conclusion: Gravity is a nasty mess.

Definitely not a good approach.

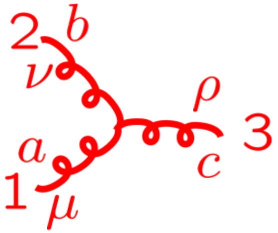
Simplicity of Gravity Amplitudes

People were looking at gravity the wrong way. On-shell formalism much more powerful.

***On-shell* three vertices contains all information:**

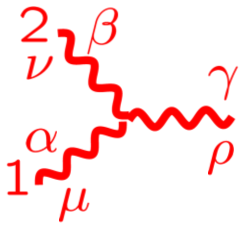
$$k_i^2 = 0$$

gauge theory:



$$-gf^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$

gravity:



$$i\kappa(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$
$$\times\,(\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic})$$

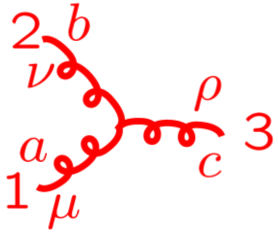
double copy of Yang-Mills vertex.

- **Using modern on-shell methods, any gravity scattering amplitude constructible solely from *on-shell* 3 vertex.**
- **Higher-point vertices irrelevant! BCFW recursion for trees, BDDK unitarity method for loops.**

Duality Between Color and Kinematics

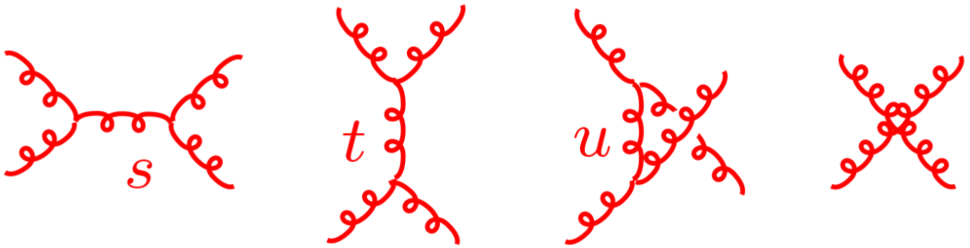
coupling constant **color factor** **momentum dependent kinematic factor**

$$-gf^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$



Color factors based on a Lie algebra: $[T^a, T^b] = if^{abc}T^c$

Jacobi Identity $f^{a_1a_2b}f^{ba_4a_3} + f^{a_4a_2b}f^{ba_3a_1} + f^{a_4a_1b}f^{ba_2a_3} = 0$



Use 1 = *s/s* = *t/t* = *u/u* to assign 4-point diagram to others.

$$\mathcal{A}_4^{\text{tree}} = g^2\left(\frac{n_sc_s}{s} + \frac{n_tc_t}{t} + \frac{n_uc_u}{u}\right)$$

$s = (k_1 + k_2)^2$

$t = (k_1 + k_4)^2$

$u = (k_1 + k_3)^2$

Color factors satisfy Jacobi identity: $c_u = c_s - c_t$

Numerator factors satisfy similar identity: $n_u = n_s - n_t$

Color and kinematics satisfy the same identity

Duality Between Color and Kinematics

Consider five-point tree amplitude:

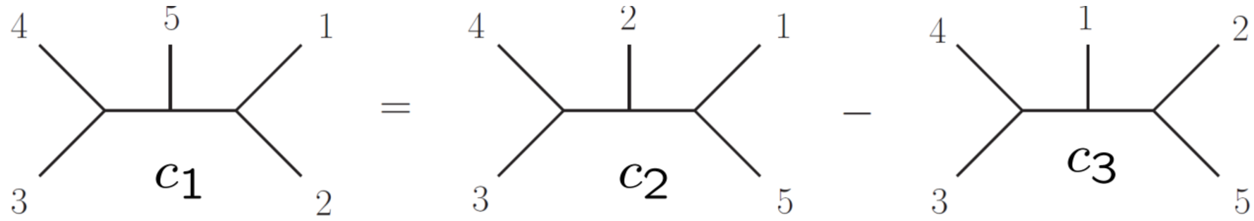
ZB, Carrasco, Johansson (BCJ)

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i\, n_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

color factor

kinematic numerator factor

Feynman propagators



$$c_1 \equiv f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2}, \quad c_2 \equiv f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5}, \quad c_3 \equiv f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

$$\boxed{c_1 - c_2 + c_3 = 0 \Leftrightarrow n_1 - n_2 + n_3 = 0}$$

Claim: At n-points we can always find a rearrangement so color and kinematics satisfy the same algebraic constraint equations.

