Spectrum of the open QCD flux tube in d=2+1 and its effective string description

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20.07.2013

Contents

1. Introduction

Confinement, flux tubes and strings

Confinement and flux tubes

Confinement in QCD:

Only color singuletts can appear as free particles in nature!

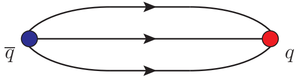
Possibility to show that QCD is confining: [see: Greensite, PPNP 51, 1 (2003)]

Show that for $m_q \to \infty$ and $R \to \infty$ the $\bar{q}q$ potential $V(R) \to \sigma R$!

R: $\bar{q}q$ separation
σ: String tension

Possible mechanism to explain this:

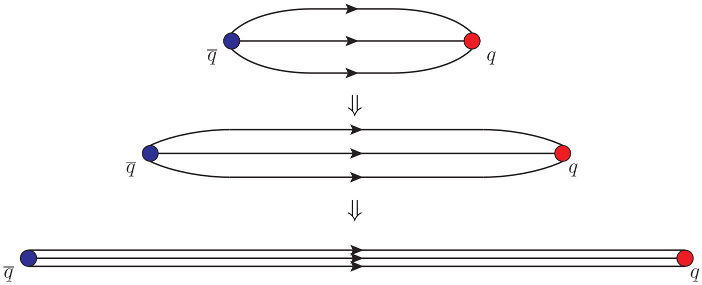
Formation of a flux tube between quark and antiquark!



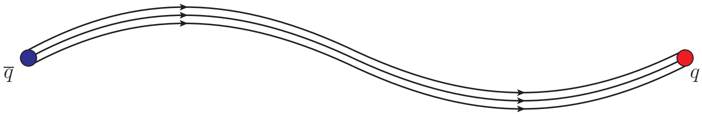
String picture of confinement

Pulling the quarks appart:

$\Rightarrow$ The flux tube stretches and effectively looks like a string.



In this regime: Excitation spectrum dominated by string excitations!



String picture of confinement

⇒ Formulation of effective string theories for the flux tube.

[Nambu, PLB 80, 372 (1979); Lüscher, Symanzik, Weisz, NPB 173, 365 (1980), Polyakov, NPB 164, 171 (1980)]

Universal results:

$$V(R) = \sigma\, R - \frac{\pi\,(d-2)}{24}\,\frac{1}{R} + \mathcal{O}\left(\frac{1}{R^2}\right) \quad ; \quad w^2_{\mathrm{mid}} = \frac{(d-2)}{2\pi\,\sigma}\,\log\left(\frac{R}{R_0}\right)$$

Free bosonic string: Nambu-Goto action

Spectrum: [Arvis, PLB 127, 106 (1983)]

$$E_n(R) = \sigma\, R\,\sqrt{1 + \frac{2\pi}{\sigma\, R^2}\,\left(n - \frac{(d-2)}{24}\right)}$$

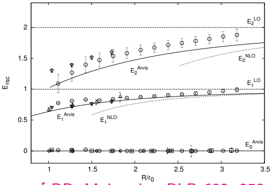
But: Quantisation only consistent for $d = 26$!

(Weyl anomaly [Goddard *et al*, NPB 56, 109 (1973)])

String picture of confinement

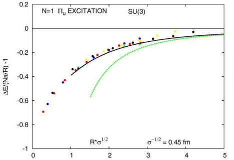
Good agreement of numerical data with NG string theory also for small R!

3d, $SU(2)$ gauge theory:



[BB, Majumdar, PLB 682, 253 (2009)]

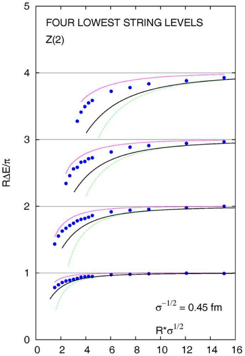
4d, $SU(3)$ gauge theory:



[Kuti, PoS LAT2005, 001]

[Data: Juge *et al*, PRL 90, 161601 (2003)]

3d, $Z(2)$ gauge theory:



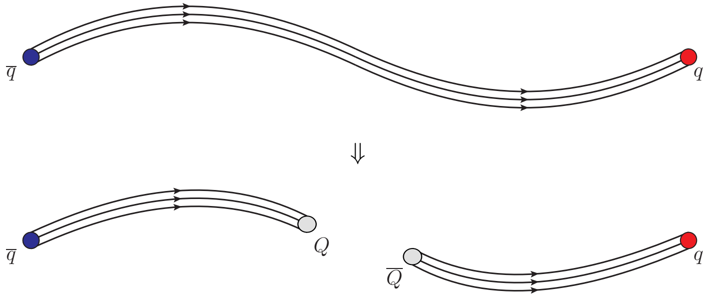
[Kuti, PoS LAT2005, 001]

[Data: Juge *et al*, hep-lat/0401032]

Quarks and string breaking

So far: Quarks infinitely heavy!
For dynamical quarks:

String breaking



For a study in full QCD see [Bali *et al*, PRD 71, 114513 (2005)] .

2. Effective string theories for the flux tube

Effective string theories

Historically: Two main approaches to construct the effective action order for order in derivatives of the string-field coordinates (derivative $\sim 1/R$):

Polchinski-Strominger string theory

Conformal worldsheet field theory consisting of all terms that avoid the Weyl anomaly in d dimensions.

[Polchinski, Strominger, PRL 67, 1681 (1991)]

Shown to be iso-spectral to NG string theory up to $\mathcal{O}(1/R^3)$!

[Series of papers: Drummond, (2004-2006); Hari Dass *et al*, (2005-2007)]

Lüscher-Weisz string theory

Worldsheet field theory in the $(d-2)$ transverse degrees of freedom.

[Lüscher, Weisz, JHEP 0207, 049 (2002); JHEP 0407, 014 (2004)]

Action constrained by $SO(1|d-1)$ Lorentz symmetry

[Lüscher, Weisz, JHEP 0407, 014 (2004); Meyer, JHEP 0605, 066 (2006)]

Also iso-spectral to NG to $\mathcal{O}(1/R^3)$

Effective string theories

Recently: Both string theories might be representation of an underlying effective string theory.
(conformal ⇔ static/unitary gauge) [Aharony *et al*, JHEP 1305, 118 (2013)]

Conjecture: Fundamental string theory is the one comming from a generalisation of the AdS/CFT correspondence for gauge theories.

⇒ Fundamental string moving in weakly curved background.

Spectrum of the effective string theory

- Closed string spectrum:

  [Aharony *et al*, JHEP 0906, 012 (2009); JHEP 1012, 058 (2010); JHEP 1101, 065 (2011)]

  - Iso-spectral to NG to $\mathcal{O}(1/R^5)$.
  - First correction: $\mathcal{O}(1/R^7)$ for $d = 2+1$
    $\mathcal{O}(1/R^5)$ for $d > 2+1$

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Spectrum of the effective string theory

- Open string spectrum:

  [Aharony *et al*, JHEP 0906, 012 (2009); JHEP 1012, 058 (2010); JHEP 1101, 065 (2011)]

  - Iso-spectral to NG to $\mathcal{O}(1/R^3)$.
  - First correction: Boundary term at $\mathcal{O}(1/R^4)$.
    Free coefficient: b_2
  - The next corrections are at $\mathcal{O}(1/R^5)$ for $n > 0$ and $d > 2+1$.
  - For $d = 2+1$ all corrections can first ocur at $\mathcal{O}(1/R^6)$.

Effective string theories: Conclusions

- Advantageous to investigate corrections in 3d open string spectrum. ($\mathcal{O}(1/R^4)$ corrections and $\mathcal{O}(1/R^6)$ h.o. corrections)
- Correction to NG takes the form:

$$\delta V(R) = -\frac{b_2\ \pi^3}{60\ R^4} \qquad \delta E_1(R) = -\frac{4\ b_2\ \pi^3}{R^4}$$

- Details of the properties of the EST are interesting to constrain and check the properties of the fundamental string theory.
- Closed strings in $d = 2 + 1$ and $d = 3 + 1$ were also studied

  [Athenodorou, Bringoltz, Teper, PLB 656, 132 (2007); JHEP 1102, 030 (2011); JHEP 1105, 042 (2011)]
- and the $\bar{q}q$ potential at finite temperature.

