B meson decays
$B_s \to J/\psi + \eta^{(\prime)}$ $\quad B \to K^{(*)} + 2\nu$
in the framework of covariant quark model

A. Liptaj IP SAS Bratislava, Slovakia
M. A. Ivanov LTF JINR Dubna, Russia
S. Dubnička IP SAS Bratislava, Slovakia
A. Z. Dubničková FMFI UK Bratislava, Slovakia

HEP EPS 2013 conference
18.07.2013 – 24.07.2013
Stockholm, Sweden

Outline

- Motivation
  - Hadron physics: success for experimentalists, challenge for theorists
  - Covariant quark model – an appropriate theoretical framework
- Model in a nutshell
  - Lagrangian (density) and parameters
  - Compositeness condition, calculation methods and IR cut-off
  - Form factors, decay constants and factorization
- Outcomes
  - Decay $B_s \to J/\psi + \eta^{(\prime)}$: form factors, decay widths and comparison with experiment
  - Decay $B \to K^{(*)} + 2\nu$: form factors, decay widths and comparison with experiment
- Summary, conclusions and outlook

Motivation & Introduction

- Hadron physics
  - Past & ongoing experiments: big quantity of high-precision data ⇒ Challenge!
  - However:
    - Quark bound states: low momenta ⇒ pQCD looses its applicability
    - Lattice QCD results promising but still not "good enough"
  - Models needed by theory & experiment (hadrons in detectors)

- Experimental situation
  - Heavy quark factories: new experimental results, increased statistics & precision
  - Studied decays $B_s \to J/\psi + \eta^{(\prime)}$ & $B \to K^{(*)} + 2\nu$ measured by BELLE[1], LHCb[2] & BABAR[3]

[1] Phys.Rev.Lett. 108, 181808 (2012) [2] Nucl.Phys.B 867, 547 (2013)

[3] Phys. Rev. D 87, 112005 (2013) [1] arXiv:1303.3719

- Covariant Quark Model
  - Lagrangian-based formulation ⇒ full Lorentz invariance
  - Straightforward inclusion of higher-number quark states (baryons, tetraquarks,...)
  - One free parameter per hadron (for large number of hadrons)
  - ...wide application spectra, standard QFT techniques and convincing results

Lagrangian and parameters

- Lagrangian (density)
  - Meson - quark interaction $\mathcal{L}_{\mathbf{int}} = g_M \cdot M(x) \cdot J_M(x)$
  - With

$$J_M(x) = \int dx_1 \int dx_2 \, F_M(x; x_1, x_2) \cdot \bar{q}_1^a(x_1) \, \Gamma_M \, q_2^a(x_2)$$

$$F_M(x, x_1, x_2) = \delta^{(4)}\Big(x - w_1 x_1 - w_2 x_2\Big) \, \Phi_M\Big[(x_1 - x_2)^2\Big]$$

$$w_i = \frac{m_i}{m_1 + m_2}, \quad i = 1, 2 \qquad \tilde{\Phi}_H\left(-p^2\right) = \exp\left(\frac{p^2}{\Lambda_M^2}\right)$$

- Parameters
  - Constituent quark masses [4], hadron size parameters [N], universal cut-off [1]

    $\Rightarrow$ N+5 in total
  - Numerical values: fitting basic observables [lepto. decay const., EM decay widths]

    $m_{u,d}$=0.235, m_s=0.424, m_c=2.16, m_b=5.09, $\lambda_{\text{cut-off}}$=0.181, Λ_π = 0.87, Λ_K=... in GeV
  - Couplings g_M eliminated as free parameters [compositeness condition]

Compositeness condition & calculations methods

- Compositeness condition
  - $\mathcal{L}_{int}$ → quarks & hadrons elementary; Nature → hadrons compound of quarks
  - Compositeness condition: renormalization constant $Z_H^{1/2}$ can be interpreted as the matrix element between a physical state and the corresponding bare state.
  - Z_H=0 ⇒ Physical state does not contain the bare state ⇒ Is properly described as a bound state. A. Salam, Nuovo Cim. 25, 224 (1962); S. Weinberg, Phys. Rev. 130, 776 (1963);

$$Z_H = 1 - \frac{3g_H^2}{4\pi^2}\tilde{\Pi}_H'\left(m_H^2\right) = 0$$

- Calculation methods
  - Schwinger quark propagator representation & Feynman diagram general form

$$\tilde{S}_q(k) = \left(m + \hat{k}\right)\int_0^\infty d\alpha\ e^{\left[-\alpha\left(m^2 - k^2\right)\right]}$$

$$\Pi = \int\limits_0^\infty d^n\alpha \int [d^4k]^\ell\,\Phi\,\exp[-\sum_{i=1}^n \alpha_i(m_i^2 - (K_i + P_i)^2)]$$