Nontrivial constraints on amplitudes in field theory and string theory

BCJ, Bjerrum-Bohr, Feng,Damgaard, Vanhove, ; Mafra, Stieberger, Schlotterer; Cachazo; Tye and Zhang; Feng, Huang, Jia; Chen, Du, Feng; Du, Feng, Fu; Naculich, Nastase, Schnitzer

BCJ

Gravity and Gauge Theory

kinematic numerator

color factor

gauge theory:

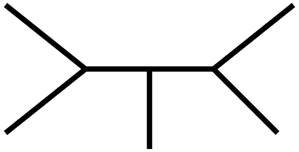
$$\frac{1}{g^{n-2}}\mathcal{A}_n^{\text{tree}}(1,2,3,\ldots,n)=\sum_i \frac{n_i\, c_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

sum over diagrams with only 3 vertices

$$c_i \sim f^{a_1 a_2 b_1} f^{b_1 b_2 a_5} f^{b_2 a_4 a_5}$$

Assume we have:

$$c_1+c_2+c_3=0 \quad \Leftrightarrow \quad n_1+n_2+n_3=0$$



Then: $c_i \Rightarrow \tilde{n}_i$ **kinematic numerator of second gauge theory**

gravity:

$$-i\left(\frac{2}{\kappa}\right)^{(n-2)}\mathcal{M}_n^{\text{tree}}(1,2,\ldots,n)=\sum_i \frac{n_i\, \tilde{n}_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

Gravity numerators are a double copy of gauge-theory ones!

This works for ordinary Einstein gravity and susy versions!

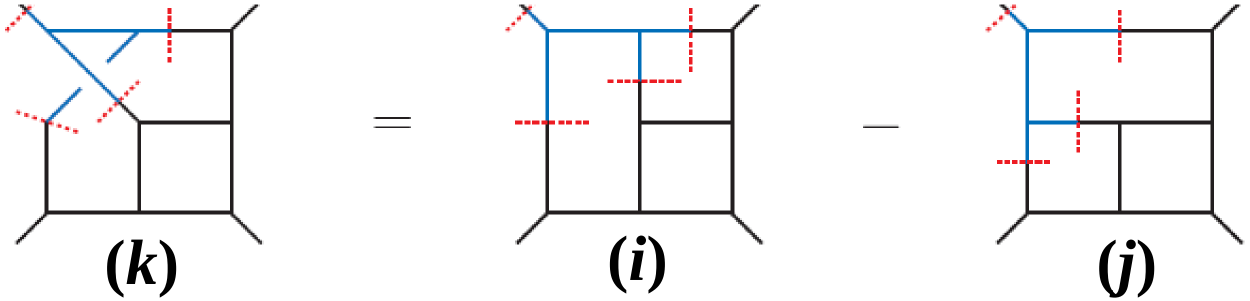
Cries out for a unified description of the sort given by string theory!

Tree level proof, ZB, Dennen, Huang, Kiermaier; Bjerrum Bohr, Damgaard, Vanhove

BCJ

Gravity loop integrands are free!

Ideas generalize to loops:



color factor

$$c_k = c_i - c_j$$

$$n_k = n_i - n_j$$

kinematic numerator

If you can find a set of duality satisfying numerators.

To get:

gauge theory → gravity theory

simply take

color factor → kinematic numerator

$$c_k \longrightarrow n_k$$

Application: UV Properties of Gravity

Quantum Gravity

Often repeated statement:

"Einstein's theory of General Relativity is incompatible with quantum mechanics."

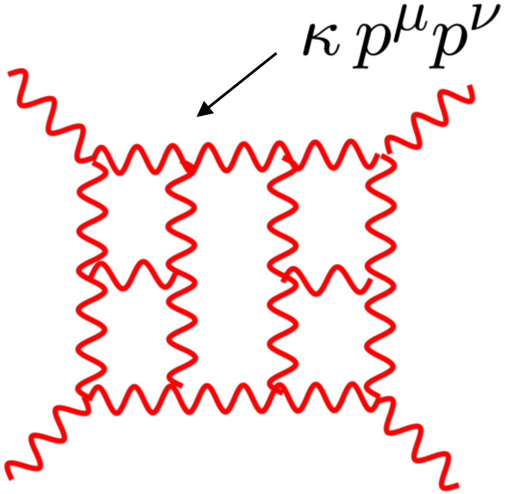
To a large extent this is based on another often repeated statement:

"All point-like quantum theories of gravity are ultraviolet divergent and non-renormalizable."