[Caselle *et al*, JHEP 0206, 061 (2002); Bialas *et al*, NPB 836, 91 (2010); Caselle *et al*, JHEP 1104, 020 (2011)]

Similar string theories also appear for string-like objects in other systems:
E.g. interface free energy in $Z(2)$ gauge theories

[Caselle, Hasenbusch, Panero, (2003-2007) - series of papers]

⋮

3. Flux tube spectrum in pure gauge theory

Simulations of pure gauge theories

Monte-Carlo simulations of $SU(N)$ gauge theories:

- Discretise Yang-Mills theory on a spacetime lattice.
  $(A_\mu(x) \in su(N) \quad \to \quad U_\mu(x) \in SU(N))$
- Stochastic evaluation of the (regularised) path integral

$$\langle O \rangle = \frac{1}{Z} \int d[U]\; O(U)\; \exp\{-S_G(U)\}\ .$$

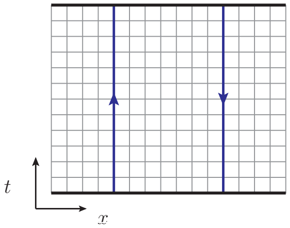
  ($S_G(U)$: e.g. Wilson's gauge action [Wilson, PRD 10, 2445 (1974)])
- Use: Markov chains with heatbath updates.

[Creutz, PRD 21, 2308 (1980); Kennedy, Pendleton, PLB 156, 393 (1985)]

[Cabibbo, Marinari, PLB 119, 387 (1982)]

Extracting the groundstate

Polyakov loop:



- Spectral representation:

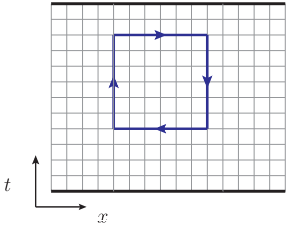
$$\left\langle P^\dagger(0)P(R)\right\rangle = \sum_{m=0}^{\infty} b_m \; \exp\{-E_m(R)\; T\}$$

- Excited states exponentially suppressed by temporal extent of the lattice.
  $\Rightarrow$ Good operator to extract the groundstate energy.
- From the potential one can also set the lattice scale.
  Here: Use the string tension σ.
  Can be extracted from the large R region via a fit to the form

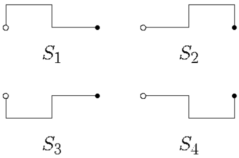
$$V(R) = \sigma\; R + \frac{\pi\;(d-2)}{24\; R}$$

Extracting excited states

Wilson loops:



& Spatial operators



- Spectral representation in channel n:

$$\langle W_n(R,t)\rangle = \sum_{m=0}^{\infty} c_m^n \, \exp\{-E_m^n(R)\, t\}$$

  $\Rightarrow$ Groundstate is an excited string state, E_n.

- Use a number of different operators:
  $\Rightarrow$ Possible to extract excited states in the channels.
- Excited states are usually NOT sufficiently suppressed.
  $\Rightarrow$ Need to take the $t \to \infty$ limit.
- Consequence:
  Extrapolated results lose accuracy and simulations become more expensive!

Error reduction for large loops

Extraction of the energy spectrum demands the measurement of large loops.

Problem: Wilson and Polyakov loop expectation values follow an area law

$$\langle O(r,t)\rangle \sim \exp\{-C\ r\ t\}$$

$\Rightarrow$ Exponential decay of signal-to-noise ratio!

Possibility to overcome this:

- Lüscher-Weisz algorithm.

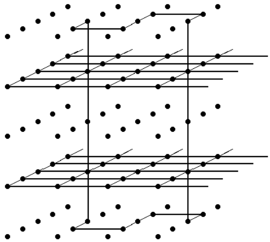
  [Lüscher, Weisz, JHEP 0109, 010 (2001)]

- LW for Wilson loops with spatial operators.