Φ: numerator product of propagators and vertex functions
P_i: linear combination of external momenta
K_i: linear combination of loop momenta

Calculation methods and confinement

- Operator identities
  - An intelligent way of loop momenta integration and trace evaluation

$$\int d^4k \, P(k) e^{2kr} = \int d^4k \, P\left(\frac{1}{2}\frac{\partial}{\partial r}\right) e^{2kr} = P\left(\frac{1}{2}\frac{\partial}{\partial r}\right) \int d^4k \, e^{2kr}$$

$$\int_0^\infty d^n\alpha \, P\left(\frac{1}{2}\frac{\partial}{\partial r}\right) e^{-\frac{r^2}{a}} = \int_0^\infty d^n\alpha \, e^{-\frac{r^2}{a}} P\left(\frac{1}{2}\frac{\partial}{\partial r} - \frac{r}{a}\right), \; r = r(\alpha_i), \; a = a(\Lambda_H, \alpha_i)$$

- Infrared cut-off
  - Cut-off introduced in the integration over Schwinger parameters
    $\Rightarrow \Pi$ becomes a smooth function, thresholds in the quark loop diagrams and corresponding branch points removed.

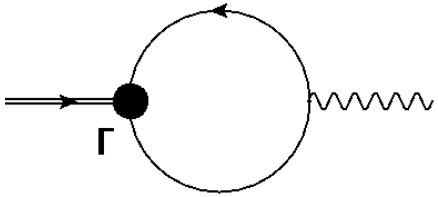
$$\Pi = \int_0^\infty d^n\alpha \, F(\alpha_1, \cdots, \alpha_n) \longrightarrow \Pi = \int_0^{1/\lambda^2} dt \, t^{n-1} \int_0^1 d^n\alpha \, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big) F(t\alpha_1, \ldots, t\alpha_n)$$

$$1 = \int_0^\infty dt \, \delta(t - \sum_{i=1}^n \alpha_i) \Rightarrow \Pi = \int_0^\infty dt \, t^{n-1} \int_0^1 d^n\alpha \, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big) F(t\alpha_1, \ldots, t\alpha_n)$$

  - Integration done numerically (and cross-checked independently Fortran vs. Java)

Decay constants and form factors

- DC & FF: most straightforward to calculate
  - Meson decay constants (pseudoscalar example)



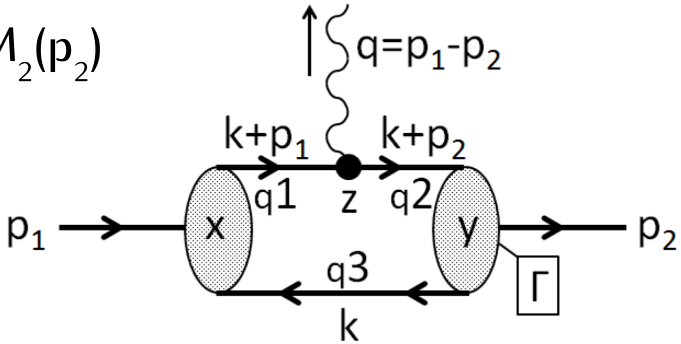
$$f_P p^\mu = N_c g_P \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}_P(-k^2) \mathrm{Tr}\left[\gamma^\mu S_1\left(k + w_1 p\right)\gamma^5 S_2\left(k - w_2 p\right)\right]$$

$$f_P\left(\alpha_1, \alpha_2\right) = \frac{3g_P}{4\pi^2} \iint_0^\infty d\alpha_1 d\alpha_2 \left[\frac{m_1 - m_2}{a^3}\left(\omega_1\alpha_1 - \omega_2\alpha_2\right) + \frac{m_1\omega_2 + m_2\omega_1}{a^2}\right]$$

$$\times \exp\left[-\alpha_1 m_1^2 - \alpha_2 m_2^2 + \left(\alpha_1\omega_1^2 + \alpha_2\omega_2^2\right)p^2 - a^{-1}\left(\alpha_1\omega_1 - \alpha_2\omega_2\right)^2 p^2\right]$$

$$f_P(z, x) = \frac{1}{\lambda^4}\int_0^1 dz\, z \int_0^1 dx\, f\left[\frac{zx}{\lambda^2}, \frac{z(1-x)}{\lambda^2}\right] \quad \Leftrightarrow \quad f_\pi = 0.1285\ \mathrm{GeV}$$