Where do these statements come from and are they true?

Power Counting at High Loop Orders

$\kappa = \sqrt{32\pi G_N}$ ⟵ Dimensionful coupling



Gravity: $$\int \prod_{i=1}^{L} \frac{dp_i^D}{(2\pi)^D} \frac{(\kappa\, p_j^\mu p_j^\nu)\cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(g\, p_j^\nu)\cdots}{\text{propagators}}$$

Extra powers of loop momenta in numerator means integrals are badly behaved in the UV.

Non-renormalizable by power counting.

Reasons to focus on $N = 8$ supegravity:

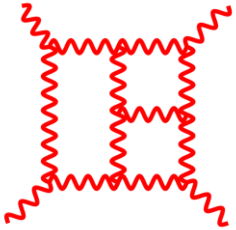
- **With more susy expect better UV properties.**
- **High symmetry implies technical simplicity.**

Finiteness of *N* = 8 Supergravity?

If $N = 8$ supergravity is finite it would imply a new symmetry or non-trivial dynamical mechanism. No known symmetry can render a $D = 4$ gravity theory finite.
The discovery of such a mechanism would have a fundamental impact on our understanding of gravity.

Note: Perturbative finiteness is not the only issue for consistent gravity: Nonperturbative completions? High energy behavior of theory? Realistic models?

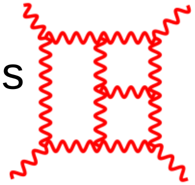
Consensus opinion for the late 1970's and early 1980's: All supergravities would diverge by three loops and therefore not viable as fundamental theories.



Feynman Diagrams for Gravity

SUPPOSE WE WANT TO CHECK IF CONSENSUS OPINION IS TRUE

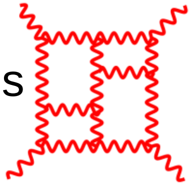
3 loops



$\sim 10^{20}$ TERMS

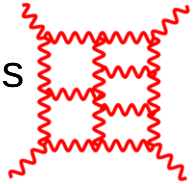
No surprise it has never been calculated via Feynman diagrams.

4 loops



$\sim 10^{26}$ TERMS

5 loops



$\sim 10^{31}$ TERMS

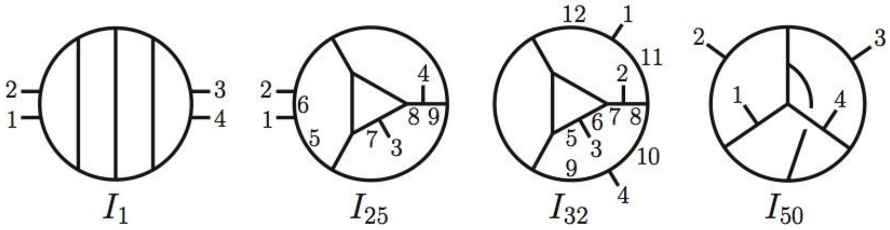
More terms than atoms in your brain!

- **Calculations to settle this seemed utterly hopeless!**
- **Seemed destined for dustbin of undecidable questions.**

3- and 4-Loop Amplitude Construction

ZB, Carrasco, Dixon, Johansson, Roiban

In 2007 and 2010 using unitarity method. 3, 4 loop amplitudes constructed in N = 8 supergravity



3 loops: UV finite for $D < 6$
4 loops: UV finite for $D < 11/2$
These are very finite in $D = 4$

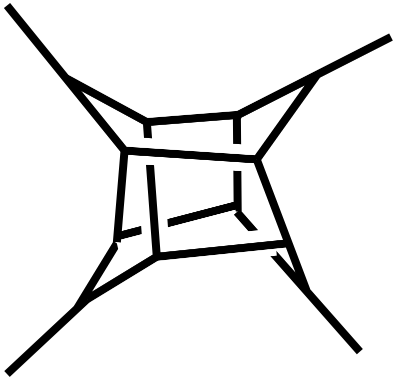
4 loops originally took more than a year.
Today with the double copy we can reproduce it in a few days!

Current Status

More recent papers argue that trouble starts at 5 loops and by 7 loops we have valid UV divergence in *D* = 4 under known symmetries.

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Bjornsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, , Freedman, Kiermaier, Morales, Stieberger

On the other hand duality between color and kinematics implies new constraints on the amplitudes.