  [BB, Majumdar, PoS LAT2007, 027; PLB 682, 253 (2009)]

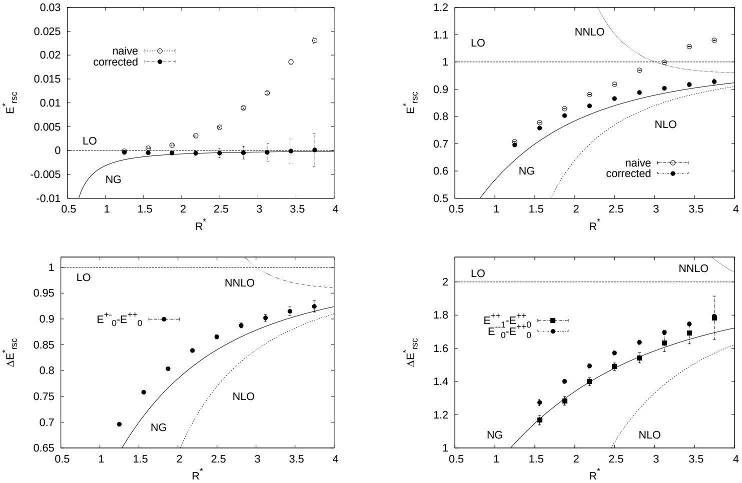
- Further error reduction and suppression of excited states:
  Combination with correlation matrix methods.

  [BB, JHEP 1102, 040 (2011)]



Results in 3d $SU(2)$ gauge theory

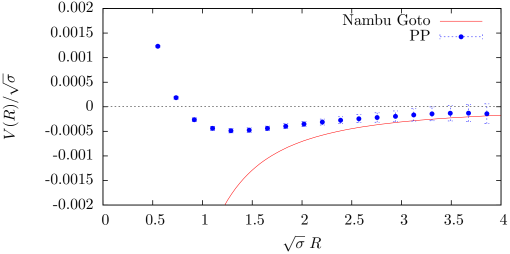
$\beta = 5.0$, $a \sim 0.125$ fm [BB, JHEP 1102, 040 (2011)]



Results in 3d $SU(3)$ gauge theory ($\beta = 20.0$)

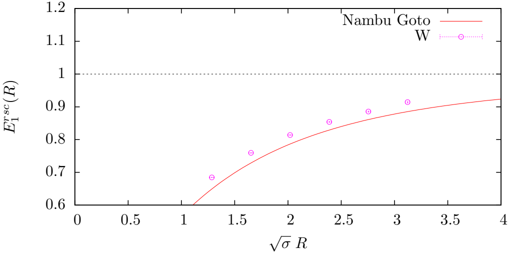
Groundstate:

- Lat: 64^3
- $a\sqrt{\sigma} \approx 0.184$ (or: $a \approx 0.08$ fm)
- meas: 2300



Excited states:
(Preliminary)

- Wilson loops: $t = 6,\ 10,\ 14,\ 18$
- Lat: 36^3, 40^3, 54×42^2, 56×42^2
- meas: 1700, 2800, 6100, 4300



4. Boundary corrections

Extraction of b_2

Two options to fix the boundary term:

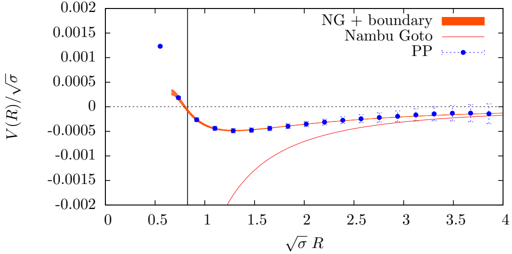
1. From high accuracy simulations of the groundstate:
   - Advantages:
     - Simulations are (relatively) cheap!
     - The expansion of the NG energy levels converges for all $R > 0$.
     - Results can be compared to results from excited states.
   - Problem: Boundary contribution 1/240 suppressed!
2. From groundstate and first excited state:
   - Advantages: Boundary contributions large.
   - Problems:
     - Excited states more difficult to extract.
     - Simulations more expensive.
     - Window (in R) for comparison to NG shrinks with n.
     - Results cannot be compared to high precision data (as for E_1).

Here: Use the first option! (in contrast to [BB, JHEP 1102, 040 (2011)])

Results for b_2: 3d $SU(3)$; $\beta = 20.0$

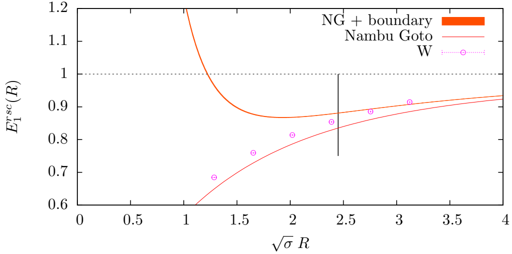
Fit to the groundstate:

- $b_2 = -0.0169(6)$
- Very good description of data down to $R\sqrt{\sigma} \sim 0.7$.
- No sign of higher order terms!?