  - Meson form factors
    - Model predicts transition form factors $M_1(p_1) \to M_2(p_2)$
    - Effective theory (Wilson coefficients) used to describe quark transition at **z** (weak decays)
    - Factorization of the amplitude to form form factor and the rest



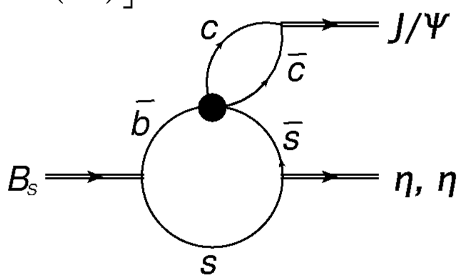
Decay $B_s \to J/\psi + \eta^{(\prime)}$

- Quark content & mixing

$$\eta: \ \frac{1}{\sqrt{2}} \sin\delta(u\bar{u} + d\bar{d}) - \cos\delta(s\bar{s}) \approx \frac{1}{\sqrt{6}} \left[(u\bar{u} + d\bar{d}) - 2(s\bar{s})\right]$$

$$B_S^0: \ s\bar{b} \qquad \eta': \ \frac{1}{\sqrt{2}} \cos\delta(u\bar{u} + d\bar{d}) + \sin\delta(s\bar{s}) \approx \frac{1}{\sqrt{3}} \left[(u\bar{u} + d\bar{d}) + (s\bar{s})\right]$$

$$\delta = \theta_I + \theta_P = \arctan\frac{1}{\sqrt{2}} + \theta_P \qquad \theta_P \approx 15^\circ, 20^\circ$$



- Lagrangian examples: $\mathcal{L}_\eta$ and $\mathcal{L}_{eff.}$

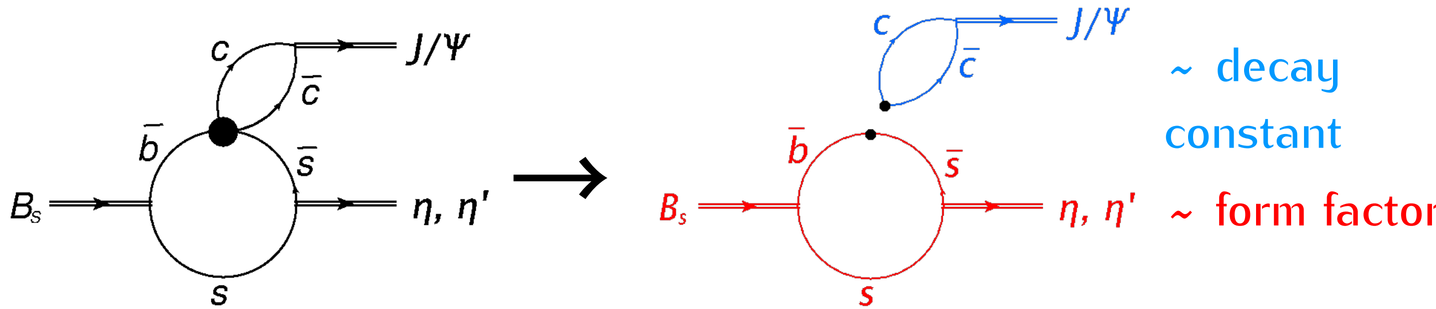
$$\mathcal{L}_\eta(x) = g_\eta \eta(x) \iint dx_1 dx_2 \delta\left(x - \frac{1}{2}x_1 - \frac{1}{2}x_2\right) \phi_\eta\left[(x_1 - x_2)^2\right]$$
$$\times \left\{ \frac{1}{\sqrt{2}} \cos(\delta) \left[\bar{u}(x_1) i\gamma^5 u(x_2) + \bar{d}(x_1) i\gamma^5 d(x_2)\right] - \sin(\delta) \left[\bar{s}(x_1) i\gamma^5 s(x_2)\right] \right\}$$