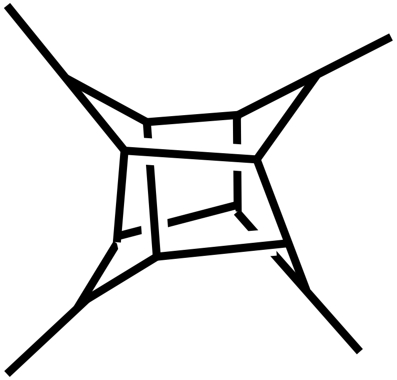


To settle the question it's time to to calculate again.

"Shut up and calculate!"

N = 8 Sugra 5 Loop Calculation

ZB, Carrasco, Dixon, Johannson, Roiban



A reasonable person would bet on divergences.
But is a reasonable person right?

Place your bets:

- **At 5 loops in *D* = 24/5 does *N* = 8 supergravity diverge?**
- **At 7 loops in *D* = 4 does *N* = 8 supergravity diverge?**

5 loops

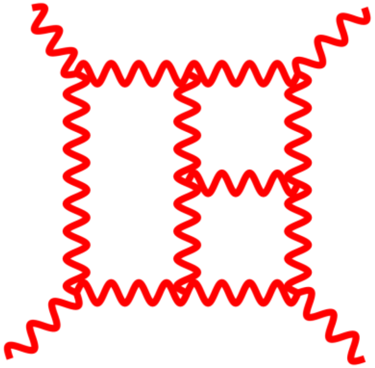
Kelly Stelle:
English wine
"It will diverge"

Zvi Bern:
California wine
"It won't diverge"

N = 4 Supergravity in *D* = 4

$N = 4$ sugra at 3 loops ideal $D = 4$ case to study.

Cremmer, Scherk, Ferrara (1978)



Consensus has $N = 4$ supergravity has a valid UV divergences in $D = 4$ under all known symmetries.
Similar to $N = 8$ supergravity at 7 loops.

Bossard, Howe, Stelle; Bossard, Howe, Stelle, Vanhove

Is the consensus opinion true?

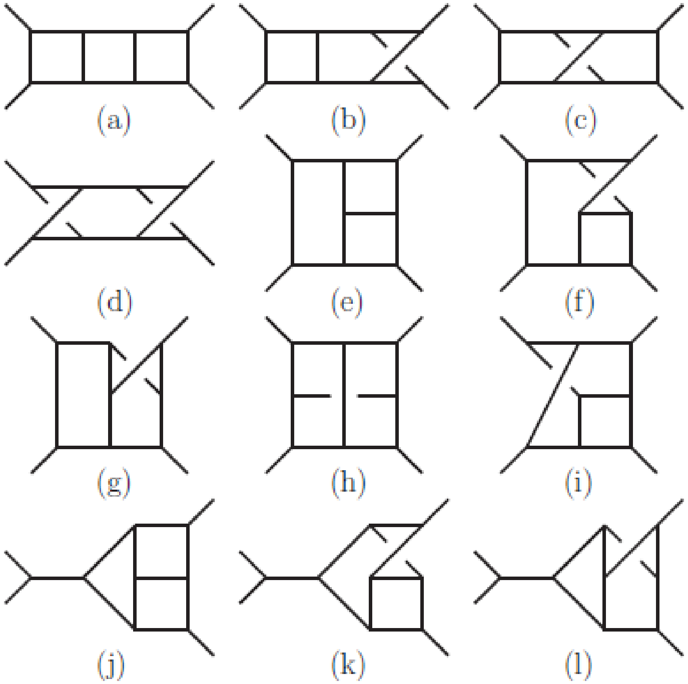
BCJ representation exists for $N = 4$ sYM 3-loop 4-pt Amplitude.