Comparison to E_1:

- Qualitative agreement.
- But: Higher order terms can be large!



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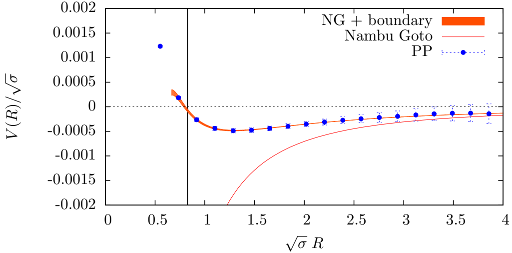

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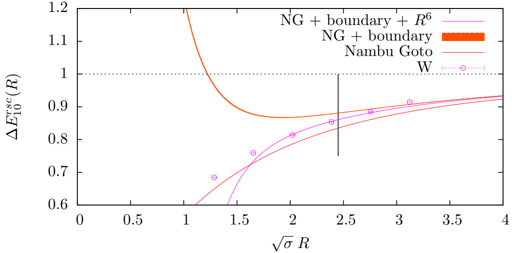


Check: 3d $SU(2)$; $\beta = 5.0$

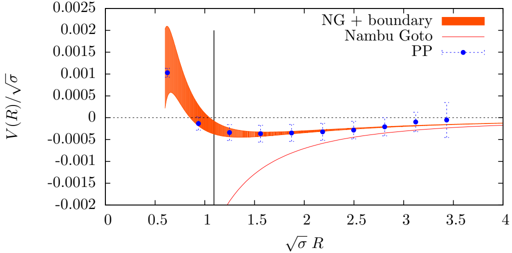
Same procedure in the $SU(2)$ case:

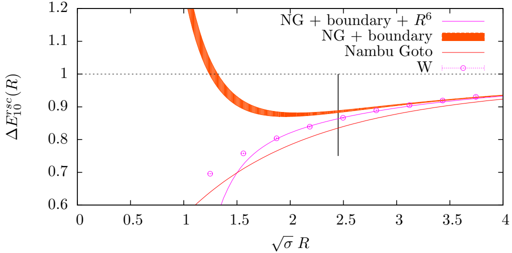
Use parameters from [BB, JHEP 1102, 040 (2011)] .

Much better quality of excited states data!

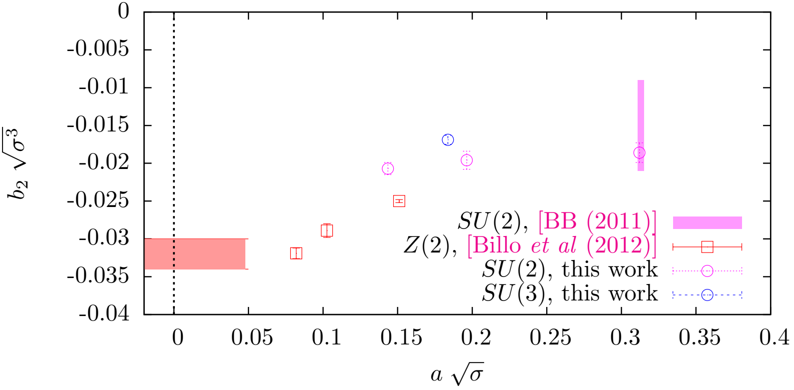
Results:

- $b_2 = -0.186(12)$
- Similar good description of groundstate data.
- Here: Also good agreement with E_1!





Universality of b_2



Conclusions

- Results in lattice simulations support the flux tube picture of confinement.
- Results are mostly consistent with NG string theory.
  New results with increasing precision:
  Deviations from NG!
- Associated effective string theory predicts deviations from NG.
- High accuracy constraints for EST from lattice.

  $$\Updownarrow$$

  Constraints on fundamental ST from string/gauge duality.
- The deviations in the case of the open flux tube are well described by the boundary terms in the effective string theory.
- Results from different theories give similar, but not identical results.
  $\Rightarrow$ At the moment conclusion about universality difficult!
  However: No universality expected!

Thank you for your attention!