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_i C_i Q_i \qquad Q_1 = (\bar{c}_{a_1} b_{a_2})_{V-A} (\bar{s}_{a_2} c_{a_1})_{V-A} \qquad Q_2 = \ldots$$

$$(\bar{\psi}\psi)_{V-A} = \bar{\psi} O^\mu \psi, \ O^\mu = \gamma^\mu (1 - \gamma^5) \qquad (\bar{\psi}\psi)_{V+A} = \bar{\psi} O_+^\mu \psi, \ O_+^\mu = \gamma^\mu (1 + \gamma^5)$$

Decay $B_s \to J/\psi + \eta^{(\prime)}$

- **Factorization:** Amplitude ~ form factor x coupling constant



- **Data Fitting**
  - Prediction, not parameter determination ⇒ model is over-constrained
  - Chosen processes (already studied within CQM): $\eta \to \gamma\gamma$ $\quad \eta' \to \gamma\gamma$ $\quad \rho \to \eta\gamma$ $\quad \varphi \to \eta\gamma$ $\quad \varphi \to \eta'\gamma$ $\quad \mathrm{B_d} \to \mathrm{J}/\psi\ \eta$ $\quad \mathrm{B_d} \to \mathrm{J}/\psi\ \eta'$ $\quad \omega \to \eta\gamma$ $\quad \eta' \to \omega\gamma$

- **Results**
  - Branching fractions x 10^4

$$\mathcal{B}_{\mathrm{CQM}}\,(\mathrm{J}/\psi\ \eta) = 4.67$$
$$\mathcal{B}_{\mathrm{Belle}}\,(\mathrm{J}/\psi\ \eta) = 5.10 \pm 1.12$$

$$\mathcal{B}_{\mathrm{CQM}}\,(\mathrm{J}/\psi\ \eta') = 4.04$$
$$\mathcal{B}_{\mathrm{Belle}}\,(\mathrm{J}/\psi\ \eta') = 3.71 \pm 0.95$$

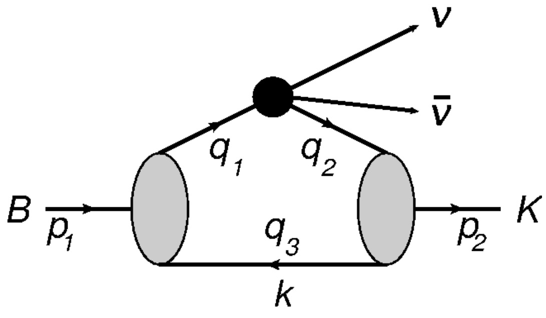
  - Relative branching fractions
    (**q**: momentum in the decay rest frame)

$$R = \frac{\Gamma(J/\psi + \eta')}{\Gamma(J/\psi + \eta)} = \begin{cases} 0.73 \pm 0.14 \pm 0.02 & \text{Belle} \\ 0.90 \pm 0.09^{+0.06}_{-0.02} & \text{LHCb} \end{cases}$$

$$R^{\mathrm{theor}} = \underbrace{\frac{|\mathbf{q}_{\eta'}|^3}{|\mathbf{q}_{\eta}|^3}\tan^2\delta}_{\approx 1.04} \times \underbrace{\left(\frac{F_+^{B_s\eta'}}{F_+^{B_s\eta}}\right)^2}_{\approx 0.83} \approx 0.86.$$

Decay B→K^(*)+2ν

- Amplitude calculation
  - Effective theory (Wilson coefficients)
  - Factorization
  - Different form factors
    - K (scalar particle)



$$\left\langle P'_{[\bar{q}_3,q_2]}(p_2)\right|\bar{q}_2 O^{\mu} q_1 \left|P'_{[\bar{q}_3,q_1]}(p_1)\right\rangle = F_+(q^2)P^{\mu} + F_-(q^2)q^{\mu}$$

$$\left\langle P'_{[\bar{q}_3,q_2]}(p_2)\right|\bar{q}_2(\sigma^{\mu\nu}q_{\nu})q_1\left|P'_{[\bar{q}_3,q_1]}(p_1)\right\rangle = \frac{i}{m_1+m_2}\left(q^2P^{\mu} - q\cdot P q^{\mu}\right)F_T(q^2)$$