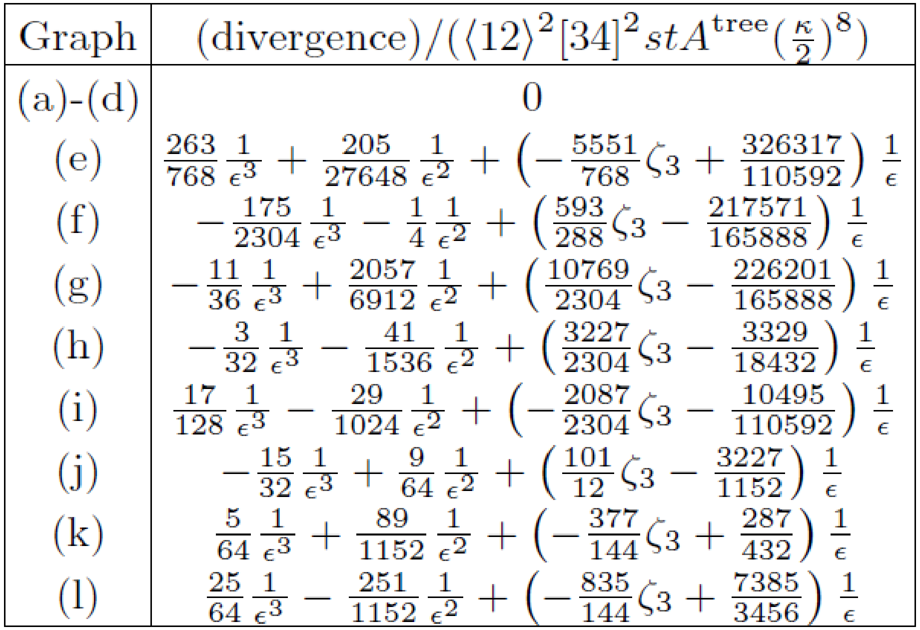
ZB, Carrasco, Johansson (2010)

The *N* = 4 Supergravity UV Cancellation

ZB, Davies, Dennen, Huang (2012)

N = 4 sugra = (*N* = 4 super-Yang-Mills)x(ordinary Yang-Mills)





Graph	$(\text{divergence})/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3}+\frac{205}{27648}\frac{1}{\epsilon^2}+\left(-\frac{5551}{768}\zeta_3+\frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3}-\frac{1}{4}\frac{1}{\epsilon^2}+\left(\frac{593}{288}\zeta_3-\frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3}+\frac{2057}{6912}\frac{1}{\epsilon^2}+\left(\frac{10769}{2304}\zeta_3-\frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3}-\frac{41}{1536}\frac{1}{\epsilon^2}+\left(\frac{3227}{2304}\zeta_3-\frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3}-\frac{29}{1024}\frac{1}{\epsilon^2}+\left(-\frac{2087}{2304}\zeta_3-\frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3}+\frac{9}{64}\frac{1}{\epsilon^2}+\left(\frac{101}{12}\zeta_3-\frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3}+\frac{89}{1152}\frac{1}{\epsilon^2}+\left(-\frac{377}{144}\zeta_3+\frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3}-\frac{251}{1152}\frac{1}{\epsilon^2}+\left(-\frac{835}{144}\zeta_3+\frac{7385}{3456}\right)\frac{1}{\epsilon}$

$D = 4 - 2\epsilon$

All divergences cancel completely!

UV finite contrary to expectations based on standard symmetries.

Explanations?

Key Question:

Is there an ordinary symmetry explanation for this UV finiteness? Or is something extraordinary happening?

Standard symmetry arguments have failed to explain three-loop finiteness of N = 4 supergravity.

Bossard, Howe, Stelle (2013); ZB, Davies, Dennen (2013)

This is very puzzling to our supergravity friends!

What might the magic be?

- **In a relatively simple case (half-maximal supergravity at two loops in D = 5) source of the magic is same magic found by 't Hooft and Veltman 40 years ago in their proof of renormalizability of gauge theory.** ZB, Davies, Dennen (2013)
- **Other attempts based on either string theory or appealing to a hidden superconformal symmetry.** Tourkine and Vanhove (2012) Ferrara, Kallosh, van Proeyen (2012)

New calculations underway at 4 and 5 loops will clarify this.

ZB, Davies, Dennen ; ZB, Carrasco, Johansson, Roiban

Summary

- Remarkable progress in understanding and computing scattering amplitudes: "Impossible calculations" are now commonplace.
- Example: *W*+4, 5 jet production at LHC evaluated in NLO QCD.
- Example: Duality between color and kinematics provides a powerful way to explore the UV properties of (super)gravity theories.
- Example: Supergravity theories have a better UV behavior than apparent from standard symmetry considerations. Constructing a perturbative point-like UV finite of (super)gravity is still an open challenge.

Given the remarkable advances of the past years in understanding and computing scattering amplitudes, we can expect to see many more new exciting developments in the coming years.

Further Reading

If you wish to read more see following non-technical descriptions.

Hermann Nicolai, *PRL Physics Viewpoint*, "Vanquishing Infinity"
http://physics.aps.org/articles/v2/70

Z. Bern, L. Dixon, D. Kosower,
May 2012 *Scientific American*, Cover Story
"Loops, Trees and the Search for New Physics"

Anthony Zee, *Quantum Field Theory in a Nutshell*,
2nd Edition is first textbook to contain modern formulation of scattering and commentary on new developments. 4 new chapters.