- K* (vector particle)

$$\left\langle V_{[\bar{q}_3,q_2]}(p_2,\epsilon_2)\right|\bar{q}_2 O^{\mu} q_1\left|P_{[\bar{q}_3,q_1]}(p_1)\right\rangle = \frac{\epsilon_{\nu}^{\dagger}}{m_1+m_2}\Big[-g^{\mu\nu}P\cdot q A_0(q^2) + P^{\mu}P^{\nu}A_+(q^2) + q^{\mu}P^{\nu}A_-(q^2) + i\varepsilon^{\mu\nu\alpha\beta}P_{\alpha}q_{\beta}V(q^2)\Big]$$

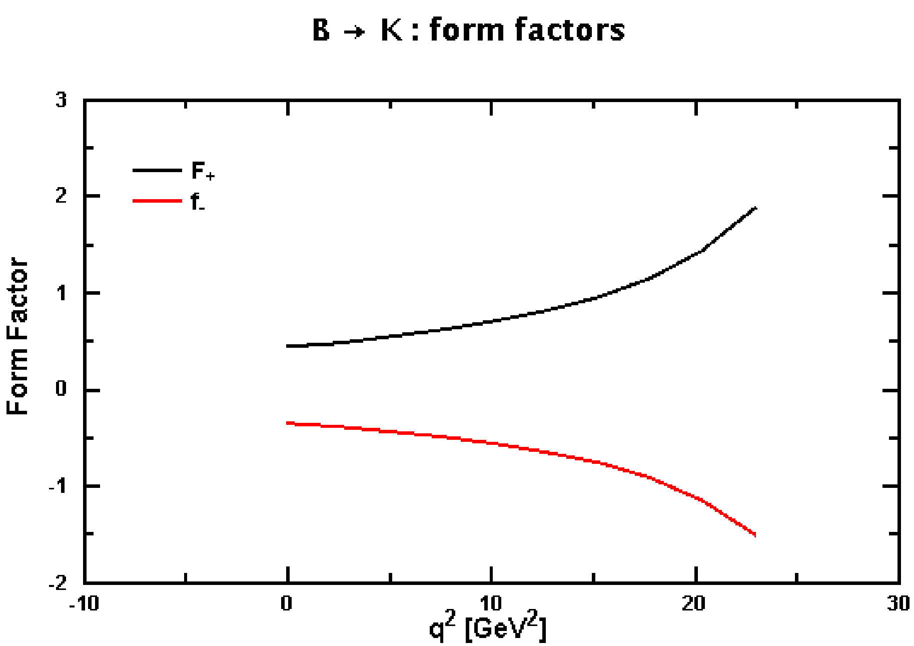
$$\left\langle V_{[\bar{q}_3,q_2]}(p_2,\epsilon_2)\right|\bar{q}_2\left[\sigma^{\mu\nu}q_{\nu}(1+\gamma^5)\right]q_1\left|P_{[\bar{q}_3,q_1]}(p_1)\right\rangle = \epsilon_{\nu}^{\dagger}\Big[-\left(g^{\mu\nu}-\frac{q_{\mu}q_{\nu}}{q^2}\right)P\cdot q\, a_0(q^2) + \left(P^{\mu}P^{\nu} - q^{\mu}P^{\nu}\frac{p\cdot q}{q^2}\right)a_+(q^2) + i\varepsilon^{\mu\nu\alpha\beta}P_{\alpha}q_{\beta}\, g(q^2)\Big]$$

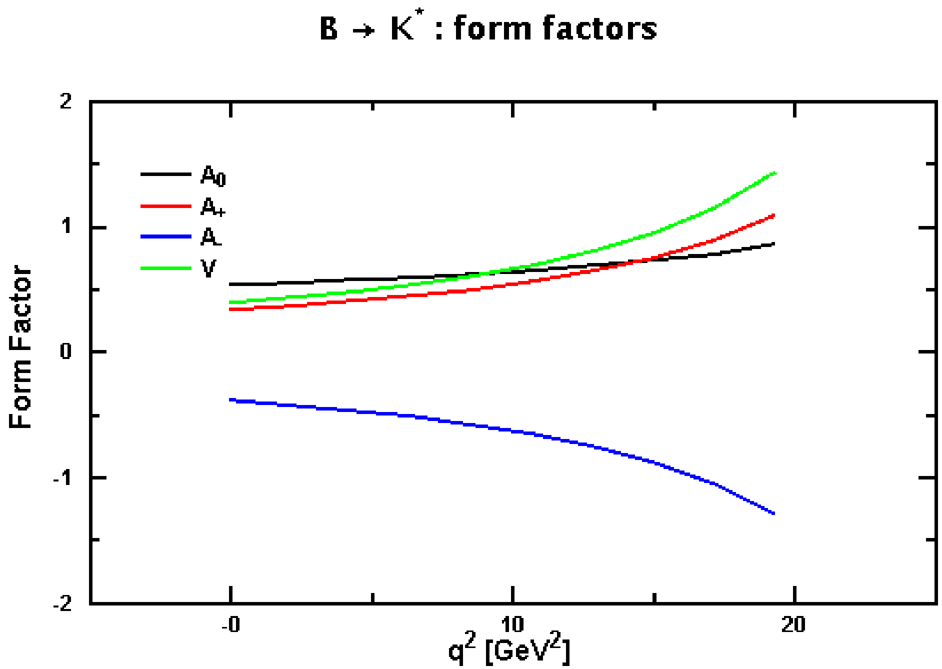
Decay $B \to K^{(*)} + 2\nu$

- Size parameter Λ_{K^*} determination
  - All other parameters previously settled
  - Λ_{K^*} determined by fitting K^* decay constant from tau decay

$$\tau \to K^* + \nu_\tau \implies f_{K^*} = 0.217\,\text{GeV}$$

- Form factors



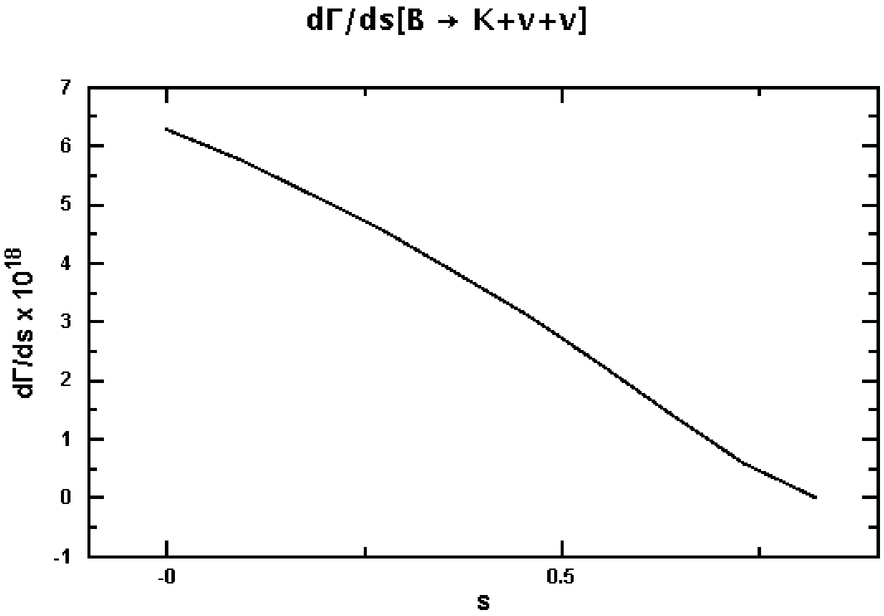


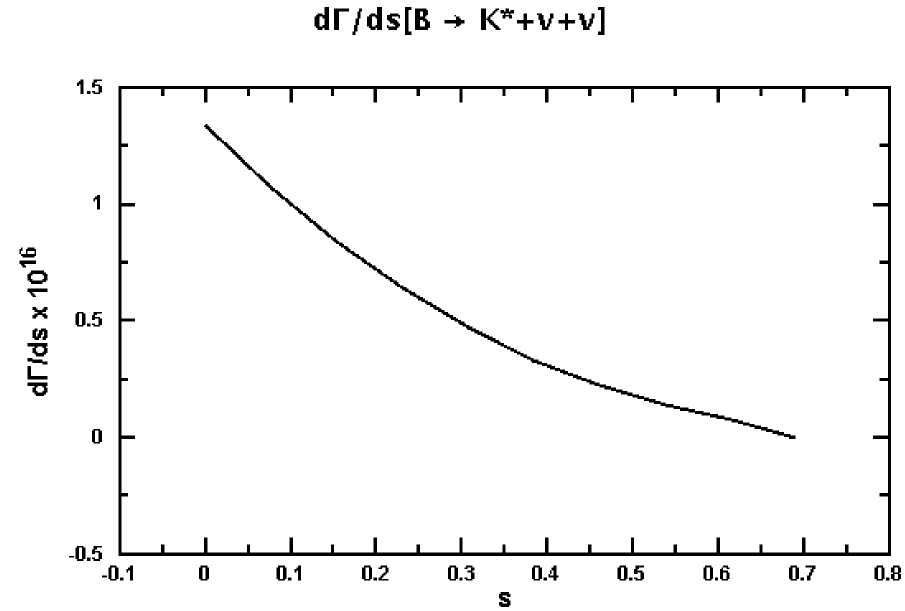
$B \to K^{(*)} + 2\nu$

- $d\Gamma/ds$ distributions [Geng, Hwang, Liu, Phys. Rev. D. 65 094037]

$$\frac{d\Gamma\left[P \to P'\nu\bar{\nu}\right]}{ds} = \frac{G_F^2 m_P^5 |\lambda_t|^2 \alpha_{em}^2 |D(x_t)|^2}{2^8 \pi^5 \sin^4\theta_W} |F_+|^2 \Phi_{P'}^{\frac{3}{2}}, \qquad s = \frac{q^2}{m_P^2}$$

$$\frac{d\Gamma\left[P \to V\nu\bar{\nu}\right]}{ds} = \frac{3G_F^2 m_P^5 |\lambda_t|^2 \alpha_{em}^2 |D(x_t)|^2}{2^8 \pi^5 \sin^4\theta_W} \Phi_V^{\frac{1}{2}} \left[s\alpha_1\left(A_0, V\right) + \frac{\Phi_V}{3}\beta_1\left(A_0, A_+, V\right) \right]$$





$B \to K^{(*)} + 2\nu$

- Results
  - Numerical integration ⇒ partial decay widths

$$\frac{\Gamma\left[B \to K\nu\bar{\nu}\right]}{\Gamma_{tot}} \begin{cases} < 5.5 \times 10^{-5}\ \textit{Belle} \\ = 0.63 \times 10^{-5}\ \mathrm{CQM} \\ < 3.2 \times 10^{-5}\ \textit{BaBar} \end{cases} \qquad \frac{\Gamma\left[B \to K^{*}\nu\bar{\nu}\right]}{\Gamma_{tot}} \begin{cases} < 4.0 \times 10^{-5}\ \textit{Belle} \\ = 7.9 \times 10^{-5}\ \mathrm{CQM} \\ < 7.9 \times 10^{-5}\ \textit{BaBar} \end{cases}$$

  - Still preliminary, we want to cross-check by an independent calculation
  - Results on $B \to K^{(*)}+2ll$ are about to come!

Summary & outlook

- Covariant Quark Model
  - Clear Lagrangian–based formulation with standard computation techniques
  - Heavily over-constrained with unambiguous predictions
  - Highly actual with respect to ongoing experiments

- Results
  - Overall very nice agreement with experiment
  - Deviations (one here $B \to K^* + 2\nu$) still within one order of magnitude: wait for more data with error bars and cross-check our result

- Outlook

  AMany new result coming (recently measured or in the near future): wide application field for the model (predictions and testing)

  ALHCb, Belle: $B_s \to J/\Psi\, f_0(980)$

  BCDF, LHCb, Belle: $B_s^0 \to \pi^+\pi^-,\ B^0 \to K^+K^-$

  CLHCb: $B_{(s)}^0 \to \mu^+\mu^-,\ B_s^0 \to J/\Psi K_S^0$

  Dand many others....

Conclusion & further reading

- Conclusion
  - CQM – an appropriate theoretical framework to be used in various contexts (decay widths calculations, baryons and tetraquarks, light and heavy hadrons)

- More details about the covariant quark model
  - "Relativistic constituent quark model with infrared confinement", M. A. Ivanov *et al.*, Phys.Rev. D81 (2010) 034010
  - "Form factors for semileptonic, nonleptonic and rare meson decays", M. A. Ivanov *et al.*, Phys.Rev. D85 (2012) 034004

Thank you for your attention!